CS5800: Algorithms Spring 2018
Assignment 8.3

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We have to prove that a graph (not tree) has at least one cycle of length at most 2 * diam(G) + 1.
Now, let's assume the graph G has a shortest cycle in it SC. If we can prove length of $SC <= 2 * diam(G) + 1$, then we are done.
Let's assume that SC has a length $>= 2 * diam(g) + 2$. Then SC must have two vertices u and v, whose distance is atleast diam(G) + 1. But in graph G, distance between u and v must be less than diam(G)+1. [Since diam(G) is max of shortest paths] Therefore, the shortest path (say SP) between u and v in G is not part of SC.
Thus,SP between u and v contains a path in SC as (u–SP–v).So, we can construct another shorter cycle with minimum of u-v paths in SC and (u–SP–v), which is a contradiction to original assumption that SC is the shortest cycle.
Hence, proved.
Basically, for even length cycle it will be $<= 2 * diam(G)$. But for odd length cycle, one path will have length diam(G) and another diam(G)+1. So we need +1 for odd length cycle.