CS5800: Algorithms Spring 2018
Assignment 4

Out: 13 February 2018 **Due: 23 February 2018, 8:59pm**

Instructions:

- The assignment is due at the time and date specified. Late assignments will not be accepted.
- You must work on all the problems and write the solutions by yourself! Finding solutions to homework problems on the web, or by asking students in and outside the course is strictly prohibited. This would be defeat the purpose of learning by doing the assignment.
- You must submit typed solutions. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. You may hand-sketch and scan any diagrams that you support your answer.
- If you are not comfortable with Word or Latex, please solve these problems by hand before devoting time to typing them. Do not waste precious time investigating typesetting up front!

1. (30 points)

Suppose you are a high-level manager in a software firm and you are managing n software projects. You are asked to assign m of the programmers in your firm among these n projects. Assume that all of the programmers are equally competent.

After some careful thought, you have figured out how much benefit i programmers will bring to project j. View this benefit as a number. Formally put, for each project j, you have computed an array $A_j[0..m]$ where $A_j[i]$ is the benefit obtained by assigning i programmers to project j. Assume that $A_j[i]$ is nondecreasing with increasing i. Further make the economically sound assumption that the marginal benefit obtained by assigning an ith programmer to a project is non-increasing as i increases. Thus, for all j and $i \geq 1$, $A_j[i+1] - A_j[i] \leq A_j[i] - A_j[i-1]$.

(a) Design a greedy algorithm to determine how many programmers you will assign to each project such that the total benefit obtained over all projects is maximized. Your answer should be in the form of a sequence of clearly stated steps. Pseudo-code is optional.

(b) Justify the correctness of your algorithm and analyze its efficiency in space and time.

2. (30 points)

The Museum of Fine Art is planning to construct a new wing to showcase paintings of contemporary art. The new wing consists of a single, long corridor. Paintings roughly of size 2x2 feet will be hung along both walls of this corridor. Their centers are placed along distances $x_1, x_2, ...x_n$ from the start of their respective walls, all at the same height.

An architect is trying to design how to light this corridor. She has to fit linear panels of light (fluorescent tube lights) above each wall, along it. Each panel of light reliably provides light within a horizontal span of m feet at the height at which the paintings are hung (same for all panels). Each painting is lit if it is within the horizontal span of at least one panel. Panels of lights are significantly longer than the span of each painting ($m \gg 2$), so she is ready to assume that each painting is a dot at its center. Her problem is to place a minimum number of panels of lights along and above each wall so that each painting is lit.

(a) State the technical problem (i.e. state the actual computational problem without context, in a way that it can be applied to any other context).

(b) Provide an efficient algorithm to compute the centers of the panels of light along one wall, obeying the above constraints. Your algorithm should be in the form of a sequence of clear and precise steps. Pseudo-code is optional.

(c) Justify the correctness of your algorithm and analyze its efficiency in space and time.

3. (30 points)

"Elixir of Life" is a milk bank that provides mother's milk to newborn babies in their critical first few months of life. They accept donations from mothers, homogenize and pasteurize it and then package them into vials of 1, 5, 10, 20 and 50 ounces. Then they supply to local hospitals for a fee to cover their costs for processing and packaging, which must be done in extremely hygienic conditions. As the milk bank is sustained only through donations, there are a limited number of vials of each size.

(a) When new parents come to the bank with a prescription of m ounces, the bank must dispense the amount to them. Note that due to hygiene issues the bank is not allowed to open the packaged vial to dispense part of an amount. However you may assume that m is an integral number. Design an algorithm to dispense exactly m ounces using the minimum number of vials. Your answer should include a short description about why your algorithm returns the optimal answer. Your answer must be in the form of a sequence of clear and precise steps. Pseudo-code is optional.

(b) Justify the correctness of your algorithm and analyze its efficiency in space and time.

(c) How will you know if dispensing the amount is even possible?