CS5800: Algorithms Spring 2018
Assignment 2.1

Saptaparna Das

January 29, 2018

1.(a) $T(n) = 3T(\frac{n}{4}) + \sqrt{n}, T(1) = 1$

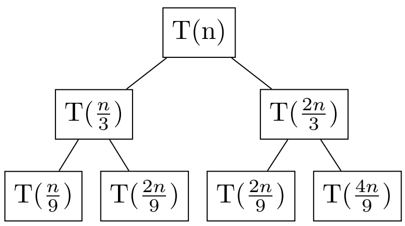
Since, this is in the form $T(n) = aT(\frac{n}{b}) + f(n)$, we can try to apply Master theorem.
Here, $a = 3$,$b = 4$ and $f(n) = \sqrt{n} = n^{0.5}$.
Now, $n^{\log_b a} = n^{\log_4 3} = n^{0.79}$. $f(n)$ can be written as, $O(n^{0.79-0.29}) = O(n^{\log_4 3-\varepsilon})$.
So Case 1 applies. Hence, we can write, $T(n) = \Theta(n^{\log_4 3})$=$T(n) = \Theta(n^{0.79})$.[**Solved**]

(b) $T(n) = 9T(\frac{n}{3}) + 5n^2, T(1) = 1$

Since, this is in the form $T(n) = aT(\frac{n}{b}) + f(n)$, we can try to apply Master theorem.
Here, $a = 9$, $b = 3$ and $f(n) = 5n^2 = O(n^2)$
Now, $n^{\log_b a} = n^{\log_3 9} = n^2$ (same as f(n)).
So, Case 2 applies. Hence we can write $T(n) = \Theta(n^{\log_3 9}. \log n) = \Theta(n^2 \log n)$.[**Solved**]

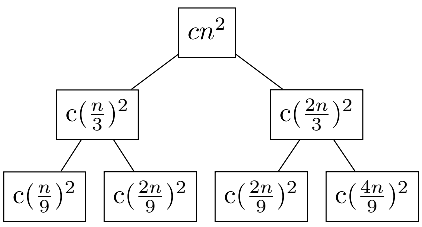
(c) $T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + cn^2, T(1) = 1$

The recursion tree of T(n) looks like,



.
.
and so on...

So, we can see that left-most path in T(n) tree, will reach to T(1) first. Because n gets divided by 3 here. Similarly, right-most path will take longest time to reach base (n gets divided by $\frac{3}{2}$). Clearly, the depth of the left-most path is $\log_3 n$ and the right-most path is $\log_{\frac{3}{2}} n$.Now for the cn^2, the recurion tree looks like..



.
.
and so on..

So, Level 0 can be expressed as... $cn^2((\frac{1}{3})^2 + (\frac{2}{3})^2))^2$=$cn^2(\frac{5}{9})^0$

Level1 can be expressed as... $cn^2((\frac{1}{3}^2)^1 * (\frac{2^2}{3^2})^0 + (\frac{1}{3^2})^0 * (\frac{2^2}{3^2})^1)$=$\sum_{i=0}^{1} {}_1\mathrm{C}_i(\frac{1}{3^2})^{1-i}(\frac{2^2}{3^2})^i$=$cn^2(\frac{5}{9})^1$

Level2 can be expressed as...$cn^2((\frac{1}{3}^2)^2 + 2(\frac{2}{3^2})^2 + (\frac{2^2}{3^2})^2))$=$\sum_{i=0}^{2} {}_2\mathrm{C}_i(\frac{1}{3^2})^{2-i}(\frac{2^2}{3^2})^i$=$cn^2(\frac{5}{9})^2$

.

.
Same way, Level m can be expressed as, $cn^2 \sum_{i=0}^{m} {}_m\mathrm{C}_i(\frac{1}{3^2})^{2-i}(\frac{2^2}{3^2})^i$=$((\frac{1}{3^2})+(\frac{2^2}{3^2}))^m$=$cn^2(\frac{5}{9})^m$

Now, we have already seen depth of left-most path is $\log_3 n$

So we can write, $T(n) >= T(1)2^{no.ofleavesatlastlevel} + \sum_{i=0}^{\log_3 n-1} cn^2(\frac{5}{9})^i$=$T(1)2^{log_3 n} + cn^2\frac{1}{1-5/9}$=$1 * n^{log_3 2} + cn^2\frac{1}{1-5/9}$ =$\Omega(n^2)$
Similarly, for the right most path, the depth is $log_{3/2}n$. We can write, $T(n) <= T(1)2^{no.ofleavesatlastlevel}$+$\sum_{i=0}^{\log_{3/2} n-1} cn^2(\frac{5}{9})^i$=$1* n^{log_{3/2}2} + cn^2\frac{1}{1-5/9}$=$O(n^2)$. Hence, n^2 is a tight bound for T(n). So, T(n)=$\Theta(n^2)$**[Solved]**