ASSIGNMENT 02 : LINEAR AND CIRCULAR CONVOLUTION

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1. PROBLEM 1(LINEAR CONVOLUTION)

The sampled version of the following signal is created:

$$x(t) = sin(2\pi f t) + 0.8\eta \tag{1}$$

for $t \in [0,1]$ with sampling frequency $f_s = \frac{1}{T} = 200Hz$ given $f = 20Hz$ and $\eta \in \mathcal{N}(\eta; \mu = 0, \sigma^2 = 0)$ is the noise. The filter is given by:

$$h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2]) \tag{2}$$

We are required to compute : $y[n] = (x * h)[n]$
The above signal $y[n]$ is the linear convolution of $x[n]$ and $h[n]$ Linear Convolution has been implement in three ways as given in the following subsections.

1.1. Brute Force

This is the brute force method of computing linear convolution between two discrete signals which utilizes the following formula:

$$y[n] = \sum_k x[k]h[n-k] \tag{3}$$

1.2. Toeplitz matrix

Linear Convolution can also be done by multiplying the teoplitz matrix with the input signal created from h[n]

Lets assume: x = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and h = $\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$. Then convolution operation can be done as shown in 4.

$$y = Hx = \begin{bmatrix} h_1 & 0 & 0 & 0 \\ h_2 & h_1 & 0 & 0 \\ h_3 & h_2 & h_1 & 0 \\ 0 & h_3 & h_2 & h_1 \\ 0 & 0 & h_3 & h_2 \\ 0 & 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{4}$$

1.3. DFT and IDFT

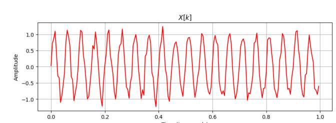
In this method (L + M - 1)-point DFTs of x[n] and h[n] are computed and IDFT of Y [k].

The Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) are given in (5) and (6) respectively.

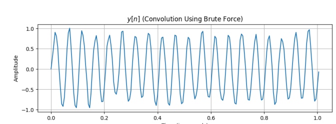
$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}} \tag{5}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}} \tag{6}$$

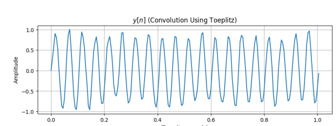
In Fig.1 we can see that the result of both types of method of circular convolution yields the same results



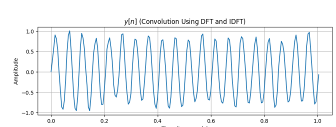
(a) x(t)



(b) Using Brute Force

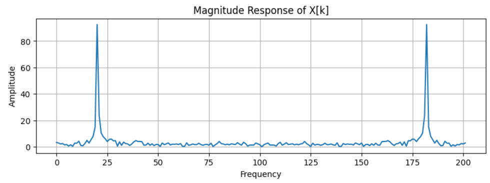


(c) Using Toeplitz

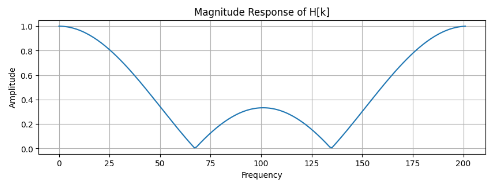


(d) Using DFT and IDFT

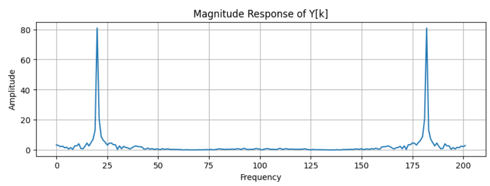
Fig. 1: $x[k]$ and Convolved signal $y[k]$ using different methods



(a) Magnitude Response of X(k)



(b) Magnitude Response of H(k)



(c) Magnitude Response of Y(k)

Fig. 2: Magnitude Response of $x[k]$, $h[k]$ and Convolved signal $y[k]$

2. PROBLEM 2 (CIRCULAR CONVOLUTION)

We are rquired to compute $y[n] = (x * h)[n]$ for the same $x[n]$ and $h[n]$ as Problem 1. Circular Convolution was implemented in two ways as given in the following subsections.

2.1. Circulant Matrix

Circular Convolution can be done by multiplying the circular matrix created from h[n] with the input signal x[k]

Lets assume: $x = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$ and $h = \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[M-1] \end{bmatrix}$.

Then circular convolution can computed as given in

$$y = Hx$$

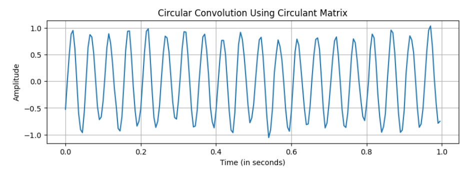
$$= \begin{bmatrix} h[0] & 0 & & h[1] \\ h[1] & h[0] & & \vdots \\ \vdots & h[1] & \cdots & h[M-1] \\ h[M-1] & \vdots & & \vdots \\ 0 & h[M-1] & & 0 \\ \vdots & 0 & & h[0] \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

2.2. DFT and IDFT

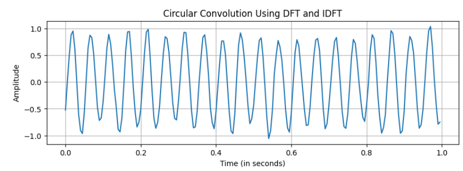
In this method N-point DFTs of x[n] and h[n] are computed and IDFT of Y[k].

The Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) are given in (5) and (6) respectively.

In Fig.3 we can see that the result of both types of method of circular convolution yields the same results

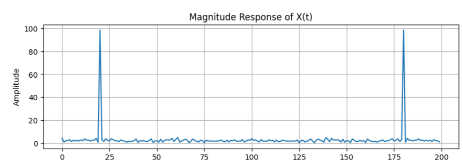


(a) Using Circular Matrix

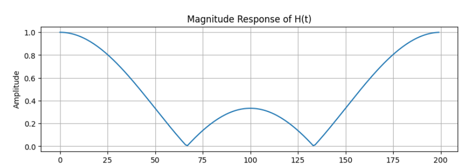


(b) Using DFT and IDFT

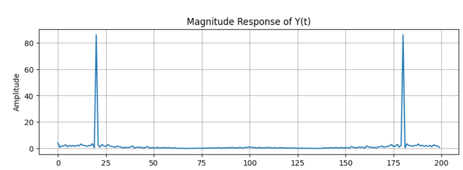
Fig. 3: $x[k]$ and Circular Convolved Signal $y[k]$ using different methods



(a) Magnitude Response of X(k)



(b) Magnitude Response of H(k)



(c) Magnitude Response of Y(k)

Fig. 4: Magnitude Response of $x[k]$, $h[k]$ and Circular Convolved signal $y[k]$