

Inverse Distance Weighted Image Interpolation

Signal Processing in Practice

Assignment 9

Report

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March 2024

1 Introduction

Understanding Inverse Distance Weighted Image Interpolation

Inverse distance weighted image interpolation (IDWI) is a technique used in image processing to estimate the values of missing pixels by considering the known values of neighboring pixels. This method assigns higher weights to nearby pixels and lower weights to pixels that are farther away, hence the term "inverse distance weighted." The interpolation process involves averaging the known pixel values in the neighborhood of each missing pixel, weighted by their distances.

Suppose you are given with an $N \times N$ corrupted image $f(r)$ with some missing pixel values replaced by zeros. Let Y denote the set of pixel locations at which pixel values are missing. Let X denote the set of pixel locations at which pixels values are available. Note that $X \cup Y = \{1, N\}^2$. For each $y_i \in Y$, it is required to estimate the missing pixel value as

$$\hat{f}(y_i) = \frac{\sum_{z_j \in X_i} w_{j,i} f(z_j)}{\sum_{z_j \in X_i} w_{j,i}},$$

where $X_i = X \cap R_i$ with R_i denoting the set of pixel location within a square box centered at y_i , and where $w_{j,i} = \frac{1}{1 + \alpha \|y_i - z_j\|_p^2}$ with α denoting a positive real number.

We have selected 5 images $g(r)$ and generated 5 corrupted images $f(r)$ by randomly setting 30% of pixels to zero. Let $\hat{f}(r)$ denote the reconstructed image

obtained by the above method. Let $E = \sum_{y_i \in Y} (\hat{f}(y_i) - g(y_i))^2$. It is to be noted that E and MSE when computed will result in the same value.

The following images and their corrupted versions are generated:

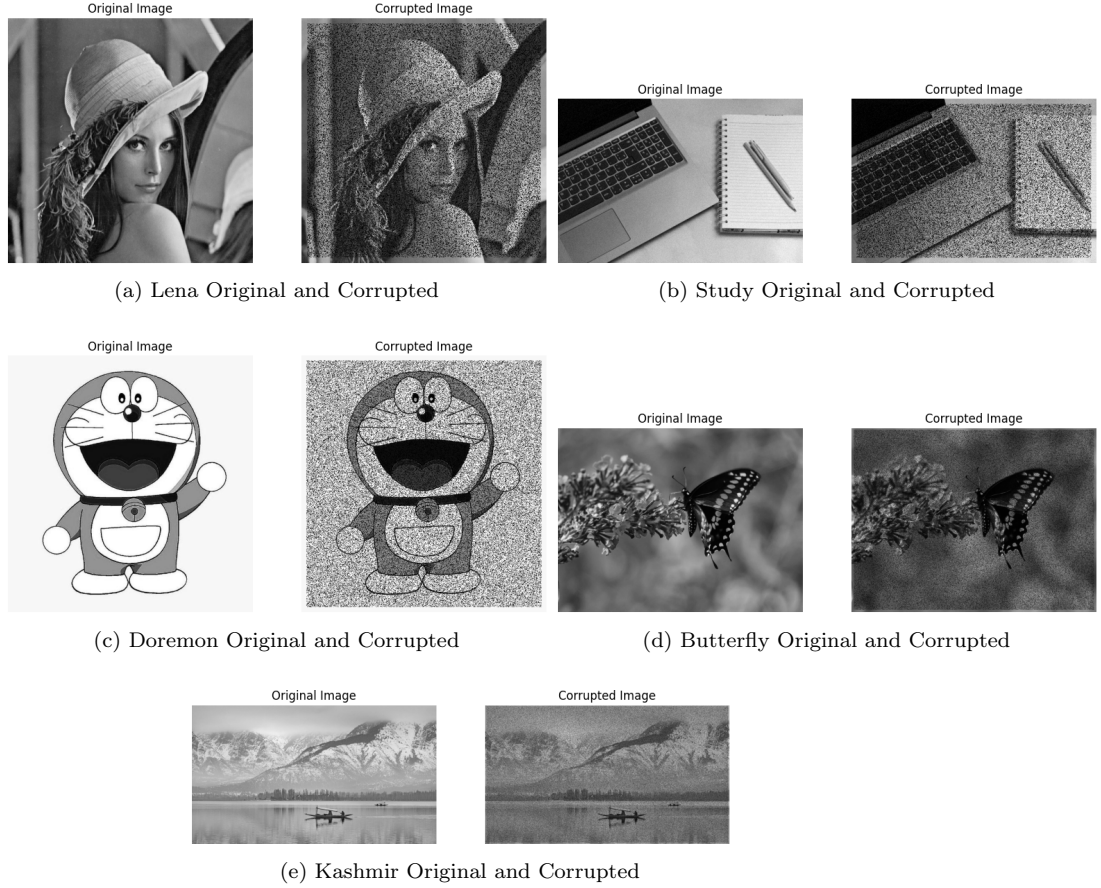
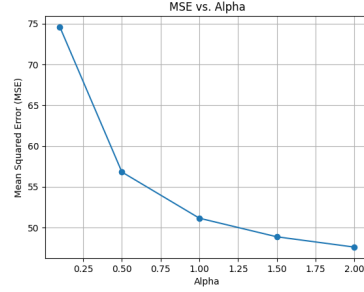


Figure 1: Original and Corrupted Images

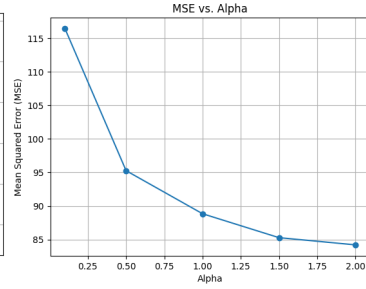
2 Results

2.1 Choosing Optimal α

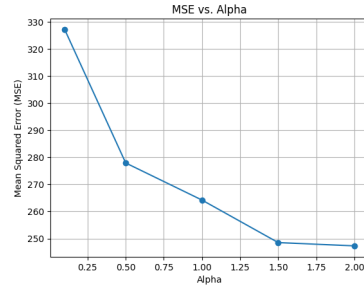
MSE is plotted for range of values of α by fixing $p = 2$, and fixing the window size at 25×25 pixels. The best value of α is chosen.



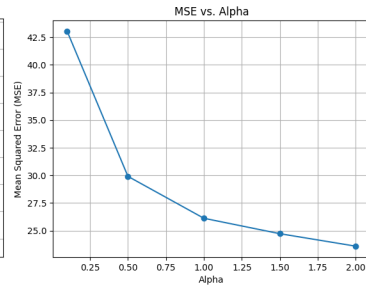
(a) Lena



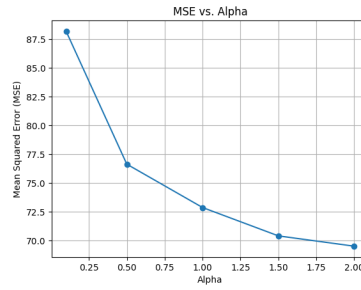
(b) Study



(c) Doremon



(d) Butterfly



(e) Kashmir

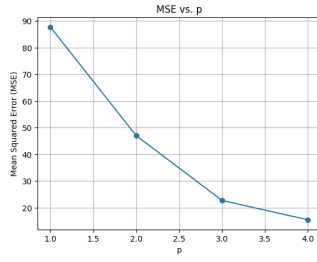
Figure 2: E vs α

The optimal alphas are found as follows :

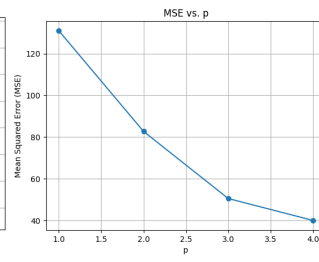
| Image | Optimal α |
|--------------|------------------------------------|
| Lena | 2 |
| Study | 2 |
| Doremon | 2 |
| Butterfly | 2 |
| Kashmir | 2 |

2.2 Choosing Optimal p

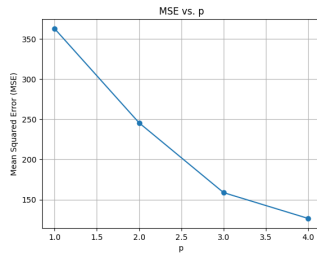
With best value for α chosen, E is for a range of values of $p \in [1, 4]$. The best value for p is chosen.



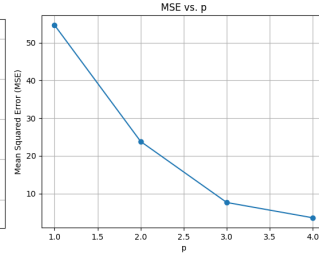
(a) Lena



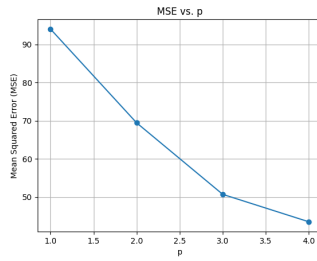
(b) Study



(c) Doremon



(d) Butterfly



(e) Kashmir

Figure 3: E vs p

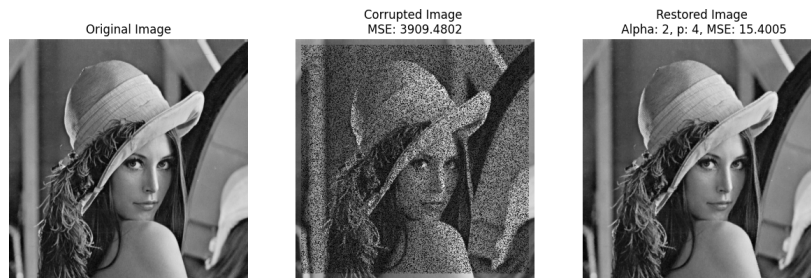
The optimal \mathbf{ps} are found as follows :

| Image | Optimal α | Optimal p |
|--------------|------------------------------------|-------------------------------|
| Lena | 2 | 4 |
| Study | 2 | 4 |
| Doremon | 2 | 4 |
| Butterfly | 2 | 4 |
| Kashmir | 2 | 4 |

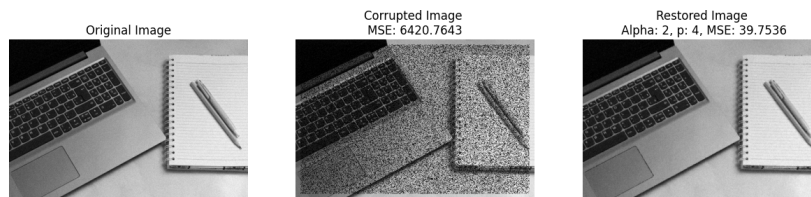
The reconstructed images are displayed along with the model images and corrupted images.

2.3 Observations

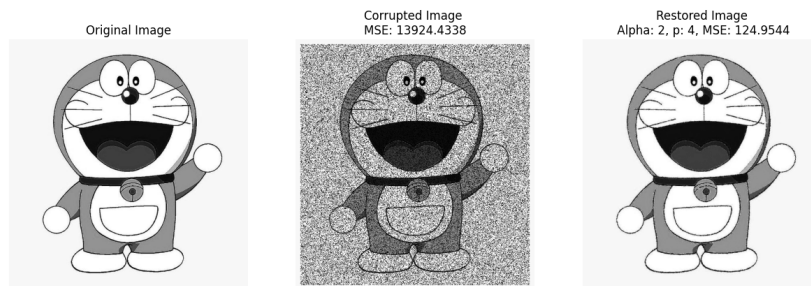
1. For a fixed Window Size, larger α values typically lead to lower Error.
2. For a fixed α , increase in the exponent p leads to significant reduction in Error.



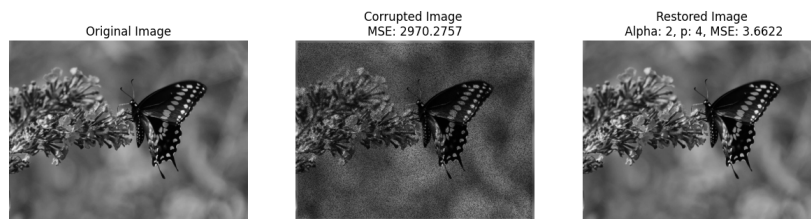
(a) Lena Original, Corrupted and Restored



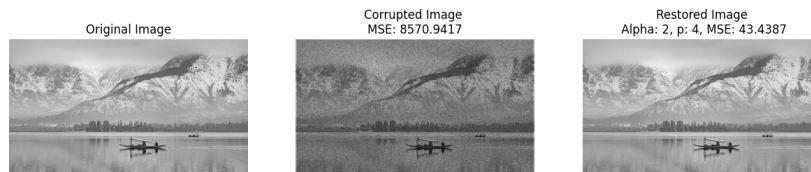
(b) Study Original, Corrupted and Restored



(c) Doremon Original, Corrupted and Restored



(d) Butterfly Original, Corrupted and Restored



(e) Kashmir Original, Corrupted and Restored

Figure 4: Original, Corrupted and Restored Images