E9 222 Signal Processing in Practice

Assignment 10 - Image Denoising (Due Feb 7, 2024)

1 Experiment 1 - Low pass Gaussian filter

Images corrupted with Gaussian noise are often denoised using a simple Gaussian low pass filter. If the noisy image is $y(m,n)$ and the desired denoised image is $\hat{x}(m,n)$, the denoised image is obtained as

$$\hat{x}(m,n) = \sum_{k=-P}^{P}\sum_{l=-P}^{P} w(k,l)y(m+k,n+l), \text{ where}$$

$$w(k,l) = C\exp\left(-\frac{k^2+l^2}{2\sigma^2}\right) \text{ and } \sum_{k=-P}^{P}\sum_{l=-P}^{P} w(k,l) = 1.$$

Here P corresponds to the window size and σ is the standard deviation of the Gaussian filter.

1. For each of given noisy images (except img167.bmp), find the optimal Gaussian filter of size 11×11 with standard deviation from the set $\{0.1, 1, 2, 4, 8\}$ with the best
   (a) mean squared error (MSE).
   (b) structural similarity index (SSIM).

   Present the MSE/SSIM corresponding to different sigma for all the images in a single table. To compute MSE/SSIM, use img167.bmp as the reference image. Show the images denoised with the best Gaussian filter for each of the noisy images in a single figure along with the noisy images.
2. Plot a curve between the image index (sorted visually in terms of increasing noise levels) and the standard deviation of the optimal Gaussian filter. Comment on the curve.

2 Experiment 2 - Bilateral filter

The bilateral filter is defined as

$$g(m,n) = \frac{1}{C(m,n)}\sum_{k=-P}^{P}\sum_{l=-P}^{P} G(k,l)H(f(m,n)-f(k,l))f(m+k,n+l),$$

where

$$H(x) = \exp\left(-\frac{x^2}{2\sigma_H^2}\right) \text{ and } x \text{ is the intensity difference,}$$

$$G(k,l) = \exp\left(-\frac{k^2+l^2}{2\sigma_G^2}\right)$$

$$C(m,n) = \sum_{k=-P}^{P}\sum_{l=-P}^{P} G(k,l)H(f(m,n)-f(k,l)).$$

Find a bilateral filter to denoise the imagenoisybook.png corrupted by the Gaussian noise and compare the results with a Gaussian filter that also does equally well visually. Choose the parameters of the bilateral filter to give a visually good result.