E9 222 SIGNAL PROCESSING IN PRACTICE
FINAL EXAMINATION

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Question 2: Analysis of Image Magnitude and Phase Spectra

Given two images, we need to verify the following properties:

1. The equality:

$$\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} x_1[m,n]x_2^*[m,n] = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} X_1[k,l]X_2[k,l] \tag{1}$$

Results: After computing the above LHS and RHS for the given two images, the following was observed:

- LHS = 14062.126
- RHS = 14062.126

So, we can see that LHS = RHS. So, this property is verified.

2. Parseval's theorem:

$$\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} x[m,n]x^*[m,n] = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} X[k,l]X[k,l] \tag{2}$$

Results: After computing the above LHS and RHS for the given two images, the following was observed:

For Cameraman image

- LHS = 18123.245
- RHS = 18123.245

For Barbara image

- LHS = 16706.449
- RHS = 16706.449

So, we can see that for both the images LHS = RHS. So, this property is verified.

Fig. 1: Image: Barbara

Fig. 2: Image: Cameraman

Next, Magnitude-phase swapping is done for the two images. The images observed are given in Figure.3

- New Image 1: Contains the Magnitude Spectrum of the "Cameraman" Image and the Phase Spectrum of the "Barbara" Image.
- New Image 2: Contains the Magnitude Spectrum of the "Barbara" Image and the Phase Spectrum of the "Cameraman" Image.

We can see the resemblance is more for the image from which the phase spectrum plot is taken.

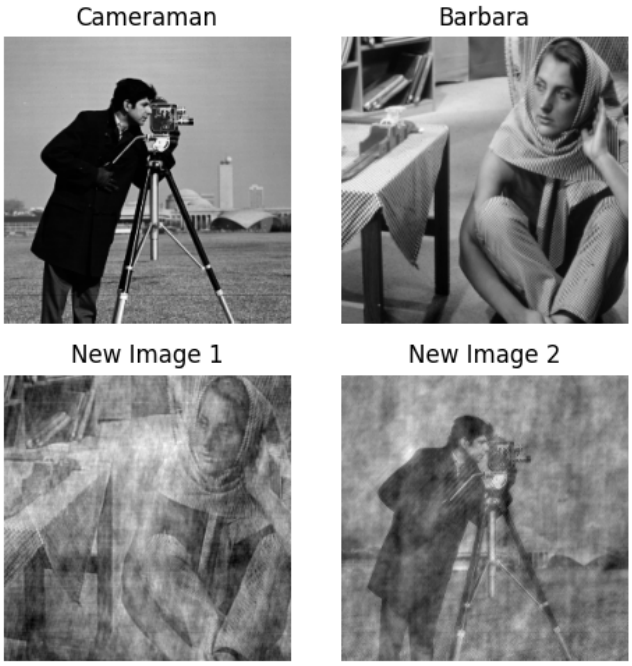


Fig. 3: Magnitude-Phase Swapping Comparison

Possible Explanations: Since the phase spectrum captures the spatial details and texture patterns of the image, the resemblance between the hybrid image and the image from which the phase spectrum is taken is stronger. Therefore, in this case, New Image 1 will resemble the ”Barbara” Image more closely, and New Image 2 will resemble the ”Cameraman” Image more closely.

Question 3: Analysis of Sinusoids in Noise

The Sum of Sinusoids Signal (x[n]) is generated and is plotted in Figure.4

The Magnitude Spectrum overlaid on the idea spectrum of continuous spectrum is shown in Figure.5. Both Hamming Window and Rectangular Window has been considered.

We can observe, that the peaks are observed at the frequencies that are present in x[n] and it matches the peaks of the ideal spectrum. Note that the ideal spectrum should have infinite amplitude as Fourier Transform of a Continuous Sine is a delta function. In the image the infinite peaks have been truncated for plotting convenience.

Next we consider the sequence y[n] as,

$$y[n] = x[n] + w[n]$$
$$where, w[n] \sim \mathcal{N}(0, \sigma^2)$$

The Magnitude spectrum of y[n] for σ = 1,5,10,15,20,25 is given in Figure.6

As the Noise Variance, Increases we can see it induces more frequencies in the signal which can be observed in many small peaks in the Magnitude Spectrum of y[n]

Now, in another set of plots the auto-correlation of y[n] is plotted overlaid on auto-correlation of x[n] for various values of σ in Figure.7

As x[n] is periodic, so the auto-correlation is has periodic peaks and it tapers down at both ends since, it is a finite duration signal. Now, at less values of sigma, auto-correlation of y[n] will have similar periodic peaks with tapering ends like characteristics like auto-correlation of x[n]. With more variance in noise, y[n] will lose its periodicity and hence, the auto-correlation will also reflect that.

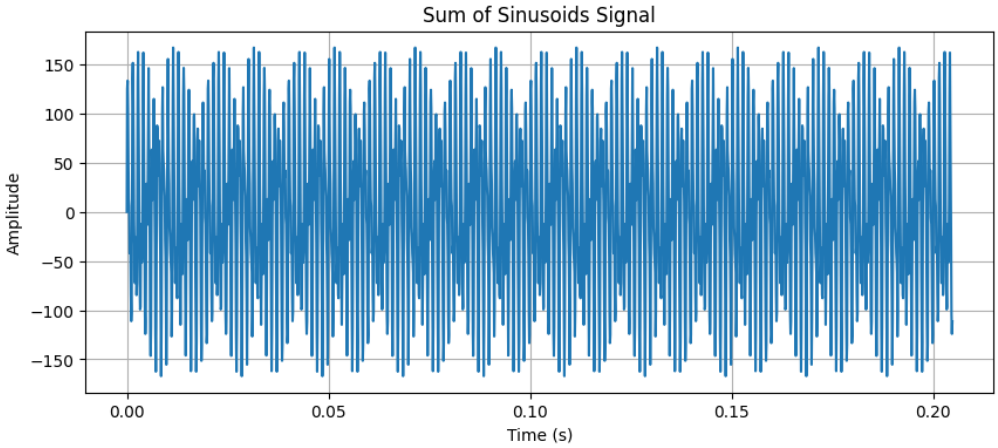


Fig. 4: Image: Sum of Sinusoids (x[n])

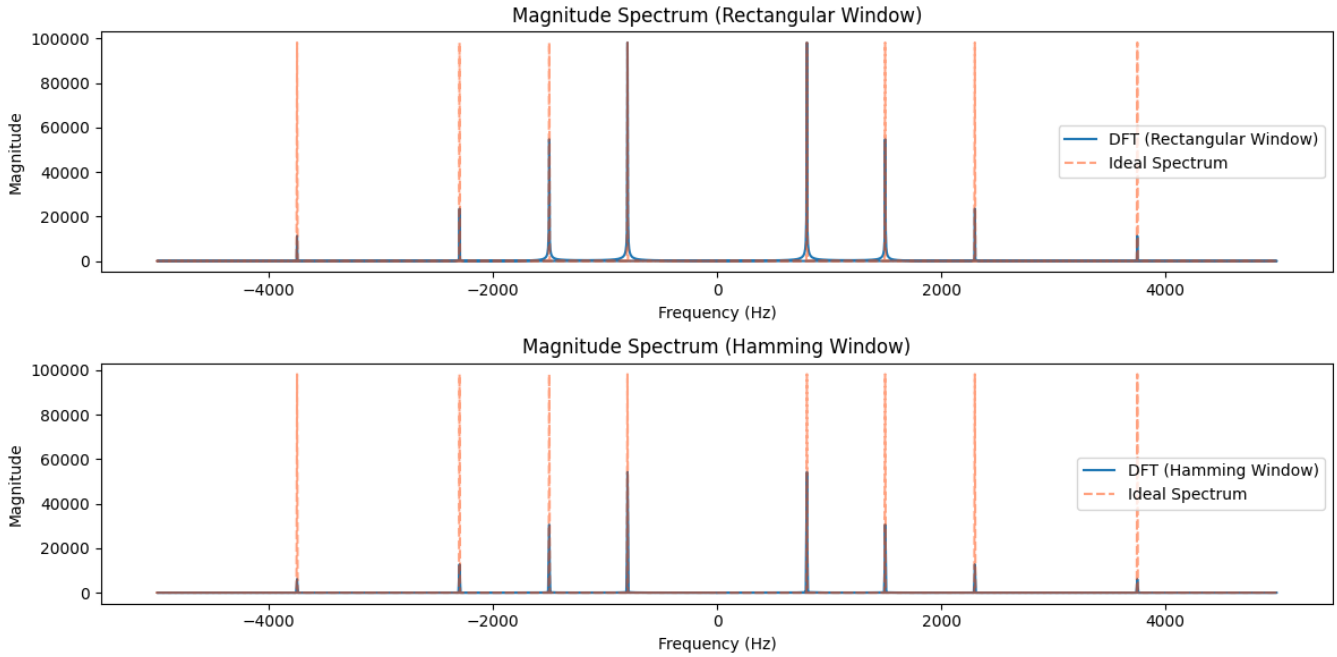


Fig. 5: Magnitude Spectrum of x[n] for different Windows

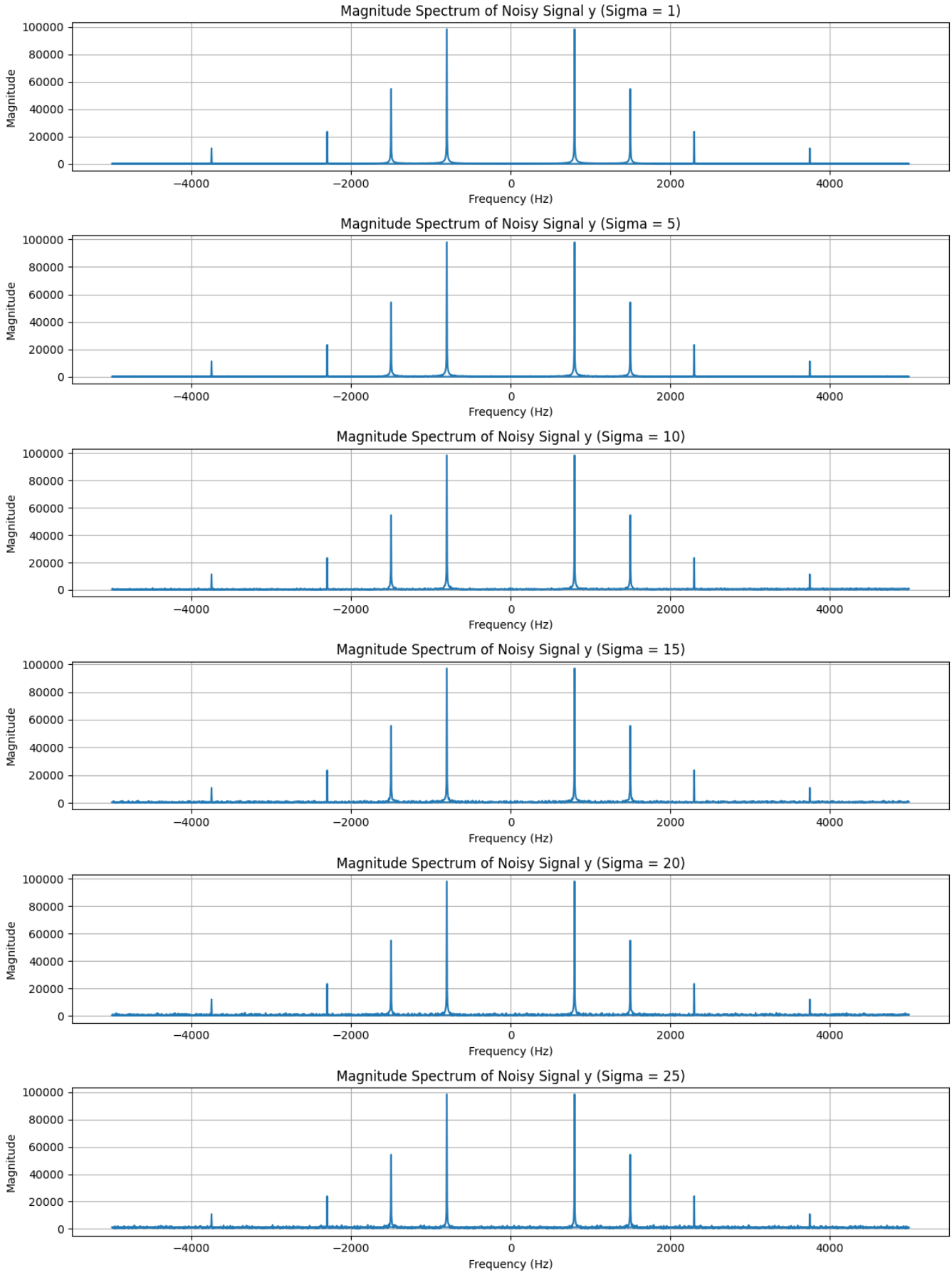


Fig. 6: Magnitude Spectrum of y[n] for varying σ

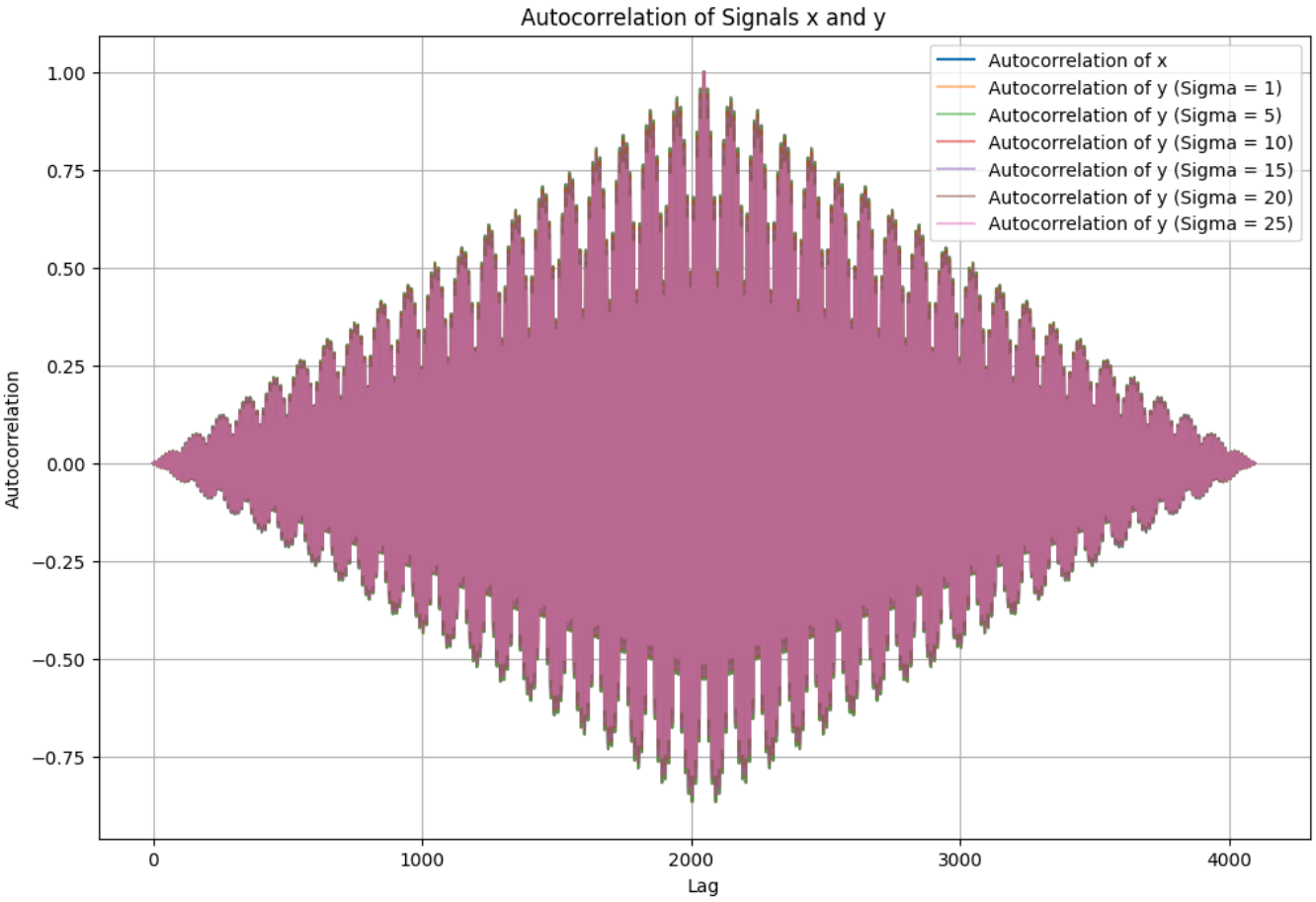
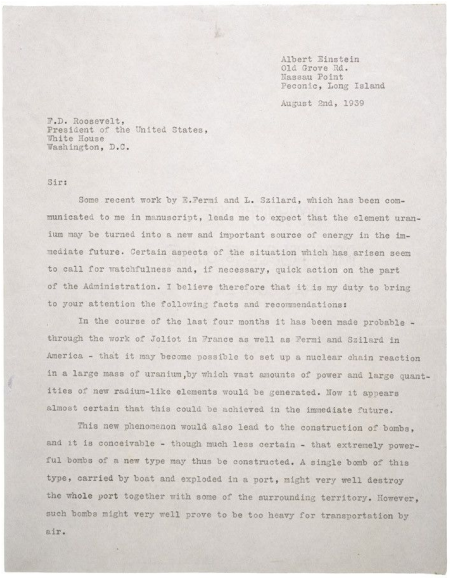


Fig. 7: Magnitude Spectrum of y[n] with varying σ

Question 4: Optical Character Recognition

Scanned Images of Einstein's letters to President Roosevelt are displayed in Figure 8 and 9.



Albert Einstein
Old Grove Rd.
Nassau Point
Peconic, Long Island

August 2nd, 1939

F.D. Roosevelt,
President of the United States,
White House
Washington, D.C.

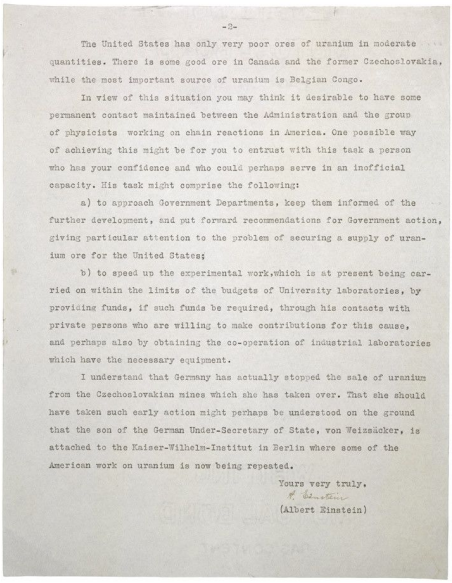
Sir:

Some recent work by E.Fermi and L. Szilard, which has been communicated to me in manuscript, leads me to expect that the element uranium may be turned into a new and important source of energy in the immediate future. Certain aspects of the situation which has arisen seem to call for watchfulness and, if necessary, quick action on the part of the Administration. I believe therefore that it is my duty to bring to your attention the following facts and recommendations:

In the course of the last four months it has been made probable - through the work of Joliot in France as well as Fermi and Szilard in America - that it may become possible to set up a nuclear chain reaction in a large mass of uranium,by which vast amounts of power and large quantities of new radium-like elements would be generated. Now it appears almost certain that this could be achieved in the immediate future.

This new phenomenon would also lead to the construction of bombs, and it is conceivable - though much less certain - that extremely powerful bombs of a new type may thus be constructed. A single bomb of this type, carried by boat and exploded in a port, might very well destroy the whole port together with some of the surrounding territory. However, such bombs might very well prove to be too heavy for transportation by air.

Fig. 8: Letter Page 1



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The United States has only very poor ores of uranium in moderate quantities. There is some good ore in Canada and the former Czechoslovakia, while the most important source of uranium is Belgian Congo.

In view of this situation you may think it desirable to have some permanent contact maintained between the Administration and the group of physicists working on chain reactions in America. One possible way of achieving this might be for you to entrust with this task a person who has your confidence and who could perhaps serve in an inofficial capacity. His task might comprise the following:

a) to approach Government Departments, keep them informed of the further development, and put forward recommendations for Government action, giving particular attention to the problem of securing a supply of uranium ore for the United States;

b) to speed up the experimental work,which is at present being carried on within the limits of the budgets of University laboratories, by providing funds, if such funds be required, through his contacts with private persons who are willing to make contributions for this cause, and perhaps also by obtaining the co-operation of industrial laboratories which have the necessary equipment.

I understand that Germany has actually stopped the sale of uranium from the Czechoslovakian mines which she has taken over. That she should have taken such early action might perhaps be understood on the ground that the son of the German Under-Secretary of State, von Weizsäcker, is attached to the Kaiser-Wilhelm-Institut in Berlin where some of the American work on uranium is now being repeated.

Yours very truly,

(Albert Einstein)

Fig. 9: Letter Page 2

K-means Clustering is done to identify 30 clusters of alphabets and the cluster Centers are shown in Figure.10. As we can see, that the cluster centers are different alphabets. So, we have done unsupervised learning on the alphabets.

Fig. 10: Cluster Centers