Signal Processing in Practice: Jan'2024

Assignment I:

Suppose you are given with an $N \times N$ corrupted image $f(\mathbf{r})$ with some missing pixel values replaced by zeros. Let $\mathcal{Y}$ denote the set of pixel locations at which pixel values are missing. Let $\mathcal{X}$ denote the set of pixel locations at which pixels values are available. Note that $\mathcal{X} \cup \mathcal{Y} = [1, N]^2$. For each $\mathbf{y}_i \in \mathcal{Y}$, it is required to estimate the missing pixel value as

$$\hat{f}(\mathbf{y}_i) = \frac{\sum_{\mathbf{z}_j \in \mathcal{X}_i} w_{j,i} f(\mathbf{z}_j)}{\sum_{\mathbf{z}_j \in \mathcal{X}_i} w_{j,i}},$$

where $\mathcal{X}_i = \mathcal{X} \cap R_i$ with R_i denoting the set of pixel location within a square box centered at $\mathbf{y}_i$, and where $w_{j,i} = \frac{1}{1+\alpha \|\mathbf{y}_i - \mathbf{z}_j\|_2^p}$ with α denoting a positive real number. After implementing this method, run the following numerical experiments.

1. Select a model image $g(\mathbf{r})$ and generate a corrupted image $f(\mathbf{r})$ by randomly setting 30% of pixels to zero. let $\hat{f}(\mathbf{r})$ denote the reconstructed image obtained by the above method. Let $E = \sum_{\mathbf{y}_i \in \mathcal{Y}} (\hat{f}(\mathbf{y}_i) - g(\mathbf{y}_i))^2$. Plot E for range of values of α by fixing $p = 2$, and fixing the window size at 25×25 pixels. Choose the best value for α.

2. With best value for α chosen, plot E for a range of values of $p \in [1, 4]$. Choose the best value for p.

3. Select at least five model images, and repeat the above experiments. Tabulate best values for α and p for each image with corresponding value of E. Display the reconstructed images along with the model images and corrupted images.

Assignment II:

Let $f^{(1)}(\mathbf{r})$ be the image reconstructed from the corrupted image $f(\mathbf{r})$. Using this, we reconstruct a further improved image as

$$\hat{f}^{(2)}(\mathbf{y}_i) = \frac{\sum_{\mathbf{z}_j \in \mathcal{X}_i} v_{j,i} w_{j,i} f(\mathbf{z}_j)}{\sum_{\mathbf{z}_j \in \mathcal{X}_i} v_{j,i} w_{j,i}},$$

where $v_{j,i} = \phi(|(\mathbf{y}_i - \mathbf{z}_j)^T \nabla f^{(1)}(\mathbf{y}_i)|)$ with ϕ being some monotonically decreasing function. The idea here is to smooth less along the direction of the gradient of preliminary reconstruction $f^{(1)}(\mathbf{r})$ to get the improve reconstruction $\hat{f}^{(2)}(\mathbf{y}_i)$. This naturally leads to an iteration defined by

$$\hat{f}^{(k+1)}(\mathbf{y}_i) = \frac{\sum_{\mathbf{z}_j \in \mathcal{X}_i} v_{j,i} w_{j,i} f(\mathbf{z}_j)}{\sum_{\mathbf{z}_j \in \mathcal{X}_i} v_{j,i} w_{j,i}},$$

where $v_{j,i} = \phi(|(\mathbf{y}_i - \mathbf{z}_j)^T \nabla f^{(k)}(\mathbf{y}_i)|)$. After implementing this iteration, use the reconstructed images of Part (3) of the Assignment I, as initialization and run this iteration. Report the results after running for 5, 10, 15, and 20 iteration. Display the reconstructed images after running for 5, 10, 15, and 20 iterations.