General Information:

- Write MATLAB **or** Python code for each of the following problems. Ensure you add comments for better readability.
- Provide a file for each problem with the name `run_Problem1.m` or `run_Problem1.py` as the solutions to Problem 1, and so on.
- Prepare a less than 2-page LaTeX report for the assignment using the IEEE `spconf` template(shared on the Teams channel). Include the `.tex` file and supporting files in the main folder.
- Save plots as `.pdf` files before adding them to the `.tex` file as figures. All plots must be labelled with appropriate titles. Poorly made plots will be penalised.
- Submit a zipped file with the name `A3_FirstNameLastName.zip` via Teams before the deadline. Late submissions will be penalised.

Problem 1. (Discrete cosine transform, *10 points*)

1. Write a function that computes the DCT of an N-length sequence $x[n]$. Use this function to compute the DCT of

$$x[n] = \cos(2\pi k_0 n/N),\ 0 \leq n \leq N-1,\ k_0 = 5, N = 32.$$

2. Compare the result using the inbuilt DCT function.

Plot the sequence in subplot 1, and the DCT obtained using the two methods in subplot 2. Observe the index p_0 for which the DCT has its maximum value. How is it related to k_0? Briefly explain the relation.

Problem 2. (2D DCT, *10 points*)

1. Use the 1D-DCT function from Problem 1 to compute the 2D-DCT of the image

$$x[m,n] = \cos(2\pi k_1 m/M)\cos(2\pi k_2 n/N),\ 0 \leq m \leq M-1,\ 0 \leq n \leq N-1.$$

   where $k_1 = 10, k_2 = 8, M = 48, N = 32$.

2. Compare the result using the inbuilt DCT2 function. Plot the image in subplot 1, and the DCTs obtained using the two methods in subplots 2 and 3. Observe the indices (p_1 and p_2) for which the DCT has its peak value. How are they related to k_1 and k_2? Briefly explain the relation.

3. Use your 2D-DCT function to compute the 2D-DCT of Cameraman. Plot the image in subplot 1 and the 2D-DCT in log scale in subplot 2.

Problem 3. (Block 2D DCT, *10 points*)
Use the 1D-DCT function from Problem 1 to compute the block 2D-DCT of Peppers. Use the following steps.

1. Write a function to extract 8×8 patches from an image, and a complementary function to stitch patches into an image.

2. Compute 2D-DCT on each patch, and stitch the 2D-DCT patches to display the block 2D-DCT. Display the image in subplot 1 and the block 2D-DCT in subplot 2.

Problem 4. (Inverse 2D DCT, *10 points*)
Write a function to compute the inverse 2D-DCT.

1. Use the inverse 2D-DCT (2D-IDCT) function to invert the 2D-DCT of the cosine image and Cameraman from Problem 2.
2. Obtain partial image reconstructions by retaining the top m 2D-DCT coefficients for $0 \leq m \leq MN$.
3. Repeat the procedure to invert the block 2D-DCT with $0 \leq m \leq 64$.

Report the reconstruction error as a function of m in all cases.

Recommended Reading

- Digital Signal Processing by John Proakis, Dimitris Manolakis (4^{th} edition): Section 7.5
- Discrete-Time Signal Processing by Alan Oppenheim, Ronald Schafer and John Buck (2^{nd} edition): Section 8.8
- Digital Image Processing by Rafael C. Gonzalez and Richard E. Woods (3^{rd} edition): Section 8.2