ASSIGNMENT 04 : DISCRETE COSINE TRANSFORM

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1. PROBLEM 1.

We are required to write a function that computes the DCT of an N-length sequence x[n] given in Eq.1.

$$x[n] = \cos\left(\frac{2\pi k_0 n}{N}\right) \quad (1)$$

Where, $\quad 0 \leq n \leq N-1, \quad k_0 = 5, \quad N = 32$
The equation used to compute 1-D DCT in given in Eq.2

$$X[k] = \sum_{n=0}^{N-1} 2x[n]\cos\left(\frac{2\pi k(2n+1)}{2N}\right) \quad (2)$$

The sequence is plotted and the Discrete Cosine Transform (DCT) is obtained using the two methods (Manual Implementation and Library Function) in Figure.1
It can be observed that both the implementation is giving the same output. In Discrete Cosine Transform the peaks are observed at twice the frequency of the cosines present in the signal and it can be observed that the peak is observed to be at $p_0 = 10$ which is double the value of k_0.

2. PROBLEM 2.

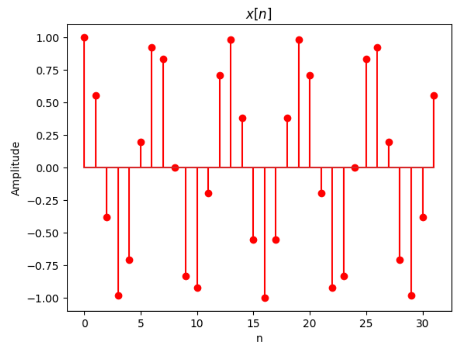
We are required to use the 1D-DCT function from Problem 1 to compute the 2D-DCT of the image given by Eq.3

$$x[m,n] = \cos\left(\frac{2\pi k_1 m}{M}\right)\cos\left(\frac{2\pi k_2 n}{N}\right) \quad (3)$$

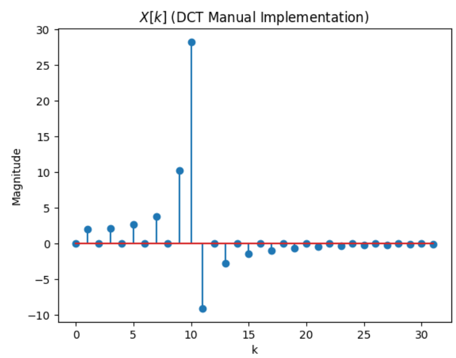
Where, $\quad 0 \leq m \leq M-1, \quad 0 \leq n \leq N-1 \quad k_1 = k1, \quad k_2 = k2, \quad M = M, \quad N = N$
For computation of 2D-DCT we use the 1D-DCT function from Problem 1. At first 1D DCT is computed Row-wise by transposing the image.Then 1D DCT is computed column wise by again transposing the image. This gives us the 2D-DCT of the Image
The Results obtained by computing the DCT of the given image is given in Figure.2

2.1. Comparison of Results

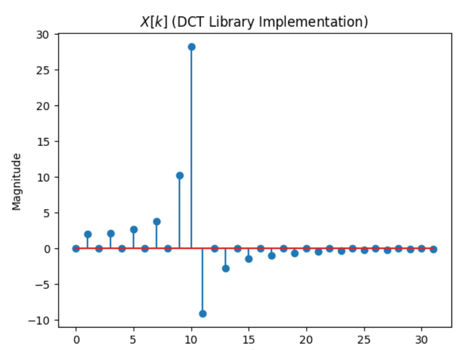
It can be seen that both the manual implementation of 2D DCT and the library function gives identical Results. The peaks are observed at (20,16) as expected as the peaks for



(a) x(k)

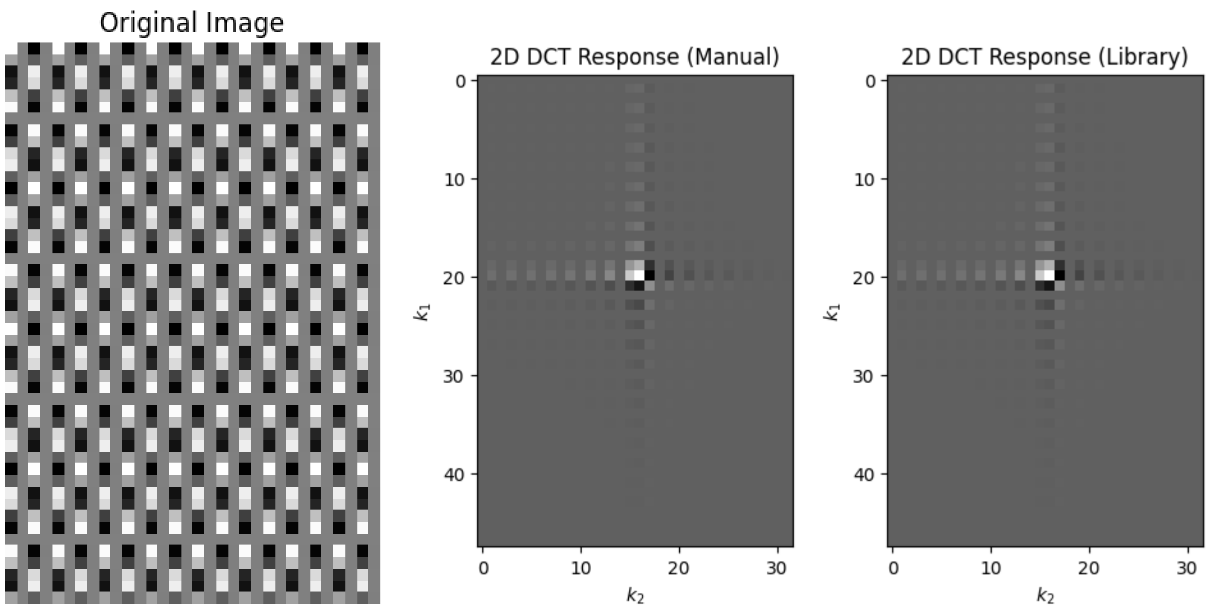


(b) 1D-DCT Response (Manual)



(c) 1D-DCT Response (Library)

Fig. 1: Sequence x[n], 1D-DCT of x[n] using Manual Implementation using Library Function



(a) x(k) (b) 2D DCT Response (Manual) (c) 2D DCT Response (Library)

Fig. 2: Original Image], 2D-DCT (Manual Implementation) and (Library Implementation)

a DCT are found at double the frequency present at original signal.

k_1(Observed) = 2 * k_1
k_2(Observed) = 2 * k_2

The DCT function is now used to compute the DCT of the given image *'cameraman.tif'*. The Results is given in Figure.3

3. PROBLEM 3.

In this problem, the 1D-DCT function from Problem 1 is used to compute the block 2D-DCT of the image *Peppers.png*
The following steps are followed:

- A function is written to extract 8 × 8 patches from an image, and a complementary function to stitch patches into an image.
- 2D-DCT is computed on each patch, and the 2D-DCT patches are stitched back to display the block 2D-DCT.

The original *peppers.png* and it's block 2D DCT response is shown in Figure.4

4. PROBLEM 4.

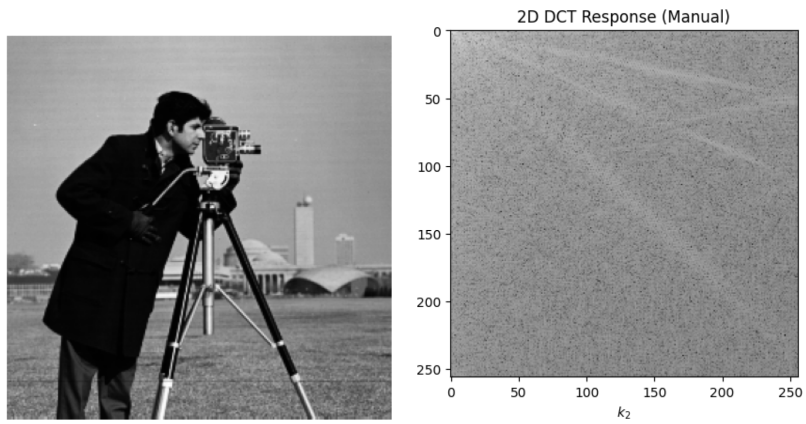
In this Problem the following has been done:

1. A function is written to compute the inverse 2D-DCT.
2. The inverse 2D-DCT (2D-IDCT) function is used to invert the 2D-DCT of the cosine image and Cameraman from Problem 2.
3. Partial image reconstructions are obtained by retaining the top m 2D-DCT coefficients for $0 \leq m \leq MN$.
4. The procedure is repeated to invert the block 2D-DCT with $0 \leq m \leq 64$.

The equation used to compute 1D-Idct is given in Eq.4. 2D-Inverse DCT is carried out by computing 1D IDCT row-wise and then column wise

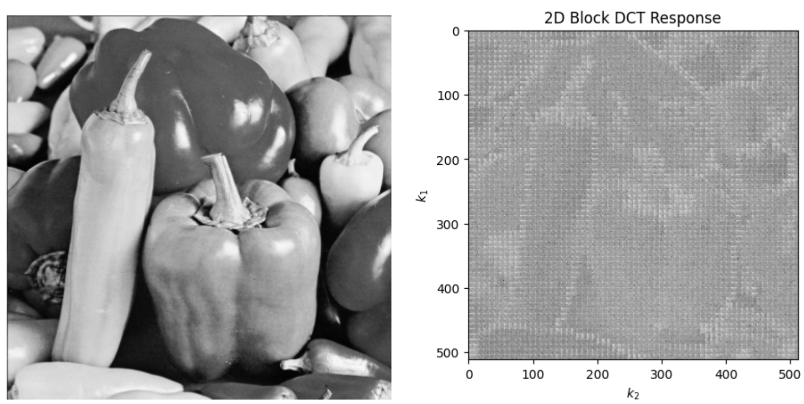
$$x[k] = x_0 + \sum_{n=1}^{N-1} 2x[n] \cos\left(\frac{2\pi k(2n+1)}{2N}\right) \tag{4}$$

The Required Images are given Figure 5, Figure 6, Figure 7, Figure 8, Figure 9, Figure 10



(a) Original Image (b) DCT Response (Manual)

Fig. 3: Original Image and DCT (Manual Implementation)



(a) Original Image (b) 2D Block DCT Response

Fig. 4: Original Imageand DCT (Manual Implementation)

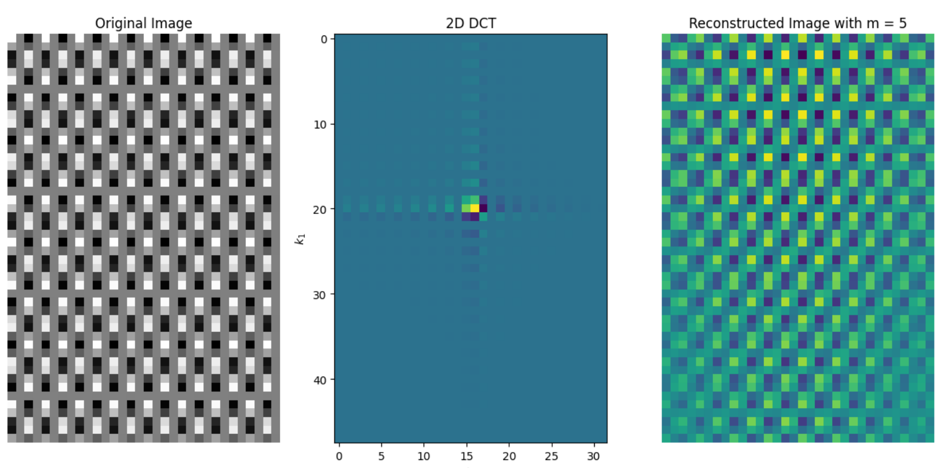


Fig. 5: Cosine Image, Its 2D DCT and Reconstructed Image retaining top m coefficients

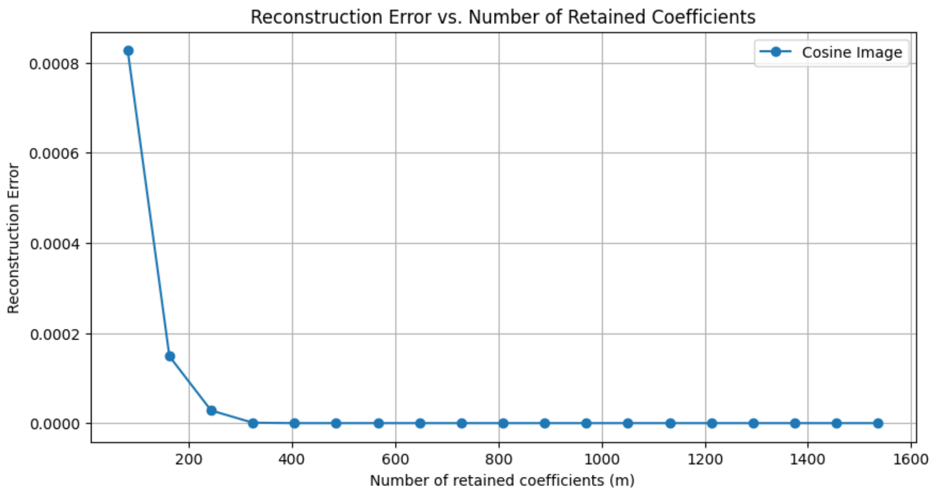


Fig. 6: Reconstruction Error of Cosine Image vs Retained number of components

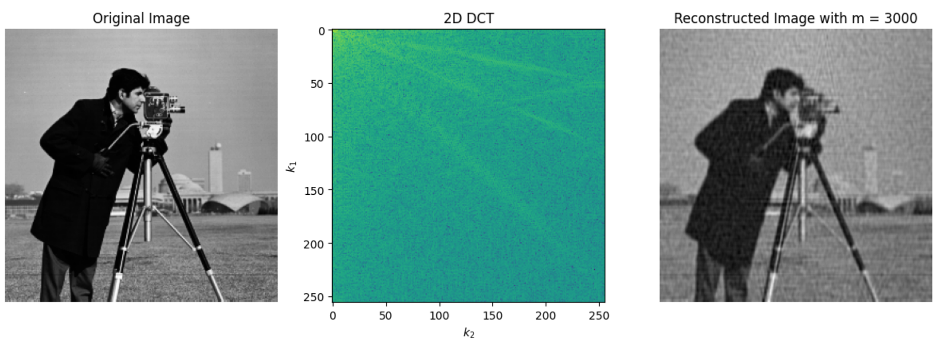


Fig. 7: Cameraman Image, Its 2D DCT and Reconstructed Image retaining top m coefficients

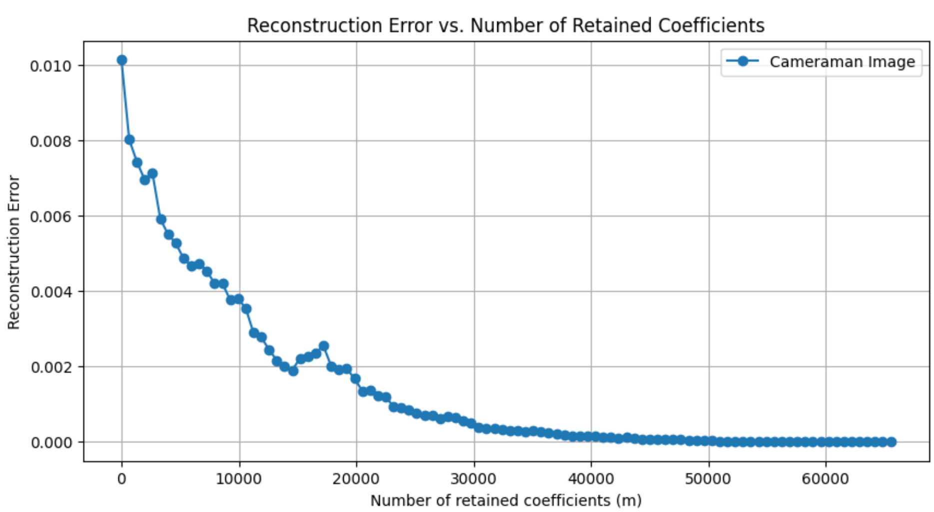


Fig. 8: Reconstruction Error of Cameraman Image vs Retained number of components

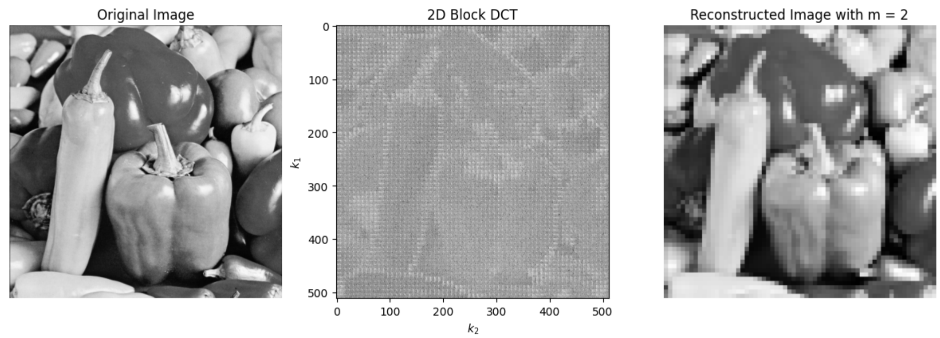


Fig. 9: Peppers Image, Its 2D DCT and Reconstructed Image retaining top m coefficients

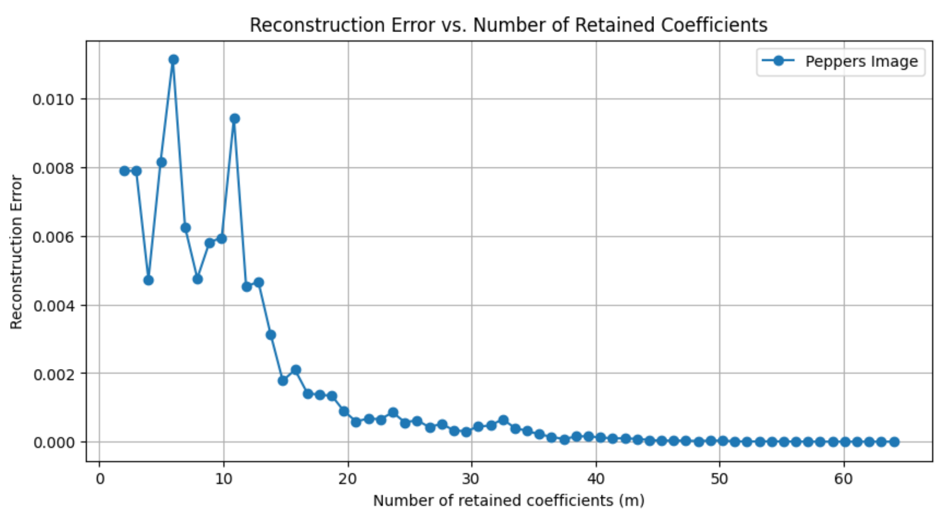


Fig. 10: Reconstruction Error of Peppers Image vs Retained number of components