1. Solve the following recurrerence relations:

(a) $x(n) = x(n-1) + 5$ for $n \geq 1$ $x(1) = 0$

$$x(n) = x(n-1) + 5$$

By subsitituting method

$$x(1) = 0 \rightarrow (1)$$

if $n=2 \Rightarrow x(2) = x(2-1)+5$

$$x(2) = x(1)+5$$

$$x(2) = 5 \rightarrow (4)$$

if $n=3 \Rightarrow x(3) = x(3-1)+5$

$$= x(2)+5$$

$\therefore$ Sub (3) in (4)

$$x(3) = 5+5 = 10$$

$\therefore$ $x(n) = 5n$ for $n \geq 1$

b) $x(n) = 3x(n-1)$ for $x(1) = 4$

$$x(1) = 4 \rightarrow (1)$$

If $x(2) = 3x(2-1) = 3x(1)$

$$x(2) = 3x(1) \rightarrow (2)$$

Sub (1) in (2)

$$x(2) = 3(4) = 12 \rightarrow (3)$$

if $n=3 \Rightarrow x(3) = 3x(3-1) = 3x(2)$

$$x(3) = 3x(2) \rightarrow (4)$$

Sub (3) in (4)

$$x(3) = 3(12) = 36$$

$\therefore \quad x(n) = 4(3)^{n-1}$

(c) $x(n) = x(n/2)+n$ for $n>1$ $x(1) = 1$ (solve $n = 2^k$).

$$x(n) = x(n/2)+n$$

Sub $n = 2^k$

$$x(2^k) = x\left(\frac{2^k}{2}\right)+2^k$$

$$x(2^k) = x(2^{k-1})+2^k$$

$$x(2^0) = 1$$

$$x(2^1) = x(2^{1-1})+2^1$$

$$= x(2^0)+2^1 = 1+2 = 3$$

$$x(2^1) = 3$$

$$x(2^2) = x(2^{2-1})+2^2$$

$$= x(2^1)+4 = 3+4 = 7$$

$$x(2^2) = 7$$

$\therefore \quad x(2^k) = 2^{k+1}-1$

(d) $x(n) = x(n/3) + 1$ for $n>1$ $x(1) = 1$ (solve for $n = 3^k$)

$$x(n) = x(n/3) + 1 \rightarrow (1)$$

$$x(1) = 1 \rightarrow (1)$$

Sub $x = 3^k$ in (1)

$$x(3^k) = x\left(\frac{3^k}{3}\right) + 1$$

$$= x(3^{k-1}) + 1)$$

$$x(3^0) = x(1) = 1$$

$$x(3^1) = x(3^{1-1}) = x(3^0) + 1 = 2$$

$$x(3^1) = 2$$

$$x(3^2) = x(3^{2-1}) + 1 = x(3^1) + 1) = 3$$

$$\therefore x(3^k) = k + 1$$

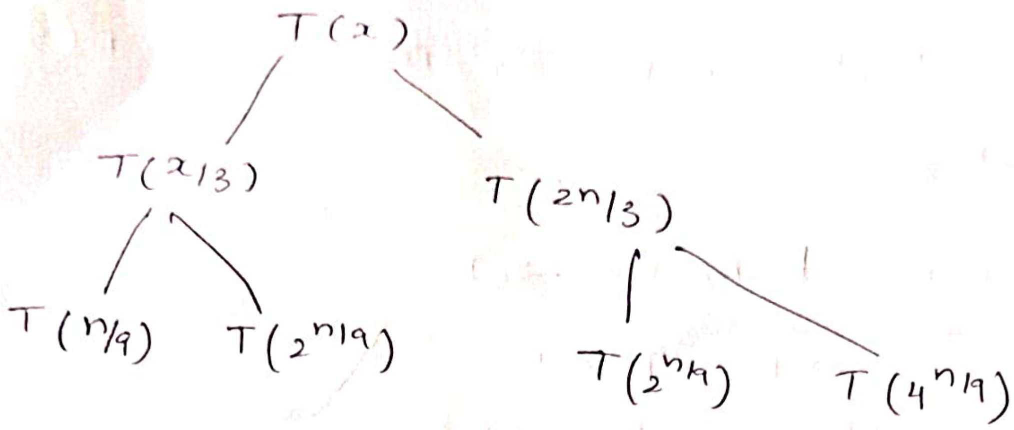
2) Evaluate the following recurrence completely.

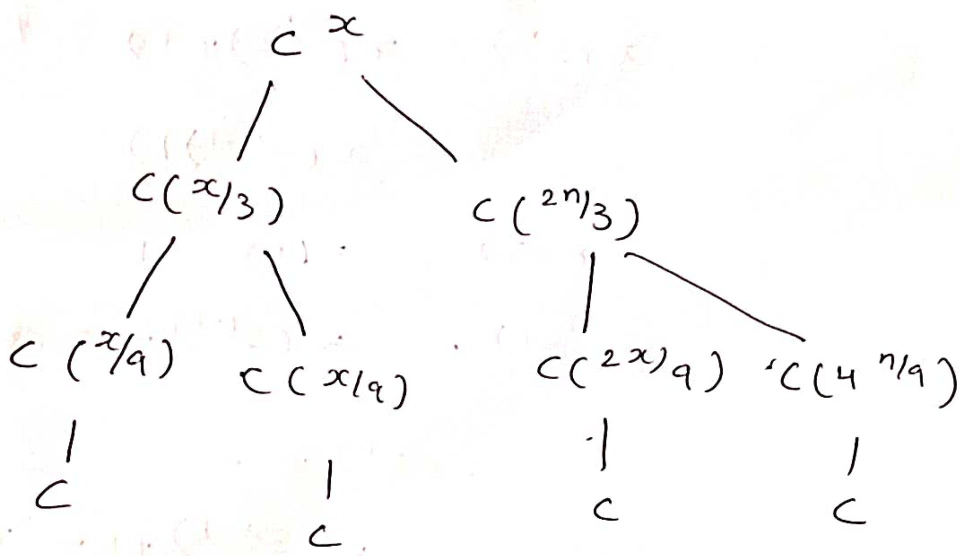
i) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$

$$T(n) = T(n/2) + 1$$

$$T(2^k) = T(2^k/2) + 1$$

$$T(2^k) = T(2^{k-1}) + 1$$





Length $= \log_3 x$ (div by 3)

$T(n) = cx \cdot \log_3 x \Rightarrow \omega(n \log n)$

3) Consider the following recurrision algorithm

min1 (A[0 ... n-1])

if n=1 return A[0]

Else temp = Min1(A[0 ... n-2])

temp <= A[n-1] return temp

else Return A[n-1]

a) What does this algorithm compute?

This algorithm computers minimum element in an array A of sige n.

If $i<n$, $A[i]$ is smaller than all element, then

$A[j]$ ∃ $j = i+1$ to $n+1$, then it returns $A[i]$.

It also returns the leftmost minimum element

b) Setup a recurrence relation for the algorithm basic operation count and solve it.

mainly comparision occurs during recursion

So, $T(n) = T(n-1) + 1$, where $n>1$ (one comparion at every step except $n=1$)

$T(1) = 0$ (when $n=1$ no comparison)

$T(n) = T(1) + (n-1) + 1$

$= 0 + (n-1)$

$T(n) = n-1$

④ Analyse the order of growth

(i) $F(x) - 2x^2 + 5$ and $g(x) - 7n$

use the $\Omega\, g(n)$ notation.

$F(x) = 2x^2 + 5$

$c \cdot g(n) = 7n$

$F(x) \geq c \cdot g(n)$

$n = 1$

$F(1) = 2(1)^2 + 5$

$= 2 + 5$

$= 7$

$c \cdot g(x) = 7n$

$= 7(1)$

$= 7$

$n = 2$

$F(2) = (2)^2 + 5$

$= 8 + 5$

$= 13$

$c \cdot g(n) = 7n$

$= 7(2)$

$= 14$

$n = 3$

$F(3) = 2(3^2) + 5$

$= 18 + 5$

$= 23$

$c \cdot g(n) = 7n$

$= 7(3$

$= 21$

$\therefore$ $n = 1$; $7 = 7$

$n = 2$; $13 = 14$

$n = 3$; $23 = 21$

$n \geq 3$; $F(n) \geq c \cdot g(n)$