**Part4:**

**Q: The project argues that "the Manhattan distances are consistent in gridworlds in which the agent can move only in the four main compass directions." Prove that this is indeed the case.**

The Manhattan distances hold true in gridworlds where an agent can only move in the four main compass directions. This is due to the fact that no matter what order the cells are visited, the Manhattan distance between them is constant. For example, if the agent visits the cells in the following order: (0, 0), (1, 1), (2, 2), or (2, 2), (1, 1), (0, 0), the Manhattan distance between the cells (0, 0) and (2, 2) is 4 .

**Here is evidence that gridworlds consistently use Manhattan distances:**

Let s and g represent two cells in a gridworld, and let d represent their distance in Manhattan. Make p the route/path from s to g. The Manhattan distance between cells s and g is then equal to the total of the Manhattan distances along the path p.

For example, the Manhattan distance between the cells (0, 0) and (2, 2) is equal to the sum of the Manhattan distances between the cells (0, 0) and (1, 0), (1, 0) and (1, 1), and (1, 1) and (2, 2).

Now Let p represent a path from s to g that travels through the cells in the reverse order. The Manhattan distance between s and g is then also equal to the total of the Manhattan distances between adjacent cells along path p′.

**Here is a more detailed explanation of the proof:**

The Manhattan distance between two cells is the sum of the absolute values of the differences between the coordinates of the two cells. For example, the Manhattan distance between the cells (0, 0) and (2, 2) is 4, because the absolute value of the difference between the x-coordinates is 2 and the absolute value of the difference between the y-coordinates is 2.

The sum of the absolute values of the differences between the cells' coordinates is unaffected by the order in which they are visited. The sum of the absolute values of the differences, for instance, between the coordinates of the cells (0, 0), (1, 1), and (2, 2) is equal to the sum between the coordinates of the cells (2, 2), (1, 1), and (0, 0).

Therefore, the Manhattan distance between two cells is the same regardless of the order in which the cells are visited.

**Q: Furthermore, it is argued that "The h-values h(s) are not only admissible but also consistent." Prove that Adaptive A leaves initially consistent h-values consistent even if action costs can increase.**

**Initially Consistent h-values:**

The h-values assigned to each state in adaptive A\* are initially derived from a consistent heuristic function. For any state s and its successor state s', the estimated cost of moving from s to a goal state, h(s), should not be greater than the estimated cost of moving from s to s' plus the estimated cost from s' to the goal, h(s'). This is known as the consistency condition.

**Proof by Contradiction:**

To prove that Adaptive A\* maintains consistency, we will assume the opposite: that Adaptive A\* can produce inconsistent h-values during the search process when action costs increase.

**Inconsistent h-values:**

Consider a state s for which the estimated cost to reach a goal state, h(s), during the search process changes as action costs rise. This would imply that there is a successor state s' of s, where the cost of moving from s to s' plus the estimated cost of moving from s to the goal, h(s), is less than the total cost of moving from s to s'.

**Reconsidering g-values:**

Now let's think about the g-values of states. The price of the path from the start state to a given state is represented by its g-value. The costs of the path discovered during the search are used to dynamically update the g-values in Adaptive A\*.

**Path through s to s':**

It follows that the estimated cost from s' to the goal is less than the actual cost of the path from the start state to s' passing through s because h(s') is less than the cost of moving from s to s' plus h(s). The definition of the g-values, which should reflect the actual costs of the path discovered during the search, is in conflict with this.

**Consistency is maintained:**

We can infer that the assumption that inconsistent h-values exist in Adaptive A\* is untrue from the contradiction in step 5. Thus, even though action costs may rise, adaptive A\* maintains initially consistent h-values.

We have demonstrated that Adaptive A\* maintains consistency in the h-values throughout the search process, even when action costs can rise, by following this proof by contradiction. This characteristic guarantees the algorithm's efficacy and dependability in locating the best answers in cost-variable domains.

**Additional information:**

* The Adaptive A algorithm is a type of incremental heuristic search algorithm. This means that the algorithm can be used to solve a series of similar search problems by updating the h-values from previous searches.
* The Adaptive A algorithm is admissible, which means that it never overestimates the cost of a path.
* The Adaptive A algorithm is consistent, which means that it does not change if the order in which the states are visited is changed.