DE:

$$M(x,y)\,dx + N(x,y)\,dy = 0 \longrightarrow (1)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M(x,y)\,dx + \int (\text{Terms in } N(x,y) \text{ free from } x)\,dy = 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\downarrow$ y not present	$\downarrow$ x not present
$\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N(x,y)} = P(x)$	$\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M(x,y)} = Q(y)$
$\mu(x) = e^{\int p\,dx}$	$\mu(y) = e^{\int Q\,dy}$

2. Homogeneous differential eq.

$$y\,dx + x(\ln x - \ln y - 1)\,dy = 0 \longrightarrow (1)$$

$$y(1) = e.$$

$$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 1\ln x + \cancel{1} - \ln y - \cancel{1}$$

$$= \ln x - \ln y$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$= \frac{N_x - M_y}{M(x,y)}$$

$$= \frac{\ln x - \ln y - 1}{\cancel{x}\, y} \quad X \quad \text{(Because } x \text{ is present}$$

Rule 2:

$$\frac{M_y - N_x}{N(x,y)} = \frac{1 - (\ln x - \ln y)}{x(\ln x - \ln y - 1)}$$

$$= \frac{-[\cancel{\ln x - \ln y - 1}]}{x(\cancel{\ln x - \ln y - 1})} = -\frac{1}{x}$$

(no y in it)

$$p(x) = -\frac{1}{x}$$

$$\mu(x) = e^{\int p\,dx} = e^{-\int \frac{1}{x} dx}$$

$$= e^{-\ln|x|} = e^{\ln|x|^{-1}}$$

$$\boxed{\mu(x) = \frac{1}{x}}$$

Multiply eq ① by $1/x$

$$\frac{y}{x}\,dx + (\ln x - \ln y - 1)\,dy = 0$$

$$\int \frac{y}{x}\,dx + \int (-\ln y - 1)\,dy = 0$$

$$y \ln|x| + - \int (\ln y + 1)\,dy = 0$$

$$y \ln|x| - \ln y \cdot y - \int \frac{1}{y} \cdot y\,dy + y = C$$

x

$$y \ln|x| - y \ln|y| + y - y = c$$

$$y(\ln|x - \ln|y) = c$$

$$y \ln\frac{x}{y} = c$$

$$e \ln\frac{1}{e} = c$$

$$e[\ln 1 - \ln e] = c \qquad \because \ln 1 = 0$$

$$-e \ln e = c \qquad \because \ln e = 1$$

$$\boxed{c = -e}$$

Bernoulli's Equation: (1st order non-linear)

An equation of the form

$$\frac{dy}{dx} + p(x)y = Q(x)y^n \longrightarrow (1)$$

is called Bernoulli's eq. where $p(x)$ and $Q(x)$ are function of x and n is a real number.

suppose $n \neq 0$ or $n \neq 1$ so to reduce eq (1) to linear form we have following procedure.

Ex 2.5
15-20 odd

(21) IVP

Dividing by y^n

$$y^{-n}\frac{dy}{dx} + p(x)\,y^{1-n} = Q(x) \rightarrow (2)$$

put $y^{1-n} = V$

differentiate:

$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dv}{dx}$$

Eqn (2) is now

$$\frac{1}{1-n}\frac{dv}{dx} + p(x)V = Q(x)$$

$$\boxed{\frac{dv}{dx} + (1-n)P(x)V = (1-n)Q(x)}$$

→ Linear in V

Ex 2.5

16) $\frac{dy}{dx} - y = e^x y^2$

Dividing by y^2

$$y^{-2}\frac{dy}{dx} - y^{-1} = e^x$$

Put $y^{-1} = V$

$$-y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

$$\frac{dv}{dx} + V = -e^{x} \longrightarrow (iii)$$

$$P(x) = 1$$

$$\mu(x) = e^{\int P dx} = e^{\int dx} = e^{x}$$

- Multiply eq (iii) by e^{x}

$$e^{x}\frac{dv}{dx} + Ve^{x} = -e^{2x}$$

$$\int \frac{d}{dx}(Ve^{x})dx = -\int e^{2x}dx$$

$$Ve^{x} = -\frac{1}{2}e^{2x} + C$$

$$\boxed{\frac{1}{y} = -\frac{1}{2}e^{x} + Ce^{-x}}$$