P.D.E.   er Series

$y = f(x) = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

$y = g(x, t) = x^2 - xt + t^2$

$\frac{\partial y}{\partial x} = 2x - t$

$\frac{\partial y}{\partial t} = -x + 2t$

Consider a few PD's

$\left(\frac{\partial^2 u}{\partial t^2}\right) = c^2 \frac{\partial^2 u}{\partial x^2}$ 1-dim wave eq.

$\left(\frac{\partial u}{\partial t}\right) = c^2 \cdot \frac{\partial^2 y}{\partial x^2}$ " " heat "

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ 2 " laplace eq

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ 2 " Poisson eq.

$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ 2 " wave "

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad 3" \text{ laplace "}$$

All these equations are linear and of order 2.

ex Show that $U(x, y) = x^2 - y^2$ satisfies laplace eq.,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad u = e^x \sin y$$

$$U(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \quad \text{satisfies } \boxed{\nabla^2 u = 0}$$

$$U(x, y, z) = (x^2+y^2+z^2)^{-1/2}$$

$$U_x = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \cdot 2x$$

$$= \frac{-x}{(x^2+y^2+z^2)^{3/2}}$$

$$U_{xx} = -\left[\frac{(x^2+y^2+z^2)^{3/2} \cdot 1 - x \cdot \frac{3}{2}(x^2+y^2+z^2)^{1/2}(2x)}{(x^2+y^2+z^2)^3}\right]$$

$$U_{xx} = \frac{-\left[x^2+y^2+z^2-3x^2\right]}{(x^2+y^2+z^2)^{5/2}}$$

$$U_{xx} = \frac{-(y^2+z^2-2x^2)}{\text{"}} = \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}$$

$U_{yy} = -$ - - - - - - - .

Sol. of a PDE's by method of separating variables (product method):

Ex Solve the PDE's

$$U_x + U_y = 0 \quad \text{—} \quad (1)$$

let,

$U(x,y) = F(x)\,G(y)$ be the solution of eq. (1)

$$U_x = F'(x)\,G(y)$$

$U_y = F(x).\,\dot{G}(y)$, where dot is the derivative of w.r.t 'y'

Putting the values of U_x and U_y in eq (1),

$$F'(x)\,G(y) + F(x)\,\dot{G}(y) = 0$$

Dividing by $F(x)\,G(y)$.

$$\frac{F'(x)}{F(x)} = -\frac{\dot{G}(y)}{G(y)} = k$$

$$\frac{F'(x)}{F(x)} = k \qquad \frac{G'(y)}{G(y)} = -k$$

Integrating,

$$\ln|F(x)| = kx + \ln c_1$$
$$\ln|F(x)| = \ln c_1 e^{kx}$$
$$F(x) = c_1 e^{kx}$$

$$\ln|G(y)| = -ky + \ln c_2$$
$$\ln|G(y)| = \ln c_2 e^{-ky}$$
$$G(y) = c_2 e^{-ky}$$

The sol. is,

$$U(x,y) = c e^{kx} \cdot e^{-ky}$$
$$= c e^{k(x-y)}$$

Questions

2) $U_{xy} = U$

3) $U_x + U_y = (x+y)U$

4) $y^2 U_x - x^2 U_y = 0$

5) $U_x = 4U_y$

6) $U_{xy} + U_x + x = 0$

7) $U_{xy} + U_y + y = 0$

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