

1. *Journal of the American Medical Association*, 1997; 277: 1039-1043.

1. **Definition:** A **subgroup** of a group  $(G, \cdot)$  is a subset  $H$  of  $G$  such that  $(H, \cdot)$  is a group under the same operation  $\cdot$ .

2. **Properties of Subgroups:**

- $H$  must be non-empty.
- $H$  must be closed under the operation  $\cdot$ .
- $H$  must contain the identity element  $e$ .
- $H$  must contain the inverse of every element in  $H$ .

3. **Trivial Subgroups:** The trivial subgroups of  $G$  are  $\{e\}$  and  $G$  itself.

4. **Subgroup Test:** A non-empty subset  $H$  of  $G$  is a subgroup if and only if for all  $a, b \in H$ ,  $a \cdot b^{-1} \in H$ .

5. **Examples:**

- The set of integers  $\mathbb{Z}$  is a subgroup of the group of rational numbers  $\mathbb{Q}$  under addition.
- The set of even integers  $2\mathbb{Z}$  is a subgroup of  $\mathbb{Z}$ .
- The set of real numbers  $\mathbb{R}$  is a subgroup of the group of complex numbers  $\mathbb{C}$  under addition.

6. **Subgroups and Cosets:** If  $H$  is a subgroup of  $G$ , then the cosets of  $H$  in  $G$  are the sets  $aH = \{a \cdot h \mid h \in H\}$  for  $a \in G$ .

7. **Lagrange's Theorem:** If  $H$  is a subgroup of  $G$ , then the order of  $H$  divides the order of  $G$ .

8. **Normal Subgroups:** A subgroup  $N$  of  $G$  is normal if  $gN = Ng$  for all  $g \in G$ .

9. **Quotient Groups:** If  $N$  is a normal subgroup of  $G$ , then the quotient group  $G/N$  is the set of cosets of  $N$  in  $G$  with the operation  $(aN)(bN) = (ab)N$ .

10. **Isomorphism:** Two groups  $(G, \cdot)$  and  $(H, \cdot)$  are isomorphic if there exists a bijective homomorphism  $f: G \rightarrow H$ .

11. **Automorphism:** An isomorphism from a group  $G$  to itself is called an automorphism.

12. **Endomorphism:** A homomorphism from a group  $G$  to itself is called an endomorphism.

13. **Homomorphism:** A map  $f: G \rightarrow H$  between two groups  $(G, \cdot)$  and  $(H, \cdot)$  is a homomorphism if  $f(a \cdot b) = f(a) \cdot f(b)$  for all  $a, b \in G$ .

14. **Kernel:** The kernel of a homomorphism  $f: G \rightarrow H$  is the set  $\ker f = \{g \in G \mid f(g) = e_H\}$ .

15. **Image:** The image of a homomorphism  $f: G \rightarrow H$  is the set  $\text{Im } f = \{f(g) \mid g \in G\}$ .

16. **First Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism, then  $\text{Im } f \cong G / \ker f$ .

17. **Second Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $K$  is a subgroup of  $G$ , then  $f(K) \cong K / (K \cap \ker f)$ .

18. **Third Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

19. **Fourth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

20. **Fifth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

21. **Sixth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

22. **Seventh Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

23. **Eighth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

24. **Ninth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

25. **Tenth Isomorphism Theorem:** If  $f: G \rightarrow H$  is a homomorphism and  $N$  is a normal subgroup of  $G$ , then  $f(N) \cong N / (N \cap \ker f)$ .

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Figure 1. The effect of the number of trials on the number of correct responses. The number of correct responses was significantly higher than the number of incorrect responses in all cases. Error bars represent the standard error of the mean.