

COMSM1201 : Data Structures & Algorithms

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Built : November 26, 2024



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- Let's look at some toy examples to begin with.

```
1  #include <stdio.h>
2  #include <string.h>
3
4  #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
5
6  void strrev(char* s, int n);
7
8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, strlen(str));
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Iterative Inplace String Reverse */
17 void strrev(char* s, int n)
18 {
19     for(int i=0, j=n-1; i<j; i++, j--){
20         SWAP(s[i], s[j]);
21     }
22 }
```

Execution :

!dlroW olleH

Recursion for *strrev()*

```
1  #include <stdio.h>
2  #include <string.h>
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4  #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
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6  void strrev(char* s, int start, int end);
7
8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, 0, strlen(str)-1);
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Recursive : Inplace String Reverse */
17 void strrev(char* s, int start, int end)
18 {
19     if(start >= end){
20         return;
21     }
22     SWAP(s[start], s[end]);
23     strrev(s, start+1, end-1);
24 }
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23     strrev(s, start+1, end-1);
24 }
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- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

$\text{fib}(1)$ is 1

$\text{fib}(2)$ is 1

$\text{fib}(n)$ is $\text{fib}(n-1) + \text{fib}(n-2)$

1, 1, 2, 3, 5, 8, 13, 21, ...

Iterative & Recursive Fibonacci

```
1  #include <stdio.h>
2
3  #define MAXFIB 24
4
5  int fibonacci(int n);
6
7  int main(void)
8  {
9
10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15 }
16
17
18 int fibonacci(int n)
19 {
20     if(n <= 2){
21         return 1;
22     }
23     int a = 1;
24     int b = 1;
25     int next;
26     for(int i=3; i<=n; i++){
27         next = a + b;
28         a = b;
29         b = next;
30     }
31     return b;
32 }
```

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12     }
13
14     return 0;
15 }
16
17 int fibonacci(int n)
18 {
19     if(n <= 2){
20         return 1;
21     }
22     int a = 1;
23     int b = 1;
24     int next;
25     for(int i=3; i<=n; i++){
26         next = a + b;
27         a = b;
28         b = next;
29     }
30     return b;
31 }
32 }
```

Execution :

```
1 = 1
2 = 1
3 = 2
4 = 3
5 = 5
6 = 8
7 = 13
8 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

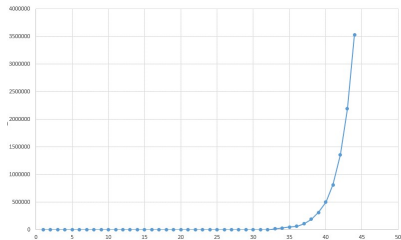
Iterative & Recursive Fibonacci

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7  int main(void)
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10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15
16 }
17
18 int fibonacci(int n)
19 {
20     if(n == 1) return 1;
21     if(n == 2) return 1;
22     return( fibonacci(n-1)+fibonacci(n-2));
23 }
```

Iterative & Recursive Fibonacci

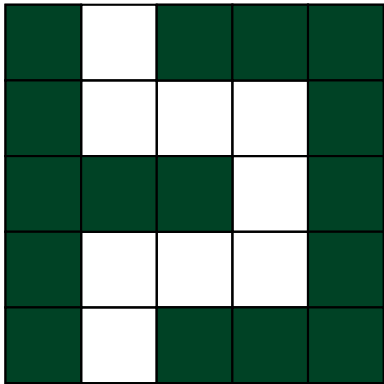
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1  #include <stdio.h>
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7  int main(void)
8  {
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10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
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14     return 0;
15 }
16
17
18 int fibonacci(int n)
19 {
20     if(n == 1) return 1;
21     if(n == 2) return 1;
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23 }
```

It's interesting to see how run-time increases as the length of the sequence is raised.



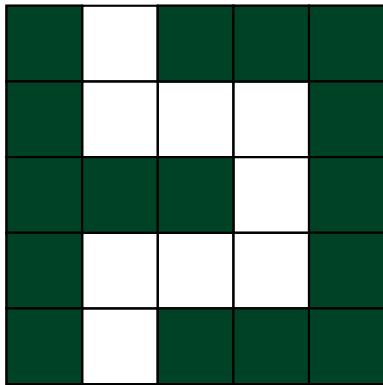
Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



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```
bool explore(int x, int y, char mz[YS][XS])
{
    if mz[y][x] is exit return true;

    Mark mz[y][x] so we don't return here

    if we can go up :
        if(explore(x, y+1, mz)) return true

    if we can go right :
        if(explore(x+1, y, mz)) return true

    Do left & down in a similar manner

    return false; // Failed to find route
}
```

Permuting

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- Permutations are all possible ways of rearranging the positions of the characters.

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BAC
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Execution :

ABC
ACB
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```
1 // From e.g. http://www.geeksforgeeks.org
2 #include <stdio.h>
3 #include <string.h>
4
5 #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
6
7 void permute(char* a, int s, int e);
8
9 int main()
10 {
11     char str[] = "ABC";
12     int n = strlen(str);
13     permute(str, 0, n-1);
14     return 0;
15 }
16
17 void permute(char* a, int s, int e)
18 {
19     if (s == e){
20         printf("%s\n", a);
21         return;
22     }
23     for (int i = s; i <= e; i++){
24         SWAP((a+s), (a+i)); // Bring one char to the front
25         permute(a, s+1, e);
26         SWAP((a+s), (a+i)); // Backtrack
27     }
28 }
```

Self-test : Power

- Raising a number to a power $n = 2^5$ is the same as multiple multiplications
 $n = 2*2*2*2*2$.

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 $n = 2*2*2*2*2$.
- Or, thinking recursively, $n = 2 * (2^4)$.

```
1  /* Try to write power(a,b) to computer a^b
2  without using any maths functions other than
3  multiplication :
4  Try (1) iterative then (2) recursive
5  (3) Trick that for  $n\%2==0$ ,  $x^n = x^{(n/2)}*x^{(n/2)}$ 
6
7  */
8
9  #include <stdio.h>
10
11 int power(unsigned int a, unsigned int b);
12
13 int main(void)
14 {
15
16     int x = 2;
17     int y = 16;
18
19     printf("%d^%d = %d\n", x, y, power(x,y));
20
21 }
22
23 int power(unsigned int a, unsigned int b)
24 {
25 }
```

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- Simply move through the array from beginning to end, stopping when you have found the value you require.

```
1  #include <stdio.h>
2  #include <string.h>
3  #include <assert.h>
4
5  #define NOTFOUND -1
6  #define NUMPEOPLE 6
7  typedef struct person{
8      char* name; int age;
9  } person;
10
11 int findAge(const char* name, const person* p, int n);
12
13 int main(void)
14 {
15     person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
16                                {"Chumley", 26}, {"Dalton", 25},
17                                {"Eggson", 22}, {"Fulton", 41} };
18
19     assert(findAge("Eggson", ppl, NUMPEOPLE)==22);
20     assert(findAge("Campbell", ppl, NUMPEOPLE)==NOTFOUND);
21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         if(strcmp(name, p[j].name) == 0){
28             return p[j].age;
29         }
30     }
31     return NOTFOUND;
32 }
```

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11 int findAge(const char* name, const person* p, int n);
12
13 int main(void)
14 {
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18
19     assert(findAge("Eggson", ppl, NUMPEOPLE)==22);
20     assert(findAge("Campbell", ppl, NUMPEOPLE)==NOTFOUND);
21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         int m = strcmp(name, p[j].name);
28         if(m == 0) // Braces!
29             return p[j].age;
30         if(m < 0)
31             return NOTFOUND;
32     }
33     return NOTFOUND;
34 }
```

Binary Search for *101*

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Binary Search for 101

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4 7 19 25 36 37 50 100 101 205 220 270 301 321

↑

↑

↑

↑

↑

↑

↑

↑

↑

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5  #define NMBS 1000000
6
7  int bin_it(int k, const int* a, int l, int r);
8
9  int main(void)
10 {
11     int a[NMBS];
12     srand(time(NULL));
13
14     // Put even numbers into array
15     for(int i=0; i<NMBS; i++){
16         a[i] = 2*i;
17     }
18
19     // Do many searches for a random number
20     for(int i=0; i<10*NMBS; i++){
21         int n = rand()%NMBS;
22         if((n%2) == 0){
23             assert(bin_it(n, a, 0, NMBS-1) == n/2);
24         }
25         else{ // No odd numbers in this list
26             assert(bin_it(n, a, 0, NMBS-1) < 0);
27         }
28     }
29     return 0;
30 }
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int l, int r)
{
    while(l <= r){
        int m = (l+r)/2;
        if(k == a[m]){
            return m;
        }
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

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            }
            else{
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            }
        }
    }
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```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(l > r) return -1;
    int m = (l+r)/2;
    if(k == a[m]){
        return m;
    }
    else{
        if (k > a[m]){
            return bin_rec(k, a, m+1, r);
        }
        else{
            return bin_rec(k, a, l, m-1);
        }
    }
}
```

Interpolation Search

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

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- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - l[0]) / (l[n - 1] - l[0]) * n$$

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- when searching for '15' :

0 4 5 9 10 12 15 20
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- when searching for '15' :

0 4 5 9 10 12 15 20
 ↑↑

```
int interp(int k, const int* a, int l, int r)
{
    int m;
    double md;

    while(l <= r){
        md = ((double)(k-a[l])/
              (double)(a[r]-a[l]))*
              (double)(r-l)
            )
            +(double)(l);
        m = 0.5 + md;
        if((m > r) || (m < l)){
            return -1;
        }
        if(k == a[m])
            return m;
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
}
```

Algorithmic Complexity

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <time.h>
4
5  #define CSEC (double)(CLOCKS_PER_SEC)
6  #define BIGLOOP 1000000000
7
8  int main(void)
9  {
10
11     clock_t c1 = clock();
12     for(int i=0; i<BIGLOOP; i++){
13         int j = i * 2;
14     }
15     clock_t c2 = clock();
16     printf("%f\n", (double)(c2-c1)/CSEC);
17     return 0;
18 }
19 }
```

- This code on an old Dell laptop took:

Algorithmic Complexity

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <time.h>
4
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Algorithmic Complexity

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- We'll discuss the dream of a **$O(1)$** search later in "Hashing".

Binary vs. Interpolation Timing

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5
6  int bin_it(int k, const int *a, int l, int r);
7  int bin_rec(int k, const int *a, int l, int r);
8  int interp(int k, const int *a, int l, int r);
9  int* parse_args(int argc, char* argv[], int* n, int* srch);
10
11 int main(int argc, char* argv[])
12 {
13
14     int i, n, srch;
15     int* a;
16     int (*p[3])(int k, const int*a, int l, int r) =
17         {bin_it, bin_rec, interp};
18
19     a = parse_args(argc, argv, &n, &srch);
20
21     srand(time(NULL));
22     for(i=0; i<n; i++){
23         a[i] = 2*i;
24     }
25     for(i=0; i<5000000; i++){
26         assert((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
27     }
28
29     free(a);
30     return 0;
31 }
32 }
```

Binary vs. Interpolation Timing

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```

Execution :

Binary Search : Iterative

n = 100000 = 0.39

n = 800000 = 0.57

n = 6400000 = 1.00

n = 51200000 = 2.46

Binary Search : Recursive

n = 100000 = 0.40

n = 800000 = 0.56

n = 6400000 = 0.97

n = 51200000 = 2.42

Interpolation

n = 100000 = 0.05

n = 800000 = 0.05

n = 6400000 = 0.10

n = 51200000 = 0.13

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- By following pointers one after another, we can travel right along the structure.

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include "general.h"
4
5  typedef struct data{
6      int i;
7      struct data* next;
8  } Data;
9
10 Data* allocateData(int i);
11 void printList(Data* l);
12
13 int main(void)
14 {
15     int i;
16     Data* start, *current;
17     start = current = NULL;
18     printf("Enter the first number: ");
19     if(scanf("%i", &i) == 1){
20         start = current = allocateData(i);
21     }
22     else{
23         on_error("Couldn't read an int");
24     }
25
26     printf("Enter more numbers: ");
27     while(scanf("%i", &i) == 1){
28         current->next = allocateData(i);
29         current = current->next;
30     }
31     printList(start);
32     return 0; // Should Free List
33 }
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) calloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* l)
{
    printf("\n");
    do{
        printf("Number : %i\n", l->i);
        l = l->next;
    }while(l != NULL);
    printf("END\n");
}
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) malloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* l)
{
    printf("\n");
    do{
        printf("Number : %i\n", l->i);
        l = l->next;
    }while(l != NULL);
    printf("END\n");
}
```

Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
        }
        n = n->next;
    }while(n != NULL);
    return NULL;
}

void printList_r(Data* l)
{
    // Recursive Base-Case
    if(l == NULL) return;

    printf("Number: %i\n", l->i);
    printList_r(l->next);
}
```

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- The **user** of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)
- Actually, I'll sometimes blur the boundaries of Data Structures (e.g. a linked list) with ADTs (e.g. a dictionary) themselves.

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```
1  #include "../General/general.h"
2
3  typedef int colltype;
4
5  typedef struct coll coll;
6
7  #include <stdio.h>
8  #include <stdlib.h>
9  #include <assert.h>
10
11 // Create an empty coll
12 coll* coll_init(void);
13 // Add element onto top
14 void coll_add(coll* c, colltype i);
15 // Take element out
16 bool coll_remove(coll* c, colltype d);
17 // Does this exist ?
18 bool coll_isin(coll* c, colltype i);
19 // Return size of coll
20 int coll_size(coll* c);
21 // Clears all space used
22 bool coll_free(coll* c);
```

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Fixed/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7      // Underlying array
8      colltype a[FIXEDSIZE];
9      int size;
10 };
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) ncalloc(1, sizeof(coll));
7      c->size = 0;
8      return c;
9  }
10
11 int coll_size(coll* c)
12 {
13     if(c==NULL){
14         return 0;
15     }
16     return c->size;
17 }
18
19 bool coll_isin(coll* c, colltype d)
20 {
21     for(int i=0; i<coll_size(c); i++){
22         if(c->a[i] == d){
23             return true;
24         }
25     }
26     return false;
27 }
```

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7      c->size = 0;
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```

```
void coll_add(coll* c, colltype d)
{
    if(c){
        if(c->size >= FIXEDSIZE){
            on_error("Collection overflow");
        }
        c->a[c->size] = d;
        c->size = c->size + 1;
    }
}

bool coll_remove(coll* c, colltype d)
{
    for(int i=0; i<coll_size(c); i++){
        if(c->a[i] == d){
            // Shuffle end of array left one
            for(int j=i; j<coll_size(c); j++){
                c->a[j] = c->a[j+1];
            }
            c->size = c->size - 1;
            return true;
        }
    }
    return false;
}

bool coll_free(coll* c)
{
    free(c);
    return true;
}
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Realloc"
4
5  #define INITSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
1  #pragma once
2
3  #define COLLTTYPE "Realloc"
4
5  #define INITSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

Realloc/realloc.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) ncalloc(1, sizeof(coll));
7      c->a = (colltype*) ncalloc(INITSIZE, sizeof(colltype));
8      c->size = 0;
9      c->capacity = INITSIZE;
10     return c;
11 }
12
13 void coll_add(coll* c, colltype d)
14 {
15     if(c){
16         if(c->size >= c->capacity){
17             c->a = (colltype*) nrealloc(c->a,
18                                     sizeof(colltype)*c->capacity*SCALEFACTOR);
19             c->capacity = c->capacity*SCALEFACTOR;
20         }
21         c->a[c->size] = d;
22         c->size = c->size + 1;
23     }
24 }
```


Collection ADT via a Linked List

Linked/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Linked"
4
5  struct dataframe {
6      colltype i;
7      struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

Collection ADT via a Linked List

Linked/specific.h:

```
1  #pragma once
2
3  #define COLLYTYPE "Linked"
4
5  struct dataframe {
6      colltype i;
7      struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

Linked/linked.c:

```
#include "../coll.h"
#include "specific.h"

coll* coll_init(void)
{
    coll* c = (coll*) nalloc(1, sizeof(coll));
    return c;
}

int coll_size(coll* c)
{
    if(c==NULL){
        return 0;
    }
    return c->size;
}

bool coll_isin(coll* c, colltype d)
{
    if(c == NULL || c->start==NULL){
        return false;
    }
    dataframe* f = c->start;
    do{
        if(f->i == d){
            return true;
        }
        f = f->next;
    }while(f != NULL);
    return false;
}
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(1, sizeof(dataframe));
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
{
    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(1, sizeof(dataframe));
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
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    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

```
bool coll_remove(coll* c, colltype d)
{
    dataframe* f1, *f2;
    if((c==NULL) || (c->start==NULL)){
        return false;
    }

    // If Front
    if(c->start->i == d){
        f1 = c->start->next;
        free(c->start);
        c->start = f1;
        c->size = c->size - 1;
        return true;
    }

    f1 = c->start;
    f2 = c->start->next;
    do{
        if(f2->i == d){
            f1->next = f2->next;
            free(f2);
            c->size = c->size - 1;
            return true;
        }
        f1 = f2;
        f2 = f1->next;
    }while(f2 != NULL);
    return false;
}
```

Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (`testcoll.c`) code.

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- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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 - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.

Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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Collection Summary

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 - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.
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 - fast to delete an element.

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Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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 - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array : Implementation fairly simple. Deletion expensive. Every realloc() is very **expensive**. Need to tune SCALEFACTOR.
 - Linked : Slightly fiddly implementation - fast to delete an element.

Task	Fixed Array	Realloc Array	Linked List
Insert new element	$O(1)$ at end <i>if space</i>	$O(1)$ at end <i>but realloc()</i>	$O(1)$ at front
Search for an element	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>
Search + delete	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(1)$ <i>delete 'free'</i>

- If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to $O(\log n)$ or $O(\log \log n)$ search times.

ADTs Making Coding Simpler

Linked List code from the previous Chapter :

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include "general.h"
4
5  typedef struct data{
6      int i;
7      struct data* next;
8  } Data;
9
10 Data* allocateData(int i);
11 void printList(Data* l);
12
13 int main(void)
14 {
15     int i;
16     Data* start, *current;
17     start = current = NULL;
18     printf("Enter the first number: ");
19     if(scanf("%i", &i) == 1){
20         start = current = allocateData(i);
21     }
22     else{
23         on_error("Couldn't read an int");
24     }
25
26     printf("Enter more numbers: ");
27     while(scanf("%i", &i) == 1){
28         current->next = allocateData(i);
29         current = current->next;
30     }
31     printList(start);
32     return 0; // Should Free List
33 }
```

ADTs Making Coding Simpler

Linked List code from the previous Chapter :

```
1  #include <stdio.h>
2  #include <stdlib.h>
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4
5  typedef struct data{
6      int i;
7      struct data* next;
8  } Data;
9
10 Data* allocateData(int i);
11 void printList(Data* l);
12
13 int main(void)
14 {
15     int i;
16     Data* start, *current;
17     start = current = NULL;
18     printf("Enter the first number: ");
19     if(scanf("%i", &i) == 1){
20         start = current = allocateData(i);
21     }
22     else{
23         on_error("Couldn't read an int");
24     }
25
26     printf("Enter more numbers: ");
27     while(scanf("%i", &i) == 1){
28         current->next = allocateData(i);
29         current = current->next;
30     }
31     printList(start);
32     return 0; // Should Free List
33 }
```

Becomes :

```
1  #include "coll.h"
2  #include "Fixed/specific.h"
3
4  int main(void)
5  {
6      printf("Please type some numbers :");
7      coll* c = coll_init();
8      int i;
9      while(scanf("%i", &i) == 1){
10         coll_add(c, i);
11     }
12     // Do print etc.
13     coll_free(c);
14     return 0;
15 }
```

Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W : Algorithms III - Huffman/Strings

X : ADTs - Graphs

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

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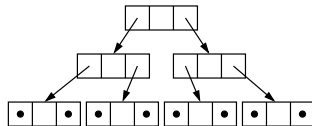
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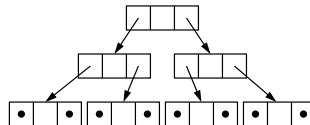
Binary Trees:



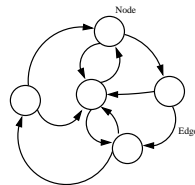
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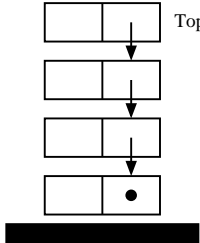


Unidirectional Graph:



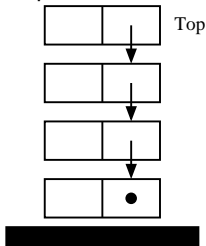
Stacks

The push-down stack:

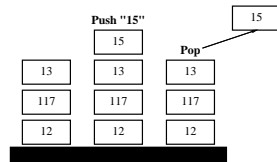


Stacks

The push-down stack:



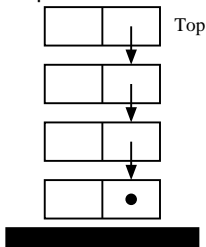
LIFO (Last in, First out):



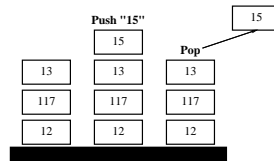
- Operations include push and pop.

Stacks

The push-down stack:



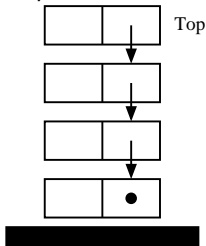
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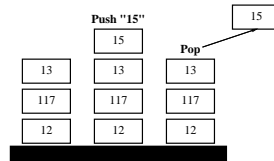
- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.

Stacks

The push-down stack:

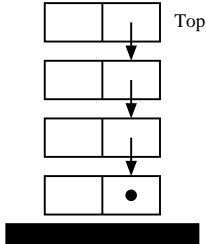


LIFO (Last in, First out):

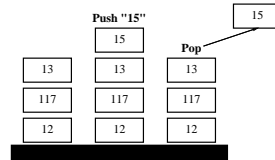


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question : How best to implement such a data type ?

ADT:Stacks Arrays (Realloc) I

stack.h:

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int stacktype;
6
7  typedef struct stack stack;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
19 bool stack_pop(stack* s, stacktype* d);
20 /* Clears all space used */
21 bool stack_free(stack* s);
22
23 /* Optional? */
24
25 /* Copy top element into d (but don't pop it) */
26 bool stack_peek(stack* s, stacktype* d);
27 /* Make a string version - keep .dot in mind */
28 void stack_tostring(stack* s, char* str);
```

ADT:Stacks Arrays (Realloc) I

stack.h:

```
1  #pragma once
2
3  #include "../General/general.h"
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5  typedef int stacktype;
6
7  typedef struct stack stack;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
19 bool stack_pop(stack* s, stacktype* d);
20 /* Clears all space used */
21 bool stack_free(stack* s);
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23 /* Optional? */
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25 /* Copy top element into d (but don't pop it) */
26 bool stack_peek(stack* s, stacktype* d);
27 /* Make a string version - keep .dot in mind */
28 void stack_tostring(stack* s, char* str);
```

Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5
6  #define STACKTYPE "Realloc"
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11 struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16 };
```

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack *s = (stack*) ncalloc(1, sizeof(stack));
9      /* Some implementations would allow you to pass
10       a hint about the initial size of the stack */
11      s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
12      s->size = 0;
13      s->capacity = FIXEDSIZE;
14      return s;
15  }
16
17 void stack_push(stack* s, stacktype d)
18 {
19     if(s==NULL){
20         return;
21     }
22     if(s->size >= s->capacity){
23         s->a = (stacktype*) nrealloc(s->a,
24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25         s->capacity = s->capacity*SCALEFACTOR;
26     }
27     s->a[s->size] = d;
28     s->size = s->size + 1;
29 }
```


ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
1  #include "../stack.h"
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4  #define DOTFILE 5000
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6  stack* stack_init(void)
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8      stack *s = (stack*) ncalloc(1, sizeof(stack));
9      /* Some implementations would allow you to pass
10       a hint about the initial size of the stack */
11      s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
12      s->size = 0;
13      s->capacity = FIXEDSIZE;
14      return s;
15  }
16
17  void stack_push(stack* s, stacktype d)
18  {
19      if(s==NULL){
20          return;
21      }
22      if(s->size >= s->capacity){
23          s->a = (stacktype*) nrealloc(s->a,
24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25          s->capacity = s->capacity*SCALEFACTOR;
26      }
27      s->a[s->size] = d;
28      s->size = s->size + 1;
29  }
```

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s == NULL) || (s->size < 1)){
4          return false;
5      }
6      s->size = s->size - 1;
7      *d = s->a[s->size];
8      return true;
9  }
10
11  bool stack_peek(stack* s, stacktype* d)
12  {
13      if((s==NULL) || (s->size <= 0)){
14          /* Stack is Empty */
15          return false;
16      }
17      *d = s->a[s->size - 1];
18      return true;
19  }
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
1 void stack_tostring(stack* s, char* str)
2 {
3     char tmp[ELEMSIZE];
4     str[0] = '\0';
5     if((s==NULL) || (s->size <1)){
6         return;
7     }
8     for(int i=s->size-1; i>=0; i--){
9         sprintf(tmp, FORMATSTR, s->a[i]);
10        strcat(str, tmp);
11        strcat(str, "|");
12    }
13    str[strlen(str)-1] = '\0';
14 }
15
16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
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2 {
3     char tmp[ELEMSIZE];
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7     }
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16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

- We need a thorough testing program
teststack.c

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
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10        strcat(str, tmp);
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12    }
13    str[strlen(str)-1] = '\0';
14 }
15
16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

- We need a thorough testing program teststack.c
- See also revstr.c : a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

ADT:Stacks Linked I

Linked/specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Linked"
6
7  struct dataframe {
8      stacktype i;
9      struct dataframe* next;
10 };
11 typedef struct dataframe dataframe;
12
13 struct stack {
14     /* Underlying array */
15     dataframe* start;
16     int size;
17 };
```

ADT:Stacks Linked I

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10 };
11 typedef struct dataframe dataframe;
12
13 struct stack {
14     /* Underlying array */
15     dataframe* start;
16     int size;
17 };
```

Linked/linked.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack* s = (stack*) nalloc(1, sizeof(stack));
9      return s;
10 }
11
12 void stack_push(stack* s, stacktype d)
13 {
14     if(s){
15         dataframe* f = nalloc(1, sizeof(dataframe));
16         f->i = d;
17         f->next = s->start;
18         s->start = f;
19         s->size = s->size + 1;
20     }
21 }
```

ADT:Stacks Linked II

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s==NULL) || (s->start==NULL)){
4          return false;
5      }
6
7      dataframe* f = s->start->next;
8      *d = s->start->i;
9      free(s->start);
10     s->start = f;
11     s->size = s->size - 1;
12     return true;
13 }
14
15 bool stack_peek(stack* s, stacktype* d)
16 {
17     if((s==NULL) || (s->start==NULL)){
18         return false;
19     }
20     *d = s->start->i;
21     return true;
22 }
```

ADT:Stacks Linked II

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s==NULL) || (s->start==NULL)){
4          return false;
5      }
6
7      dataframe* f = s->start->next;
8      *d = s->start->i;
9      free(s->start);
10     s->start = f;
11     s->size = s->size - 1;
12     return true;
13 }
14
15 bool stack_peek(stack* s, stacktype* d)
16 {
17     if((s==NULL) || (s->start==NULL)){
18         return false;
19     }
20     *d = s->start->i;
21     return true;
22 }
```

```
1  void stack_tostring(stack* s, char* str)
2  {
3      char tmp[ELEMSIZE];
4      str[0] = '\0';
5      if((s==NULL) || (s->size < 1)){
6          return;
7      }
8      dataframe* p = s->start;
9      while(p){
10         sprintf(tmp, FORMATSIR, p->i);
11         strcat(str, tmp);
12         strcat(str, "|");
13         p = p->next;
14     }
15     str[strlen(str)-1] = '\0';
16 }
17
18 bool stack_free(stack* s)
19 {
20     if(s){
21         dataframe* p = s->start;
22         while(p!=NULL){
23             dataframe* tmp = p->next;
24             free(p);
25             p = tmp;
26         }
27         free(s);
28     }
29     return true;
30 }
```


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N : Recursion

O : Algorithms I - Search

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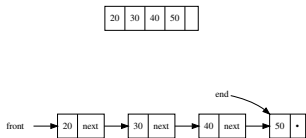
V : Algorithms II - Sort

W : Algorithms III - Huffman/Strings

X : ADTs - Graphs

ADTs : Queues

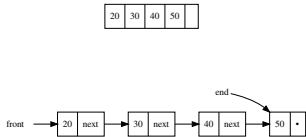
FIFO (First in, First out):



- Intuitively more “useful” than a stack.

ADTs : Queues

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- Think of implementing any kind of service (printer, web etc.)

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FIFO (First in, First out):



- Intuitively more “useful” than a stack.
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- Operations include enqueue, dequeue and size.

queue.h

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int queue_type;
6
7  typedef struct queue queue;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <string.h>
12 #include <assert.h>
13
14 /* Create an empty queue */
15 queue* queue_init(void);
16 /* Add element on end */
17 void queue_enqueue(queue* q, queue_type v);
18 /* Take element off front */
19 bool queue_dequeue(queue* q, queue_type* d);
20 /* Return size of queue */
21 int queue_size(queue* q);
22 /* Clears all space used */
23 bool queue_free(queue* q);
24
25 /* Helps with visualisation & testing */
26 void queue_tostring(queue* q, char* str);
```

ADTs : Queues (Fixed) I

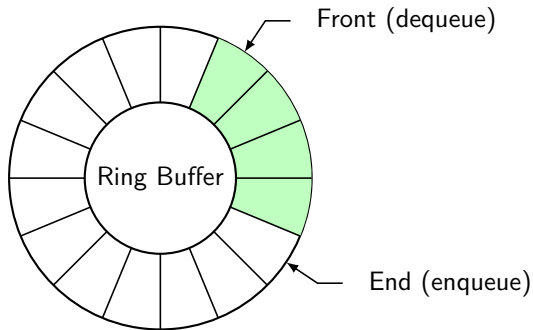
specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Fixed"
7
8  #define BOUNDED 5000
9
10 struct queue {
11     /* Underlying array */
12     queuetype a[BOUNDED];
13     int front;
14     int end;
15 };
16
17 #define DOTFILE 5000
```

ADTs : Queues (Fixed) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
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10 struct queue {
11     /* Underlying array */
12     queuetype a[BOUNDED];
13     int front;
14     int end;
15 };
16
17 #define DOTFILE 5000
```



ADTs : Queues (Fixed) II

fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void __inc(int* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) ncalloc(1, sizeof(queue));
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         __inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

ADTs : Queues (Fixed) II

fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void _inc(int* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) ncalloc(1, sizeof(queue));
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         _inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

```
1  bool queue_dequeue(queue* q, queuetype* d)
2  {
3      if((q==NULL) || (q->front==q->end)){
4          return false;
5      }
6      *d = q->a[q->front];
7      _inc(&q->front);
8      return true;
9  }
10
11 void queue_tostring(queue* q, char* str)
12 {
13     char tmp[ELEMSIZE];
14     str[0] = '\0';
15     if((q==NULL) || (queue_size(q)==0)){
16         return;
17     }
18     for(int i=q->front; i != q->end;){
19         sprintf(tmp, FORMATSTR, q->a[i]);
20         strcat(str, tmp);
21         strcat(str, "|");
22         _inc(&i);
23     }
24     str[strlen(str)-1] = '\0';
25 }
```

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end-q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void __inc(int* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end - q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void __inc(int* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

- We need a thorough testing program

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
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5      }
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8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
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21 }
```

- We need a thorough testing program
- We'll see queues again for traversing trees

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
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12 bool queue_free(queue* q)
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14     free(q);
15     return true;
16 }
17
18 void __inc(int* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

ADTs : Queues (Linked) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

ADTs : Queues (Linked) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

linked.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  queue* queue_init(void)
5  {
6      queue* q = (queue*) nalloc(1, sizeof(queue));
7      return q;
8  }
9
10 void queue_enqueue(queue* q, queuetype d)
11 {
12     dataframe* f;
13     if(q == NULL){
14         return;
15     }
16
17     /* Copy the data */
18     f = nalloc(1, sizeof(dataframe));
19     f->i = d;
20
21     /* 1st one */
22     if(q->front == NULL){
23         q->front = f;
24         q->end = f;
25         q->size = q->size + 1;
26         return;
27     }
28     /* Not 1st */
29     q->end->next = f;
30     q->end = f;
31     q->size = q->size + 1;
32 }
```


ADTs : Queues (Linked) II

```
1  bool queue_dequeue(queue* q, queue_t* d)
2  {
3      dataframe* f;
4      if((q==NULL) || (q->front==NULL) || (q->end==NULL)){
5          return false;
6      }
7      f = q->front->next;
8      *d = q->front->i;
9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
12     return true;
13 }
14
15 bool queue_free(queue* q)
16 {
17     if(q){
18         dataframe* tmp;
19         dataframe* p = q->front;
20         while(p!=NULL){
21             tmp = p->next;
22             free(p);
23             p = tmp;
24         }
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27     return true;
28 }
```

ADTs : Queues (Linked) II

```
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7      f = q->front->next;
8      *d = q->front->i;
9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
12     return true;
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14
15 bool queue_free(queue* q)
16 {
17     if(q){
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22             free(p);
23             p = tmp;
24         }
25         free(q);
26     }
27     return true;
28 }
```

```
1  void queue_tostring(queue* q, char* str)
2  {
3      dataframe *p;
4      char tmp[ELEMSIZE];
5      str[0] = '\0';
6      if((q==NULL) || (q->front == NULL)){
7          return;
8      }
9      p = q->front;
10     while(p){
11         sprintf(tmp, FORMATSTR, p->i);
12         strcat(str, tmp);
13         strcat(str, "|");
14         p = p->next;
15     }
16     str[strlen(str)-1] = '\0';
17 }
18
19 int queue_size(queue* q)
20 {
21     if((q==NULL) || (q->front==NULL)){
22
23         return 0;
24     }
25     return q->size;
26 }
```

Detour : Graphviz

- There exists a nice package, called Graphviz:

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- To create a .pdf:

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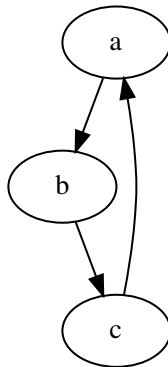


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T : ADTs - Trees

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Binary Trees : Data Structures

- Binary trees are used extensively in computer science

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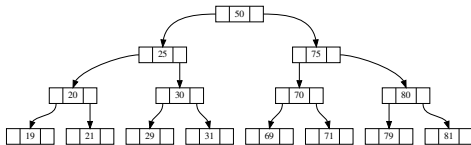
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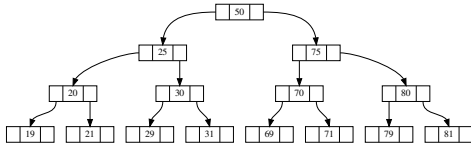
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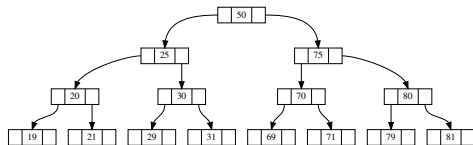
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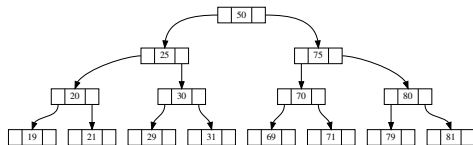
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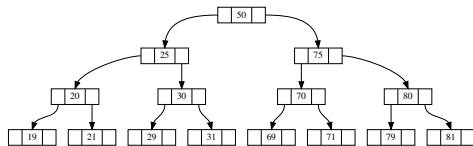
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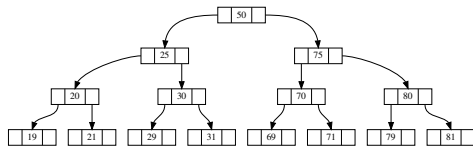
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- In a tree, there is only one path from the root to any child



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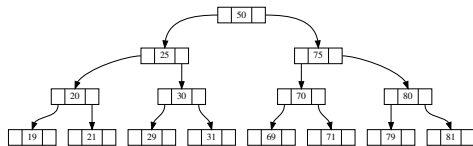
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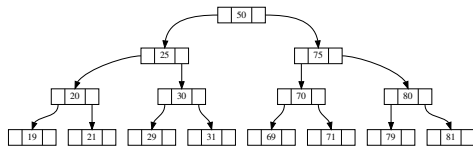
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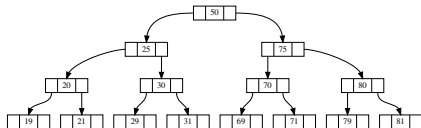
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- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



bst.h

```
1  #include "../General/general.h"
2  #include "../Queue/queue.h"
3
4  #include <stdio.h>
5  #include <stdlib.h>
6  #include <assert.h>
7
8  bst* bst_init(void);
9
10 /* Insert 1 item into the tree */
11 bool bst_insert(bst* b, treetype d);
12
13 /* Return number of nodes in tree */
14 int bst_size(bst* b);
15
16 /* Whether the data d is stored in the tree */
17 bool bst_isin(bst* b, treetype d);
18
19 /* Bulk insert n items from an array a into an initialised tree */
20 bool bst_insertarray(bst* b, treetype* a, int n);
21
22 /* Clear all memory associated with tree, & set pointer to NULL */
23 bool bst_free(bst* b);
24
25 /* Optional ? */
26
27 char* bst_preorder(bst* b);
28 void bst_printlevel(bst* b);
29 /* Create string with tree as ((head)(left)(right)) */
30 char* bst_printlisp(bst* b);
31 /* Use Graphviz via a .dot file */
32 void bst_todot(bst* b, char* dotname);
```

Binary Search Trees : Linked I

specific.h

```
1  #include <string.h>
2
3  typedef int treetype;
4  #define FORMATSIR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Linked"
7
8  struct dataframe {
9      treetype d;
10     struct dataframe* left;
11     struct dataframe* right;
12 };
13 typedef struct dataframe dataframe;
14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```


Binary Search Trees : Linked I

specific.h

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3  typedef int treetype;
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12 };
13 typedef struct dataframe dataframe;
14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```

```
/* Based on geekforgeeks.org */
dataframe* __insert(dataframe* t, treetype d)
{
    dataframe* f;
    /* If the tree is empty, return a new frame */
    if (t == NULL){
        f = ncalloc(sizeof(dataframe), 1);
        f->d = d;
        return f;
    }
    /* Otherwise, recurs down the tree */
    if (d < t->d){
        t->left = __insert(t->left, d);
    }
    else if(d > t->d){
        t->right = __insert(t->right, d);
    }
    /* return the (unchanged) dataframe pointer */
    return t;
}
```

Binary Search Trees : Linked II

```
bool __isin(dataframe* t, treetype d)
{
    if(t==NULL){
        return false;
    }
    if(t->d == d){
        return true;
    }
    if(d < t->d){
        return __isin(t->left, d);
    }
    else{
        return __isin(t->right, d);
    }
    return false;
}
```

Binary Search Trees : Linked II

```
bool __isin(dataframe* t, treetype d)
{
    if(t==NULL){
        return false;
    }
    if(t->d == d){
        return true;
    }
    if(d < t->d){
        return __isin(t->left, d);
    }
    else{
        return __isin(t->right, d);
    }
    return false;
}
```

```
char* __printlisp(dataframe* t)
{
    char tmp[ELEMSIZE];
    char *s1, *s2, *p;

    if(t==NULL){
        /* \0 string */
        p = ncalloc(1,1);
        return p;
    }
    sprintf(tmp, FORMATSTR, t->d);
    s1 = __printlisp(t->left);
    s2 = __printlisp(t->right);
    p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
        strlen(" ( ) "), 1);
    sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
    free(s1);
    free(s2);
    return p;
}
```

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Counting from cell 1, for a tree with n nodes:

To find	Use	Iff
The root	$A[1]$	A is nonempty
The left child of $A[i]$	$A[2i]$	$2i \leq n$
The parent of $A[i]$	$A[i/2]$	$i > 1$
Is $A[i]$ a leaf ?	True	$2i > n$

Binary Search Trees : Realloc

specific.h

```
1  #include <stdbool.h>
2
3  typedef int treetype;
4  #define FORMATSTR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Realloc"
7
8  // Probably (2^n) -1
9  #define INITSIZE 31
10 #define SCALEFACTOR 2
11
12 struct dataframe {
13     treetype d;
14     bool isValid;
15 };
16 typedef struct dataframe dataframe;
17
18 struct bst {
19     dataframe* a;
20     int capacity;
21 };
22 typedef struct bst bst;
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21 };
22 typedef struct bst bst;
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Using a queue for Level-Order traversal:

```
void bst_printlevel(bst* b)
{
    treetype n;
    if((b==NULL) || (! __isvalid(b, 0))){
        return;
    }
    /* Make a queue of cell indices */
    queue* q = queue_init();
    queue_enqueue(q, 0);
    while(queue_dequeue(q, &n) && __isvalid(b, (int)n)){
        printf(FORMATSTR, b->a[n].d);
        putchar(' ');
        queue_enqueue(q, __leftchild((int)n));
        queue_enqueue(q, __rightchild((int)n));
    }
}
```

Binary Search Trees : Complexity

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- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list !
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- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees - Red-Black trees, Tries, 2-3 trees, AVL trees etc.

Binary Trees : Huffman Compression I

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- To encode the string "BABBAGE":

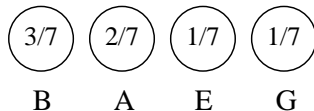
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- Keep a list of characters, ordered by their frequency

Binary Trees : Huffman Compression II

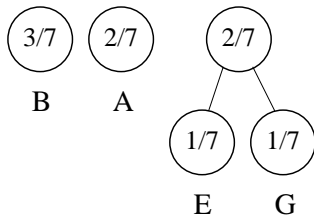
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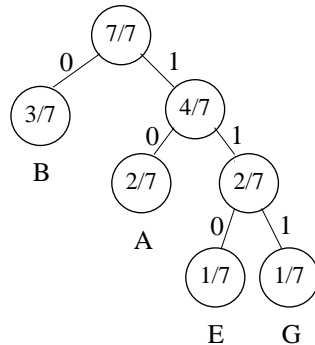
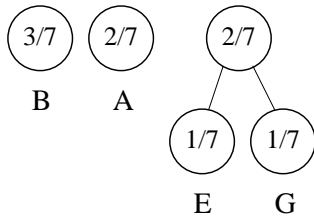
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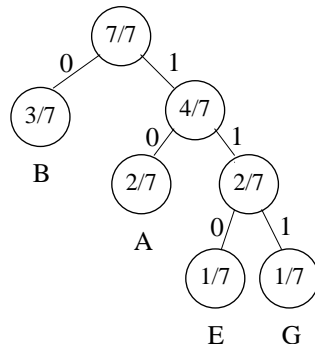
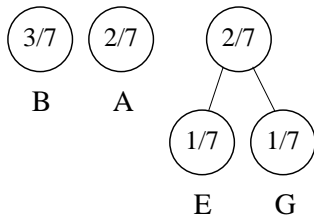
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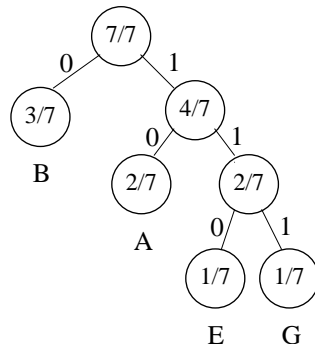
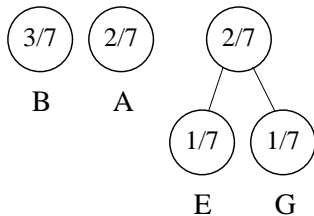
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- $A = 10$, $B = 0$, $E = 110$, $G = 111$

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- Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :



- A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

Table of Contents

N : Recursion

O : Algorithms I - Search

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Q : ADTs - Collection

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S : ADTs - Queues

T : ADTs - Trees

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V : Algorithms II - Sort

W : Algorithms III - Huffman/Strings

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 - A black-box into which we insert a key (e.g. NI number) and out pops an array index.
 - As an example lets use an array of size 11 to store some airport codes, e.g. PEK, BRS, FRA.

Hashing : Airport Codes

- In a three letter string $X_2X_1X_0$ the letter 'A' has the value 0, 'B' has the value 1 etc.

Hashing : Aiport Codes

- In a three letter string $X_2X_1X_0$ the letter 'A' has the value 0, 'B' has the value 1 etc.
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- Inserting "PHL", "ORY" and "GCM":

	0
	1
	2
	3
PHL	4
	5
GCM	6
	7
ORY	8
	9
	10

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- However, inserting "HKG" causes a collision.

	0
	1
	2
	3
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HKG ?

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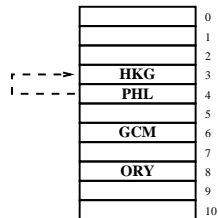
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- The function is chosen so that two keys which collide at the same address will have different probe decrements, e.g. :

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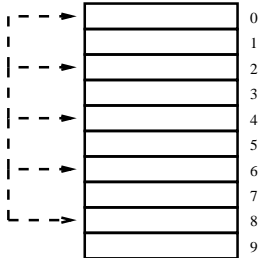
- Although "PHL" and "HKG" share the same primary hash value of $h(K) = 4$, they have different probe decrements:

$$p(\text{"PHL"}) = 4$$

$$p(\text{"HKG"}) = 3$$

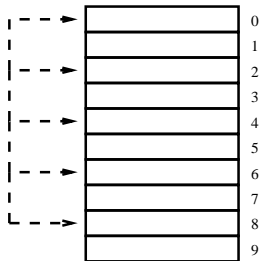
Hashing : Primes and Chaining

- If the size of our array, M , was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



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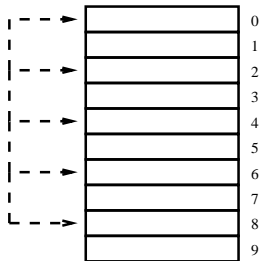
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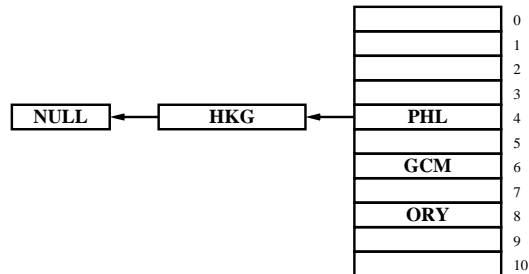
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Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



A Practical Hash Function

```
1  #include <stdio.h>
2
3  int hash(unsigned int sz, char *s);
4
5  int main(void)
6  {
7
8      char str[] = "Hello World!";
9      // Hash modulus 7919
10     printf("%d\n", hash(7919, str));
11     return 0;
12 }
13
14 /*
15  Modified Bernstein hashing
16  5381 & 33 are magic numbers required by the algorithm
17  */
18 int hash(unsigned int sz, char *s)
19 {
20     unsigned long hash = 5381;
21     int c;
22     while((c = (*s++))) {
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Execution :

5479

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Execution :

5479

Has similarities to the implementation of rand() :

```
int rand_r(unsigned int* seed);

int main(void)
{
    unsigned int seed = 0;
    printf("%d\n", rand_r(&seed));
    return 0;
}

/* This algorithm is mentioned in the ISO C standard,
   here extended for 32 bits. */
int rand_r(unsigned int* seed)
{
    unsigned int next = *seed;
    int result;
    next *= 1103515245;
    next += 12345;
    result = (unsigned int) (next / 65536) % 2048;
    next *= 1103515245;
    next += 12345;
    result <<= 10;
    result ^= (unsigned int) (next / 65536) % 1024;
    next *= 1103515245;
    next += 12345;
    result <<= 10;
    result ^= (unsigned int) (next / 65536) % 1024;
    *seed = next;
    return result;
}
```

Execution :

1012484

Cuckoo Hashing

- We have two tables, each with their **own** hash function.

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Table getting full -> rehashed old sz =16
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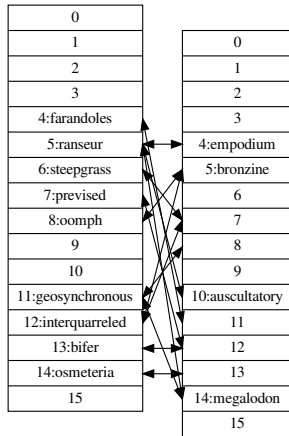


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V : Algorithms II - Sort

W : Algorithms III - Huffman/Strings

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Algorithms : Sorting

```
#define NUMS 6

void bubble_sort(int b[], int s);

int main(void)
{
    int a[] = {3, 4, 1, 2, 9, 0};
    bubble_sort(a, NUMS);
    for(int i=0; i<NUMS; i++){
        printf("%i ", a[i]);
    }
    printf("\n");
    return 0;
}

void bubble_sort(int b[], int s)
{
    bool changes;
    do{
        changes = false;
        for(int i=0; i<s-1; i++){
            if(b[i] > b[i+1]){
                SWAP(b[i], b[i+1]);
                changes = true;
            }
        }
    }while(changes);
}
```

Execution :

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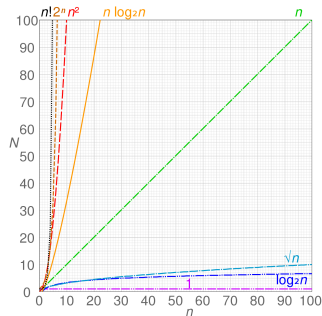
int main(void)
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1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4
5  void mergesort(int *src, int *spare, int l, int r);
6  void merge(int *src, int *spare, int l, int m, int r);
7
8  #define NUM 5000
9
10 int main(void)
11 {
12     int a[NUM];
13     int spare[NUM];
14
15     for(int i=0; i<NUM; i++){
16         a[i] = rand()%100;
17     }
18
19     mergesort(a, spare, 0, NUM-1);
20
21     for(int i=0; i<NUM; i++){
22         printf("%4d ==> %d\n", i, a[i]);
23     }
24
25     return 0;
26 }
```

Merge Sort II

```
void mergesort(int *src, int *spare, int l, int r)
{
    int m = (l+r)/2;
    if(l != r){
        mergesort(src, spare, l, m);
        mergesort(src, spare, m+1, r);
        merge(src, spare, l, m, r);
    }
}

void merge(int *src, int *spare, int l, int m, int r)
{
    int s1 = l;
    int s2 = m+1;
    int d = l;

    do{
        if(src[s1] < src[s2]){
            spare[d++] = src[s1++];
        }
        else{
            spare[d++] = src[s2++];
        }
    }while((s1 <= m) && (s2 <= r));

    if(s1 > m){
        memcpy(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
    }
    else{
        memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
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- Once again, the sort is then applied recursively.

Algorithms : Quicksort

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <math.h>
4
5  int partition(int *a, int l, int r);
6  void quicksort(int *a, int l, int r);
7
8  #define NUM 100000
9
10 int main(void)
11 {
12     int a[NUM];
13
14     for(int i=0; i<NUM; i++){
15         a[i] = rand()%100;
16     }
17     quicksort(a, 0, NUM-1);
18
19     return 0;
20 }
21
22 void quicksort(int *a, int l, int r)
23 {
24     int pivpoint = partition(a, l, r);
25     if(l < pivpoint){
26         quicksort(a, l, pivpoint-1);
27     }
28     if(r > pivpoint){
29         quicksort(a, pivpoint+1, r);
30     }
31 }
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27     }
28     if(r > pivpoint){
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30     }
31 }
```

```
int partition(int *a, int l, int r)
{
    int piv = a[l];
    while(l<r){
        /* Right -> Left Scan */
        while(piv < a[r] && l<r) r--;
        if(r!=l){
            a[l] = a[r];
            l++;
        }
        /* Left -> Right Scan */
        while(piv > a[l] && l<r) l++;
        if(r!=l){
            a[r] = a[l];
            r--;
        }
    }
    a[r] = piv;
    return r;
}
```

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```
1  #include <stdio.h>
2  #include <stdlib.h>
3
4  int intcompare(const void *a, const void *b);
5
6  int main(void)
7  {
8      int a[10];
9
10     for(int i=0; i<10; i++){
11         a[i] = 9 - i;
12     }
13
14     qsort(a, 10, sizeof(int), intcompare);
15
16     for (int i=0; i<10; i++){
17         printf(" %d",a[i]);
18     }
19     printf("\n");
20     return 0;
21 }
22
23
24 int intcompare(const void *a, const void *b)
25 {
26     const int *ia = (const int *)a;
27     const int *ib = (const int *)b;
28     return *ia - *ib;
29 }
```

Algorithms : The Radix Sort

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- For integer data, repeated passes of radix sort focus on the right digit (the units), then the second digit (the tens) and so on.

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459 254 472 534 649 239 432 654 477

0

1

2 472 432

3

4 254 534 654

5

6

7 477

8

9 459 649 239

Read out the new list:

472 432 254 534 654 477 459 649 239

Radix Sort II

472 432 254 534 654 477 459 649 239

0

1

2

3 432 534 239

4 649

5 254 654 459

6

7 472 477

8

9

432 534 239 649 254 654 459 472 477

Radix Sort II

472 432 254 534 654 477 459 649 239

0

1

2

3 432 534 239

4 649

5 254 654 459

6

7 472 477

8

9

432 534 239 649 254 654 459 472 477

432 534 239 649 254 654 459 472 477

0

1

2 239 254

3

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- Now: `gprof ./executable gmon.out` shows the function-call profile of your code.

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U : ADTs - Hashing

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X : ADTs - Graphs

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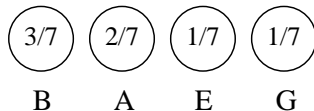
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- Keep a list of characters, ordered by their frequency

Huffman Compression II

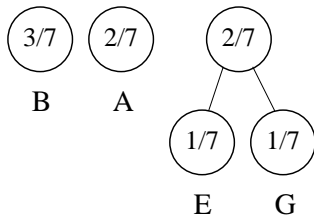
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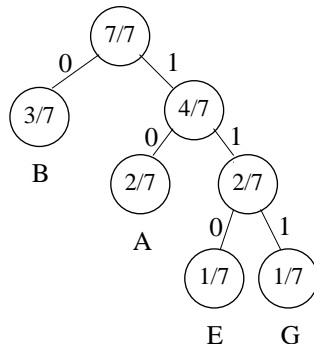
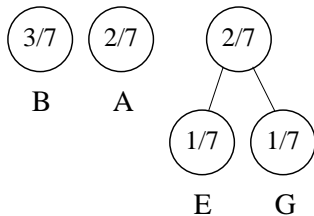
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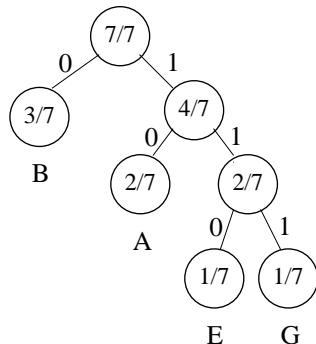
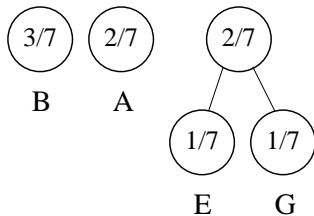
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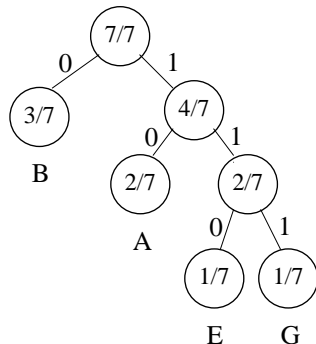
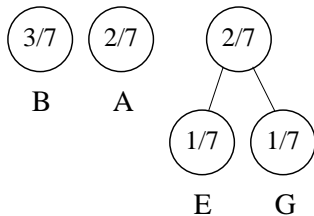
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- A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

Algorithm : Rabin-Karp String Searching

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.

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Substring : AAAAAAH

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$$h("NEILL") =$$

$$(13 \times 26^4 + 4 \times 26^3 + 8 \times 26^2 + 11 \times 26 + 11) \% P$$

where P is a big prime number.

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- This can be expanded by Horner's method to:

$$(((((((13 \times 26) + 4) \times 26) + 8) \times 26) + 11) \times 26 + 11) \% P$$

Rabin-Karp II

- For a large search string, overflow can occur. We therefore move the *mod* operation inside the brackets:

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```
1  #include <string.h>
2  #include <assert.h>
3
4  #define Q 33554393
5  #define D 26
6  #define index(C) (C - 'A')
7
8  int rk(char *p, char *a);
9
10 int main(void)
11 {
12     assert(rk("STING",
13              "A STRING EXAMPLE CONSISTING OF ...") == 22);
14     return 0;
15 }
16
17 int rk(char *p, char *a)
18 {
19     int i, dM = 1, h1 = 0, h2 = 0;
20     int m = strlen(p);
21     int n = strlen(a);
22     for(i = 1; i < m; i++) dM = (D * dM) % Q;
23     for(i = 0; i < m; i++){
24         h1 = (h1 * D + index(p[i])) % Q;
25         h2 = (h2 * D + index(a[i])) % Q;
26     }
27     // h1 = search string hash, h2 = master string hash
28     for(i = 0; h1 != h2; i++){
29         h2 = (h2 * D + Q - index(a[i]) * dM) % Q;
30         h2 = (h2 * D + index(a[i + m])) % Q;
31         if(i > n - m) return n;
32     }
33     return i;
34 }
```

Algorithm : Boyer-Moore String Searching

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution :

```
A STRING SEARCHING EXAMPLE CONSISTING OF ...  
  |      |  
STING    |  
    STING  
      STING
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- The next comparison is between the G and the S. We can slide the search string right until it matches the S in the master.

Boyer-Moore II

Execution :

A STRING SEARCHING EXAMPLE CONSISTING OF ...

STING							
	STING						
		STING					
			STING				
				STING			

- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.

Boyer-Moore II

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- After 3 more full slides right we arrive at the T in CONSISTING.

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  STING |   |   |   |  
        |   |   |   |  
      STING |   |   |  
            |   |   |  
          STING |   |  
                |   |  
              STING
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- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.
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- We align the T's, and have found our match using 7 compares (plus 5 to verify the match).

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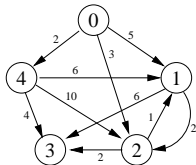
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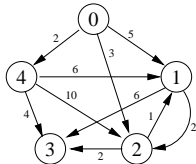
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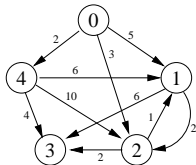
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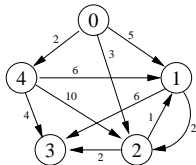
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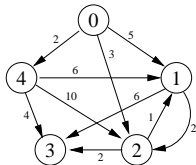
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graph.h (partial)

```
#include <limits.h>
#define INF (INT_MAX)

/* Initialise an empty graph */
graph* graph_init(void);

/* Add new vertex */
int graph_addVert(graph* g, char* label);

/* Add new edge between two Vertices */
bool graph_addEdge(graph* g, int from,
                   int to, edge weight);

/* Returns NO_VERT if not already a vert
   else 0 ... (size-1) */
int graph_getVertNum(graph* g, char* label);

/* Returns label of vertex v */
char* graph_getLabel(graph* g, int v);

/* Returns edge weight - if none = INF */
edge graph_getEdgeWeight(graph* g, int from, int to);

/* Number of verts */
int graph_numVerts(graph* b);

/* Output edge weights e.g. "0->1 200 2->1 100" */
void graph_tostring(graph* g, char* str);

/* Clear all memory associated with graph */
bool graph_free(graph* g);
```

Graph ADT : 2D Realloc I

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	0	1	2	3	4
0	0	5	3	∞	2
1	∞	0	2	6	∞
2	∞	1	0	2	∞
3	∞	∞	∞	0	∞
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0	0	5	3	∞	2
1	∞	0	2	6	∞
2	∞	1	0	2	∞
3	∞	∞	∞	0	∞
4	∞	6	10	4	0

specific.h

```
1  #define GRAPHTYPE "Realloc"
2
3  #define INITSIZE 8
4  #define SCALEFACTOR 2
5
6  #define TMPSTR 1000
7
8  #define NO_VERT -1
9
10 typedef unsigned int edge;
11
12 struct graph {
13     edge** adjMat;
14     char** labels;
15     /* Actual number of vertices */
16     int size;
17     /* Max vertices before realloc() */
18     int capacity;
19 };
20 typedef struct graph graph;
```

2D Realloc II

```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(sizeof(graph), 1);
    int h = INITSIZE;
    int w = h;
    g->capacity = h;
    g->adjMat = (edge**) n2dcalloc(h, w, sizeof(edge));
    g->labels = (char**) n2dcalloc(h, MAXLABEL+1, sizeof(char));
    for(int j=0; j<h; j++){
        for(int i=0; i<w; i++){
            /* It's not clear if weight[j][i] should be 0 or INF */
            g->adjMat[j][i] = INF;
        }
    }
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    return g->adjMat[from][to];
}

int graph_numVerts(graph* g)
{
    if(g==NULL){
        return 0;
    }
    return g->size;
}
```

2D Realloc II

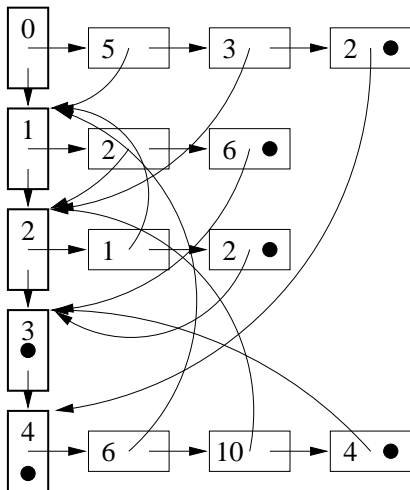
```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(sizeof(graph), 1);
    int h = INITSIZE;
    int w = h;
    g->capacity = h;
    g->adjMat = (edge**) n2dcalloc(h, w, sizeof(edge));
    g->labels = (char**) n2dcalloc(h, MAXLABEL+1, sizeof(char));
    for(int j=0; j<h; j++){
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            /* It's not clear if weight[j][i] should be 0 or INF */
            g->adjMat[j][i] = INF;
        }
    }
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    return g->adjMat[from][to];
}

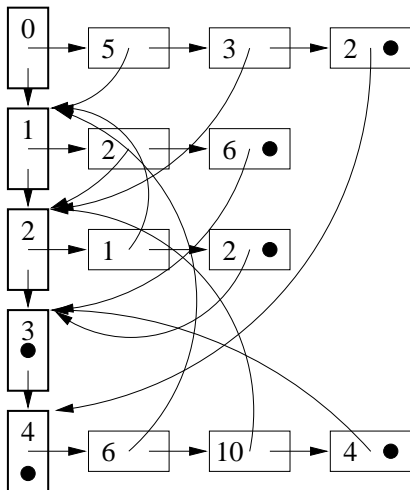
int graph_numVerts(graph* g)
{
    if(g==NULL){
        return 0;
    }
    return g->size;
}
```

```
int graph_addVert(graph* g, char* label)
{
    if(g==NULL){
        return NO_VERT;
    }
    if(graph_getVertNum(g, label) != NO_VERT){
        return NO_VERT;
    }
    /* Resize */
    if(g->size >= g->capacity){
        g->adjMat = (edge**) n2drealloc((void**)g->adjMat,
            g->capacity, g->capacity*SCALEFACTOR,
            g->capacity, g->capacity*SCALEFACTOR,
            sizeof(edge));
        g->labels = (char**) n2drealloc((void**)g->labels,
            g->capacity, g->capacity*SCALEFACTOR,
            MAXLABEL+1, MAXLABEL+1, 1);
        for(int j=0; j<g->capacity*SCALEFACTOR; j++){
            for(int i=0; i<g->capacity*SCALEFACTOR; i++){
                if((i>=g->capacity) || (j>=g->capacity)){
                    g->adjMat[j][i] = INF;
                }
            }
        }
        g->capacity = g->capacity*SCALEFACTOR;
    }
    strcpy(g->labels[g->size], label);
    g->size = g->size + 1;
    return g->size - 1;
}
```


Graph ADT - Linked



Graph ADT - Linked



specific.h

```
1  #define GRAPHTYPE "Linked"
2
3  #define INITSIZE 8
4  #define SCALEFACTOR 2
5
6  #define TMPSTR 1000
7
8  #define NO_VERT -1
9
10 typedef unsigned int edge;
11
12 struct vertex {
13     char* label;
14     struct vertex* nextv;
15     void* firste;
16     int num;
17 };
18 typedef struct vertex vertex;
19
20 struct edge {
21     edge weight;
22     vertex* v;
23     struct edge* nexte;
24 };
25 typedef struct edge edgel;
26
27 struct graph {
28     vertex* firstv;
29     vertex* endv;
30     int size;
31 };
32 typedef struct graph graph;
```

Linked II

```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(1, sizeof(graph));
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    vertex* v = g->firstv;
    for(int i=0; i<from; i++){
        v = v->nextv;
    }
    if((v==NULL) || (v->num != from)){
        return INF;
    }
    edge* e = v->firste;
    while(e != NULL){
        if(e->v->num == to){
            return e->weight;
        }
        e = e->nexte;
    }
    return INF;
}
```

Linked II

```
graph* graph_init(void)
{
    graph* g = (graph*) nalloc(1, sizeof(graph));
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    vertex* v = g->firstv;
    for(int i=0; i<from; i++){
        v = v->nextv;
    }
    if((v==NULL) || (v->num != from)){
        return INF;
    }
    edge* e = v->firste;
    while(e != NULL){
        if(e->v->num == to){
            return e->weight;
        }
        e = e->nexte;
    }
    return INF;
}
```

```
bool graph_addEdge(graph* g, int from, int to, edge w)
{
    if((g==NULL) || (g->size == 0)){
        return false;
    }
    if((from >= g->size) || (to >= g->size)){
        return false;
    }
    vertex* f = g->firstv;
    for(int i=0; i<from; i++){
        f = f->nextv;
    }
    vertex* t = g->firstv;
    for(int i=0; i<to; i++){
        t = t->nextv;
    }
    return _addEdge(f, t, w);
}
```

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- A proof either way would have profound implications for mathematics, cryptography, AI etc.

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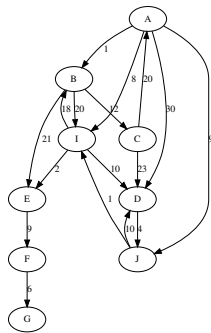
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- A -> B -> C -> D -> J -> I -> E -> F -> G



```
edge graph_salesman(graph* g, int from, char* str)
{
    bool* unvis;
    int curr, ncurr, nvs;
    edge cst, bcst, e;

    nvs = graph_numVerts(g);
    if((g==NULL) || (from >= nvs) || (str==NULL)){
        return INF;
    }
    unvis = (bool*)ncalloc(nvs, sizeof(bool));
    for(int v=0; v<nvs; v++){
        unvis[v] = true;
    }
    curr = from;
    bcst = 0;
    strcpy(str, graph_getLabel(g, from));
    do{
        unvis[curr] = false;
        cst = INF;
        ncurr = NO_VERT;
        /* Look at neighbours of curr */
```

TSP II

```
edge graph_salesman(graph* g, int from, char* str)
{
    bool* unvis;
    int curr, ncurr, nvs;
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    nvs = graph_numVerts(g);
    if((g==NULL) || (from >= nvs) || (str==NULL)){
        return INF;
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    unvis = (bool*)ncalloc(nvs, sizeof(bool));
    for(int v=0; v<nvs; v++){
        unvis[v] = true;
    }
    curr = from;
    bcst = 0;
    strcpy(str, graph_getLabel(g, from));
    do{
        unvis[curr] = false;
        cst = INF;
        ncurr = NO_VERT;
        /* Look at neighbours of curr */
```

```
        for(int v=0; v<nvs; v++){
            e = graph_getEdgeWeight(g, curr, v);
            if((v!=curr) && unvis[v] && (e!=INF)){
                if(e < cst){
                    cst = e;
                    ncurr = v;
                }
            }
        }
        /* Add in cost to go to closest */
        if(cst < INF){
            bcst += cst;
            curr = ncurr;
            strcat(str, " ");
            strcat(str, graph_getLabel(g, ncurr));
        }
    }while((cst < INF) && (curr != NO_VERT));
    free(unvis);
    return bcst;
}
```

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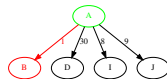
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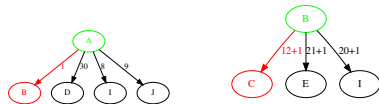
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