Assignment 3

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$[A][X] = \lambda[X]$

$[A][X] = \lambda[I][X]$

$[A][X] = [\lambda I][X]$

$[A - \lambda I][X] = 0$

→ System of Homogenous Eq.

$|A - \lambda X| = 0$ for Infinite Sol^n.

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$-1(-12 + 3 - 3\lambda) + 2(12 + 6\lambda + 6) - \lambda((-2-\lambda)(1-\lambda) - 4) = 0$$

$$12 - 3 + 3\lambda + 24 + 12\lambda + 12 + 2\lambda - \lambda^2 - \lambda^3 + 4\lambda = 0$$

$$15 + 21\lambda - \lambda^2 - \lambda^3 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$(\lambda+3)(\lambda^2-2\lambda-15)=0$

$(\lambda+3)(\lambda+3)(\lambda-5)=0$

$\lambda_1=-3 \quad \lambda_2=-3 \quad \lambda_3=5$

For $\lambda=-3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$\eta(A)=2$

let $x_2=k_1$ & $x_3=k_2$

$x_1+2x_2-3x_3=0$

$x_1=-2k_1+3k_2$

Eigen Vector $X_1=\begin{bmatrix} -2k_1+3k_2 \\ k_1 \\ k_2 \end{bmatrix}$

$$X_1=k_1\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}+k_2\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda=5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\eta(A) = 1$

let $x_3 = k$

$-x_1 - 2x_2 - 5x_3 = 0$

$-8x_2 - 16x_3 = 0$

$x_2 = -2k$

$-x_1 = k$

$x_1 = -k$

Eigen Vector $\rightarrow X = \begin{bmatrix} -k \\ -2k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

Q. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$$[A - \lambda I][X] = 0$$

$$[A - \lambda I] = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$$-2(\lambda - 1) + (1-\lambda)\left((4-\lambda)(1-\lambda)\right) = 0$$

$$(1-\lambda)\left(2 + (4-\lambda)(1-\lambda)\right) = 0$$

$$(1-\lambda)\left(6 + \lambda^2 - 5\lambda\right) = 0$$

$$(\lambda - 1)\left(\lambda^2 - 5\lambda + 6\right) = 0$$

$$(\lambda - 1)(\lambda - 3)(\lambda - 2) = 0$$

$$\lambda_1 = 1, \ \lambda_2 = 3, \ \lambda_3 = 2$$

$\lambda^2 - 3\lambda - 2\lambda + 6$

for $\lambda = 1$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $x_2 = k$

$3x_1 + x_3 = 0$

$2x_3 = 0$

$2x_3 = 0$

$\Rightarrow x_1 = 0,\ x_2 = k,\ x_3 = 0$

$$X = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$h(A) = 1$

$x_1 + x_3 = 0$

$2x_2 + 2x_3 = 0$

Let $x_3 = k$

$\Rightarrow x_1 = -k$

$x_2 = k$

$$X = k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda_3 = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$

$-x_2 + x_3 = 0$

let $x_3 = K$

$x_2 = K$

$x_1 = -\frac{K}{2}$

$$X = K \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

3. $$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

Its a lower Triangular matrix.
So, Diagonal Elements will be the Eign Values

Hence, $\lambda_1 = 5$, $\lambda_2 = 0$, $\lambda_3 = 3$

4.

For $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & -5 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - 2x_3 = 0$

$-5x_2 + 3x_3 = 0$

$\Rightarrow$ let $x_3 = k$

$x_1 = -2k$

$x_2 = \dfrac{3k}{5}$

$$\Rightarrow X = k \begin{bmatrix} -2 \\ \frac{3}{5} \\ 1 \end{bmatrix}$$

for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ -1 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 = 0$$

$$-x_1 + 3x_3 = 0$$

$\Rightarrow$ let $x_2 = k$

$x_1 = 0$ & $x_3 = 0$

$$\Rightarrow X = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$-3x_2 = 0$

$\Rightarrow$ let $x_3 = K$

$x_1 = 0$ & $x_2 = 0$

$$\Rightarrow x = K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

4.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

Eigen values $\Rightarrow$ $\lambda_1 = 0$, $\lambda_2 = 3$, $\lambda_3 = -2$

For $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_3 = 0$

$3x_2 + 4x_3 = 0$

$\Rightarrow$ let $x_1 = K$

$x_2 = 0$ & $x_3 = 0$

$$\Rightarrow X = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 = 0$

$4x_3 = 0$ let $x_2 = K$

$-5x_3 = 0$ $x_1 = 0$

$x_3 = 0$

$$X = K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$5x_2 + 4x_3 = 0$

$\Rightarrow$ let $x_3 = K$

$x_1 = 0$ & $x_2 = -\frac{4K}{5}$

$$\Rightarrow X = K \begin{bmatrix} 0 \\ \frac{4}{5} \\ 1 \end{bmatrix}$$

5. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

Because there are 3 identical rows in this matrix. So, 2 eigen values must be 0