STRUCTURE OF SCIENTIFIC DOCUMENTS

MATHEMATICAL MATERIALS

STW
EX20003
UNIT 04

WHAT ARE MATHEMATICAL MATERIALS

- Numbers
- Expressions involving basic arithmetic operations, variables, parameters, and constants used in mathematical models
- Functions that work like mathematical operators
- Subscripts and superscripts in a series of variables
- Equations

CONSISTENCY IN WRITING

- Consistency should be maintained throughout text e. g. writing 5000 in two places like this: 0.05×10^5 in one place and 0.5×10^4 in another, is inconsistent practice

  Consistently, it should be stated as 5×10^3

SI (Engineering) & Scientific Notation of Numbers

- $a \times 10^b$

a: Coefficient, 10: Base, b: Exponent

- SI (Engineering) Notation

b takes values 0, 3, 6, 9, . . and –3, –6, . . .

- Scientific Notation

$0 < a < 10$

SI Notation of Numbers

Standard Scientific Notation

- 525 is written as 0.525×10^{3}
- 0.005 is written as 5×10^{-3}

- Standard scientific notations are preferred to computer exponentials e. g. 5.0×10^{-3} instead of 5.0 E−3
  Computer exponentials (using E) appearing in computer output may be given in Appendixes, but not in main text
- 25.5 is represented as 2.55×10^{1}
- 0.055 is represented as 5.5×10^{-2}

USE OF % SYMBOL

- % symbol is used with Indo-Arabic numbers
- ‘Per cent’ is used with word version of number e. g. 5 % and five per cent (not 5 per cent)
- % symbol is not repeated with each number in a series or with each number in a range of values e. g. 5, 10, and 15 %, OR 5–10 %

COMPUND NUMBERS

- Hyphen separates individual numbers of compound number and their ordinal counterparts e. g. twenty-one, thirty-four, . . . twenty-first, thirty-fourth, . . .

NUMERALS OR WORDS?

- Numbers (nine or less) should be written in words e. g. We carried out two experiments . . .
- If a series of numbers contain numbers both less and more than 10, then only Arabic numerals should be used e. g. There are 25 samples, with a sample size of 5.

  Marks secured by the students are: 2, 5, 10, and 15.
- Exception:We assume that *x* is greater than 1.

THOUSAND OPERATOR

- For very large and very small numbers, give a space in the thousand operator e. g. 10 000 000, 0.254 25
- Always use figures when symbol of unit of measurement follows e. g. 5 A

NUMBER OF MATHEMATICAL OPERATIONS

- Use figures when mathematical operations are implied e. g. factor of 2, 3 × 3 matrix

ROUND NUMBERS

- Always spell approximate values, ‘round numbers’ e. g. about five years, two orders of magnitude, about four times larger

APPROXIMATE VALUES

- If approximate value is followed by a unit, then the unit is not expressed in symbols e. g. several kilovolts, a few tens of megahertz, a few volts

MODIFICATION OF NOUN

- Units of measurement are nouns and should not be used to modify another noun:

  Incorrect: The signal generator had a 15-kHz frequency.

  Correct: The signal generator had a frequency of 15 kHz.

  Incorrect: It is a 6-centimeter long pipe.

  Correct: The length of the pipe is 6 cm.

LIST OF SYMBOLS

- Research document should contain list of symbols in the beginning
- List of symbols helps to
  - To ensure that same symbol is neither used for more than one purpose nor defined in more than one way
  - Reader to refer to one place to find definition of a symbol
- List of symbols should be arranged with Latin letters followed by Greek letters, each arranged alphabetically
- Whenever applicable, unit of measure should be given for each symbol:

  c: Velocity (km/h)

  m: Mass (kg)

  . . .

  β: Regression coefficient (unit/rupee)

  η: Efficiency (1)

BASIC ARITHMETIC OPERATIONS

- Signs for basic arithmetic operations:
- Plus +
- Minus – (different from hyphen - look at length, vertical location, and intensity)
- Multiplication × (different from the letter x or symbol asterisk *)
- Division ⁄ (different from solidus i. e. slash or oblique / look at the inclination) and ÷

- Except plus symbol, no symbol is available on a keyboard normally available in the market
- All other symbols are to be selected from Symbols list

SCALARS, VECTORS, AND MATRICES

- Scalars, vectors, and matrices are written as follows:

scalar:	lightface italic (*v, l, m, …*)
vector:	boldface, lower-case (**v**, **l**, **m**, …)
matrix:	boldface, upper-case (**V**, **L**, . . .)
tensor:	boldface, italic (***V***, ***L***)

- Purpose of italicizing scalars is to distinguish them from SI units that are always written upright

VARIABLES & CONSTANTS

- Symbols for variables, constants, and unknown quantities are italicized e. g. *x*, *l*, *m*
- Greek letters (α, β, etc.) are always written upright
- Greek letters (α, β, etc.) are always set in Roman
- Subscripted variables follow different writing styles:

  Subscripts are numbers: x_1, x_2

  Subscripts are variables: x_i, x_{ij}
- Subscript is upright when it refers to input or output variable or entities

  other than a quantity e. g. x_i (input variable), x_o (output variable), x_e (mass of electron)
- Subscript is upright when it refers to values of upper and lower limits or maximum and minimum values e. g. x_u and x_l , x_{max} and x_{min}

FUNCTIONS & OPERATORS

- Abbreviations for standard functions, such as log, max, min, exp, sin, cos, tan, cosh, lim, avg, cov, diag, and ln, are set in Roman (upright)
- A space separates function name from expression e. g. log x, sin $(x + y)$, . . .
- Operators for expectation and transposition are in italic e. g. E and T

SUPERSCRIPTS

- Transpose of a vector

  $\mathbf{x}^T(n)$ not $\mathbf{x}(n)^T$

- Power of a scalar

  $x^2(n)$ not $x(n)^2$

- Vertical alignment of superscript and subscript

  x_i^2 not $x_i^{\ 2}$

- Indexing a subscripted variable:

  $x_i^{(2)}$ not x_i^2

USE OF ELLIPSES

- Ellipsis indicate omitted words, sentences, or symbols
- Ellipses indicate a series of variables e. g.

  $x_1, x_2, \ldots, x_n$ (not $x_1, x_2 \ldots x_n$)
- Three raised ellipses (centered dots) are used for series of additions or multiplications of variables e. g.

  $x_1 + x_2 + \cdots + x_n$ (not $x_1 + x_2 + \ldots x_n$)

  $x_1 x_2 \cdots x_n$ (not $x_1 x_2 \ldots x_n$)
- Three ellipses indicate range of variable e. g.

  $x_i, i = 1, 2, \ldots$ (not $x_i, i = 1, 2 \ldots$)

SPACE ACCOMPANYING MATHEMATICAL OPERATORS

- One space should be provided on both sides of a mathematical operator

  $z = x + y$ $\quad$ $z = x - y$

  $z = x \times y$ $\quad$ $z = x \div y$

- No space appears if division is indicated by the symbol / and also when multiplication is implied

  $z = x/y$ $\quad$ $z = xy$ $\quad$ $z = 25x$

- Negative number should not have space separating sign and number

  The number –2 (not – 2) is more than –3 (not – 3).

  $-3 < -2$ (correct) $\quad$ $- 3 < - 2$ (not correct)

- When plus sign indicates positive number then no space appears between sign and number

  $-3 \leq x \leq +3$

- No space appears between a variable and its subscripts (or superscripts) and between a variable and its power

  x_i, $\quad$ x_{ij}, $\quad$ x^2

REFERENCING EQUATIONS

- Each equation should appear immediately after it is referenced in text for first time
- Explanation for each equation should appear in text that precedes it
- Explanation should define important variables and their relationships that lead to equation
- Subsequent referencing takes place by citing equation number
- All symbols appearing in equation should be defined immediately after
- Symbols should be sequenced in order of their appearance in equation
- Unambiguous definitions should be given along with units of measure (wherever applicable)

EXPLANATION OF TERMS

EQUATION NUMBER

LOCATION OF EQUATION

- A long equation can have many terms, each representing a particular aspect of theme of equation
- Each term should be explained before or immediately after equation

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- Equations are referred to in text by equation numbers
- Equations are numbered in order of appearance (like tables and figures)
- When document is divided into chapters, equation number bears chapter number and number reflecting order of appearance

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- Equation should be separated from texts preceding and following it by a larger line spacing (compared to one in text)
- It should be set off from left margin with an indentation
- Equation numbers should be right justified and should be preceded by three raised ellipses

EXAMPLES

- Number of students (z) in class is given in Eq. 4.2 as sum of number of male students (x) and number of female students (y):

$$z = x + y \quad \cdots (4.2)$$

- This equation is second in order of appearance in Chapter 4 of document
- Note abbreviation "Eq." used in text preceding equation

Concentration = (ma' + b) × *dilution factor*

where a': unknown sample absorption

m: slope of calibration graph

b: intercept on concentration axis

Note: 'where' is not capitalized

PUNCTUATION

- Equations should be punctuated as a sentence e. g.
- Defining y as a quadratic function of x,
- $y = ax + bx^2$, ⋯ (3)
- the gradient of y is given as
- $\frac{dy}{dx} = a + 2bx$ ⋯ (4)

- **Note**: commas, full stop, raised dots, vertical alignments

PARAMETERS IN EQUATIONS

- Not numbers but parameters appear in equations (because they are estimated values and not constants):

  Not recommended: $z = x + 2$

  Recommended: $z = x + y$ $y = 2$

EQUATION IN TEXT LINE

- A numbered equation should not appear in a line in the text

  Incorrect: It thus comes out to be true that $x^2 + y = x + y^2$ ・・・(4.3)

  Correct: Thus the following equation holds:

  $$x^2 + y = x + y^2 \quad \cdots (4.3)$$

BREAKING EQUATIONS

- Long equation may not fit in one line, and may be broken in two ways:
- before verb operator (such as =, ≠, <, >, ≤, ≥) or
- before conjunction (such as +, ×) that follows an aggregation (i. e. meaningful expression within brackets)
- In first case, **run-over lines** are aligned on verb operators
- In second case, **conjunction on run-over line** is lined up with right of verb in line above

Example:

$$z = (x + y - ef + r) + 2y - ef + r - (e - f)$$

Verb Operator

conjunction on run-over line

$$(x + y - ef + r) < (2y - ef + r)$$

USE OF NUMBERS IN SENTENCES

- Provide numerical values and avoid ambiguous words

  Incorrect: The voltage across a forward-biased silicon diode is small.

  Correct: The voltage across a forward-biased silicon diode is about 0.7 V.

  Correct: The voltage across a forward-biased silicon diode is small (about 0.7 V).

- Avoid dangling comparatives and superlatives

  Incorrect: The voltage across the diode is *smaller*.

  Correct: The voltage across the diode is smaller than the voltage across resistor R1.

FRACTIONS & MIXED NUMBERS

Incorrect	Correct
3 ½	3½
3½ kilograms	3-½ kilograms
	3-1/2 kilograms

USE OF HYPHENS

Incorrect	Correct
A ten members committee	A ten-member committee
25 inch cloth	25-inch cloth
5-A current	5 A current
3, 5, 10 ampere current	3-, 5-, 10-ampere current
1 foot long wire	1-foot-long wire

ORDERS OF MAGNITUDE

- Same ‘order of magnitude’ means their ratio is less than ten e. g. 5 and 45 are of same order of magnitude
- An order of magnitude larger means ten times as large e. g. 125 is an order of magnitude larger than 10
- Two orders of magnitude larger means 100 times larger e. g. 825 is two orders of magnitude larger than 7

SPECIAL SYMBOLS

$\forall$: for all

$\exists$: there exists

$\Rightarrow$: implies

$(\cdot, \cdot)$: open interval

$\ni$: such that

$\nexists$: there does not exist

$\mathbb{R}$: the set of real numbers

$[\cdot, \cdot]$: closed interval

$[\cdot, \cdot)$: closed-open interval

$(\cdot, \cdot]$: open-closed interval

$\delta(x) = 1$ if $x = 0$, $= 0$ otherwise (Dirac-delta function)

$I(x) = 1$ if x is true, $= 0$ otherwise (Indicator function)

OTHER SYMBOLS

Set-specific Symbols

∪: union

∩: intersection

⊂: subset of

⊆: proper subset of

∅: null set

∈: an element of

Statistical

μ: population mean

σ: population standard deviation

Σ: population variance-covariance matrix

R^2: Coefficient of determination

z, *t*, and *F*: test statistics

THANK YOU!