

# Hypothesis Testing

## Cohen Chapter 5

EDUC/PSY 6600

"I'm afraid that I rather  
give myself away when I explain,"  
said he.

"Results without causes  
are much more impressive."

-- Sherlock Holmes

*The Stock-Broker's Cat*

# Two Types of Research Questions

Do **groups**  
*significantly* **differ**  
on 1 or more characteristics?

Comparing group means, counts, or proportions

- **-tests**
- **ANOVA**
- **tests**

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Comparing group means, counts, or proportions

- **$t$ -tests**
- **ANOVA**
- **$\chi^2$  tests**

Is there a  
*significant* **relationship**  
among a set of **variables**?

Testing the association or dependence

- **Correlation**
- **Regression**

# Inferential Statistics

## Descriptive statistics are limited

- Rely only on **raw** data distribution
- Generally describe **one** variable only
- Do not address **accuracy** of estimators or hypothesis testing
- How **precise** is sample mean or does it differ from a given value?
- Are there between or within **group differences** or **associations**?

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## Goals of inferential statistics

- Hypothesis testing
  - $p$ -values
- Parameter estimation
  - confidence intervals

## Repeated sampling

- Estimators will vary from sample to sample
- Sampling or random error is variability due to chance

# Causality and Statistics

Causality depends  
on evidence  
from outside statistics:

- Phenomenological (educational, behavioral, biological) credibility
- Strength of association, ruling out occurrence by chance alone
- Consistency with past research findings
- Temporality
- Dose-response relationship
- Specificity
- Prevention



According to a recent Nationwide survey:  
**MORE DOCTORS SMOKE CAMELS  
THAN ANY OTHER CIGARETTE**

DOCTORS in every branch of medicine—113,997 in all—were queried in this nationwide study of cigarette preference. Three leading research organizations made the survey. The gist of the query was—What cigarette do you smoke, Doctor?  
*The brand named must was Camel!*  
The rich, full flavor and cool mildness of Camel's superb blend of outlier tobacco seem to have the same appeal to the smoking tastes of doctors as to millions of other smokers. If you are a Camel smoker, this advertisement is for you.



Smoker's Lung

Healthy Lung

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Causality is often a judgmental evaluation  
of combined results from several studies



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Your "T-Zone" Will Tell You...

T for Taste...  
T for Throat...  
that's your  
proving ground  
for any cigarette.  
See if Camels  
don't suit your  
"T-Zone" to a "T."



Smoker's Lung

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# z-Scores and Statistical Inference

Probabilities of  $z$ -scores used to determine how **unlikely** or **unusual** a single case is relative to other cases in a sample

**Small probabilities**  
*(p-values)*  
**reflect unlikely or unusual scores**

Not frequently interested in whether **individual scores** are unusual relative to others, but whether scores from **groups of cases** are unusual.

**Sample mean**,  $\bar{x}$  or  $M$ , summarizes **central tendency** of a group or sample of subjects

# Steps of a Hypothesis test

1. State the **Hypotheses**
  - Null & Alternative
2. Select the **Statistical Test & Significance Level**
  - $\alpha$  level
  - One vs. Two tails
3. Select random sample and collect data
4. Find the **Region of Rejection**
  - Based on  $\alpha$  & # of tails
5. Calculate the **Test Statistic**
  - Examples include:  $z, t, F, \chi^2$
6. Write the **Conclusion**
  - Statistical decision must be in context!



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## Definition of a p-value:

The probability of observing  
a test statistic

**as extreme or more extreme**

**IF**

the NULL hypothesis is true.

# Stating Hypotheses

Hypotheses are always specified in terms of **population**

- Use  $\mu$  for the population mean, not  $\bar{x}$  which is for a sample

If you are comparing TWO population MEANS:

## Null Hypothesis

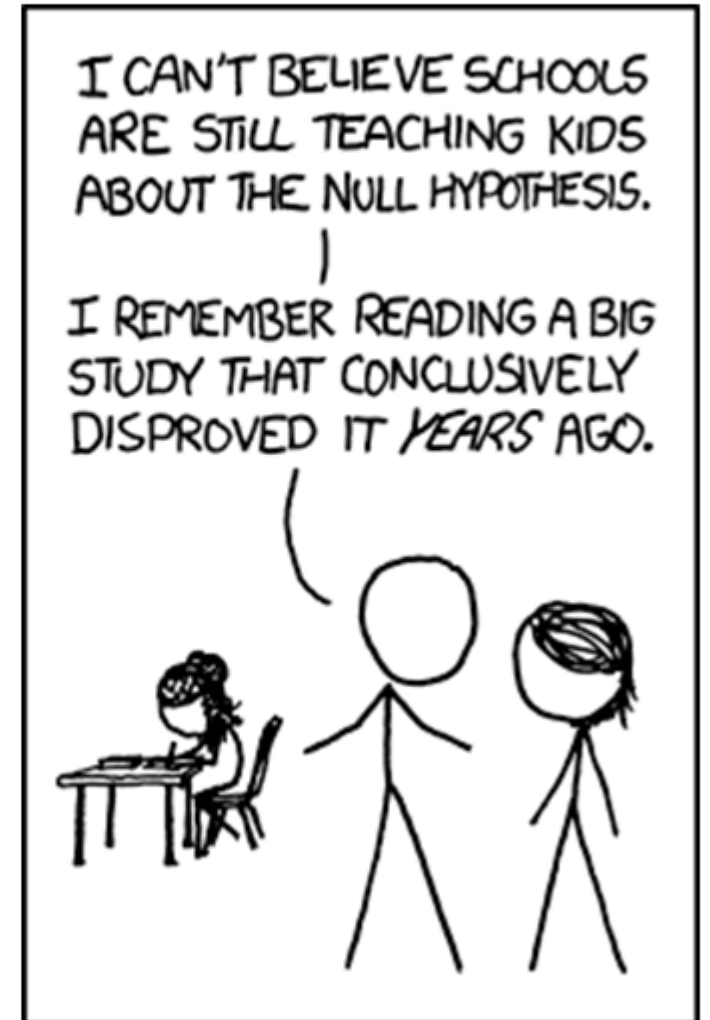
$$H_0 : \mu_1 = \mu_2$$

**Research or Alternative Hypothesis**  
options...

$$H_1 : \mu_1 \neq \mu_2$$

$$H_1 : \mu_1 < \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

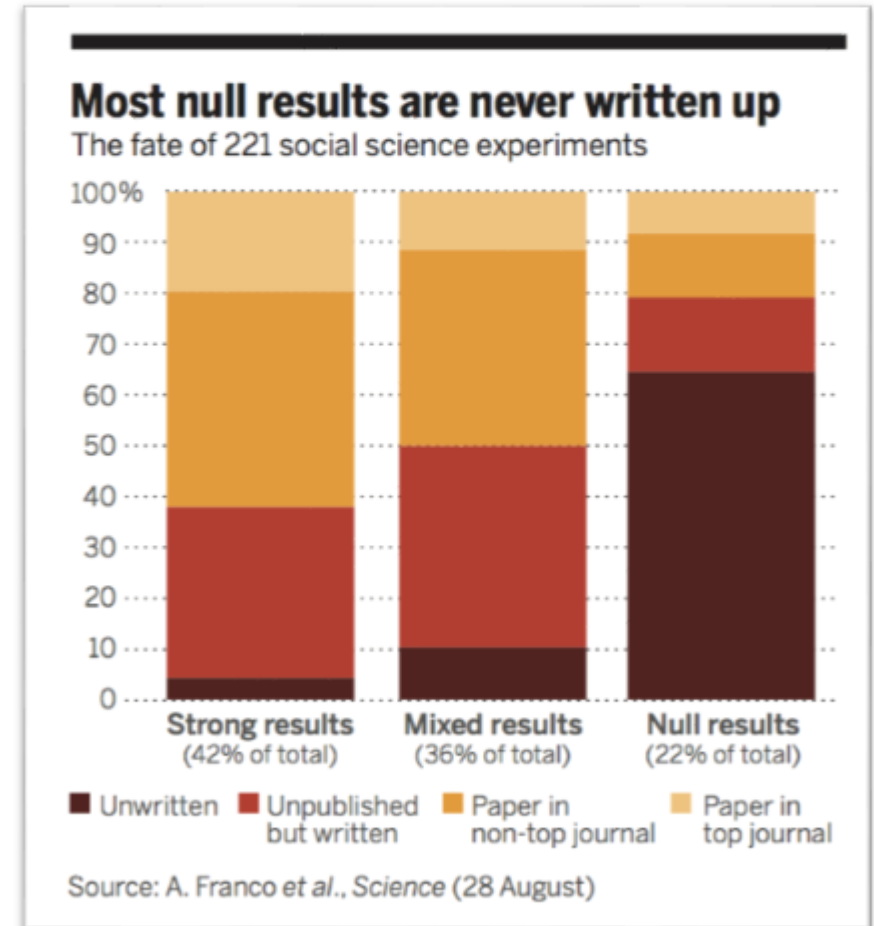


# Innocent Until Proven Guilty

IF there is Not enough statistical evidence to reject

Judgment suspended until further evidence evaluated:

- "Inconclusive"
- Larger sample?
- Insufficient data?



# Rejecting the Null Hypothesis

## Assumption:

The **NULL** hypothesis is **TRUE** in the **POPULATION**

## IF:

The p-value is very **SMALL**

- How small?  $(p\text{-value} < \alpha)$

## THEN:

We have evidence **AGAINST** the **NULL** hypothesis

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## Criteria:

May judge by either...

- the p-value  $< \alpha$
- OR-
- test statistic  $<$  Critical Value

## Conclusion:

We either **REJECT** or **FAIL TO REJECT** the **Null** hypothesis

**We NEVER ACCEPT**  
**the ALTERNATIVE hypothesis!!!**

# ONE tail or TWO?

## 2-tailed test

$$H_1 : \mu_1 \neq \mu_2$$

## 1-tailed test

**Suggests a directionality in results!**

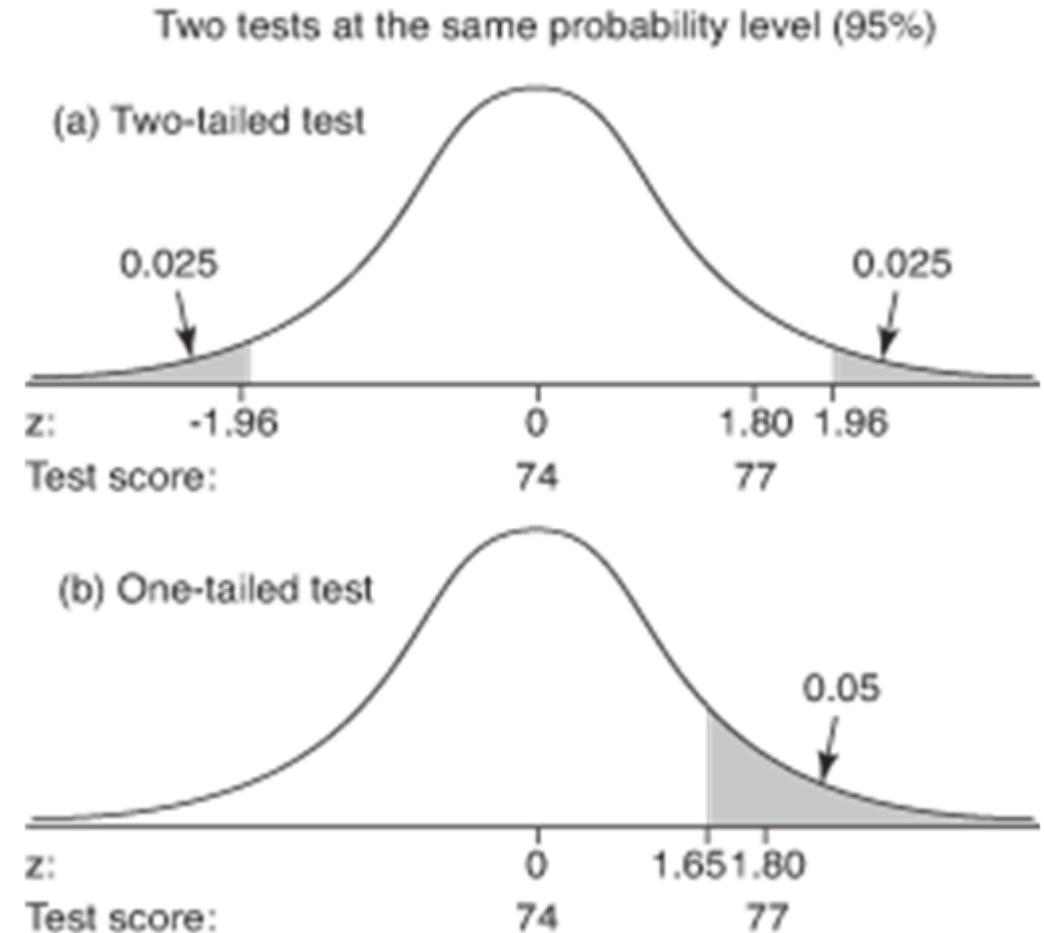
$$H_1 : \mu_1 < \mu_2 \text{ -OR- } H_1 : \mu_1 > \mu_2$$

## NO computational differences

ONLY the *p - value* differs:

$$2 \text{ tail } p - \text{value} = 2 \times 1 \text{ tail } p - \text{value}$$

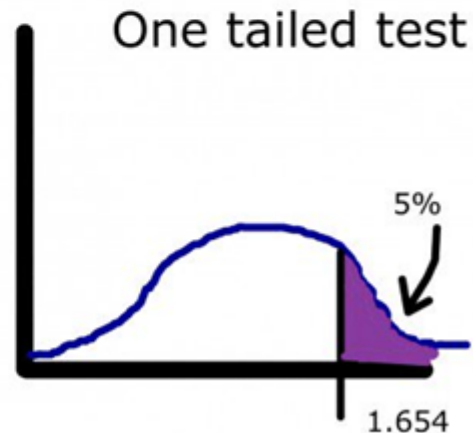
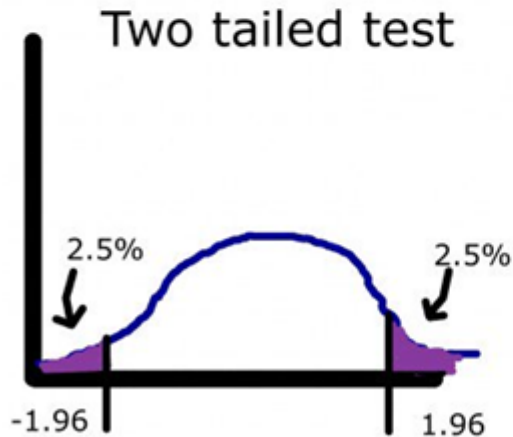
- IF: 1-sided:  $p = .03$
- THEN: 2-sided:  $p = .06$





# ONE tail or TWO?

Some circumstances may warrant a 1-tailed test, BUT...  
We generally **prefer** and default to a 2-tailed test!!!



## More conservative = 2 tails

Rejection region is distributed in both tails

- e.g.:  $\alpha = .05$  distributed across both tails
  - (2.5% in each tail)
- If we know outcome, why do study?
  - Looks suspicious to reviewer's?
  - "significant results at all costs!"

# Choosing Alpha

**Alpha** = probability of making a **type I error**

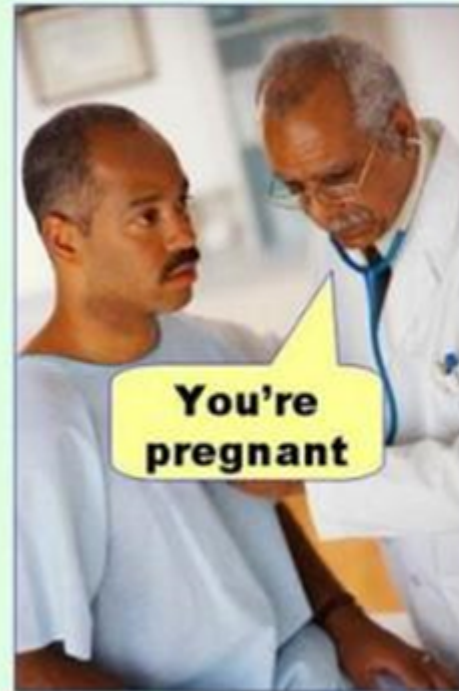
## type I error

- We reject the NULL when we should not
- The risk of "false positive" results

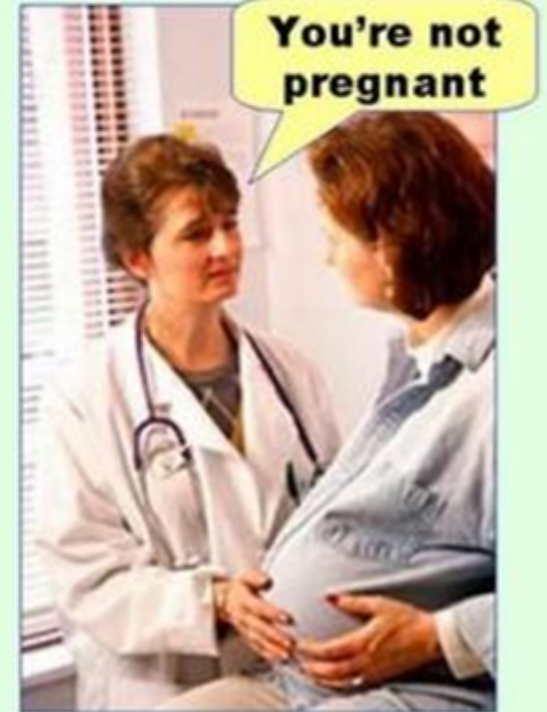
## type II error

- We FAIL to reject the NULL when we should
- The risk of "false negative" results

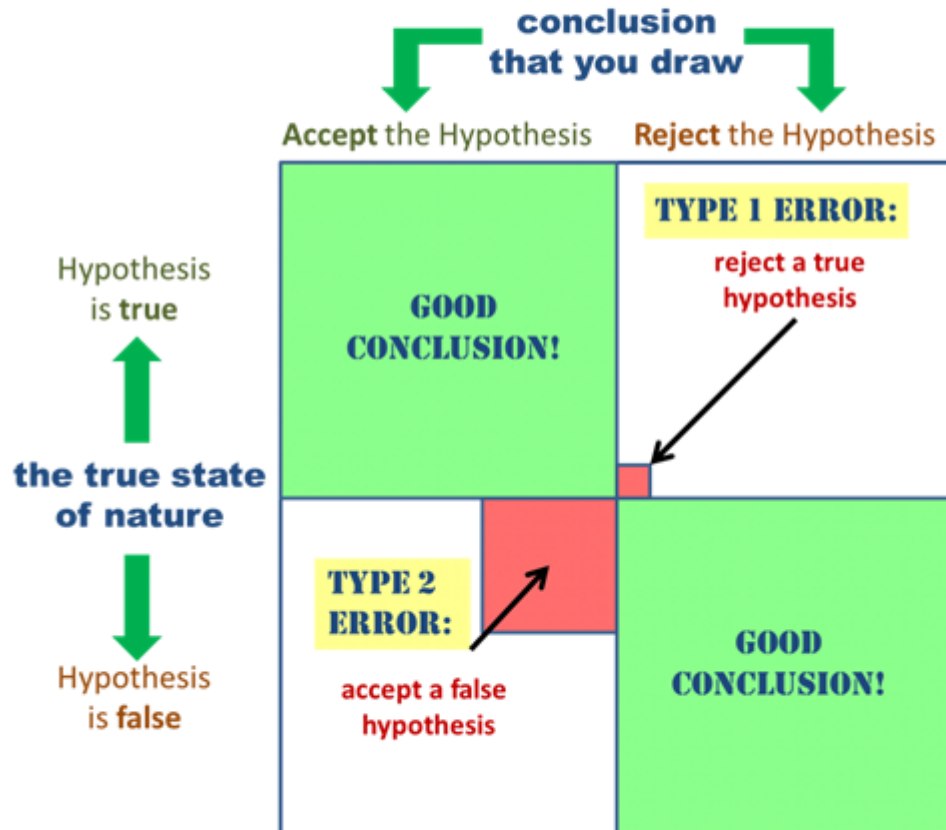
**Type I error**  
(false positive)



**Type II error**  
(false negative)



# Choosing Alpha



We want  $\alpha$  to be **SMALL**, but we can't just make too tiny, since the trade off is increasing the type II error rate

**DEFAULT** is  $\alpha = .05$  (5% = 1 in 20 & seems *rare* to humans) **BUT** there is nothing magical about it

Let it be **LARGER** value,  $\alpha = .10$ , **IF** we'd rather not miss any potential relationship and are okay with some false positives

- Ex) screening genes, early drug investigation, pilot study

Set it **SMALLER**,  $\alpha = .01$ , **IF** false positives are costly and we want to be more stringent

- Ex) changing a national policy, mortgaging the farm

# Assumptions of a 1-sample z-test

Sample was drawn at **random** (at least as representative as possible)

- Nothing can be done to fix NON-representative samples!
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Variables have a **normal** distribution

- Not as important if the sample is large (Central Limit Theorem)
- IF the sample is far from normal &/or small n, might want to transform variables
  - Look at plots: **histogram**, **boxplot**, & **QQ plot** (straight 45 degree line)
  - Skewness & Kurtosis: Divided value by its SE &  $> \pm 2$  indicates issues
  - **Shapiro-Wilks** test (small N):  $p < .05$  ??? not normal
  - Kolmogorov-Smirnov test (large N)

# APA: results of a 1-sample z-test

- State the alpha & number of tails prior to any results
- Report exact p-values (usually 2 decimal places), except for  $p < .001$

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## Example Sentence:

A one sample z test showed that the difference in the quiz scores between the current sample ( $N = 9$ ,  $M = 7.00$ ,  $SD = 1.23$ ) and the hypothesized value (6.000) were statistically significant,  $z = 2.45$ ,  $p = .040$ .



# EXAMPLE: 1-sample z-test

After an earthquake hits their town, a random sample of townspeople yields the following anxiety score:

72, 59, 54, 56, 48, 52, 57, 51, 64, 67

Assume the general population has an anxiety scale that is expressed as a T score, so that  $\mu = 50$  and  $\sigma = 10$ .

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## 1. Null/Alt Hypotheses

$$H_0: \mu = 50$$

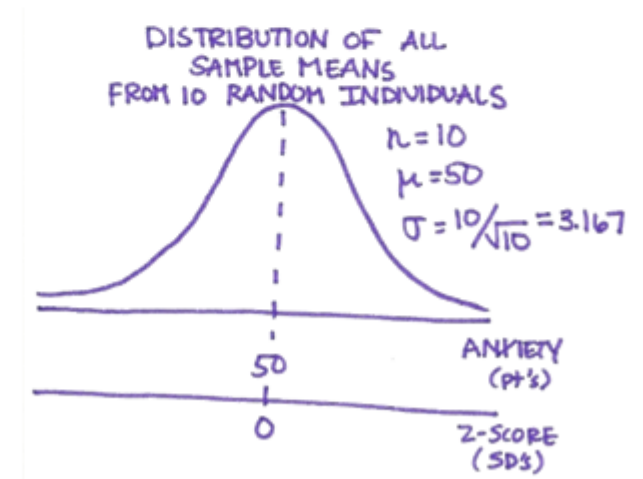
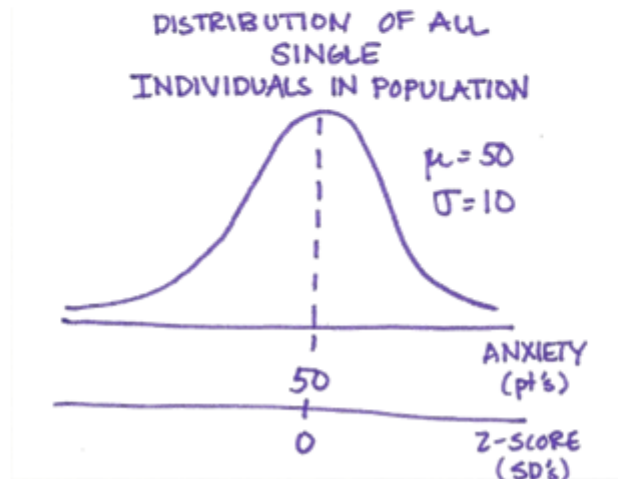
$$H_1: \mu \neq 50$$

## 2. Choose Test Stat, $\alpha$ , & # tails

CLT: mean of repeated SRS  $\rightarrow$   
normally dist.

$\rightarrow$  So use the z-stat

$\alpha = .05$  & 2 tails (default)



### 3. SRS data → Sample Mean

$$\bar{X} = \frac{\sum X}{n} = \frac{580}{10} = 58$$

### 4. Rejection Region?

.05 in **BOTH** tails, so .025 in **EACH** tail ...

→ Critical z = +/- 1.96 ...

→ Reject if Z-score is > 1.96 or < -1.96

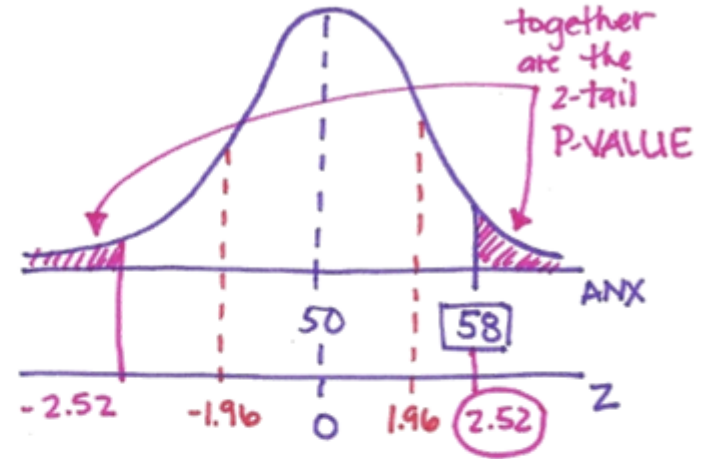
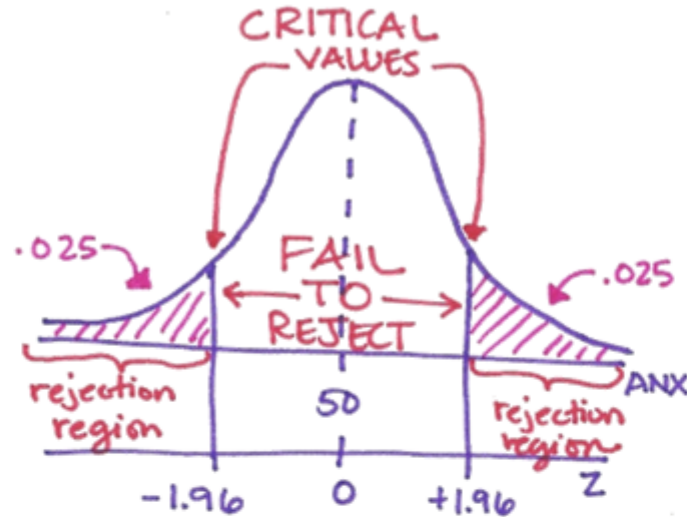
### 5. Calculate the Test Stat

Distribution of all sample means:

$$\text{Mean}_{\text{mean}} = \mu_{\bar{X}} = \mu_{\text{pop}} = 50$$

$$SE_{\text{mean}} = \sigma_{\bar{X}} = \frac{\sigma_{\text{pop}}}{\sqrt{n}} = \frac{10}{\sqrt{10}} = 3.167$$

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{58 - 50}{3.167} = 2.52$$



### 6. Conclusion

Z-stat falls in the rejection region

evidence the population's mean is not 50

"reject the Null"

**"After the earthquake, townspeople's anxiety levels are higher than 50, on average."**

# Cautions About Significance Tests

## Statistical significance

- only says whether the effect observed is likely to be **due to chance** alone, because of random sampling
- may not be **practically important**

That's because *statistical* significance doesn't tell you about the **magnitude of the effect**, only that there **is** one.

An *effect* could be too small to be **relevant**.

And with a large enough sample size, significance can be reached even for the tiniest effect.

- EX) A drug to lower temperature is found to reproducibly lower patient temperature by 0.4 degrees Celsius,  $p < 0.01$ . But clinical benefits of temperature reduction only appear for a 1 decrease or larger.

**STATISTICAL significance does NOT mean PRACTICAL significance!!!**

# Cautions About Significance Tests

Don't ignore lack of significance

**"Absence of evidence is not evidence of absence."**

Having no proof of who committed a murder  
does not imply that the murder was not committed.

Indeed, failing to find statistical significance in results is *not* rejecting the null hypothesis. This is very different from actually accepting it. The sample size, for instance, could be too small to overcome large variability in the population.

When comparing two populations, lack of significance does NOT imply that the two samples come from the same population. They could represent two very distinct populations with similar mathematical properties.

Let's Apply This to the Cancer Dataset

# Read in the Data

```
library(tidyverse)      # Loads several very helpful 'tidy' packages
library(haven)         # Read in SPSS datasets
library(psych)         # Lots of nice tid-bits
library(car)           # Companion to "Applied Regression"
```

```
cancer_raw <- haven::read_spss("cancer.sav")
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```
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```

## And Clean It

```
cancer_clean <- cancer_raw %>%
  dplyr::rename_all(tolower) %>%
  dplyr::mutate(id = factor(id)) %>%
  dplyr::mutate(trt = factor(trt,
                             labels = c("Placebo",
                                           "Aloe Juice"))) %>%
  dplyr::mutate(stage = factor(stage))
```



# Descriptive Statistics

## Skewness & Kurtosis

```
cancer_clean %>%  
  dplyr::select(age, totalcw4) %>%  
  psych::describe()
```

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew
age	1	25	59.64	12.93	60	59.95	11.86	27	86	59	-0.31
totalcw4	2	25	10.36	3.47	10	10.19	2.97	6	17	11	0.49

	kurtosis	se
age	-0.01	2.59
totalcw4	-1.00	0.69

# Tests for Normality - Shapiro-Wilks

```
cancer_clean %>%  
  dplyr::pull(age) %>%  
  shapiro.test()
```

Shapiro-Wilk normality test

```
data:  .  
W = 0.98317, p-value = 0.9399
```

```
cancer_clean %>%  
  dplyr::pull(totalcw4) %>%  
  shapiro.test()
```

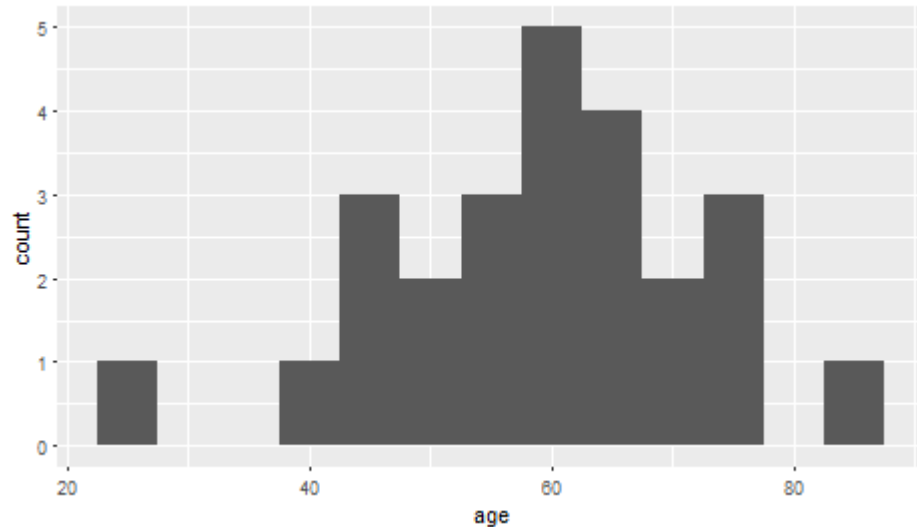
Shapiro-Wilk normality test

```
data:  .  
W = 0.9131, p-value = 0.03575
```

# Plots to Check for Normality - Age

## Histogram

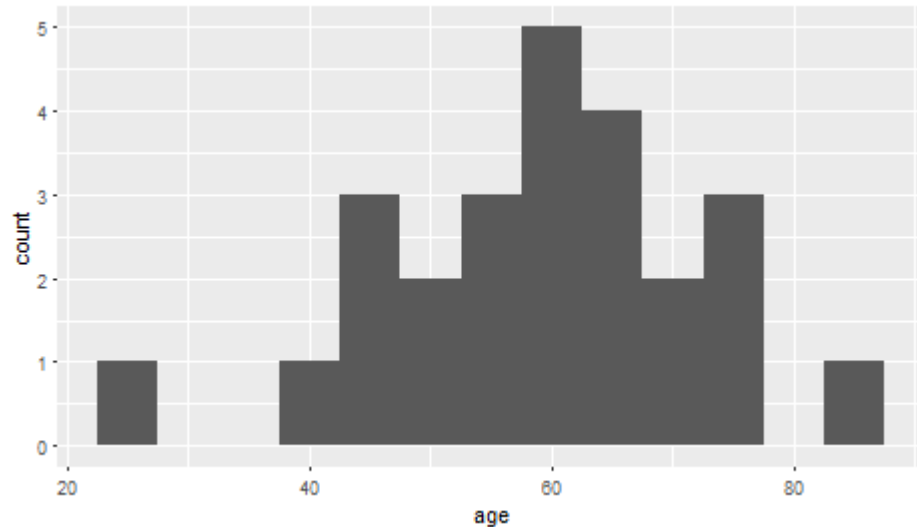
```
cancer_clean %>%  
  ggplot(aes(age)) +  
  geom_histogram(binwidth = 5)
```



# Plots to Check for Normality - Age

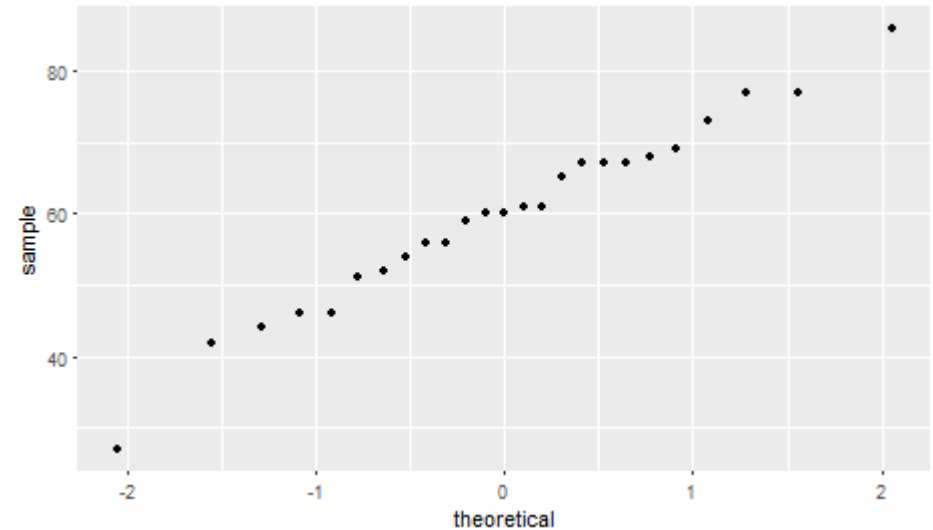
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## Q-Q Plot

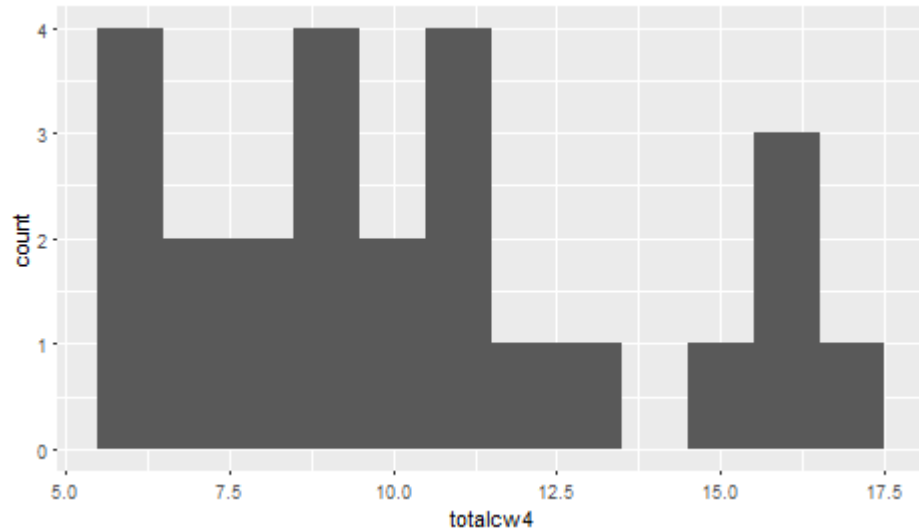
```
cancer_clean %>%  
  ggplot(aes(sample = age)) +  
  geom_qq()
```



# Plots to Check for Normality - Week 4

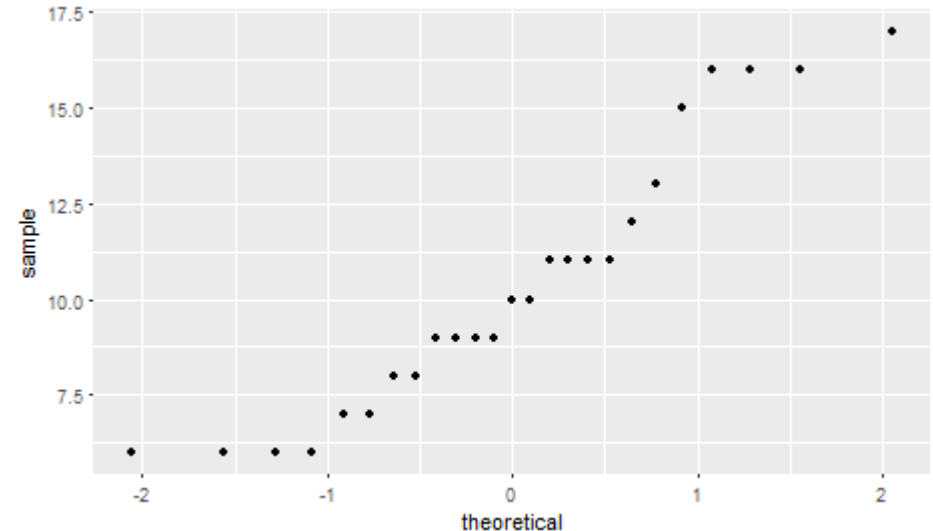
## Histogram

```
cancer_clean %>%  
  ggplot(aes(totalcw4)) +  
  geom_histogram(binwidth = 1)
```



## Q-Q Plot

```
cancer_clean %>%  
  ggplot(aes(sample = totalcw4)) +  
  geom_qq()
```



Questions?

# Next Topic

Confidence Interval Estimation &  
The t-Distribution