Review of Basic Math

Hi! I’m Barry Cohen, and I prepared this review for readers of any of the statistics textbooks that I have authored or co-authored. Users of any of my texts are expected to utilize basic statistical formulas to calculate the answers to the exercises found in those texts. In my experience, some direct practice with such calculations is necessary to develop a deep understanding of how summary statistics are related to the larger datasets that they are summarizing. I don’t believe that the same useful insight can be gained by merely entering numbers into a computer program and then reading the results, though I think it is vital that students learn that skill, as well. As a compromise, I highly recommend that anyone learning from one of my texts use a handheld electronic calculator, with which you can easily obtain both the biased and unbiased versions of the standard deviation. It is only when you are first learning how to calculate the variance or standard deviation from a formula that you should avoid using that function of your calculator. But even though you will be using a calculator, it will be very helpful for you to recall some of the basics of high school mathematics, in order to follow the more mathematical discussions and formula derivations in the text, as well as to solve some of the more complicated exercises in the text. The math review that follows contains only the mathematical procedures that you will need to make the best use of my texts.

To help you determine the extent to which you will need to review basic math, in order to make the best use of your text, I designed a two-part diagnostic quiz. The two parts of the quiz correspond to the two parts of the math review that follows (the answers to the diagnostic quiz are at the end of the math review). If you get more than two answers wrong in one part of the quiz, it is strongly recommended that you study the math review corresponding to that part. After studying the math review, and working out the practice exercises provided, you should take the post-review quiz for the part or parts you studied (the answers to the practice exercises and the post-review quiz are also at the end of the math review). If you still get two or more answers wrong in either part of the quiz, consider a more extensive review of the math concepts with which you are having some difficulty.

Basic Math Review

Diagnostic Quiz

Part I

1. 11 - 3 * 2 - 8 ÷ 2 = ?
2. (11 - 3) * 2 - 8 ÷ 2 = ?
3. 14 + (-2) + (-5) + 4 - (-1) = ?
4. (-9) * (-2) ÷ (-6) = ?
5. Convert 6 / 50 to a decimal.
6. Convert .003 to a fraction.
7. What fraction corresponds to 7%?
8. What percent of 86 is 16?
9. What is the value of 4.1 * 103?
10. Express 5.5% as a decimal.

Part II

1. (F / GX) - (LS / GX) = ?
2. T/S + L/K = ?
3. (R / QP) * (MO / S) = ?
4. (B/C)÷(X/Y) = ?
5. Simplify this fraction: (RX + PR)/QR.
6. What is the value of Y in the following equation? ½Y - 6 = 1
7. Solve the following equation for X: 2Y = CX + B.

If A = 3 and B = 5, what is the value for each of the following expressions?

(Note that the square root sign applies to the entire expression in parentheses that follows it.)

8. $(2A + B)^2$
9. $\sqrt{(2A + 2B)}$
10. $2BA^2$

Basic Math Review

Part I. Arithmetic Operations

A. Order of Arithmetical Operations

Symbols you will need:

+	addition	| |	absolute value
-	subtraction	>	greater than
*	multiplication	<	less than
÷	division	√	square root
/	another symbol for division		

If you have a string of numbers separated by various arithmetical operations, the convention is to perform the operations from left to right, except that multiplication and division take precedence over addition and subtraction. Consider the following sequence: 12 - 6 ÷ 2 + 9 - 4 * 3. If you performed the operations from left to right without regard for precedence, the sequence would be evaluated as follows: 12 - 8 ÷ 2 + 9 - 4 * 3 = 4 ÷ 2 + 9 - 4 * 3 = 2 + 9 - 4 * 3 = 11 - 4 * 3 = 7 * 3 = 21. Recognizing the precedence of multiplication and division over addition and subtraction, the sequence is evaluated differently: 12 - 8 ÷ 2 + 9 - 4 * 3 = 12 - 4 + 9 – 4 * 3 = 12 - 4 + 9 - 12 = 8 + 9 - 12 = 17 - 12 = 5.

Parentheses are used to change the conventional order of operations, or to make the order more obvious. The rule is that the operation enclosed in parentheses is performed before those surrounding it. For example, the value of the sequence above can be changed by adding parentheses in the following manner: (12 - 8) ÷ 2 + (9 - 4) * 3 = 4 ÷ 2 + (9 - 4) * 3 = 4 ÷ 2 + 5 * 3 = 2 + 5 * 3 = 2 + 15 = 17.

Practice:

a) (4 + 3)*(8 - 6) = ?

b) 7*(5 - 2) + 3 = ?

c) 12 ÷ 6 + 15 * 2 = ?

d) 10 + 3 + 2 * 8 - 5 = ?

e) 20 * 2 + 6 * (9 - 5) = ?

Basic Math Review (Part I continued)

B. Signed Numbers

Numbers that are written without plus signs in front of them are assumed to be positive (i.e., greater than zero). A minus sign in front of a number indicates that it is negative (i.e., less than zero). When positive and negative numbers are mixed together in the same sequence, it is common to use plus signs in front of the positive numbers to reduce the possibility of confusion. The conventional way to think about numbers is to imagine a horizontal number line that extends infinitely to the left and to the right. Zero is in the middle, and the numbers become more positive (i.e., larger) as you move to the right of zero, and more negative (i.e., smaller) as you move to the left of zero). Thus, larger negative numbers are said to be less than *smaller* negative numbers (e.g., -7 < -5), because they are more negative (i.e., further to the left on the number line). It can be confusing to think of -7 as being smaller than -5. It may help to think of a bank account; negative numbers mean you owe the bank. The larger the negative number the more you owe, so in that sense your bank account is smaller.

On the other hand, in some cases we are interested only in the magnitude of the number (i.e., its distance from zero) and not its sign. In those cases, we are interested in the *absolute value* of the number. The mathematical symbol for absolute value is a pair of vertical lines surrounding the number. The absolute value of a positive number is the same as the number (e.g., |+4| = +4), but the absolute value of a negative number is the same number with the sign changed from minus to plus (e.g., |-7| = +7). In terms of absolute values, |-7| > |-5|.

To add a negative number and a positive number, find the difference between the two absolute values (subtract the smaller from the larger absolute value) and then add the sign of the number that had the larger absolute value. For instance, to add -7 and +5, ignore the signs, subtract the smaller from the larger number (7 - 5 = 2), and then add a minus sign to the result (-2), because the number with the larger absolute value (-7) has a minus sign. If you need to add a whole string of positive and negative numbers, the easiest way is to add all the positive numbers together into one positive number, and then all of the negative numbers into one negative number (you can add a string of negative numbers by ignoring the minus signs, adding all the numbers, and then attaching the minus sign to the sum). Once you have a single positive number and a single negative number you can add them together according to the rule just given above. To subtract a negative number, you just change it to a positive number and add it. For instance, 5 - (-7) = 5 + 7 = 12. You can say that the two minus signs in a row "cancel each other out."

There is one simple rule that covers both the multiplication and division of signed numbers. If the two numbers have the same sign, the result will be positive; if the two numbers have different signs, the result will be negative. Ignore the signs when performing the multiplication or division, and just attach the appropriate sign to the result (Note: The result of a multiplication is called a product, and the result of a division is called a *quotient*.)

For example, (-5)(-7) = +35. [Note that a multiplication symbol is not needed when two sets of parentheses are placed next to each other; multiplication is assumed in that case.] Some additional examples should help to make the rule clear: (-4)(+6) = -24; (+8) ÷ (-2) = - 4; (-27) ÷ (-9) = +3.

Practice:

a) -18 - (-5) = ?

b) -20 + 14 = ?

c) -90 ÷ +5 = ?

d) (-14)(-7) = ?

e) (-3) + (+11) + (-8) + (-10) + (+5) = ?

C. Converting Fractions to Decimals and Decimals to Fractions

Finding the decimal that corresponds to a fraction is easy to do with a calculator. The top part of the fraction is called the *numerator*, and the bottom part is called the *denominator*. The horizontal line of a fraction means "divided by." To find the decimal corresponding to a fraction the numerator is divided by the denominator. For instance, if the fraction is 2/3, find the answer to "2 divided by 3" on your calculator; you will see that 2/3 = .6666..., etc., which can be rounded off to .67.

However, if the denominator is 10 or some power of 10 (e.g., 100, 1000, etc.), you should not need to use a calculator to convert a fraction to a decimal--you just take the numerator and insert the decimal point in the appropriate place. If the denominator is 10, insert a decimal point in the numerator (assuming the numerator is a whole number) so that there is only one digit to the right of the decimal point (e.g., 23/10 = 2.3), and then drop the denominator. In general, the number of digits to the right of the decimal point is determined by the number of zeroes following 1 in the denominator.

For instance, when dividing by 1,000 there will be three digits to the right of the decimal point, so 23/1,000 becomes .023 (you need to add as many zeroes to the left of the numerator as it takes to have the required number of digits to the right of the decimal point).

If the denominator is not a power of 10, the fraction can still be easily converted to a decimal without a calculator, if the denominator can be changed to a power of 10 by multiplying or dividing by a whole number. The trick is that whatever is done to the denominator to change it into a power of 10 must also be done to the numerator. *Multiplying or dividing both the numerator and the denominator by the same number will not change the value of the fraction*. For instance, 23/50 = 46/100 = .46. On the other hand, 74/200 = 37/100 = .37. In this last example, the denominator was divided by a factor of 2 to become a power of 10, so the numerator had to be divided by 2, as well. If dividing the numerator does not give you a whole number, do not use division at all. You can always use multiplication instead.

For example, both the numerator and denominator of 74/200 can be multiplied by 5 to yield 370/1,000. If this results in the numerator ending in zero, you
can drop the zero at the end of the numerator if you also drop the last zero in the denominator (370/1,000 becomes 37/100).

It is always easy to convert a decimal into a fraction that involves a power of 10 (e.g., tenths, hundredths). The number of digits to the right of the decimal point tells you how many zeroes should follow 1 in the denominator (this is just the reverse of the method in the previous section).

For instance, .703 has three digits to the right of the decimal point, so the denominator will be 1000. The original number goes in the numerator, except that the decimal point is removed. Therefore, .703 = 703/1,000 (which is read as 703 thousandths). Sometimes, a decimal corresponds to a simple fraction.

For example, by the method above .75 would be converted to 75/100, but this fraction is also equal to the much simpler 3/4. Reducing a fraction to its simplest form involves dividing both the numerator and denominator by the same whole number, such that the numerator and denominator both remain whole numbers. Reducing fractions to simple terms can be a useful thing to do, but because it is not a necessary step to finding a correct answer, I do not devote anymore space to it here.

Practice:

Convert each of the following to a fraction: a) .0202 b) .881

Convert each of the following to a decimal: c) 3/500 d) 437/10,000 e) 17/20

D. Converting to and from Percentages

A percentage is just a shorthand way of expressing a fraction in terms of hundredths ("percent" can be read as "out of” or “over” 100).
In fact, the percent symbol, %, is just a shorthand way of writing /100.
You can always convert a percentage to a fraction by putting the percentage in the numerator of a fraction (removing the percent sign) and putting 100 in the denominator. For instance, 83% means 83 over 100 or 83/100. If the percentage contains a decimal point (e.g., 83.5%), the fraction will not come out in a simple form (e.g., 83.5/100). In that case, it is better to

convert the percentage to a decimal first, and then convert the decimal to a fraction, if that is what is desired. To explain how to convert percentages to decimals I need to discuss multiplying and dividing decimals by a power of 10.

When you multiply a decimal by a power of 10, you move the decimal point to the right; when you divide it by a power of 10, you move the decimal point to the left. How many places (i.e., digits) you move is determined by the number of zeroes following 1. For instance, if you want to multiply .703 by 100, you move the decimal point two places to the right, so the result becomes 70.3. If you multiply 83.5 by 100, the result is 8350. Notice that a zero had to be added at the end so the decimal point could be moved two places to the right (when there are no digits to the right of the decimal point, as in 8350, it is customary to not show the decimal point). To divide 83.5 by 1000 you move the decimal point three places to the left, which results in .0835 (notice again that a zero had to be added, this time on the left, so that we could move three places to the left).

To convert a percentage to a decimal, just drop the percent sign and divide the number by 100. For example, 83.5% becomes .835, 2% becomes .02, and 0.4% becomes .004. To convert a decimal to a percentage, multiply by 100 and add the percent sign. For example, .66 becomes 66%, .066 becomes 6.6%, .0066 becomes .66%, and .00066 becomes .066%. To convert a fraction to a percentage, first convert to a decimal, and then use the method just described. For example, 3/8 = .375 = 37.5%, 3/20 = .15 = 15%, and 3/200 = .015 = 1.5%.

Practice: Convert each of the following to a percentage:

a) .062 b) 7/25 c) .0076

Convert each of the following to a fraction:

d) 99.5% e) .095%

E. Scientific Notation

If the result of a calculation has more digits to the right or left of the decimal point than can be displayed on your calculator, the number will be displayed in a form that is often called scientific notation. Most

calculators will display a number between 1 and 10 together with an *exponent* that tells you which power of 10 (e.g., how many zeroes follow the "1") you must multiply that number by. For instance, the number 200 million is too large to be shown in standard form on a calculator with an eight-digit display, so it would be expressed as 2 with an exponent of 8. In scientific notation this would be written as $2*10^8$. I will explain exponents further in the second part of this math review, but for now you can think of 10^8 as a shorthand way of expressing a number that consists of 1 followed by 8 zeroes (i.e., 100 million). To save space, most calculators use an even briefer form by leaving out the 10 (which is always part of scientific notation) and only showing its exponent (sometimes preceded by the letter E).
Converting scientific notation to a standard decimal involves multiplying by a power of 10, as explained in the previous section. For example, $3 * 10^9$ means multiply 3 by a number consisting of a 1 followed by 9 zeroes. In order to move the decimal 9 places to the right of the 3, you must add 9 zeroes to the right of the 3; the result is 3,000,000,000. For another example, $8.19 * 10^5 = 819,000$. A negative exponent means that you need to divide instead of multiply. For example, $8.19 * 10^{-3}$ means that 8.19 is divided by 1000. Moving the decimal three places to the left, the result is .00819.

If a number is not between 1 and 10, it can be converted to scientific notation by moving the decimal point until the number is between 1 and 10. The number of places you had to move the decimal point to achieve this is the exponent (e.g., the power of 10); the exponent is positive if you moved the decimal point to the left, and negative if you moved it to the right. For example, to convert 86,100 into a number between 1 and 10 multiplied by a power of 10, you must move the decimal point four places to the left, so $86,100 = 8.61 * 10^4$. On the other hand, to convert .00319 into a number between 1 and 10 multiplied by a power of 10, the decimal point must be moved three places to the *right*, so $.00319 = 3.19 * 10^{-3}$.

Practice:

Convert each of the following to scientific notation:

a) 3,877,300 b) .00301 c) .0000029.

Convert each of the following to standard notation:

d) $5.16 * 10^7$ e) $4.20 * 10^{-5}$

Basic Math Review

Summary of Part I

A. Order of Arithmetic Operations

1. Arithmetic operations proceed from left to right, except that multiplication and division take precedence over addition and subtraction.

2. Operations enclosed in parentheses are performed before adjacent operations.

B. Signed Numbers

1. Numbers without signs are assumed to be positive.

2. A large number with a minus sign is considered less than a small number with a minus sign.

3. The absolute value of a positive number is the same as the original number, but the absolute value of a negative number is the same number with the sign changed from minus to plus (e.g., $|-5| = +5$.

4. To add a positive and a negative number, find the difference of the two numbers ignoring their signs, and then attach the sign of the number with the larger absolute value.

5. To add a series of numbers, some positive and some negative, add all the negative numbers separately from all the positive numbers, so that you have just one negative number and one positive number. Then proceed as in the previous point.

6. To subtract a negative number, change the sign of the number to a plus and then add the number (e.g., $8 - (-4) = 8 + 4 = 12$).

7. When multiplying or dividing two signed numbers, the final result will be positive if the two numbers have the same sign, and negative if the two numbers have opposite signs.

C. Converting Fractions to Decimals and Decimals to Fractions

1. To convert a fraction to a decimal on your calculator, just divide the numerator of the fraction by the denominator, and round off according to your needs.

2. If the denominator of the fraction is a power of 10 (1 followed by some number of zeroes), insert a decimal point in the numerator so that the number of digits to the right of the decimal point is the same as the number of zeroes following 1 in the denominator -- then drop the denominator.

3. Multiplying or dividing both the numerator and the denominator by the same number will not change the value of the fraction.

4. To convert a decimal into a fraction, drop the decimal point and create a denominator that consists of 1 followed by a number of zeroes equal to the number of digits to the right of the decimal point in the original number (e.g., .3456 = 3456/10,000).

D. Converting to and from Percentages

1. If the percentage is a whole number, it can easily be converted to a fraction by removing the percent sign and putting 100 in the denominator.

2. To multiply a decimal by a power of 10, move the decimal point to the right a number of places that equals the number of zeroes following 1 in the multiplier, adding zeroes on the right, as necessary.

3. To divide by a power of 10 move the decimal point to the left, adding zeroes as necessary.

4. To convert a percentage to a decimal, drop the percent sign and divide the number by 100 (i.e., move the decimal point two places to the left).

5. To convert a decimal to a percentage, multiply by 100 (i.e., move the decimal point two places to the right), and add the percent sign.

E. Scientific Notation

1. Any number can be expressed in scientific notation; the number must be converted to a part that is between 1 and 10, which is multiplied by a part that consists of 10 raised to some power.

2. The number of places the decimal point had to be moved to convert the original number to a number between 1 and 10 is the exponent (i.e., power) that is attached to 10. If the decimal point had to be moved to the left, a plus sign is attached to the exponent; if the decimal point had to be moved to the right, a minus sign is attached to the exponent (e.g.,.00319 = $3.19 * 10^{-3}$; 973,600 = $9.736 * 10^{5}$).

Basic Math Review

Part II. Basic Algebra

A. Adding and Subtracting Fractions

In order to add or subtract fractions, they must have the same denominator. There are rules you can use to make the denominators the same, but if the fractions involve actual numbers (e.g., 3/4, 3/20) you can more easily convert them to decimals and add or subtract them on your calculator. If the fractions involve symbols (e.g., 2X/Y), the same rules can be used to change the way the fraction looks. Because statistical formulas frequently contain fractions, and changing the form of the fraction can make the formula look very different, it is worth exploring the various algebraic rules affecting fractions. The first rule is that when two fractions have the same denominator you can add or subtract the numerators, and put the sum or difference over the common denominator. For instance, X/Z + Y/Z = (X + Y)/Z. This rule can also be reversed; if the numerator contains addition or subtraction, the fraction can be broken apart. For example, (2A-3B)/4C = (2A/4C) – (3B/4C). However, it is important to note that this rule does not work for addition or subtraction in the denominator: A/(B + C) cannot be broken apart.

If the denominators do not match they must be changed so that they do match before the fractions can be added or subtracted. Of course, if you change the denominator of a fraction, you must also change the numerator in a compensating way, or you will change the value of the fraction. This leads to the next simple rule involving fractions: you can multiply or divide both the numerator and denominator of a fraction by the same constant or variable without changing the value of the fraction. For instance, A/B = 2A/2B, and X/Y = AX/AY. This rule can be used to make the denominators of two fractions match. The trick is to multiply the numerator and denominator of each fraction by the denominator of the other fraction. For instance, suppose you want to add: (A/B) + (X/Y). First, you multiply the top and bottom of A/B by Y to get AY/BY. Next you multiply the top and bottom of X/Y by B to get BX/BY. Now that the denominators match you can add (AY/BY) + (BX/BY) = (AY+BX)/BY.

Practice: Perform the following operations:

a) (X/4) - (3Y/4) b) (XY/RC) + (5/RC) c) (5P/Q) + (3P/T)

d) (A/5) - (B/2) e) (4/X) + (8/Y)

B. Multiplying and Dividing Fractions

The rule for multiplying fractions could not be simpler. The denominators do not even have to match. Just multiply the two numerators and multiply the two denominators. For example, (X/Z)*(B/C) = XB/ZC. When you multiply both the numerator and denominator of a fraction by the same constant or variable, say C, what you are really doing is multiplying the fraction by C/C. For instance, multiplying X/Y by C/C yields XC/YC. However, a fraction in which the numerator equals the denominator (e.g., C/C) is always equal to 1. Therefore, another way to understand how the form of a fraction can be changed without changing its value is to realize that multiplying a fraction by any other fraction that is equal to 1, will not change the first fraction. The new form is said to be *algebraically equivalent* to the first form.

One more rule concerning multiplication is worth mentioning. When a fraction is multiplied by a whole number, it is only the numerator that gets multiplied. For instance, if you want to multiply A/B by C, you can write C (A/B) or AC/B; the two forms are equivalent. The rule can be reversed; whenever there is a multiplication in the numerator, one of the terms can be moved in front (or in back of) the fraction. For example, 2X/Y can be written as 2(X/Y).

The rule for dividing fractions involves only a slight modification of the rule for multiplication. The second fraction, called the "divisor," must be inverted, and then the two fractions are multiplied. For instance, (2X/3Y) ÷ (4D/5E) = (2X/3Y)*(5E/4D) = (2X)(5E)/(3Y)(4D) = 10XE/12YD. One way that the division of fractions is expressed is to put a fraction in the numerator and/or denominator of another fraction. For instance, (X/Y)/Z is the same as X/Y ÷ Z, which according to the division rule, means X/Y * 1/Z = X/(YZ). When a fraction appears in the numerator of another fraction, as in the example above, the simple rule is to put the two denominators together (i.e., move the denominator in the numerator down so that it multiplies the denominator of the entire fraction).

Note that any whole number or variable (e.g., Z) can be expressed as a fraction by placing it over 1 as a denominator (e.g., $Z = Z/1$). Inverting a fraction means that the numerator and denominator trade places. In the case of a whole number, this means that 1 appears in the numerator instead of the denominator. Inverting the number means finding its *reciprocal* (e.g., the reciprocal of Z is $1/Z$).

When a fraction appears in the denominator of another fraction [e.g., $X/(Y/Z)$], you can again express this as a division problem. For example, $X/(Y/Z) = X \div Y/Z = X * (Z/Y) = (X/1) * (Z/Y) = XZ/Y$. The simple rule is that the denominator of the denominator is moved up to the numerator. Finally, when there are fractions in both the numerator and the denominator [e.g., $(A/B)/(C/D)$], it is just another way to show that two fractions are to be divided using the division rule described above. For example, $(A/B)/(C/D) = (A/B) \div (C/D) = (A/B) * (D/C) = AD/BC$.

Practice: Perform the following operations:

a) $X (A/B)$ b) $(5/X) * (Y/6)$ c) $(2D/E) \div (3F/G)$

d) $(3X/7)/Y$ e) $AB/(X/3)$

C. Factoring

Consider the following expression: $A(X + Y)$. You may recall that the parentheses indicate that X and Y are to be added before being multiplied by A. However, it is also permissible to multiply both X and Y by A before adding - that is, $A(X + Y) = AX + AY$. This rule is often used in the reverse manner to *factor out* a common term in a complex expression. For instance, $2BX + 3BY + 4AB = B(2X + 3Y + 4A)$. Another useful rule is that terms that appear in both the numerator and denominator of a fraction can be "canceled out." For example, AB/BC can be reduced to A/C. The numerator and denominator are being divided by the same factor, which doesn't change the value of the fraction. This is just the reverse of multiplying the numerator and denominator by the same factor, which also doesn't change the value of the fraction.

The two rules described above can be used together to simplify a complex fraction, such as the following: (AC + BC)/DC. First C is factored out of the numerator to yield: C(A + B)/DC. Then, C is canceled out of both the numerator and the denominator to leave: (A + B)/D.

Practice: Perform the following operations:

a) 3*(2A + B + 4C)   b) 5*(B + 0.2Y)

Factor out the common term in each of the following:

c) 2XY + 2AX + 2X   d) QPR - (PR/4) + 3PRS

e) (3CD - 3DY)/3AD

D. Exponents and Square Roots

You probably recall that in the expression X^2 the 2 is called the *exponent* (X, in this case, is called the "base"), and it means that X is to be multiplied by itself (X * X). An exponent of 3 would indicate that a string of three Xs are to be multiplied (X * X * X), and so on. When the exponent is 2 we say that the number is being *squared* (if 3 the number is being “cubed”). Exponents greater than 2 are rarely used in statistics text, so I will not devote space to them here.

The exponent takes precedence over the arithmetic operations that we have been dealing with so far, so if you see XY^2, it is only the Y that is being squared (i.e., the squaring occurs before the multiplication of X and Y). If you want the product of X and Y to be squared you can use parentheses to indicate that the multiplication is to be done first: $(XY)^2$. Once you are squaring both the X and the Y it doesn't matter whether you multiply before squaring, or square before multiplying -- that is, $(XY)^2 = X^2Y^2$. However, the order does make a difference when addition or subtraction is involved. For instance, $(A + B)^2$ does *not* equal $A^2 + B^2$. (You can check this with some simple numbers, such as A = 2 and B = 3). The expression $(A + B)^2$ is the same as (A + B)*(A + B), which requires the multiplication of two binomials (a binomial is just two terms being added or subtracted). The product of (A + B) times (A + B) equals $A^2 + B^2 + 2AB$, which is always larger than $A^2 + B^2$ when A and B are both positive numbers.

The square root of a number is the number that has to be squared to get back to the original number. In other words, $(\sqrt{X})^2 = X$. It is also true that: $\sqrt{(X^2)} = X$. If you are trying to remember the square root of a number (e.g., 144) and you think it might be 12, you can always check by squaring 12 (i.e., 12 * 12) to see if it equals 144. The rules for mixing multiplication with the square root are similar to the rules for exponents. First, the square root applies only to the terms under the square root sign. For example, $\sqrt{(X)}Y$ means that the square root of X is multiplied by Y. On the other hand, $\sqrt{(XY)}$ means that X and Y are multiplied first and the square root of the product is taken (I am using parentheses to indicate which terms are included under the square root sign). However, the order in that case doesn't matter: $\sqrt{(XY)} = \sqrt{X} * \sqrt{Y}$. As with exponents, the order of operations does make a difference when addition or subtraction is involved. For instance, $\sqrt{(X - Y)}$ does not equal $\sqrt{X} - \sqrt{Y}$.

The rules that concern squaring or taking the square root of a fraction are very straightforward. Squaring a fraction is equivalent to squaring both the numerator and denominator separately. For example, $(X/Y)^2 = X^2/Y^2$. Similarly, taking the square root of a fraction is equivalent to taking the square roots of both the numerator and denominator separately. For example, $\sqrt{(X/Y)} = \sqrt{X} / \sqrt{Y}$. Sometimes this rule is used in reverse. For instance, $A/\sqrt{B}$ can be rewritten as $\sqrt{A^2} / \sqrt{B}$ (recall that $\sqrt{A^2} = A$), and then the previous rule can be used in reverse to yield: $\sqrt{(A^2/B)}$.

Practice: Rewrite the following expressions without parentheses:

a) $(3FG)^2$ b) $(2X/5Y)^2$ c) $[\sqrt{(4BC)}]^2$

Rewrite the following expressions with a single square root sign and simplify:

d) $[\sqrt{(5J)}]*[\sqrt{(3K)}]$ e) $\sqrt{(XY)} / \sqrt{(AX)}$

Basic Math Review (Part II continued)

E. Solving Simple Equations

An equation is a statement that two mathematical expressions are identical in their numerical value. Equations are usually easy to solve when the mathematical expressions involved use only the four basic arithmetic operations (addition, subtraction, multiplication, and division). An equation that includes a square root or an exponent, for example, is considerably more difficult to solve, so I will confine my discussion to simple equations. If you are dealing with a simple equation with only one variable --for example, 5X - 2 = 13 -- you can use a few algebraic rules to find the value of that variable. On the other hand, if a simple equation contains several variables you won't be able to find the value of each variable, but you can use the same algebraic rules to change the appearance of the
equation, which is often desirable. The most fundamental rule when working with equations, is that whatever manipulation is performed on one side of the equation (i.e., on one side of the equals sign), the same manipulation must be performed on the other side. Otherwise, the two sides of the equation will no longer be equal.

When the equation has only one variable, the goal is to isolate the variable on one side of the equation, so that the other side of the equation can be reduced to a particular numerical value. For example, if the equation we are dealing with is 5X - 2 = 13, we want to rearrange the equation into the form: X = N, where N is some particular number. There are two terms that must be moved in order that X can be alone: the term that is multiplying X (i.e., 5) and the term that is being subtracted from X (i.e., 2). It is usually easier to deal with addition and subtraction first, so let us begin by dealing with the term being subtracted in the preceding equation. To get rid of -2 we add +2 to both sides of the equation:

$$\begin{array}{rcl} 5X - 2 & = & 13 \\ \underline{\quad +2} & \underline{=} & \underline{+2} \\ \mathbf{5X} \quad & \mathbf{=} & \mathbf{15} \end{array}$$

Then to get rid of the 5, we divide both sides of the equation by 5: $\frac{5X}{5} = \frac{15}{5}$

Therefore, X = 15/5 = 3. We can check that X = 3 by substituting 3 for X in the original equation: 5(3) - 2 = 13; 15 - 2 = 13; 13 = 13.

These same rules can be used to change the form of an equation, even if it contains no numbers at all. Consider the following equation: (AX + B)/C = DY If you are interested in the value of X, when values of the other variables have been filled in, it may be convenient to solve the equation for X, which means isolating X on one side of the equation. We begin by multiplying both sides of the equation by C, in order to get C away from the side of the equation that contains X:

$$\frac{C(AX+B)}{C} = CDY$$

Canceling out C in the left side of the equation, we are left with AX+B=CDY.

Next we subtract B from both sides:

$$\begin{array}{rcl} AX + B &=& CDY \\ \underline{\;\;\;\; - B} && \underline{-B} \\ AX &=& CDY - B \end{array}$$

Finally, we divide both sides by A: $\frac{AX}{A} = \frac{CDY - B}{A}$; $X = \frac{CDY - B}{A}$

Practice:

Find the value of Y in the following equations:

a) Y/3 + 15 = 21 b) .7Y - 7 = 7

Solve for Z:

c) (2Z + 8)/6 = X d) 3Z/5 = A/B e) BZ - 5X = 2AC

Basic Math Review

Summary of Part II

A. Adding and Subtracting Fractions

1. If two fractions have the same denominator they can be added (or subtracted) by adding (or subtracting) the numerators and placing the sum (or the difference) over the common denominator.

2. If the numerator and denominator of a fraction are both multiplied or divided by the same constant or variable, the value of the fraction will not be changed.

3. The denominators of two fractions can be made to match by multiplying both the numerator and denominator of each fraction by the denominator of the other fraction.

B. Multiplying and Dividing Fractions

1. To multiply two fractions, multiply the two numerators together and the two denominators together -- e.g., $(A/B) * (C/D) = AC / BD$.

2. When a fraction is multiplied by a whole number, only the numerator is multiplied --for example, $C(A/B) = (CA)/B$.

3. To divide two fractions, invert the second one (the divisor), and then multiply them.

4. When a fraction appears in the numerator of another fraction, the denominator of the fraction in the numerator can be moved so that it multiplies the denominator of the whole fraction. For example, $(X / Y)/Z = X/(YZ)$.

5. When a fraction appears in the denominator of another fraction, the denominator of the denominator can be moved so that it multiplies the numerator --for example, $X/(Y / Z) = (XZ)/Y$.

C. Factoring

1. If a string of terms being added and/or subtracted all contain a common term, that term can be factored out of each member of the string and placed in front of the entire expression, which is then enclosed in parentheses --for example, $AX + XY - BX = X(A + Y - B)$.

2. If the numerator and denominator of a fraction contain a term in common, that term can be "canceled out" -- that is, the common term is removed from both the numerator and denominator (e.g., $BX / AB = X/A$).

D. Exponents and Square Roots

1. Exponents and square roots take precedence over other arithmetic operations.

2. Taking the square root and squaring are opposite operations -- one can undo the other -- e.g., $\sqrt{(X^2)} = X$, and $(\sqrt{X})^2 = X$.

3. Parentheses can be used to indicate that more than one variable is to be squared -- e.g., $(XY)^2 = X^2\ Y^2$. However, the order between squaring and addition/subtraction makes a difference -- e.g., $(X + Y)^2$ does *not* equal $X^2 + Y^2$; it equals $X^2 + Y^2 + 2XY$.

4. The previous point applies in a similar way to taking square roots -- e.g., $\sqrt{(AB)} = \sqrt{A}\ \sqrt{B}$, but $\sqrt{(A+B)}$ does not equal $\sqrt{A} + \sqrt{B}$.

5. If you square a fraction, it is equivalent to squaring both the numerator and the denominator -- e.g., $(A/B)^2 = A^2/B^2$. If you take the square root of a fraction, it is equivalent to taking the square root of both the numerator and denominator -- e.g., $\sqrt{(A/B)} = \sqrt{A} / \sqrt{B})$.

E. Solving Simple Equations

1. The most fundamental rule for solving equations is that whatever operation is performed on one side of the equation must be performed on the other side, to preserve the equality.

2. If the equation consists of numbers and only one (unknown) variable, the equation can be solved numerically by isolating the variable on one side of the equation.

3. If the equation has several variables, and you want to solve for one of the variables, you can use the rules of algebra to isolate that variable on one side of the equation.

Basic Math Review

Post Review Quiz

Part I

1) $3 * 4 - 2 + 28 \div 7 - 5 = ?$

2) $3 * (4 - 2) + 28 \div (7 - 5) = ?$

3) $-17 - (-9) + (+11) + (-7) - (+4) = ?$

4) $(+5) * (-4) \div (-10) = ?$

5) Convert 46/200 to a decimal.

6) Convert .077 to a fraction.

7) What fraction corresponds to 0.66%?

8) What percent of 140 is 3?

9) Express 0.42% as a decimal.

10) What is the value of $3.3 * 10^{-3}$?

Part II

1) $(4X/17) + (2Y/17) = ?$

2) $(2A/F) + (3B/G) = ?$

3) $(7K/M) * (2J/Q) = ?$

4) $(3X/Y) \div (4A/C) = ?$

5) Simplify this fraction: $(2RT + 3ST) / TW$.

6) What is the value of B in the following equation: $(3B/2) + 7 = 19$.

7) Solve the following equation for S: $4X = (5S + 7R) / 2T$.

If X = 2 and Y = 6, what is the value for each of the following expressions?

8) $(2Y - 3X)^2$

9) $(3Y/X)^2$

10) $\sqrt{(2XY^2)}$

Basic Math Review

Answers to Diagnostic Quiz

Part I

1) +1   2) +12   3) +12   4) -3   5) .12   6) 3/1000

7) 7/100   8) 18.6%   9) 4,100   10) .055

Part II

1) (F - LS)/GX   2) (KT + LS)/KS   3) RMO/QFS   4) BY/CX

5) (X + P)/Q   6) 14   7) X = (24 - B) / C

8) 121   9) 4   10) 90

Answers to Practice Exercises

Part I

A. a) 14   b) 24   c) 32   d) 24   e) 64

B. a) -13   b) -6   c) -18   d) 98   e) -5

C. a) 202/10,000   b) 881/1000   c) .006
d) .0437   e) .85

D. a) 6.2%   b) 28%   c) 0.76%   d) 995/1000   e) 95/100,000

E. a) $3.8773 * 10^{6}$ b) $3.01 * 10^{-3}$ c) $2.9 * 10^{-6}$
d) 51,600,000   e) .000042

Part II

A. a) (X - 3Y)/4   b) (XY + 5)/RC   c) (5PT + 3PQ)/QT
d) (2A -5B)/10   e) (4Y + 8X)/XY

B. a) AX/B   b) 5Y/6X   c) 2DG/3EF   d) 3X/7Y   e) 3AB/X

C. a) 6A + 3B + 12C   b) 5B + Y   c) 2X(Y + A + 1)
d) PR(Q - 1/4 + 3S)   e) 3D(C - Y)/3AD = (C - Y)/A

D. a) $9F^2G^2$ b) $4X^2/25Y^2$ c) 4BC   d) √(15JK)   e) √(Y/A)

E. a) 18   b) 20   c) Z = 3X - 4   d) Z = 5A/3B

e) Z = (2AC + 5X)/B

Answers to Post Review Quiz

Part I

1) 9 2) 20 3) -8 4) +2 5) 0.23

6) 77/1000 7) 66/10,000 8) 2.14% 9) .0042 10) .0033

Part II

1) (4X + 2Y)/17 2) (2AG + 3BF)/FG 3) 14KJ / MQ 4) 3CX / 4AY

5) (2R + 3S)/W 6) 8 7) (8TX - 7R)/5 8) 36

9) 81 10) 12