

# CHAPTER 8

# POWER & EFFECT SIZE

FOR EDUC/PSY 6600

“Cohen (1994): “Next, I have learned and taught that the primary product of research inquiry is one or more measures of effect size, not  $p$  values.” (p. 1310).

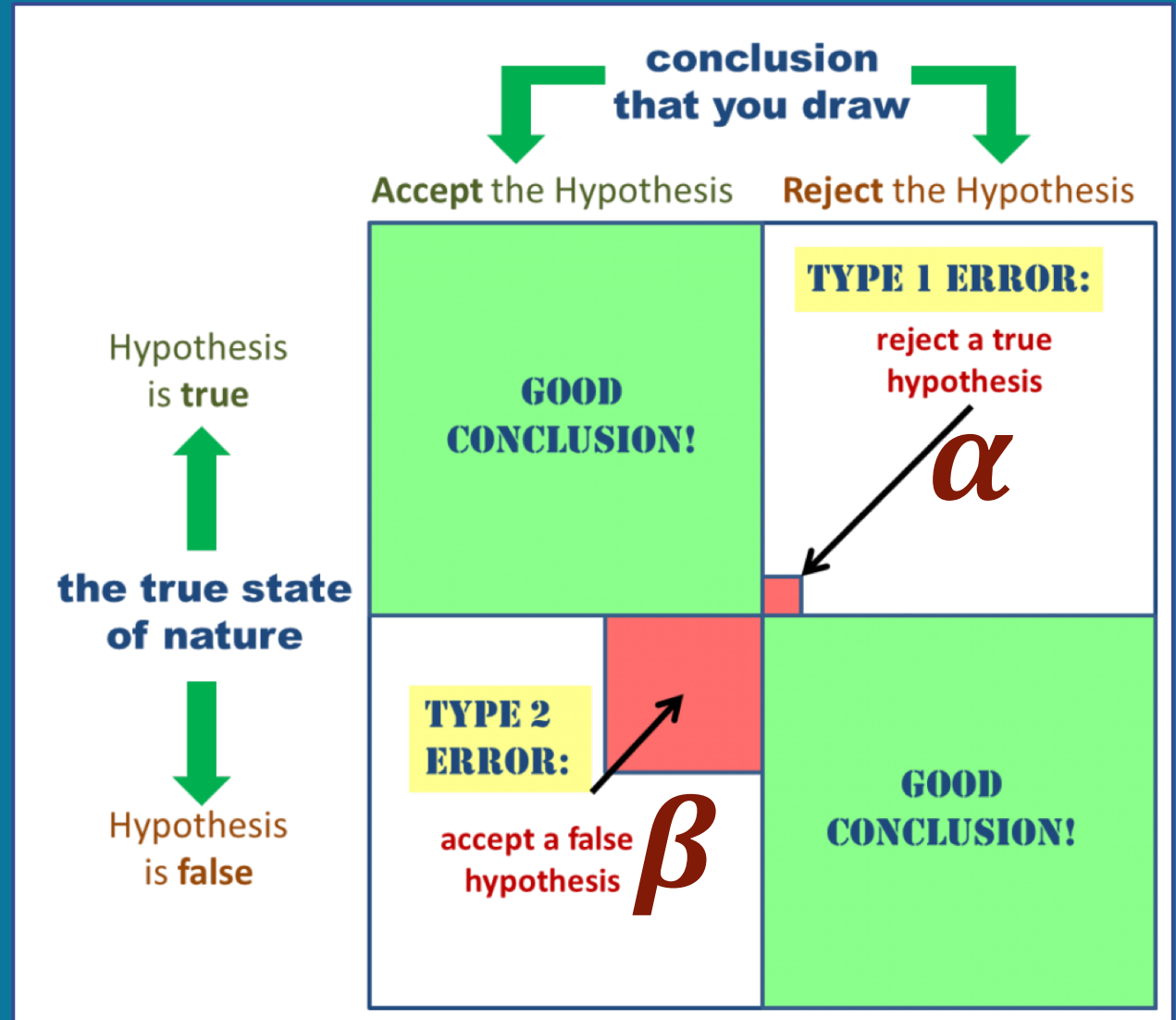
Abelson (1995): “However, as social scientists move gradually away from reliance on single studies and obsession with null hypothesis testing, effect size measures will become more and more popular” (p. 47).

”

# Types of Errors

When we conduct a hypothesis test, we either reject or fail to reject the Null Hypothesis.

Our decision usually causes four outcomes:



# Types of Errors

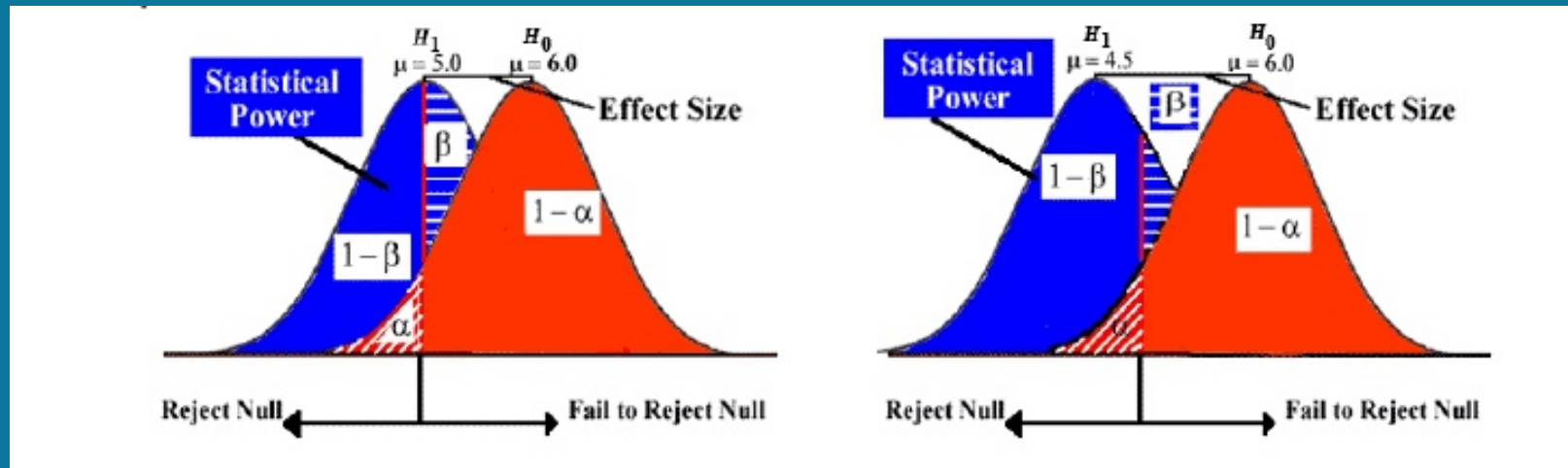
$$\underline{\text{Power} = 1 - \beta}$$

“the probability of  
correctly rejecting  
a false null hypothesis.”

# Effect Sizes

$$\text{Cohen's } d = \frac{\bar{X}_1 - \bar{X}_2}{s_p} \text{ or } t \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

$$\eta^2 = r_{pb}^2 = \frac{t^2}{t^2 + (n_1 + n_2 - 2)}$$



# Effect Sizes

$$\text{Cohen's } d = \frac{\bar{X}_1 - \bar{X}_2}{s_p} \text{ or } t \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

Cohen's d	Interpretation
.2	Small
.5	Moderate
.8	Large

# Effect Sizes

$$\eta^2 = r_{pb}^2 = \frac{t^2}{t^2 + (n_1 + n_2 - 2)}$$

$\eta^2$  (eta squared) and  $r_{pb}^2$

- **association** between grouping variable (IV) and continuous DV
- Ranges from 0 to 1
- With only 2 groups, results are same

# What affects power?

## 1. Sample Size

- Larger sample = more power

## 2. Effect Size

- Larger Effect size = more power

## 3. Alpha Level

- Higher Alphas = more power

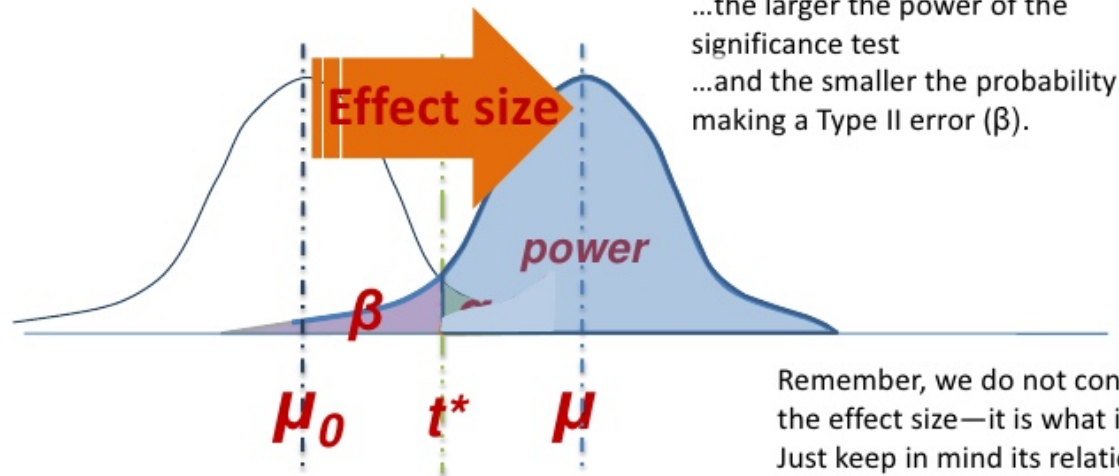
## 4. Directionality

- One tail = more power

## Types of errors and their probabilities

- How does effect size relate to power and  $\beta$ ?

The larger the effect size...  
...the larger the power of the  
significance test  
...and the smaller the probability of  
making a Type II error ( $\beta$ ).



Remember, we do not control  
the effect size—it is what it is.  
Just keep in mind its relation  
to both power and  $\beta$ .



# Power Analysis

- Non-centrality parameter is calculated by:

$$\delta = \frac{d}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- Since it's assumed that the...
  - Variances are same in 2 groups
  - $N$ 's are same in 2 groups
- ...and since  $\sigma$  is often assumed to be 1...
- ...the equation is simplified...

$\delta$	ONE-TAILED TEST ( $\alpha$ )			
	.05	.025	.01	.005
	TWO-TAILED TEST ( $\alpha$ )			
	.10	.05	.02	.01
0.5	.14	.08	.03	.02
0.6	.16	.09	.04	.02
0.7	.18	.11	.05	.03
0.8	.21	.13	.06	.04
0.9	.23	.15	.08	.05
1.0	.26	.17	.09	.06
1.1	.29	.20	.11	.07
1.2	.33	.22	.13	.08
1.3	.37	.26	.15	.10
1.4	.40	.29	.18	.12
1.5	.44	.32	.20	.14
1.6	.48	.36	.23	.16
1.7	.52	.40	.27	.19
1.8	.56	.44	.30	.22
1.9	.60	.48	.33	.25
2.0	.64	.52	.37	.28
2.1	.68	.56	.41	.32
2.2	.71	.60	.45	.35
2.3	.74	.63	.49	.39
2.4	.77	.67	.53	.43
2.5	.80	.71	.57	.47
2.6	.83	.74	.61	.51
2.7	.85	.77	.65	.55
2.8	.88	.80	.68	.59
2.9	.90	.83	.72	.63

When  $n_1 = n_2$

$$\delta = \mathbf{d} \sqrt{\frac{n_k}{2}}$$

$$n_k = 2 \left( \frac{\delta}{\mathbf{d}} \right)^2$$

When  $n_1 \neq n_2$

$$\frac{2}{\frac{1}{n_1} + \frac{1}{n_2}} = \frac{2n_1n_2}{n_1 + n_2}$$

$$\delta = \mathbf{d} \sqrt{\frac{n_h}{2}}$$

# FORMULA SHEET

$$\begin{array}{l}
 \delta = \text{"EXPECTED T OR Z"} \text{ (population parameters)} \\
 \left. \begin{array}{l}
 1 \text{ group: } \delta = \frac{\mu}{\sigma} \sqrt{n} \xrightarrow{d = \frac{\mu}{\sigma}} \delta = d \sqrt{n} \\
 2 \text{ groups: } \delta \xrightarrow{\text{equal } n's} \frac{\mu_1 - \mu_2}{\sigma} \sqrt{\frac{n}{2}} \xrightarrow{d = \frac{\mu_1 - \mu_2}{\sigma}} \delta = d \sqrt{\frac{n}{2}}
 \end{array} \right\} \begin{array}{l}
 \xleftrightarrow{\text{est. } d = g \left( 1 - \frac{3}{4 df - 1} \right)} \\
 \\
 \end{array} \left\{ \begin{array}{l}
 g = \text{"effect size"} \text{ (sample statistics)} \\
 \\
 g = \frac{\bar{X}_1 - \bar{X}_2}{s_p} \left\{ \begin{array}{l}
 = t \sqrt{\frac{2}{n}} \quad \leftarrow n_1 = n_2 \\
 = t \sqrt{\frac{n_1 + n_2}{n_1 n_2}} \quad \leftarrow n_1 \neq n_2
 \end{array} \right.
 \end{array} \right.
 \end{array}$$

# G-POWER

Download at: <http://www.gpower.hhu.de/>

G\*Power 3.1.9.2

File Edit View Tests Calculator Help

Central and noncentral distributions Protocol of power analyses

Test family: t tests

Statistical test: Correlation: Point biserial model

Type of power analysis: A priori: Compute required sample size - given  $\alpha$ , power, and effect size

Input Parameters

Determine =>

Tail(s): One

Effect size  $|p|$ : 0.3

$\alpha$  err prob: 0.05

Power ( $1-\beta$  err prob): 0.95

Output Parameters

Noncentrality parameter  $\delta$ : ?

Critical t: ?

Df: ?

Total sample size: ?

Actual power: ?

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X-Y plot for a range of values Calculate

## CHAP 8: SECTION A

- **d** is just the number of standard deviations that separate two **population** means
- **g** is the number of standard deviations (based on pooling the sample variances and taking the square-root) separating the **sample** means.
- connection between a calculated  $t$  and delta;
  - large  $t$ 's are *usually* associated with large deltas
  - small  $t$ 's *usually* with small deltas.
  - Of course, the **alternate hypothesis distribution** shows that  $t$  can occasionally come out very differently from delta

## CHAP 8: SECTION B

An estimate of **power**  
is only as good as  
the estimate of **effect size** upon which it is based

...BUT determining the effect size is usually  
the purpose (or should be) of the experiment.