1. EPIDEMICS E1

Which parameters define the time scale in the system (about a year)? Find out how it changes.

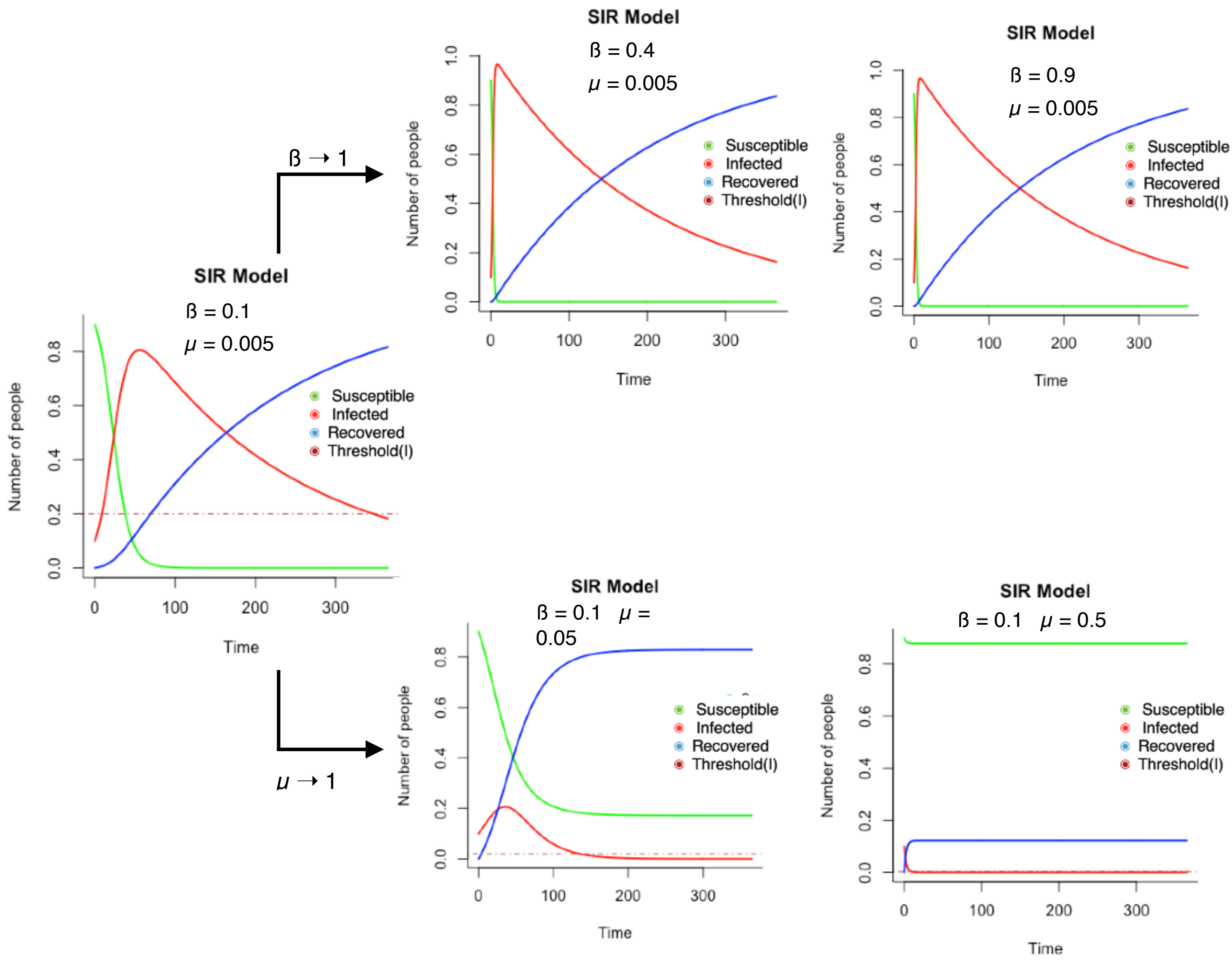
The initial conditions of the population to study this model are:

- $S_0 = 0.9$
- $I_0 = 0.1$
- $R_0 = 0$

ß and μ are the parameters that change the time scale in the system. In order to have an epidemic that ends in 1 year, the values of these parameters must be:

- ß = 0.1
- $\mu = 0.005$

The effect of these parameters can be seen in the graphs:



When ß is high, the population of susceptible drops dramatically while infected grows exponentially. The recovery is almost unaffected (except from the beginning) and the outcome for the population is the same: S=0; I=0 and R=0.

When μ is high, it can transform the infected population into recovered at a rate too fast for the ß rate to compensate and, as a result, the final state of the population is I=0 and S+R=N. The percentage of susceptible population at the end of the epidemic will be higher the higher μ is.

As a result, higher values of ß and μ accelerate the evolution of the epidemic process.

Define the end of the epidemic at a certain threshold for the infected population; separate cases above and below the epidemic threshold.

The threshold for the end of the epidemic is where the infected population drops to 20% (R0=0,2), which happens between the days 346 and 347. At the threshold, the state is:

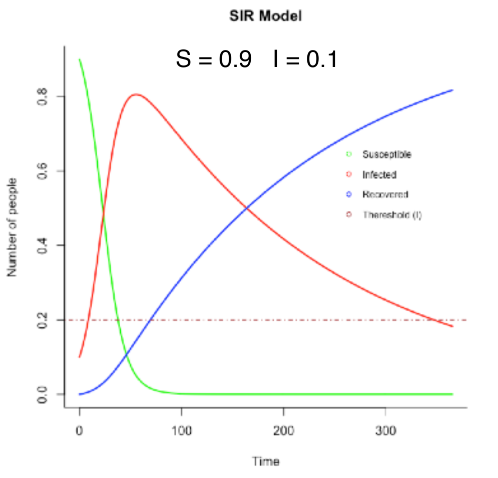
S	I	R
1.032676e-07	0.2009697	0.7990302

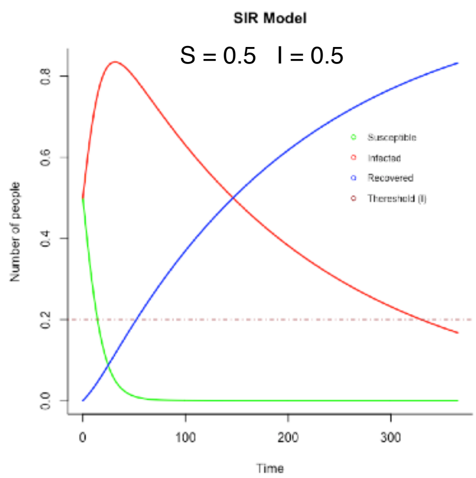
For the parameters ß and μ, the graphs above show that higher values of μ make imposible for the epidemic to happen (there is no peak of the infected population) and therefore the case ß=0.1 μ=0.5 is bellow the epidemic threshold.

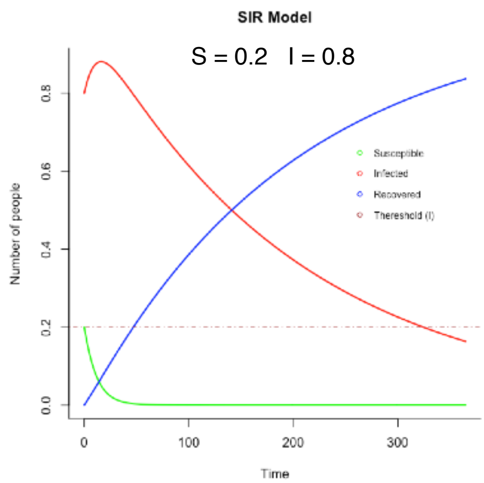
Fix parameters above the epidemic threshold and change the initial conditions. Can you find endemic states? Plot the phase space of the variables (S,I) for several initial conditions. Are these results consistent with the stability analysis?

Endemic states are those in which the infected population is always maintained at a base level, which cannot happen if the parameter μ is fixed above the epidemic threshold. If there are no births, the infected population will always, eventually, recover and an endemic state is not possible. This can be observed in the following graphs:

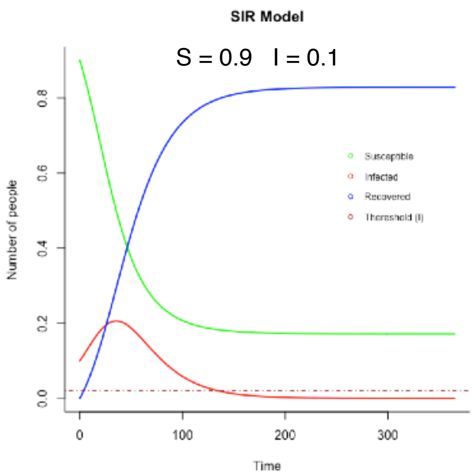
- **ß=0.1 and μ=0.005** → all the population gets infected, which means that eventually S+I=0 and R=N, reaching one of the solutions of the stability analysis.

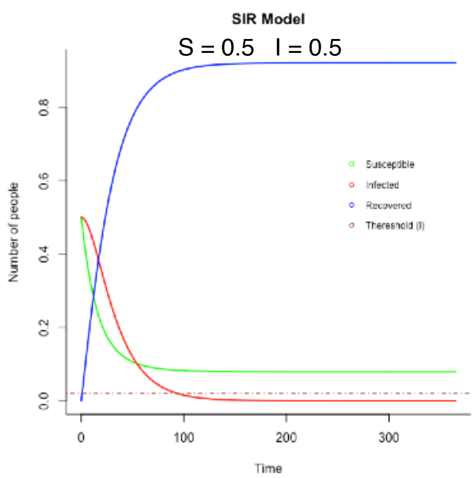


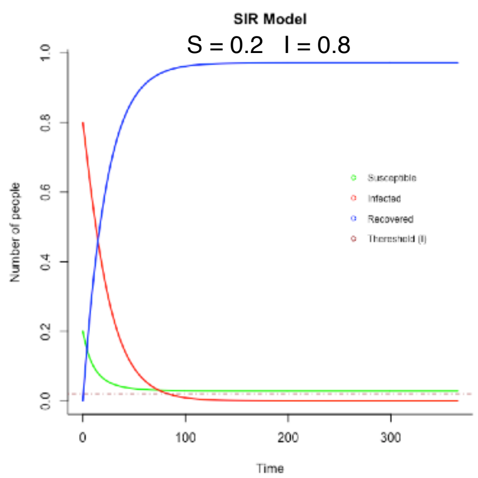




- **ß=0.1 and μ=0.05** → with a lower recovery rate, not all the population gets infected and the final state is I=0 and S+R=N, which is the other posible solution of the stability analysis. As the initial infected population grows, the final susceptible population drops until reaching the previous stability solution S+I=0 and R=N. For these fixed parameters, the model is only above the epidemic threshold when the initial infected population is not too high (lower than 50% of the population).



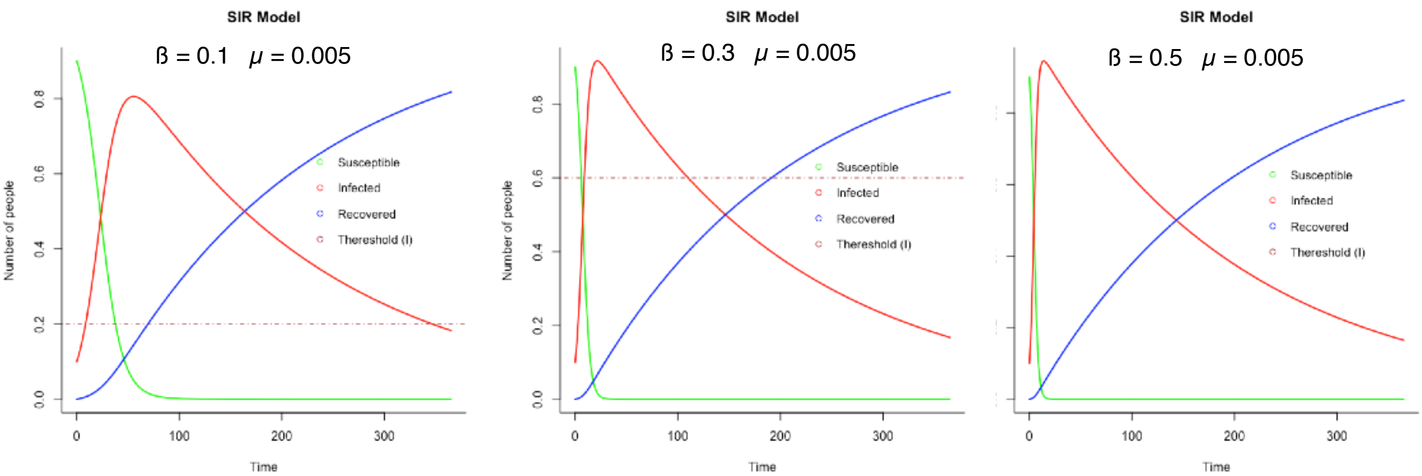




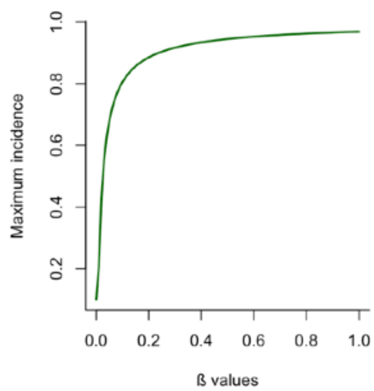
Fix initial conditions and change parameters staying above the epidemic threshold. **What is the maximum incidence (MI) of the disease**? How does the maximum incidence and its time of occurrence change with the parameters?

The incidence of a disease is the probability of occurrence over a specified period of time; therefore a higher incidence of disease will happen with higher values of ß. Lower values of μ will also raise the incidence of the disease because, by slowing down the transformation I → R, the infected population in a given time is bigger and therefore the disease can spread faster.

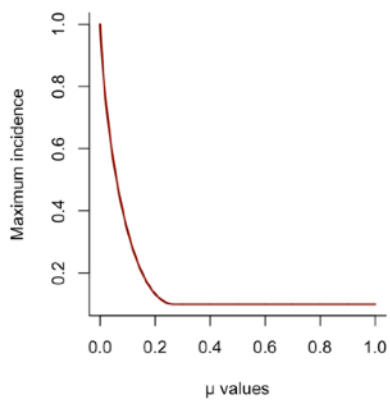
Since μ value is very low, the effect can be observed by increasing ß's value gradually while maintaining μ's value, as shown in the graphs below. This effect can be observed in the graphs of the first exercise as well, where the infected population peak grows when ß→1 and decreases when μ→1.



If we plot the maximum number of infected people when ß changes from 0—1, we see how the curve grows exponentially and reaches a plateau state at around 0.3; after this point ß might alter how fast the maximum incidence is reached, but the number of infected itself stays almost the same:.

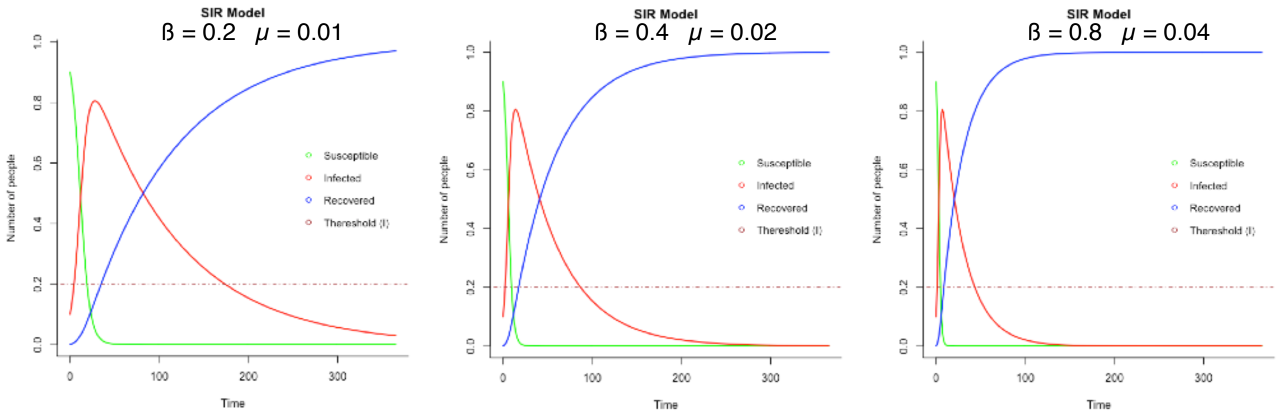


After seeing this result, I decided to see how the maximum incidence would change as μ changes, so I plotted the same as above: the function of MI as μ grows from 0—1. As expected for what the epidemic simulation plots had shown, the value of MI has the oposite trajectory that it did in the MI as a function of ß.



Fix initial conditions and the value of the epidemic threshold, and change ß and μ with this condition. Do you see differences in the trajectories? Give a qualitative interpretation to the role of different parameters.

Since the epidemic threshold has to be stable (R0=0.2), the parameters ß and μ have to grow proportionally. The simulation of the epidemic under these conditions is displayed in the graphs below, and it shows that it changes the speed of the infection but not the incidence, which stays at 0.8.



1. Epidemics E2

World War Z

Given that the incubation time is so short, for the simulation we will just assume that there is no incubation (SIR model again). We will also start by putting the D day at 300 and the following parameters:

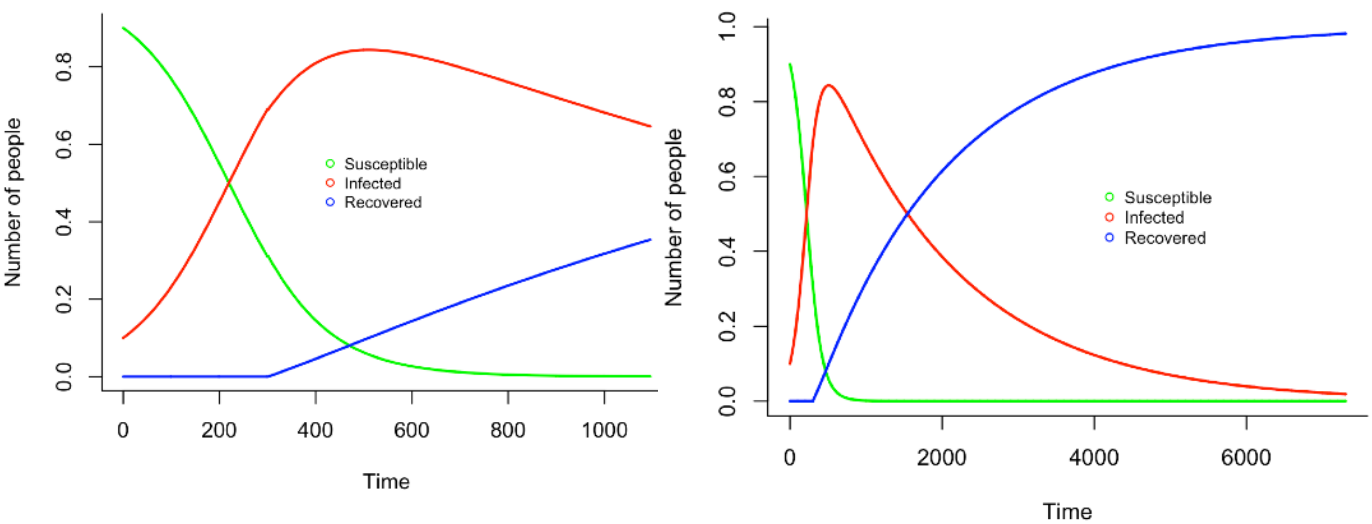
- Infection rate (ß): is 0'01, and stays the same for all the epidemic period
- Vaccination rate: is 0 until D day, and then we can estimate the rate as number of vaccines you can administer per day divided by the total number of people that need the vaccine. We assume that we have a total population of 7 billion people and we can administer 5 hundred thousand vaccines a day:

$$\frac{500000}{7000000} = 0.0007142857$$

Since the vaccination is both a vaccine and a cure, the drug has to be divided between people susceptible of getting infected, and the already infected people. We will give 20% of the vaccines to healthy people and use the other 80% to cure zombies. The main field equations would be:

dS <— -beta * S * I - 0.2 * S * vac # - infected - vaccinated
dI <— beta * S * I - 0.8 * I * vac # infected - recovered - vaccinated/cured
dR <— 0.2 * S * vac + 0.8 * I * vac # recovered + vaccinated/cured

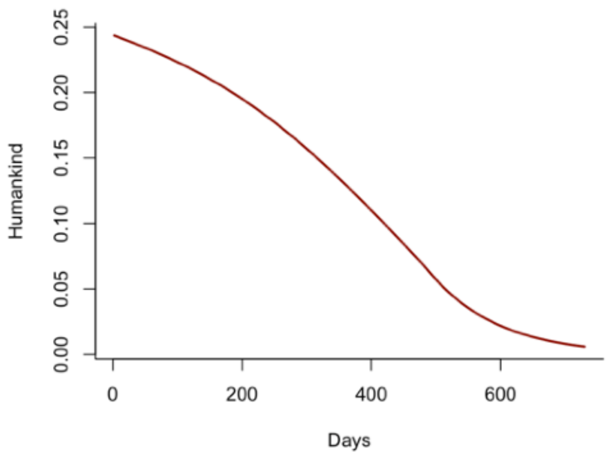
The result of this simulation is plotted in two graphs representing the same scenario, but the one to the left shows a shorter timescale (3 years) so we can see more clearly how the infection affects the population specially at the beginning, when most changes happen. The graph to the right shows a period of 20 years so we can see how the populations stabilize and see the 'final' result of the epidemic (assuming nothing changes over 20 years, which seems unlikely).



We can see clearly how at day 300 there is a sudden change in the trajectories, that is the day the cure is found and they start administering it.

According to this simulation, humankind never goes extinct, because even though the number of susceptible healthy people tends to go to 0, the percentage of recovered grows at a steady rate. According to this simulation, most people will eventually become a zombie and then revert to human. Maybe the delivery of vaccines should be different so to prevent infections more than cure the already infected ones. Although probably the best approach would be to initially reduce the number of zombies (like we did here) and after the number is significantly reduced, prioritize the vaccination of healthy people.

As for how humankind would survive the apocalypse, I calculated what would be the minimum number of non-zombies (S or R) as the days before the cure was found increased, and the result was:



Keeping the original infection parameters, if the cure takes more than 600 days to be found, humankind would go extinct.