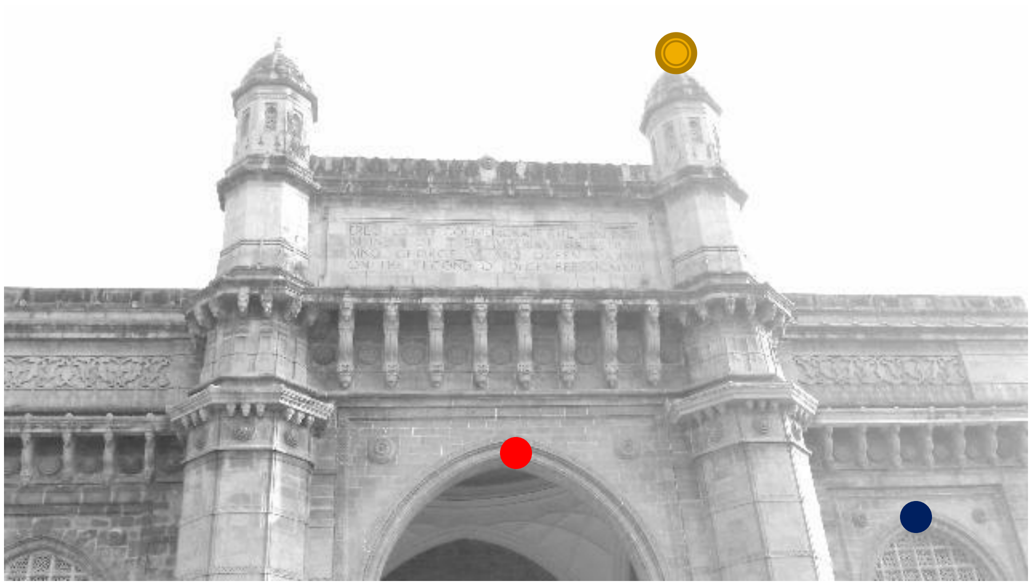
Corner Detection

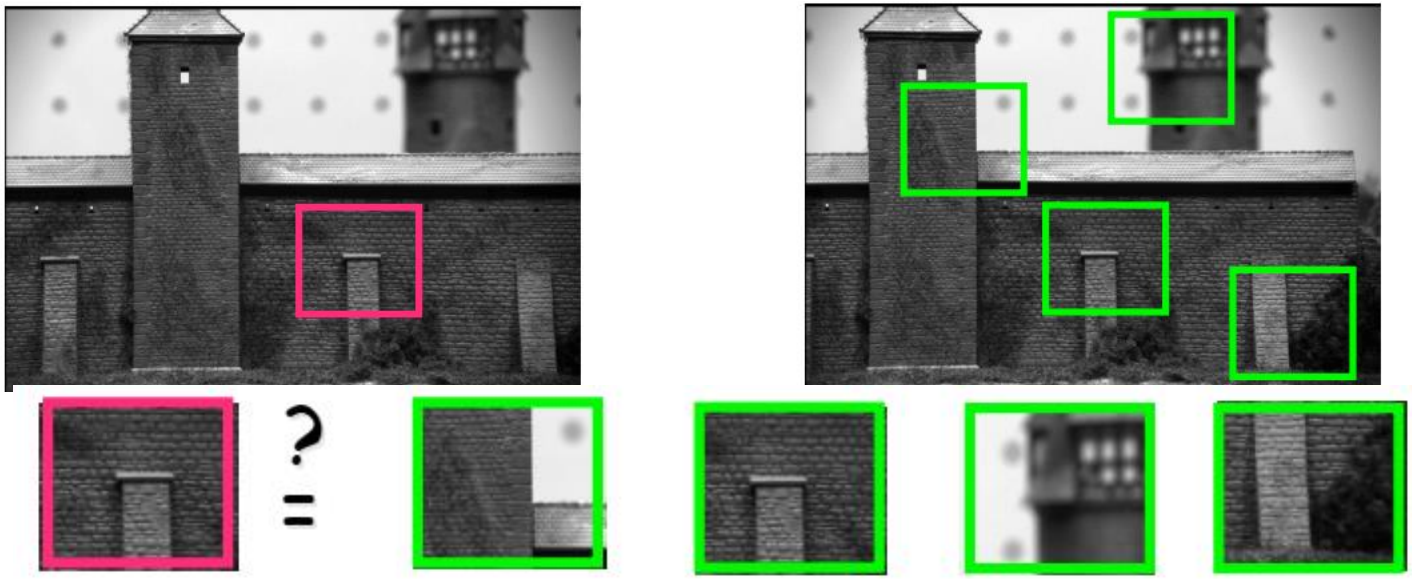
Importance of corners

- Corners are considered salient feature points.
- Useful in many tasks in computer vision and image processing
- Example: corresponding control points for image alignment

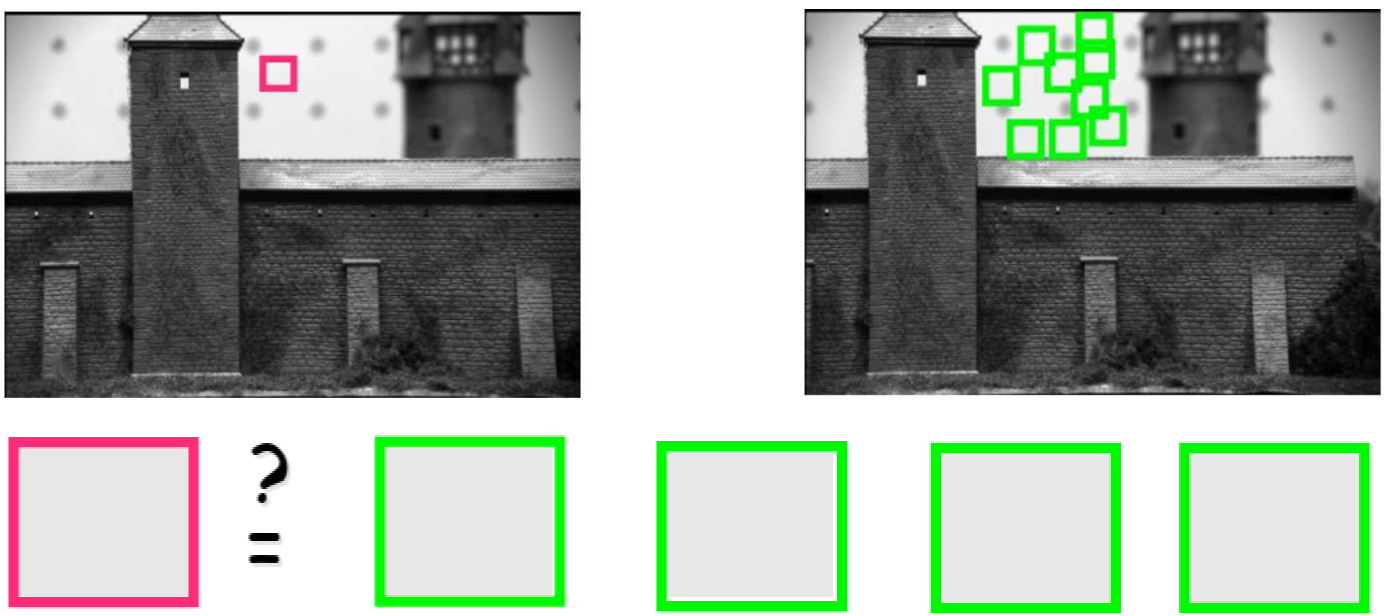


Importance of corners

- In some applications, we need to find matching patches.
- But some patch matches are easier to find than others.
- See next slide for examples.



Good patch for matching



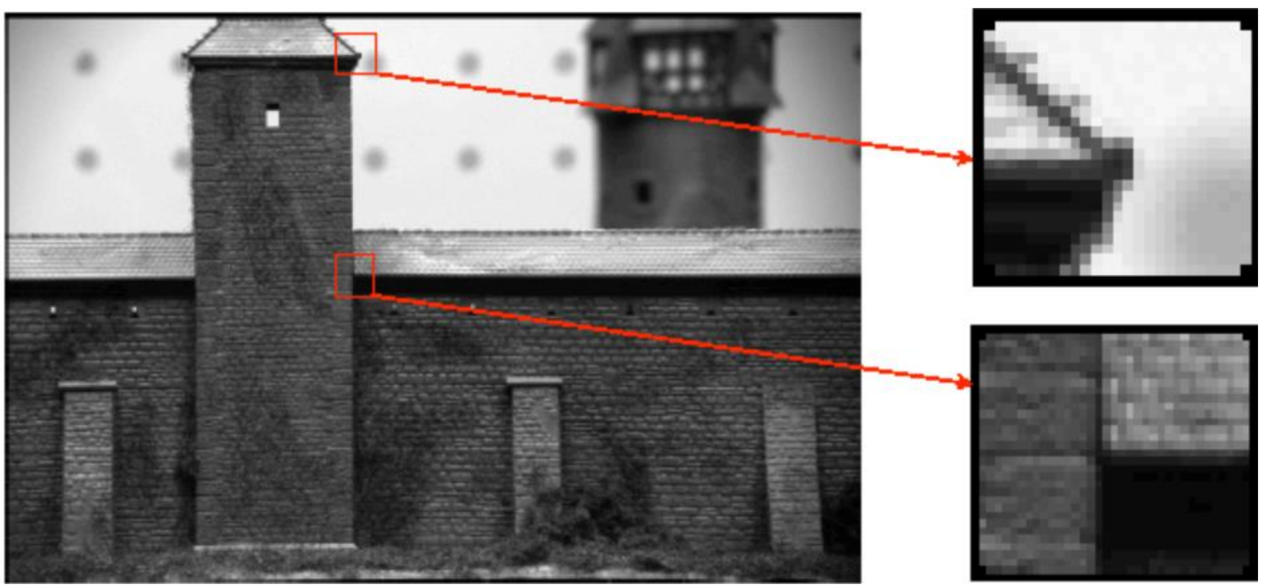
Bad patch for matching

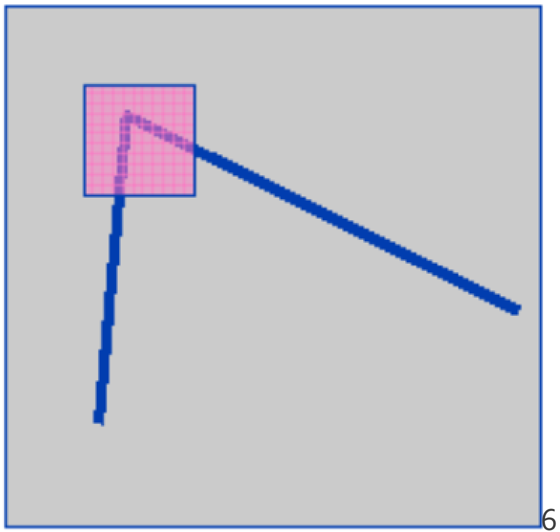
http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf

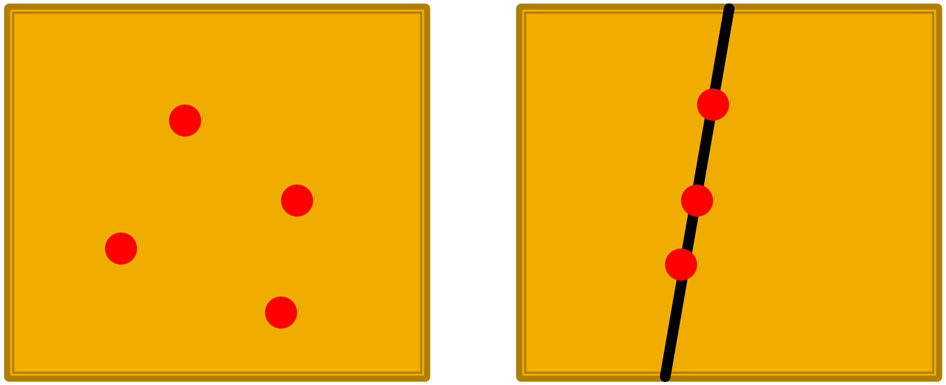
Importance of corners

- Good patches should contain corners – large intensity variations in all directions.
- Shifting the window in any direction yields large appearance change.

http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf







Ambiguity in point matching in flat intensity regions or along edges

Principle behind Harris corner detector

- Consider two patches – one each centered at (x,y) and (x+u,y+v) in two images (**patch** domain: Ω, typically rectangular, like say 7 x7 or 5 x 5) with u,v being small.

- The image have same intensity at physically corresponding locations.

- The sum of squared difference (SSD) between their intensities is given as:

$$SSD = \sum_{(x,y)\in\Omega} (I(x,y) - I(x+u,y+v))^2$$

By first order Taylor series,

$$I(x+u,y+v) \approx I(x,y) + uI_x(x,y) + vI_y(x,y)$$

$$SSD = \sum_{(x,y)\in\Omega} (I(x,y) - I(x+u, y+v))^2$$

By first order Taylor series,

$$I(x+u, y+v) \approx I(x,y) + uI_x(x,y) + vI_y(x,y)$$

$$\therefore SSD = \sum_{(x,y)\in\Omega} (uI_x(x,y) + vI_y(x,y))^2$$

$$= \begin{pmatrix} u & v \end{pmatrix} \boxed{\begin{pmatrix} \sum\limits_{x,y)\in\Omega} I_x^2(x,y) & \sum\limits_{x,y)\in\Omega} I_x(x,y) I_y(x,y) \\ \sum\limits_{x,y)\in\Omega} I_x(x,y) I_y(x,y) & \sum\limits_{x,y)\in\Omega} I_y^2(x,y) \end{pmatrix}} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$= \begin{pmatrix} u & v \end{pmatrix} A \begin{pmatrix} u \\ v \end{pmatrix}$$

Structure tensor matrix (size 2 x 2)

Principle behind Harris corner detector

- The local **A** matrix on the previous slide is called the structure tensor.

- **A** carries information about local image geometry.

$$A = \lambda_1 u_1 u_1^t + \lambda_2 u_2 u_2^t ; \text{eigenvals} : \lambda_1, \lambda_2 ; \text{eigenvecs} : u_1, u_2$$

- It is always positive semi-definite, i.e. its two eigenvalues are always non-negative.

- In locally flat regions, **A** will be close to a zero matrix and hence both its eigenvalues will be close to 0.

Proof that the structure tensor matrix is positive semi-definite: Method 1

$$A = \begin{pmatrix} \sum_{(x,y)\in\Omega} I_x^2(x,y) & \sum_{(x,y)\in\Omega} I_x(x,y)I_y(x,y) \\ \sum_{(x,y)\in\Omega} I_x(x,y)I_y(x,y) & \sum_{(x,y)\in\Omega} I_y^2(x,y) \end{pmatrix}$$

$$= \begin{pmatrix} I_x(x_1,y_1) & I_x(x_2,y_2) & . & . & I_x(x_N,y_N) \\ I_y(x_1,y_1) & I_y(x_2,y_2) & . & . & I_y(x_N,y_N) \end{pmatrix} \begin{pmatrix} I_x(x_1,y_1) & I_y(x_1,y_1) \\ I_x(x_2,y_2) & I_y(x_2,y_2) \\ . & . \\ . & . \\ I_x(x_N,y_N) & I_y(x_N,y_N) \end{pmatrix}$$

$$= ZZ^T$$

Any matrix that can be written in the form A = ZZ^T is always positive semi-definite (quoting a result from linear algebra)

Proof that the structure tensor matrix is positive semi-definite: Method 2

$$A = \begin{pmatrix} \sum_{(x,y)\in\Omega} I_x^2(x,y) & \sum_{(x,y)\in\Omega} I_x(x,y) I_y(x,y) \\ \sum_{(x,y)\in\Omega} I_x(x,y) I_y(x,y) & \sum_{(x,y)\in\Omega} I_y^2(x,y) \end{pmatrix}$$

$$trace(A) = \lambda_1 + \lambda_2 = \sum_{(x,y)\in\Omega} I_x^2(x,y) + \sum_{(x,y)\in\Omega} I_y^2(x,y)$$

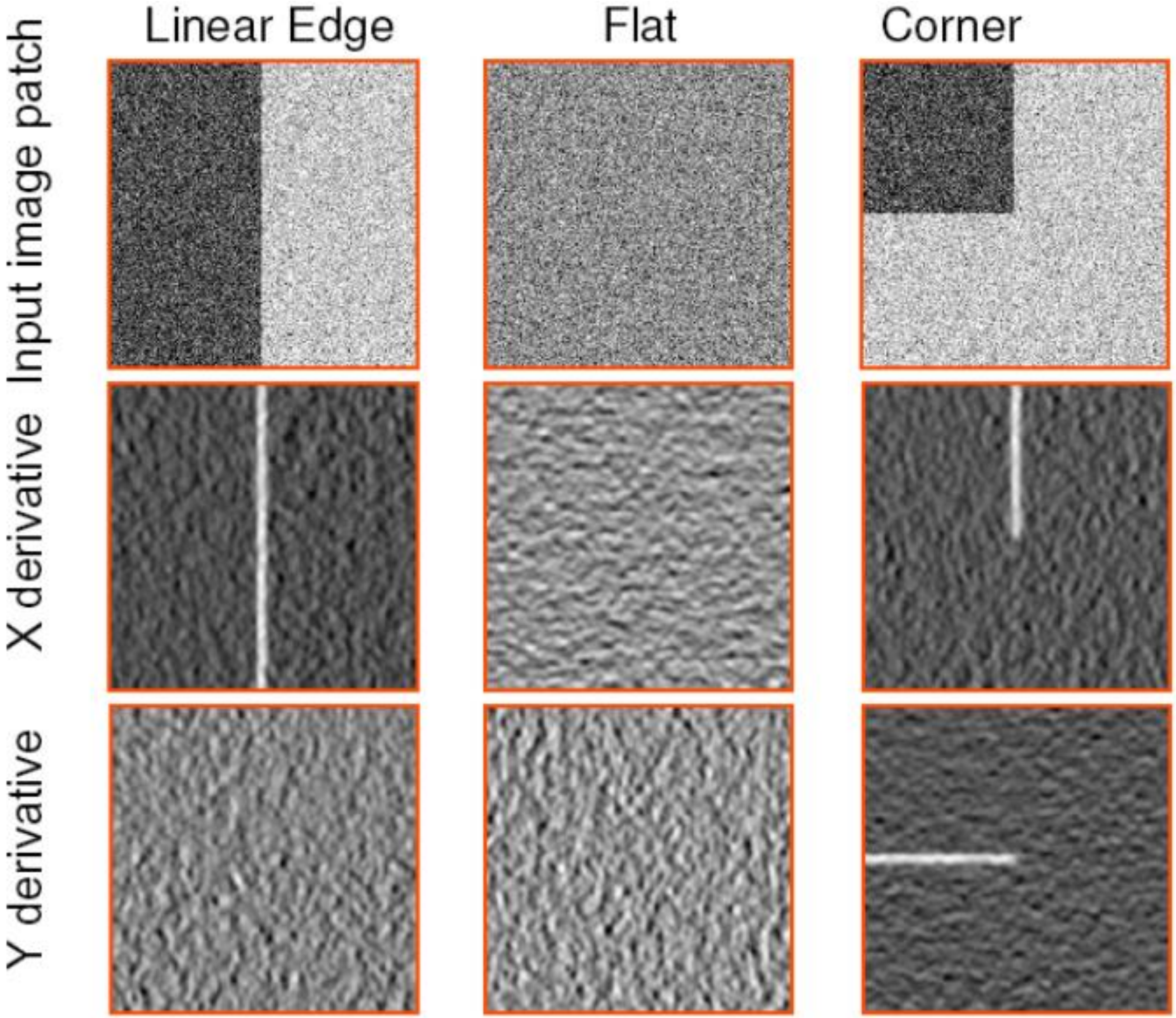
$trace(A)$ is clearly non-negative as I_x^2 and I_y^2 are both non-negative

$$|A| = \sum_{(x,y)\in\Omega} I_x^2(x,y) \sum_{(x,y)\in\Omega} I_y^2(x,y) - \left(\sum_{(x,y)\in\Omega} I_x(x,y) I_y(x,y) \right)^2$$

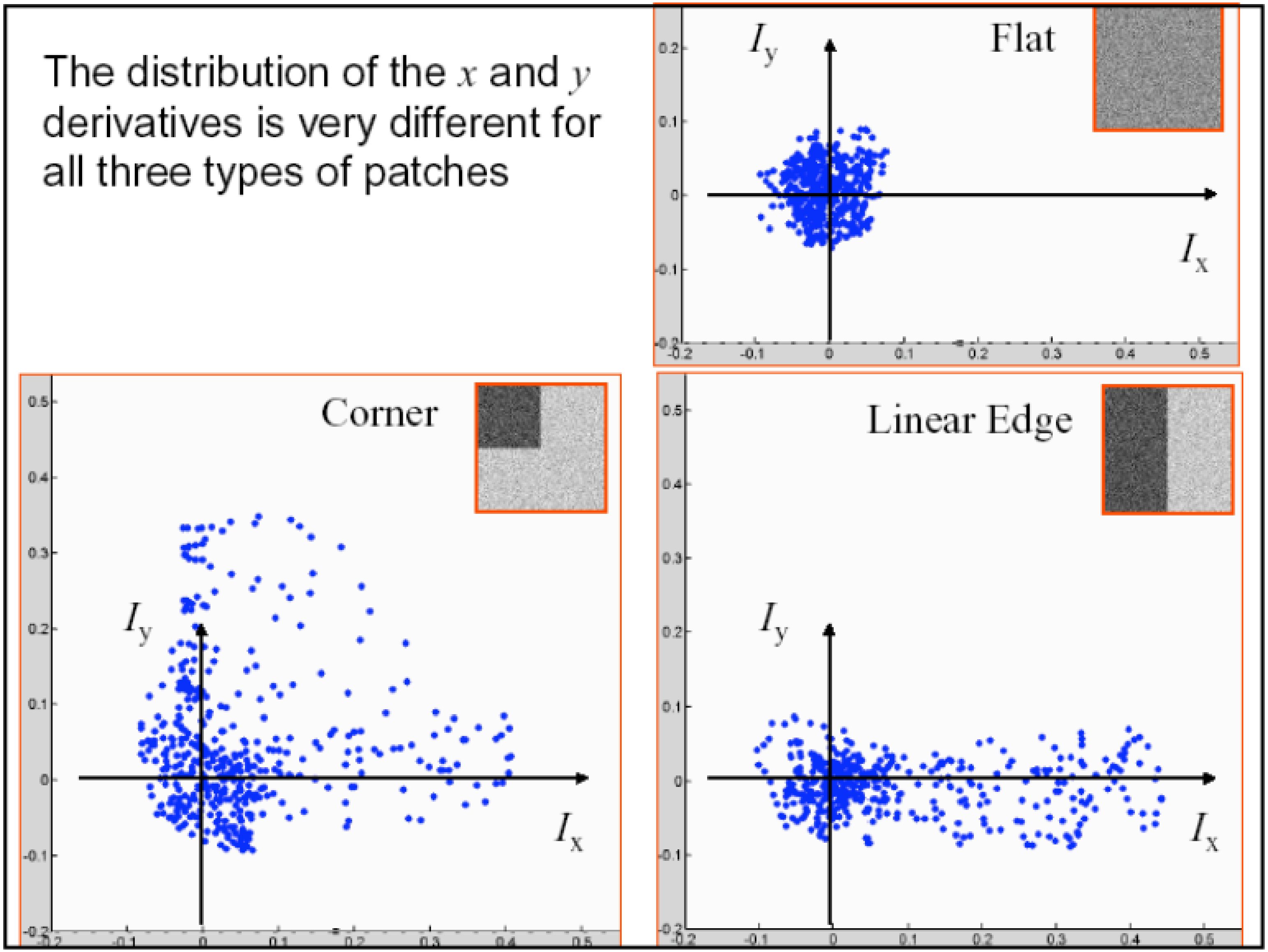
$|A| \geq 0$ by Cauchy-Schwarz inequality (the first term is $\|a\|^2 \|b\|^2$, and the second term is $(a \bullet b)^2$ for N element vectors a and b, respectively containing the I_x and I_y values)

Principle behind Harris corner detector

- At a point lying on an edge, only one of the eigenvalues is large (corresponding to the eigenvector that points across the edge) and the other is close to 0 (corresponding to the eigenvector that points along the edge)
- At a corner point, both eigenvectors will be large.
- We would like the SSD to be large for **all** non-zero shifts (u,v) so that we can allow for maximum discriminability and easier point matching.



http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf



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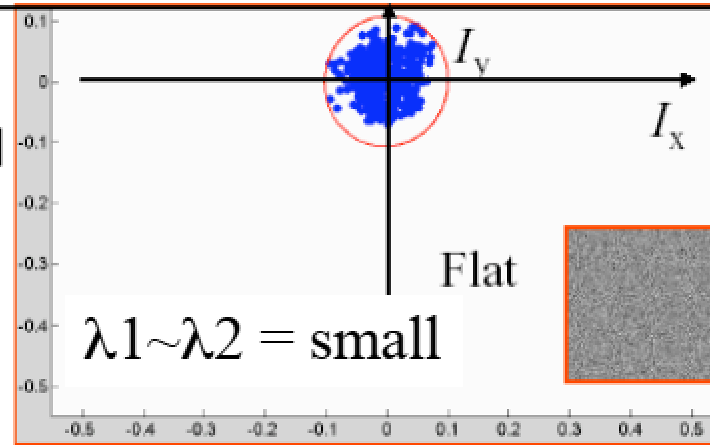
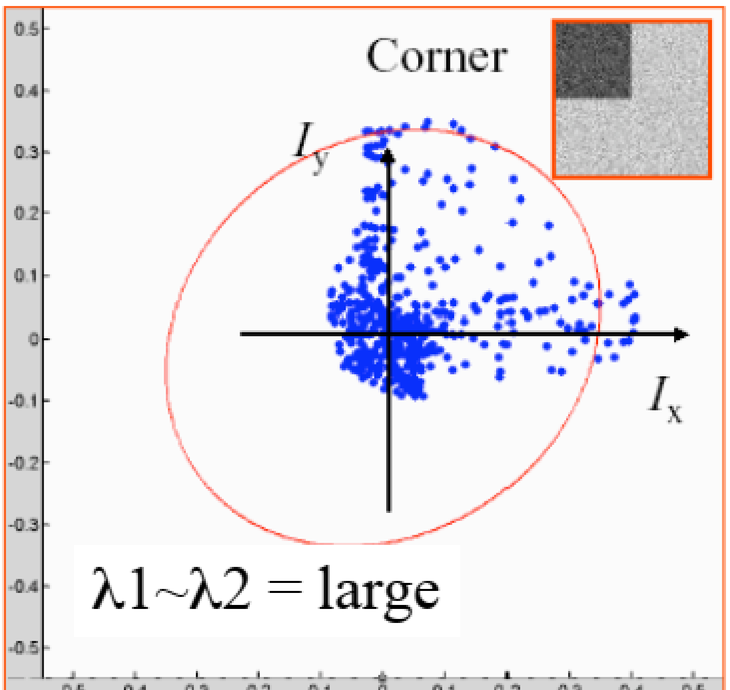
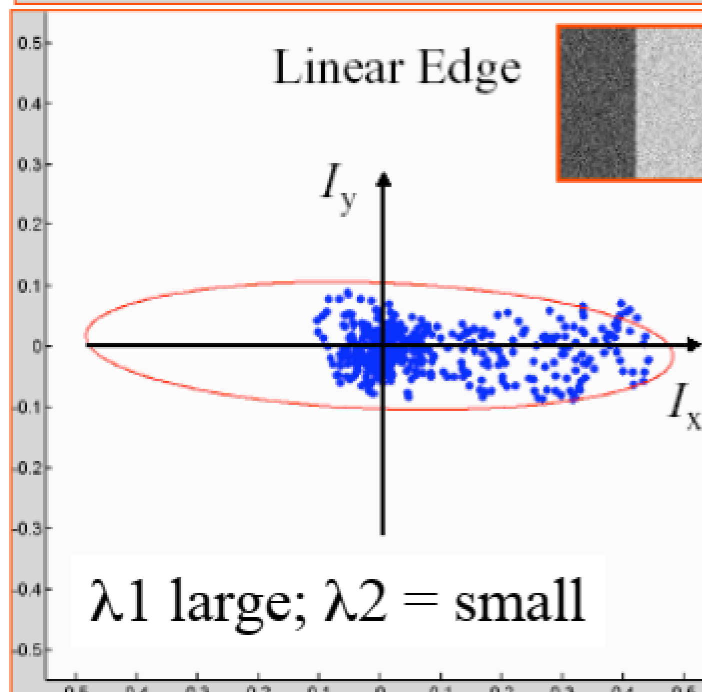
The distribution of *x* and *y* derivatives can be characterized by the shape and size of the principal component ellipse







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“Corner”-ness measure

- We would like both eigenvalues of this matrix to be large.

- Corner response or “corner”ness measure: $R_H = \det(A) - k\,(\mathrm{trace}(A))^2$, k between 0.04 to 0.06
- Does not require explicit eigenvalue/eigenvector computation:

$$|A| = \lambda_1 \lambda_2 = A_{11}A_{22} - A_{12}A_{21};$$

$$trace(A) = \lambda_1 + \lambda_2 = A_{11} + A_{22}$$

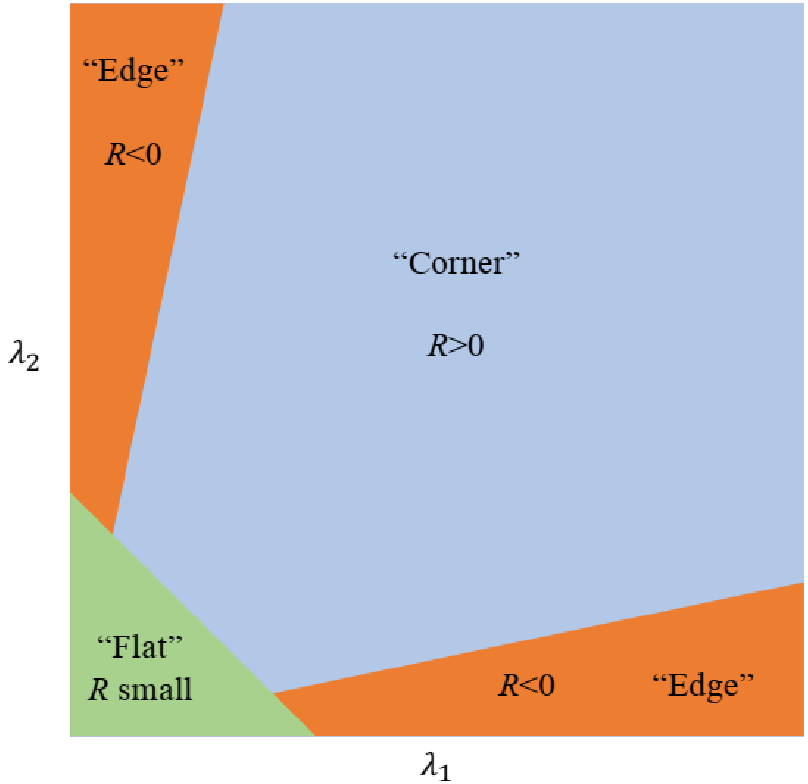
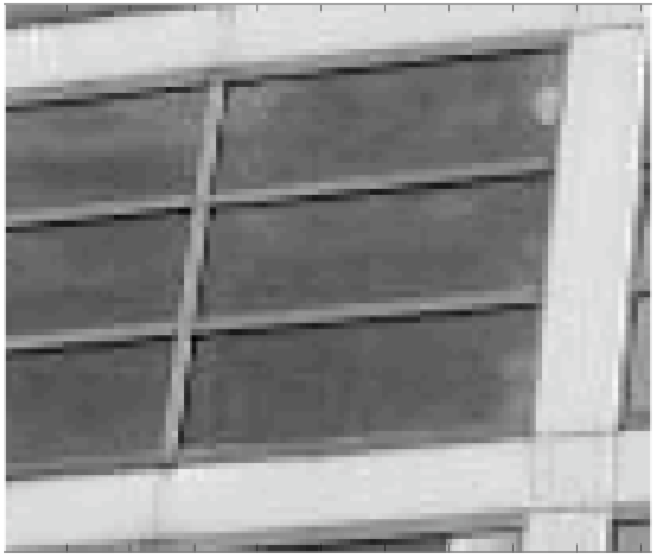
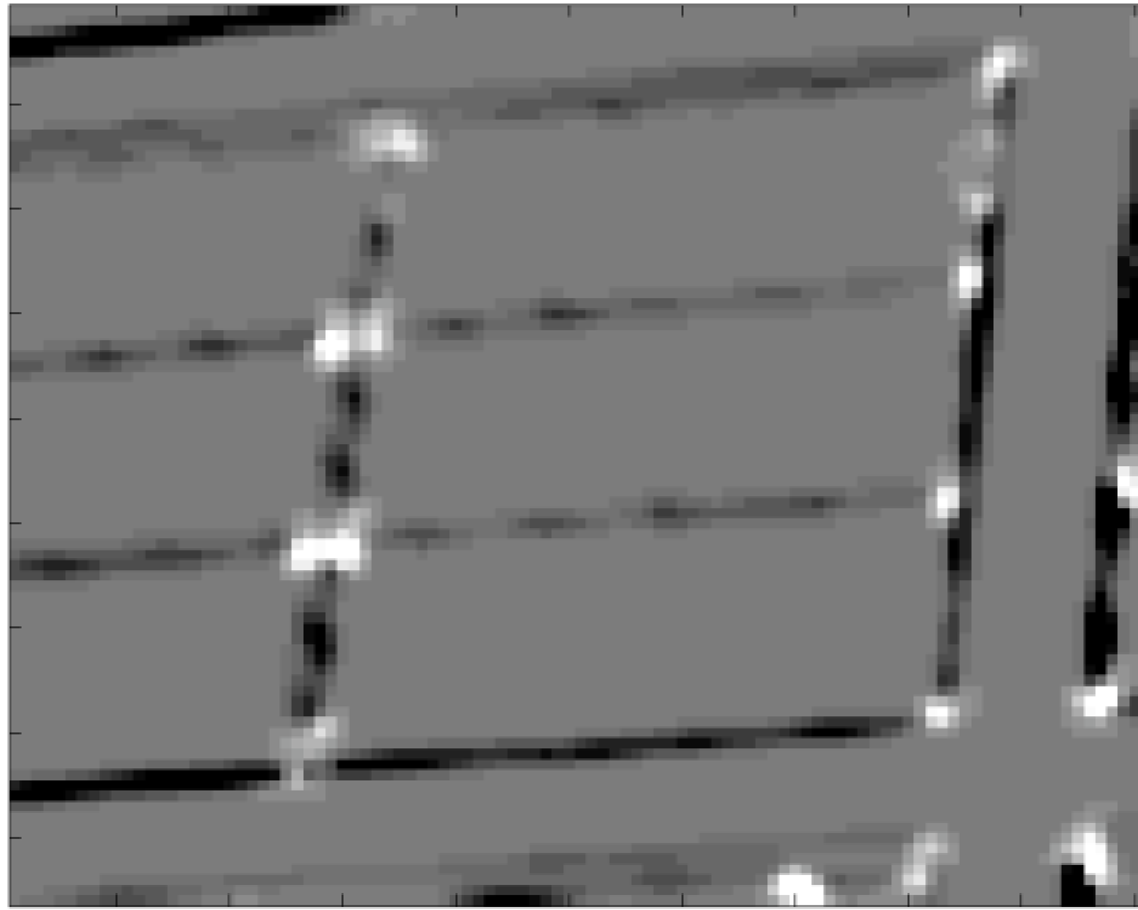


Figure 5: Harris measure. If both eigenvalues are large, then R is large and positive, providing a clue to detect a corner. If both eigenvalues are small, then R is also small and positive, which means that the point is probably part of a homogeneous region. Finally, if one of the eigenvalues is much larger than the other, R becomes negative, and the point belongs to an edge.

http://www.ipol.im/pub/art/2018/229/article_lr.pdf



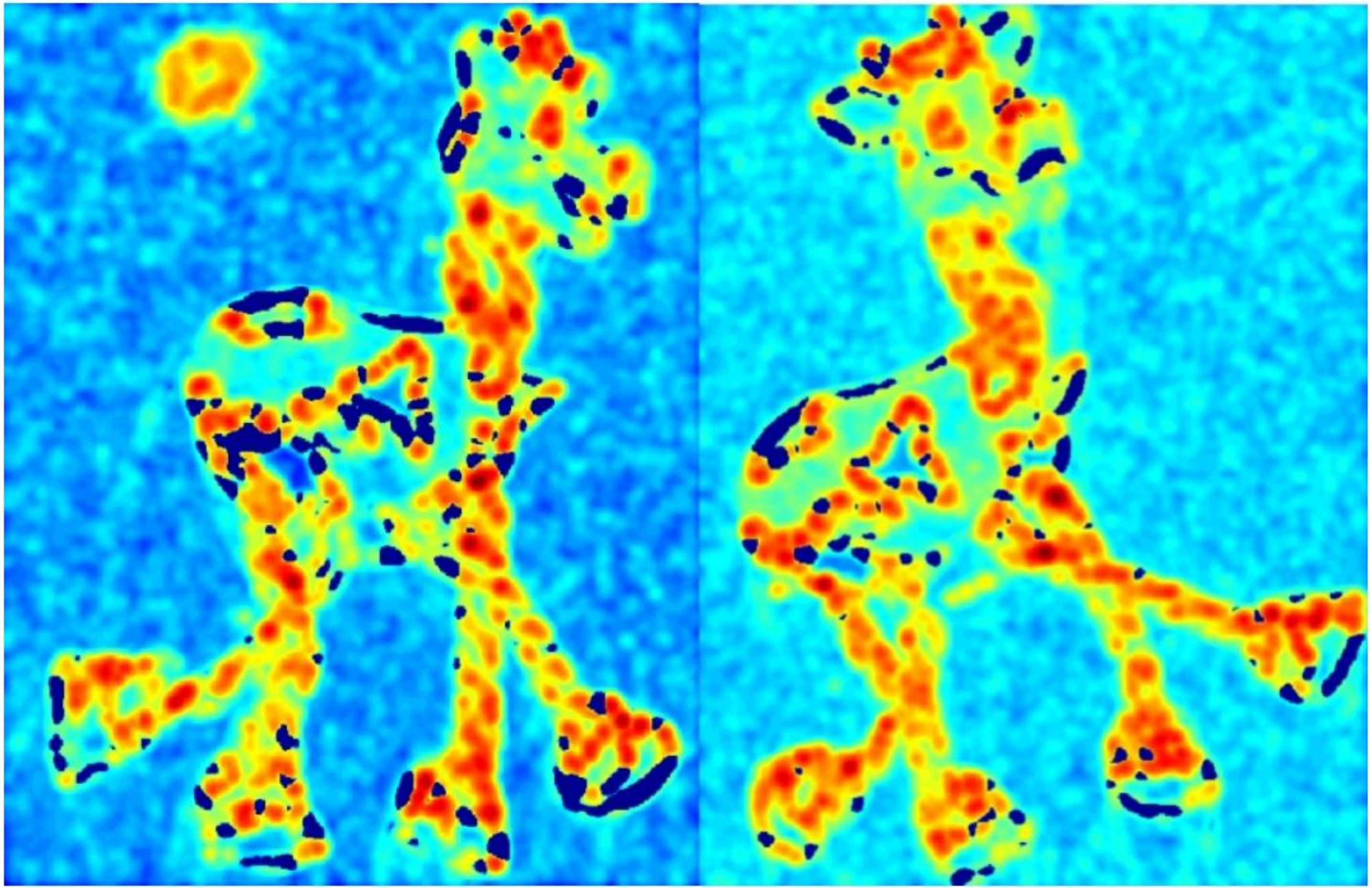


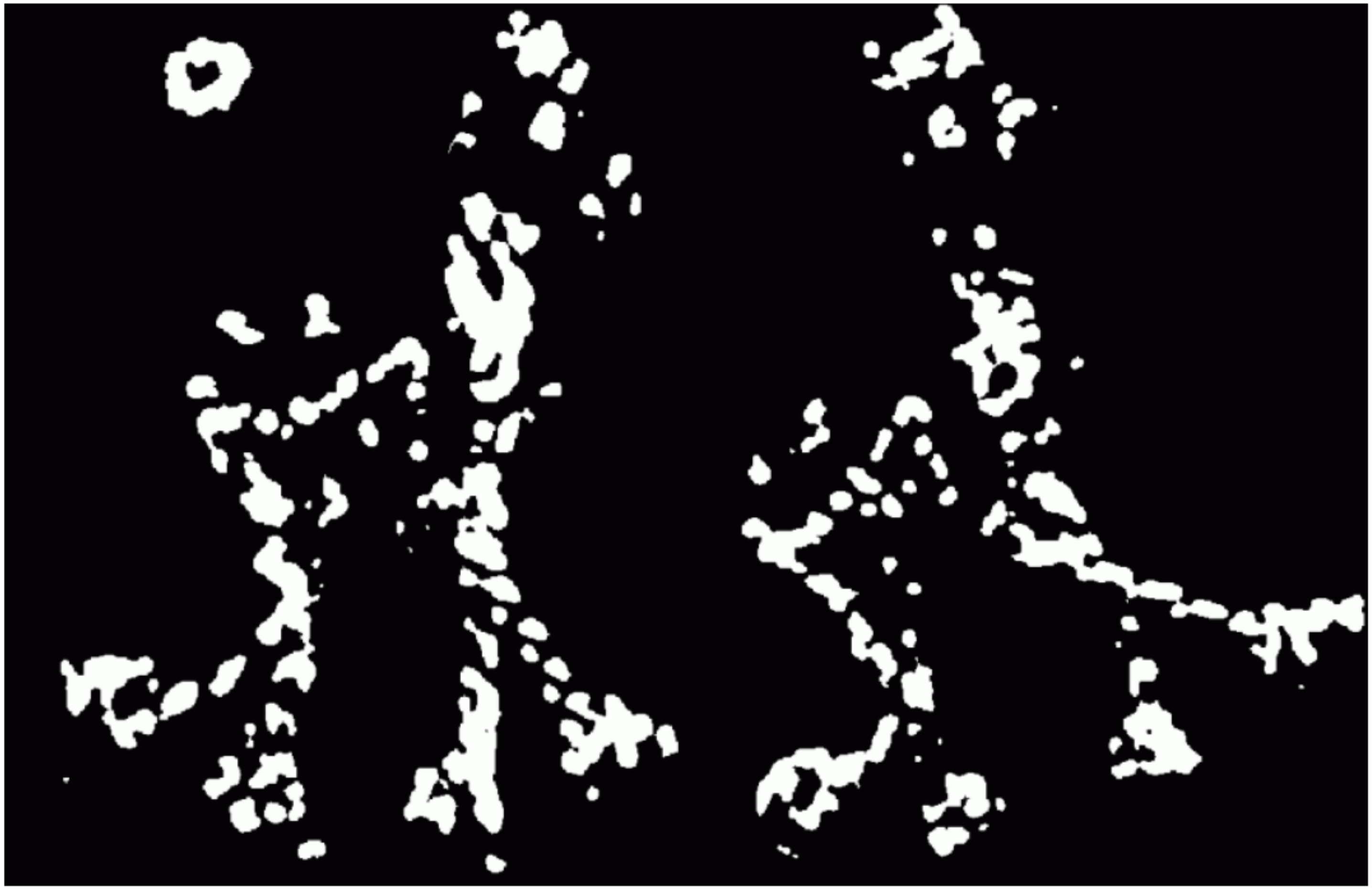
Harris R score.

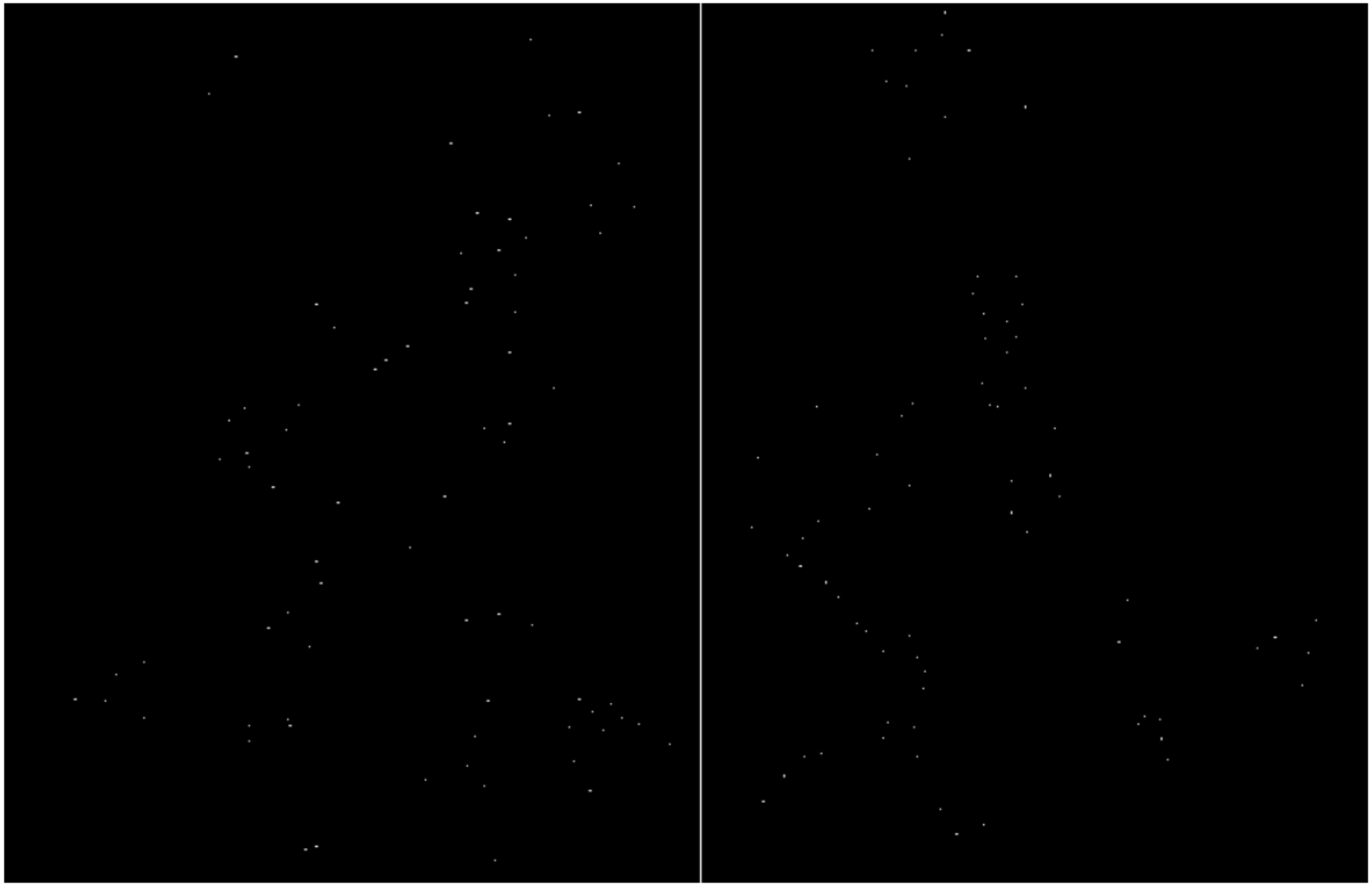
Ix, Iy computed using Sobel operator
Windowing function w = Gaussian, sigma=1

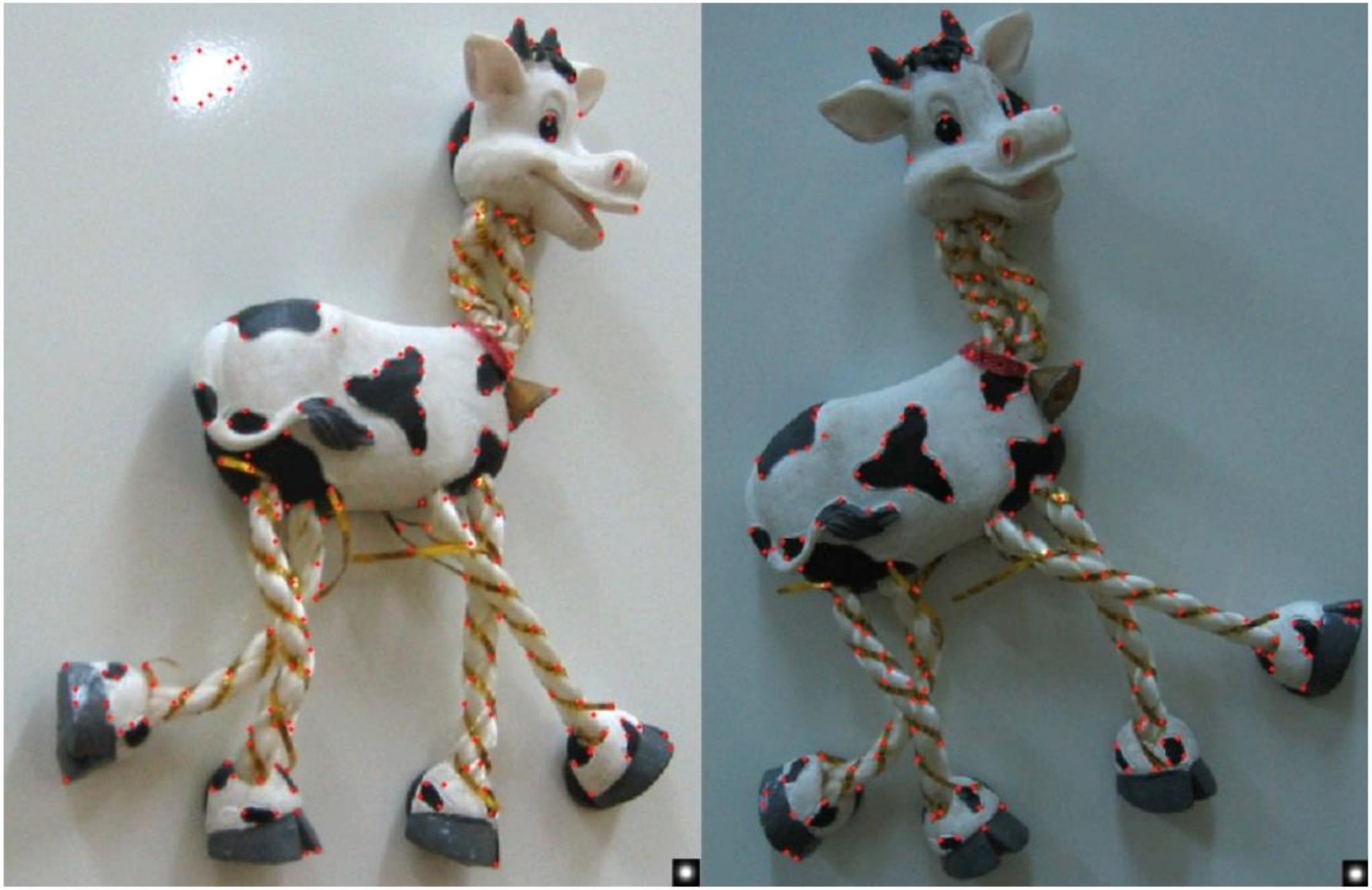
Computing corners

- Smooth image I to compute various derivatives
- Choose a window size (say 7 x 7) and compute the structure tensor at each pixel, given this window size
- Compute the corner-ness measure for each pixel
- Perform thresholding or non-maximal suppression to create a corners map.









Invariance

- The detected corners are invariant under affine intensity change, i.e. image J replaced by aJ + b for scalars a,b.
- The corners are also invariant to rotation and translation.

Other measures of corner-ness

- Based on just the minimum eigenvalue – declared a corner if the minimum eigenvalue exceeds a threshold

- Harmonic mean of eigenvalues

$$R_{HM} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{det(M)}{trace(M)}$$