Home problem 2.1
Humanoid robots
TIF160

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Contents

1 Hubert kinematics

Task: Calculate forward kinematics for the humanoid robot Hubert shown in figure 1. Using the lengths in table 1, the kinematic model should calculate the coordinates of point P (frame 3) in the coordinate system of frame 0, given the angles of the joints as input.

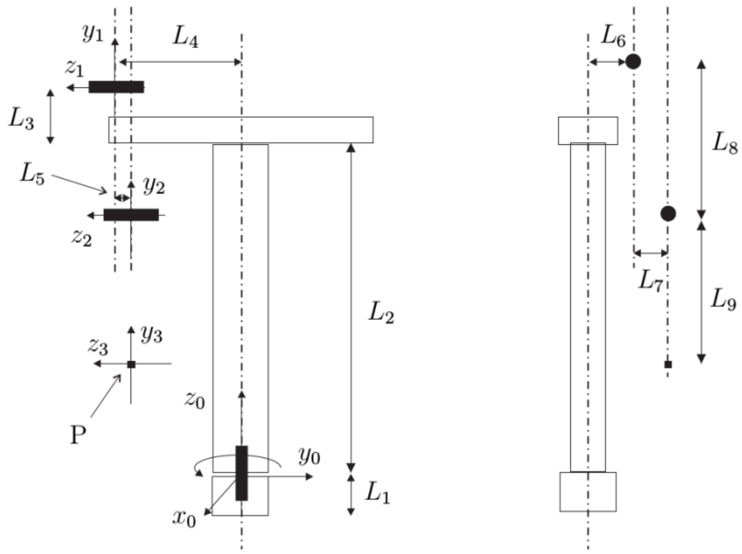

Figure 1: Drawing of Hubert with the positions of the joints, different lengths and the coordinate systems. *Source: Home problem description*

L_1: 0.055	L_4: 0.108	L_7: 0.015
L_2: 0.315	L_5: 0.005	L_8: 0.088
L_3: 0.045	L_6: 0.034	L_9: 0.204

Table 1: Values for lengths in figure 1. *Source: Home problem description*

1.1 Solution

The length L_1 is not considered in this part of the home problem since frame 0 is situated on top of L_1.

Frame 0 to frame 1

The transformation matrix is calculated by first deriving the transformations for the sub-frames a and b.

1. Frame $0 \to a$: Rotation around z_0

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_a \\ y_a \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} x_a cos\theta_1 - y_a sin\theta_1 \\ x_a sin\theta_1 + y_a cos\theta_1 \\ z_a \\ 1 \end{pmatrix} \tag{1}$$

2. Frame $a \to b$: Translation of all axes

$$\begin{pmatrix} x_a \\ y_a \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & L_6 \\ 0 & 1 & 0 & -L_4 \\ 0 & 0 & 1 & L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \\ 1 \end{pmatrix} = \begin{pmatrix} x_b + L_6 \\ y_b - L_4 \\ z_b + L_2 + L_3 \\ 1 \end{pmatrix} \tag{2}$$

3. frame Frame $b \to 1$: Rotate the coordinate system by $\pi/2$ around x_0

$$\begin{pmatrix} x_b \\ y_b \\ z_b \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 \\ -z_1 \\ y_1 \\ 1 \end{pmatrix} \tag{3}$$

Combining eq 2 and 3 we get

$$\begin{pmatrix} x_a \\ y_a \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} x_b + L_6 \\ y_b - L_4 \\ z_b + L_2 + L_3 \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 + L_6 \\ -(z_1 + L_4) \\ y_1 + L_2 + L_3 \\ 1 \end{pmatrix} \tag{4}$$

Then by putting eq 4 in 1 we get the transformation matrix for frame 0 to 1

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix} = \begin{pmatrix} x_a cos\theta_1 - y_a sin\theta_1 \\ x_a sin\theta_1 + y_a cos\theta_1 \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} (x_1 + L_6)cos\theta_1 + (z_1 + L_4)sin\theta_1 \\ (x_1 + L_6)sin\theta_1 - (z_1 + L_4)cos\theta_1 \\ y_1 + L_2 + L_3 \\ 1 \end{pmatrix} \tag{5}$$

Frame 1 to frame 2

Similarly as above, we will use a sub-frame called c.

1. Frame $1 \rightarrow c$: Rotation around z_1

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_2 & -sin\theta_2 & 0 & 0 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} x_c cos\theta_2 - y_c sin\theta_2 \\ x_c sin\theta_2 + y_c cos\theta_2 \\ z_c \\ 1 \end{pmatrix} \tag{6}$$

2. Frame $c \rightarrow 2$: Translation of all axes

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & L_7 \\ 0 & 1 & 0 & -L_8 \\ 0 & 0 & 1 & -L_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} = \begin{pmatrix} x_2 + L_7 \\ y_2 - L_8 \\ z_2 - L_5 \\ 1 \end{pmatrix} \tag{7}$$

Combining eq 6 and 7 we get the transformation matrix for frame 1 to 2

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} x_c cos\theta_2 - y_c sin\theta_2 \\ x_c sin\theta_2 + y_c cos\theta_2 \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} (x_2 + L_7)cos\theta_2 - (y_2 - L_8)sin\theta_2 \\ (x_2 + L_7)sin\theta_2 + (y_2 - L_8)cos\theta_2 \\ z_2 - L_5 \\ 1 \end{pmatrix} \tag{8}$$

Frame 2 to frame 3

Lastly we will use a sub-frame called d to calculate the last transformation matrix.

1. Frame $2 \rightarrow d$: Rotation around z_2

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_3 & -sin\theta_3 & 0 & 0 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d \\ y_d \\ z_d \\ 1 \end{pmatrix} = \begin{pmatrix} x_d cos\theta_3 - y_d sin\theta_3 \\ x_d sin\theta_3 + y_d cos\theta_3 \\ z_d \\ 1 \end{pmatrix} \tag{9}$$

2. Frame $d \rightarrow 3$: Translation along y_d

$$\begin{pmatrix} x_d \\ y_d \\ z_d \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -L_9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{pmatrix} = \begin{pmatrix} x_3 \\ y_3 - L_9 \\ z_3 \\ 1 \end{pmatrix} \tag{10}$$

Combining eq 9 and 10 we get the transformation matrix for frame 2 to 3

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} = \begin{pmatrix} x_d cos\theta_3 - y_d sin\theta_3 \\ x_d sin\theta_3 + y_d cos\theta_3 \\ z_d \\ 1 \end{pmatrix} = \begin{pmatrix} x_3 cos\theta_3 - (y_3 - L_9)sin\theta_3 \\ x_3 sin\theta_3 + (y_3 - L_9)cos\theta_3 \\ z_3 \\ 1 \end{pmatrix} \tag{11}$$

Resulting equation

By factor out the coordinates of each transformation matrix given by eq 5, 8 and 11, two matrix multiplications can be applied to get the full transformation from frame 0 to 3.

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_1 & 0 & sin\theta_1 & L_6cos\theta_1 + L_4sin\theta_1 \\ sin\theta_1 & 0 & -cos\theta_1 & L_6sin\theta_1 - L_4cos\theta_1 \\ 0 & 1 & 0 & L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_2 & -sin\theta_2 & 0 & L_7cos\theta_2 + L_8sin\theta_2 \\ sin\theta_2 & cos\theta_2 & 0 & L_7sin\theta_2 - L_8cos\theta_2 \\ 0 & 0 & 1 & -L_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} = \begin{pmatrix} cos\theta_3 & -sin\theta_3 & 0 & L_9sin\theta_3 \\ sin\theta_3 & cos\theta_3 & 0 & -L_9cos\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{pmatrix} \tag{14}$$

1.2 Matlab code

To test the derived kinematics model, a program a Matlab-script was created, "Kinematics_model_HP_2_1.m", together with the function-script "forwKinematicsModel.m"(found in the hand-in files).

The main script uses a function "checkData" that takes an angle-triplet, lengths L and a manual answer of the position in order to assert if the model in "forwKinematicsModel.m" calculates the position correctly. The model uses the derived matrix-multiplication given in eq. 12, 13 and 14. The target point in frame 3 is always located at the origin and is therefore set to [0,0,0,1].

The test function testes 5 sets of angles given in table 2 where the resulting position is easy to calculate by addition of lengths.

After the manual checks, the positions for $\theta = [\pi/4, \pi/3, \pi/6]$, which is given in the assignment, is calculated in the same script with the same model. The answer given is $P_{frame0} = [0.3003, 0.1547, 0.3290]$.

Test	**angles** (θ_1, θ_2, θ_3)	**x**	**y**	**z**
1	$0, 0, 0$	$L_6 + L_7$	$-L_4 + L_5$	$L_2 + L_3 - L_8 - L_9$
2	$\pi, 0, 0$	$-L_6 - L_7$	$L_4 - L_5$	$L_2 + L_3 - L_8 - L_9$
3	$0, \pi, 0$	$L_6 - L_7$	$-L_4 + L_5$	$L_2 + L_3 + L_8 + L_9$
4	$0, 0, \frac{\pi}{2}$	$L_6 + L_7 + L_9$	$-L_4 + L_5$	$L_2 + L_3 - L_8$
5	$\frac{\pi}{2}, 0, 0$	$L_4 - L_5$	$L_6 + L_7$	$L_2 + L_3 - L_8 - L_9$

Table 2: Positions of P in frame 0 at different θ