

Question 1)

$$a = 6 = (110)_2 = d_2 d_1 d_0, B = (5, 9)$$

We must compute  $aB$  :

step

$$\#0 \quad p = 1p \quad \rightarrow \text{initial setting} \quad p = (5, 9)$$

$$d_1 = 1$$

$$\#1a \quad p + p = 2p = (10)_2 p \quad \rightarrow \text{double}$$

$$S = (3 * 5^2 + 1) * 18^{-1} \bmod 11 = 76 * 18^9 \bmod 11 = 10 * 7^9 \bmod 11 = 10 * 5^4 * 7 \bmod 11 = 9 * 10 * 7 \bmod 11 = 3$$

$$X' = S^2 - X_1 - X_2 \bmod p = 9 - 5 - 5 \bmod 11 = 10$$

$$Y' = S(X_1 - X') - y_1 \bmod 11 = 3(5 - 10) - 9 \bmod 11 = 9 \rightarrow 2p = (10, 9)$$

$$\#2b \quad 2p + p = 3p = (11)_2 p \quad \rightarrow \text{add}$$

$$S = (9 - 9) * (5 - 10)^{-1} \bmod 11 = 0$$

$$X' = S^2 - X_1 - X_2 \bmod p = 0 - 5 - 10 \bmod 11 = 7$$

$$Y' = S(X_1 - X') - y_1 \bmod 11 = 0(5 - 7) - 9 \bmod 11 = 2 \rightarrow 3p = (7, 2)$$

$$d_0 = 1$$

$$\#3a \quad 3p + 3p = 6p = (110)_2 p \rightarrow \text{double}$$

$$S = (3 * 7^2 + 1) * 4^{-1} \bmod 11 = 5 * 4^9 \bmod 11 = 5 * 5^4 * 4 \bmod 11 = 5 * 9 * 4 \bmod 11 = 4$$

$$X' = S^2 - X_1 - X_2 \bmod p = 16 - 7 - 7 \bmod 11 = 2$$

$$Y' = S(X_1 - X') - y_1 \bmod 11 = 4(7 - 2) - 2 \bmod 11 = 7 \rightarrow 6p = (2, 7)$$

$$\#3b \quad \rightarrow \text{No add}$$

Session key = (2, 7)

Question 2)

$$y^2 = x^3 + 2x + 2 \pmod{17}$$

2.1

Elliptic curve equation :  $y^2 = x^3 + ax + b \pmod{p} \rightarrow a = 2, b = 2, p = 17$

$$4a^3 + 27b^2 \pmod{p} \neq 0 \rightarrow 4(2)^3 + 27(2)^2 \pmod{17} = 15 + 6 \pmod{17} = 4 \neq 0$$

2.2

$$(2, 7) + (5, 2) = (x', y')$$

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 7}{5 - 2} \pmod{17} = 12 \cdot 3^{-1} \pmod{17} = 12 \cdot 3^{15} \pmod{17} = 12 \cdot (3^5)^3 \pmod{17} =$$

$$12 \cdot 5^3 \pmod{17} = 4 = s$$

$$X' = s^2 - x_1 - x_2 \pmod{p} = 16 - 2 - 5 \pmod{17} = 9$$

$$Y' = s(x_1 - x') - y_1 \pmod{p} = 4(2 - 9) - 7 \pmod{17} = 16$$

$$(x', y') = (9, 16)$$

2.3

Hasse's theorem :

$$P + 1 - 2\sqrt{p} \leq \#E \leq P + 1 + 2\sqrt{p}$$

$$\#E = 19, p = 17 \rightarrow 17 + 1 - 2\sqrt{17} = 9.75, 17 + 1 + 2\sqrt{17} = 26.24$$

$$9.75 \leq 19 \leq 26.24 \text{ true}$$

2.4

Because  $\#E = 19$  is a prime number and If we have a cyclic group with  $|G|$  elements where  $|G|$  is a prime number then all the members of this group are primitive elements (generators) .

Question 3)

$P = 31$ ,  $\alpha = 3$  and  $\beta = 6$

3.1

Received message =  $x = 10$

First signature =  $(17, 5) \rightarrow r = 17, s = 5$

$t = \beta^r \cdot r^s \bmod p = 6^{17} \cdot 17^5 \bmod 31 = 26 \cdot 26 \bmod 31 = 25$ ,  $\alpha^x \bmod p = 3^{10} \bmod 31 = 25$  **signature is valid**

second signature =  $(13, 5) \rightarrow r = 13, s = 5$

$t = \beta^r \cdot r^s \bmod p = 6^{13} \cdot 13^5 \bmod 31 = 6 \cdot 6 \bmod 31 = 5$ ,  $\alpha^x \bmod p = 3^{10} \bmod 31 = 25$  **signature is not valid**

3.2

there is only one signature for every KE which select from  $\{0, 1, \dots, p - 2\}$

$$t = \beta^r \cdot r^s \bmod p = \alpha^x \bmod p$$

$$(\alpha^d)^r \cdot (\alpha^{kE})^s = \alpha^{dr + skE} \bmod p$$

$$s = (x - dr) kE^{-1} \bmod p - 1 \rightarrow s \cdot kE = x - dr \bmod p - 1 \rightarrow x = s \cdot kE + dr \bmod p - 1$$

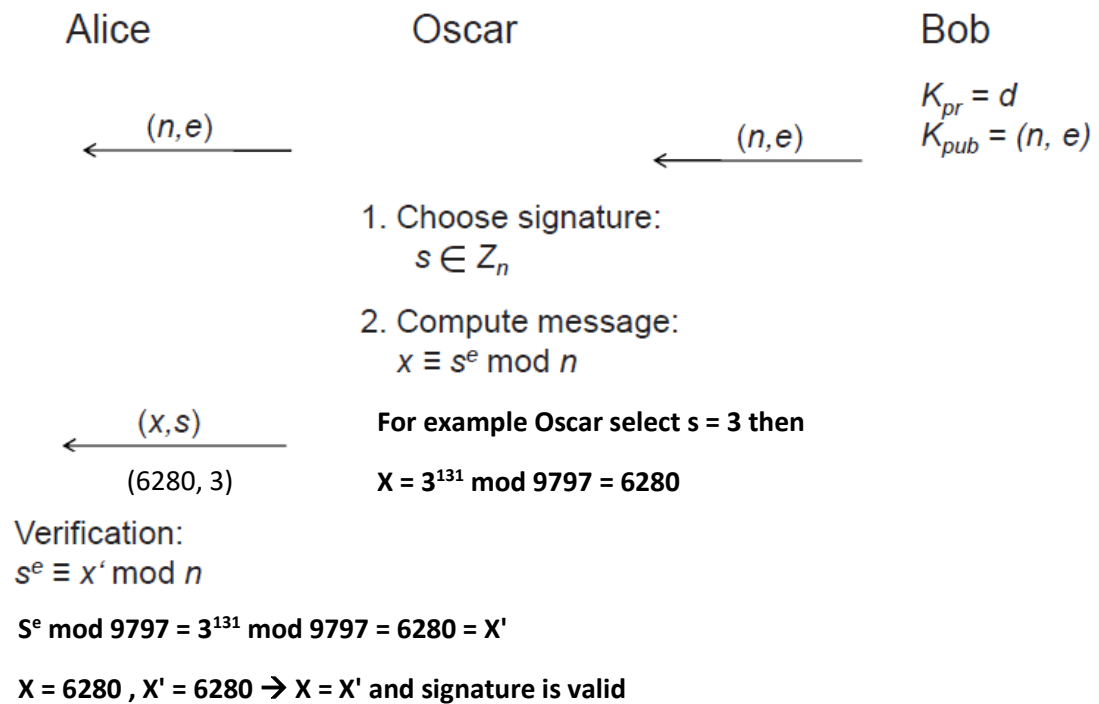
$$\rightarrow \alpha^{dr + skE} \bmod p$$

$P = 31$  so we can choose  $kE$  form  $\{0, 1, \dots, 29\} \rightarrow |S_{kE}| = 30$

So there are 30 valid signature for every  $x$  (message)

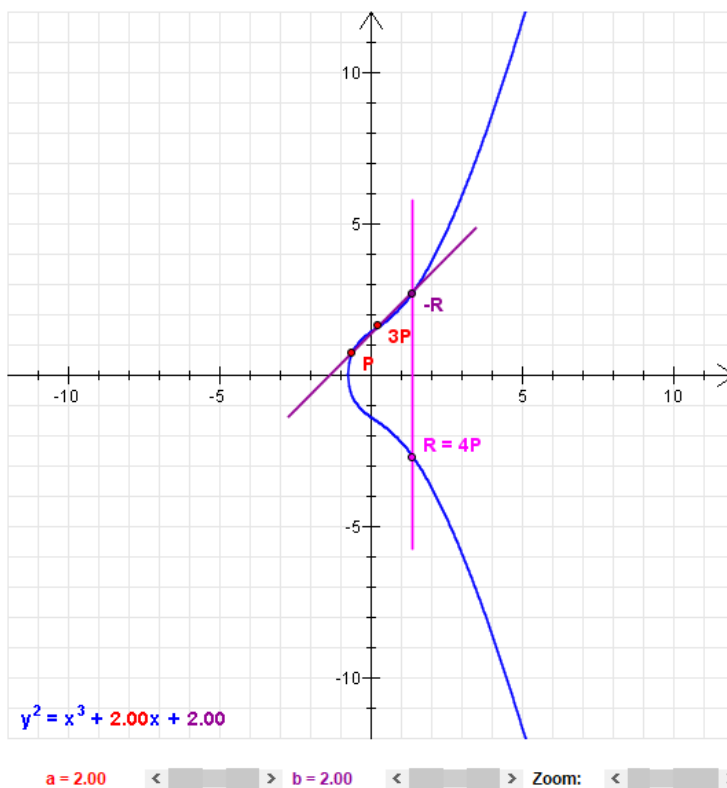
Question 4)

### ■ Existential Forgery Attack against RSA Digital Signature



# Cryptool :

1.a



## Choose the number space

- ☒ Real number space R
- ☐ Discrete group over Fp

This program allows you to generate various elliptic curves and to carry out point additions on these curves.

As number spaces you can use the real numbers or groups over the prime numbers ranging from 3 to 97.

The curve parameters a and b can be changed through the scrollbars.

The tangent of the point P intersects the curve at the point -R. The mirroring at the x-axis is the point R.

R is the result of the point addition of P.

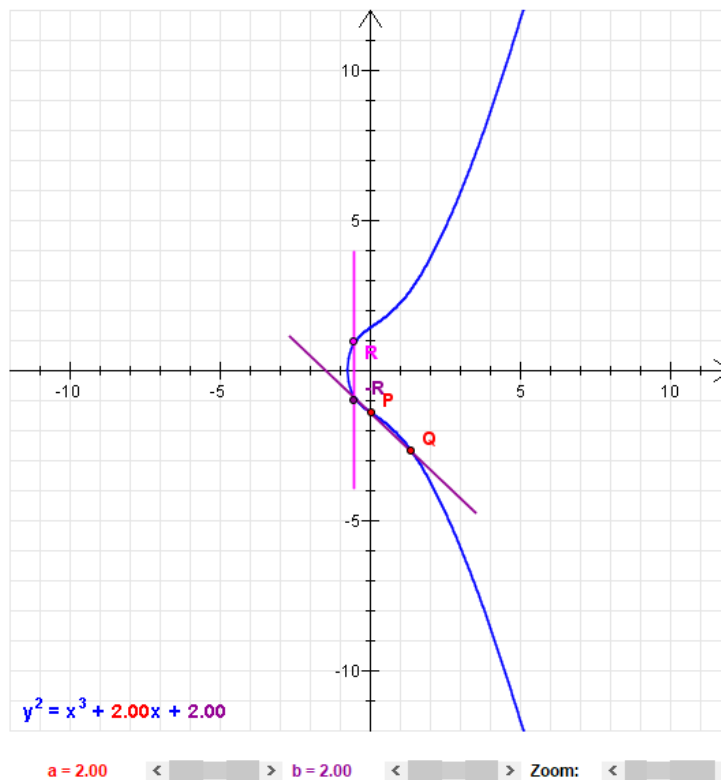
By clicking the button again, you can continue the point addition with the point P.

P = (-0.64/0.69)

3P = (0.27/1.60)

R = 3P + P = (1.39/-2.74)

1.b



## Choose the number space

- ☒ Real number space R
- ☐ Discrete group over Fp

This program allows you to generate various elliptic curves and to carry out point additions on these curves.

As number spaces you can use the real numbers or groups over the prime numbers ranging from 3 to 97.

The curve parameters a and b can be changed through the scrollbars.

The straight line through the points P and Q intersects the curve at the point -R. The mirroring at the x-axis is the point R.

R is the result of the addition of P and Q.

P = (0.05/-1.45)

Q = (1.36/-2.69)

R = (-0.51/0.92)

2.a

Generation of Asymmetric Key Pair

Algorithm

☐ RSA  
Bit length of RSA modulus: 1024

☒ DSA  
Bit length of DSA prime number: 2048

☐ Elliptic curves  
Identifier (bit length and curve parameter): prime239v1

User data

The key pair will be put in an encrypted PSE with the name shown below. The key pair will be protected by your PIN code.

Last name: Sara

First name: Baradaran

Key identifier (optional):

PIN:

PIN verification:

The domain parameter of the selected elliptic curve will be shown below.

Parameters

Value of the parameter

Bit len...

Base for presentation of numbers

☒ Octal ☒ Decimal ☐ Hexadecimal

Generate new key pair...

PKCS #12 Import

Show key pair...

Close

CrypTool

The parameters chosen by you and the new key pair have been successfully saved.  
The assigned key identifier is  
'[Sara][Baradaran][DSA-2048][1591563949]'.

Elapsed time while creating key pair: 8.361 seconds.

OK

Available Asymmetric Key Pairs

The list below shows the asymmetric key pairs that are available.  
Select the desired name by clicking its row with the left mouse button.

Last name	First name	Key type	Key identifier	Created	Internal ID no.
Baradaran	Sara	RSA-1024	1273006739	18.05.2020 15:59:16	1589801356
HybridEncrypti...	Bob	EC-prime239v1	PIN=1234	09.05.2007 13:51:14	1178702474
Sara	Baradaran	DSA-2048		08.06.2020 01:35:49	1591563949
SideChannelAt...	Bob	RSA-512	PIN=1234	06.07.2006 14:21:34	1152179494

Listed key types:

☒ RSA keys  
☒ DSA keys  
☒ EC keys

Show public parameters...

Show certificate

Delete...

Show all parameters...

Export PSE (PKCS#12)

Close

2.b → output file has been attached :

Sign a Document

Choose hash function

Algorithm:	Output length
<input type="radio"/> MD2	128 bits
<input type="radio"/> MD5	128 bits
<input type="radio"/> RIPEMD-160	160 bits
<input type="radio"/> SHA	160 bits
<input checked="" type="radio"/> SHA-1	160 bits

Choose signature algorithm

Factorization based algorithms

☐ RSA

Discrete logarithm based algorithms

☒ DSA

Elliptic curve based algorithms

☐ ECSP-DSA

☐ ECSP-NR

Presentation format

☐ Affine coordinates

☒ Projective coordinates

Choose a key/PSE to be used when signing

Last name	First name	Key type	Key identifier	Created	Internal ID no.
Sara	Baradaran	DSA-2048		08.06.2020 01:35:49	1591563949

Listed key types:

☐ RSA keys

☒ DSA keys

☐ EC keys

PIN code for chosen PSE:

xxxxxxx

☐ Display signature time

☐ Display intermediate results

Sign

Cancel

Signature Verification

Choose the signature originator from the following list:

Last name	First name	Key type	Key identifier	Created	Internal ID no.
Sara	Baradaran	DSA-2048		08.06.2020 01:35:49	1591563949

Specified data

Signature algorithm: DSA

Hash function: SHA-1

Listed key types:

☐ RSA keys
 ☒ DSA keys
 ☐ EC keys

Verification algorithm:

☒ ECSP-DSA
 ☐ ECSP-NR

Verification hash function:

☒ SHA-1
 ☐ RIPEMD-160

Presentation format:

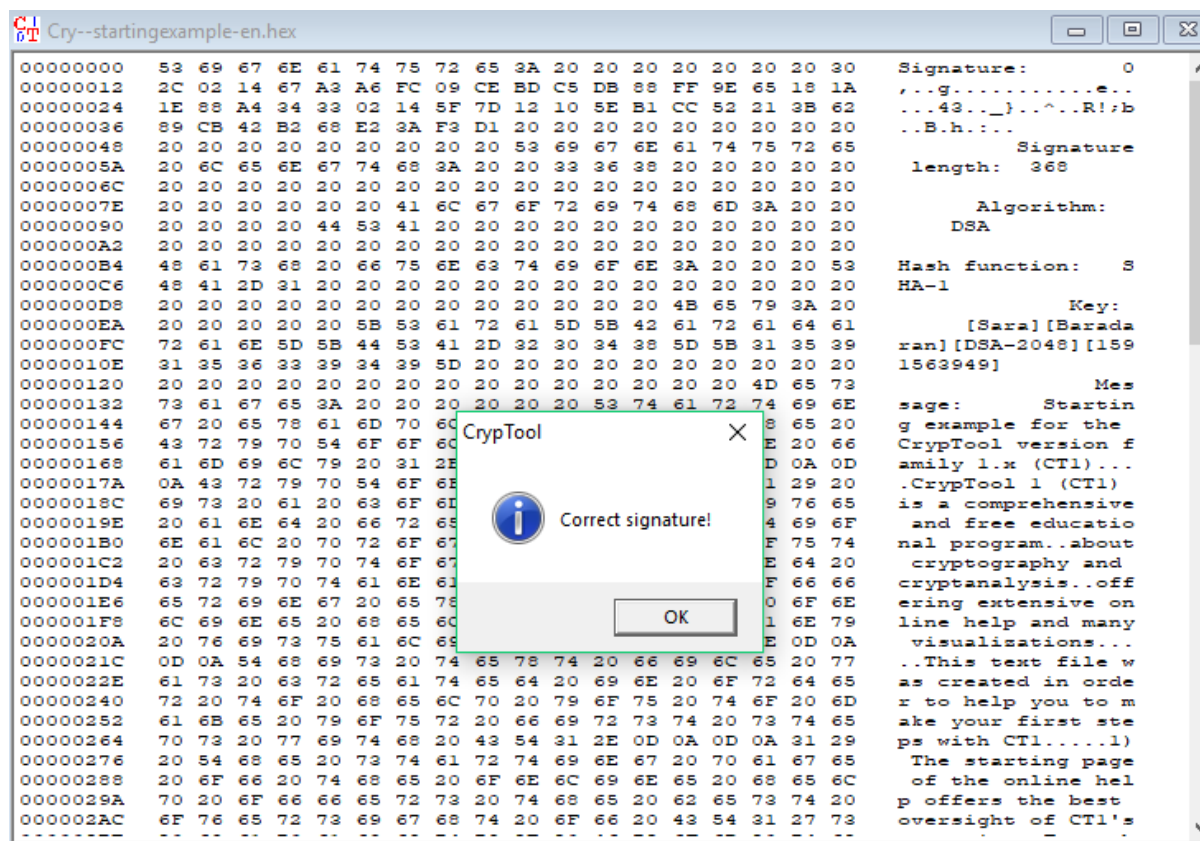
☐ Affine coord.
 ☒ Projective coord.

☐ Display verification time
 ☐ Display intermediate results

Look up key

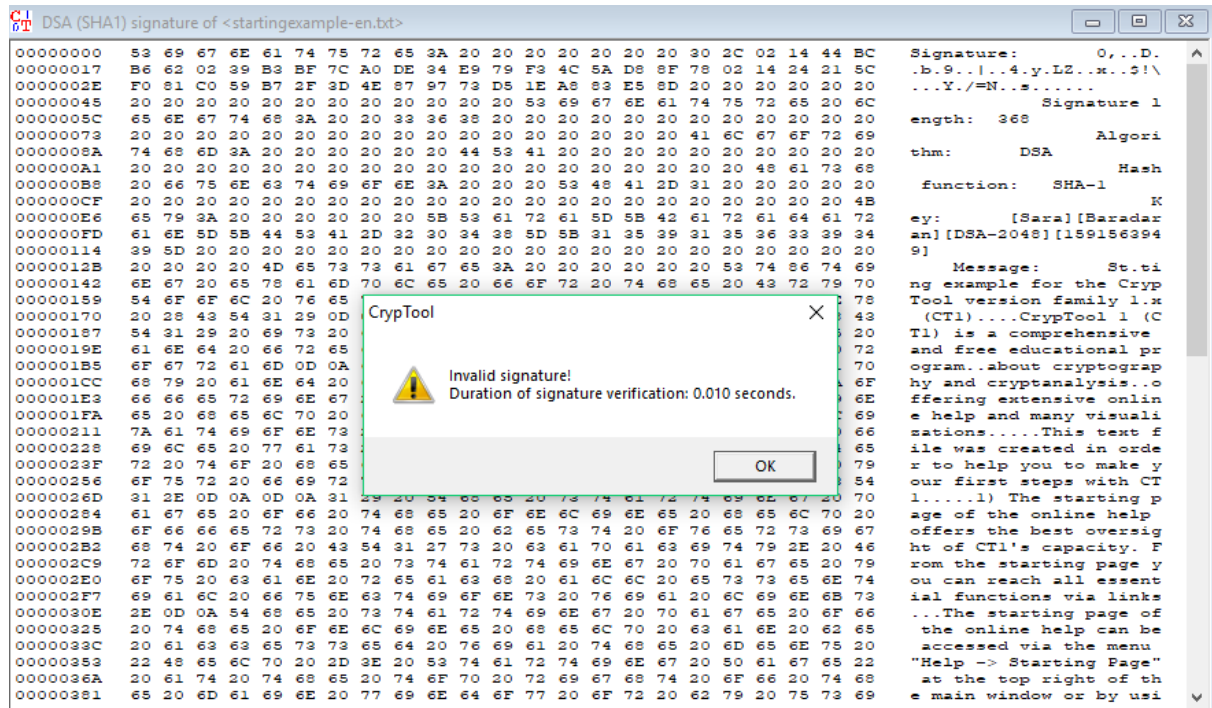
Verify signature

Cancel

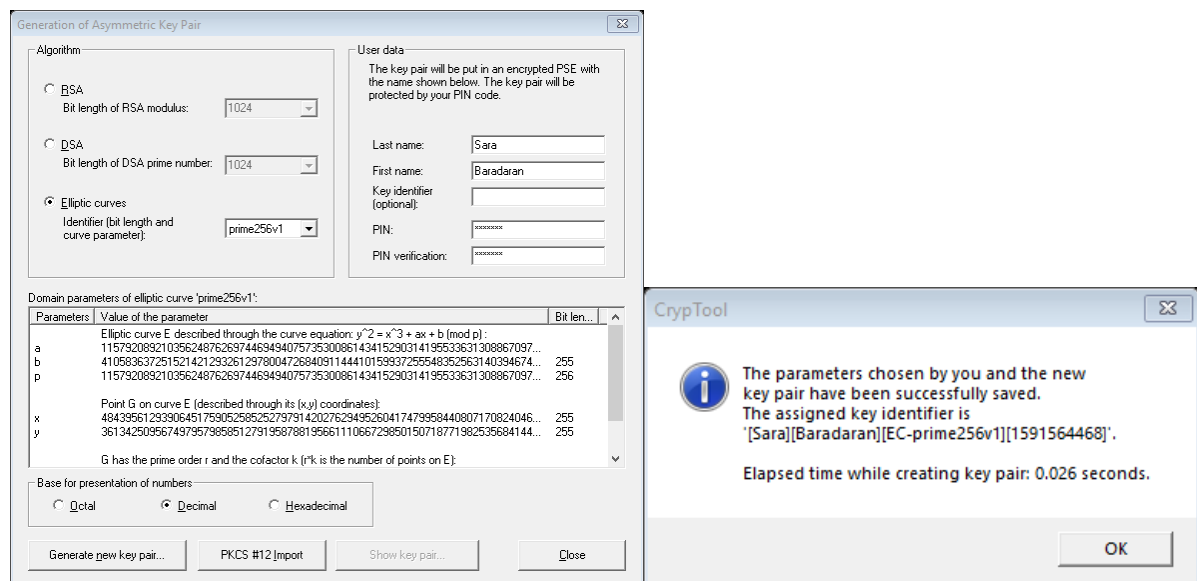




2.d → this signature is not valid



3.a



### Available Asymmetric Key Pairs



The list below shows the asymmetric key pairs that are available.  
Select the desired name by clicking its row with the left mouse button.

Last name	First name	Key type	Key identifier	Created	Internal ID no.
Baradaran	Sara	RSA-1024	1273006739	18.05.2020 15:59:16	1589801356
HybridEncrypti...	Bob	EC-prime239v1	PIN=1234	09.05.2007 13:51:14	1178702474
Sara	Baradaran	DSA-2048		08.06.2020 01:35:49	1591563949
Sara	Baradaran	EC-prime256v1		08.06.2020 01:44:28	1591564468
SideChannelAt...	Bob	RSA-512	PIN=1234	06.07.2006 14:21:34	1152179494

Listed key types:

- ☒ RSA keys
- ☒ DSA keys
- ☒ EC keys

Show public parameters...

Show all parameters...

Show certificate

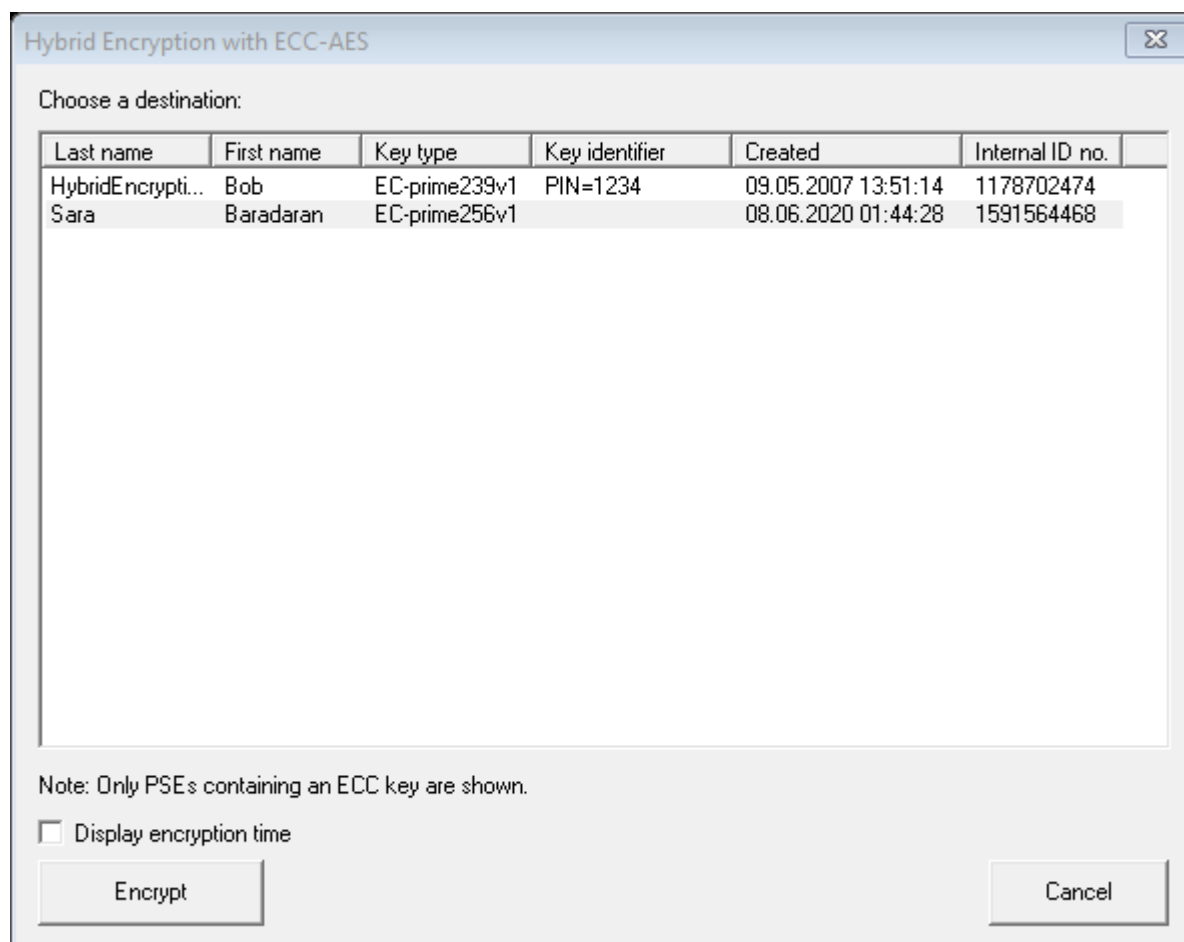
Export PSE (PKCS#12)

Delete...

Close

3.b→ output file has been attached :

عملیات توان رسانی یک عملیات سنگین و برای پیام های بزرگ بسیار هزینه بر و زمان بر است لذا نمیتوان صرفاً از رمز نامتقارن برای رمز کردن کل متن پیام استفاده کرد بلکه معمولاً برای مبادله کلید از این نوع رمزنگاری استفاده می شود و سپس پیام ها با یک روش رمز متقارن توسط کلید مذکور رمز می شوند.



3.c→ output file has been attached :

Hybrid Decryption with ECC-AES

Select your secret key from the PSE list.

Last name	First name	Key type	Key identifier	Created	Internal ID no.
HybridEncrypti...	Bob	EC-prime239v1	PIN=1234	09.05.2007 13:51:14	1178702474
Sara	Baradaran	EC-prime256v1		08.06.2020 01:44:28	1591564468

Note: Only PSEs containing an ECC key are shown.

☐ Display decryption time

Decrypt

PIN code:

Cancel

## Openssl :

1.a

```
→ Desktop openssl dsaparam -out dsaparameter.pem 2048
Generating DSA parameters, 2048 bit long prime
This could take some time
.....+.....+.....+.....+.....+.....+
+++++*
.....+.....+.....+.....+.....+.....+
...+.....+.....+.....+.....+.....+
+++++*
→ Desktop openssl genssa -out dsa.key dsaparameter.pem
Generating DSA key, 2048 bits
→ Desktop
```

1.b, c

```
→ Desktop openssl pkeyutl -sign -in file -inkey dsa.key -out sign.txt
→ Desktop openssl pkeyutl -verify -in file -sigfile sign.txt -inkey dsa.key
Signature Verified Successfully
→ Desktop
```

1.d

```
→ Desktop openssl req -x509 -sha256 -nodes -days 30 -key dsa.key -keyout privat
ekey.key -out dsa.crt
Can't load /home/sara/.rnd into RNG
140641163977152:error:2406F079:random number generator:RAND_load_file:Cannot ope
n file:../crypto/rand/randfile.c:88:Filename=/home/sara/.rnd
You are about to be asked to enter information that will be incorporated
into your certificate request.
What you are about to enter is what is called a Distinguished Name or a DN.
There are quite a few fields but you can leave some blank
For some fields there will be a default value,
If you enter '.', the field will be left blank.
-----
Country Name (2 letter code) [AU]:
State or Province Name (full name) [Some-State]:
Locality Name (eg, city) []:
Organization Name (eg, company) [Internet Widgits Pty Ltd]:
Organizational Unit Name (eg, section) []:
Common Name (e.g. server FQDN or YOUR name) []:
Email Address []:
→ Desktop
```

```
> crypto6 openssl x509 -in dsa.crt -text -noout
```

Certificate:

Data:

Version: 3 (0x2)

Serial Number:

53:c3:51:af:7a:9b:08:8e:81:3d:70:0c:ac:60:a1:b3:6f:e4:e1:dc

Signature Algorithm: dsa\_with\_SHA256

Issuer: C = AU, ST = Some-State, O = Internet Widgits Pty Ltd

Validity

Not Before: Jun 12 13:37:19 2020 GMT

Not After : Jul 12 13:37:19 2020 GMT

Subject: C = AU, ST = Some-State, O = Internet Widgits Pty Ltd

Subject Public Key Info:

Public Key Algorithm: dsaEncryption

pub:

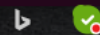
```
00:9a:fe:21:9d:5a:22:30:cc:48:84:2b:69:b5:ac:
1b:c8:dc:53:71:98:24:58:33:8a:61:dd:a5:65:e1:
3d:e1:4c:22:e8:2e:d8:b6:39:89:34:7a:a3:c7:57:
bc:d7:eb:bc:27:d6:d8:5f:af:60:21:50:a1:a1:29:
91:be:97:32:51:88:1b:7d:97:d1:31:b1:f9:f8:13:
08:0c:4c:77:bf:19:9b:55:ca:c2:d1:8d:af:c4:65:
5c:ba:d1:60:e4:33:c6:d5:c1:b7:06:6e:5e:3a:57:
05:17:04:d2:26:3c:d0:55:1f:59:03:c0:aa:1c:3d:
46:cc:48:ea:99:d3:2f:8d:41:e6:11:48:51:b3:33:
c8:34:9c:22:dc:ba:22:12:26:a7:25:08:7e:58:77:
f3:a4:61:37:a5:b5:ee:78:ba:78:22:d8:c6:9d:61:
86:a4:0b:5c:8d:82:35:fa:6d:dc:dd:84:74:da:96:
13:00:ee:d0:50:33:8c:1d:42:40:37:fb:bf:33:77:
ae:39:1e:bb:70:5b:d0:8d:7a:d0:ec:6d:1a:9c:b8:
0c:42:5e:ad:5d:f1:3f:04:e9:5a:d8:3f:e9:aa:50:
21:d2:e8:ba:26:ad:b9:e9:7a:12:eb:2f:92:5c:c9:
10:db:54:3a:74:a7:42:3d:2f:14:cb:58:0e:5d:e0:
04:f2
```

P:

```
00:b1:80:e0:23:dd:22:03:6b:cf:51:14:76:cd:02:
61:a8:da:83:aa:22:ed:e9:1f:b7:ef:cf:36:a6:03:
4c:61:94:d0:a6:7f:81:9c:be:2c:c6:c0:01:4d:2a:
a8:a1:e8:7c:96:3b:75:af:2b:61:66:aa:d2:1b:90:
49:ee:af:82:5c:af:9f:a1:62:00:79:ee:ad:aa:4d:
e8:40:08:be:12:ff:ed:6d:cf:01:66:3b:95:e1:cb:
6c:b7:bc:3f:9a:ab:62:1c:3a:8a:f9:62:36:30:82:
20:94:08:24:a9:f9:51:a3:df:60:2a:ac:b1:a7:81:
55:bc:fb:34:be:35:c0:ac:e5:1e:be:d7:36:b4:f4:
11:08:04:7d:38:a6:bd:1b:80:a7:2a:e7:3d:f2:ff:
73:20:76:7d:38:6e:7a:f1:25:2a:b8:4d:5b:38:57:
31:91:2e:9d:f8:b2:57:d0:eb:c4:c9:ac:8e:12:75:
5b:20:e9:e4:26:25:de:a4:a8:78:f3:12:b2:8e:76:
2b:4b:29:00:3e:4a:08:8b:8c:13:65:05:a0:b4:a6:
b5:22:6a:ba:0b:fb:11:0f:31:cf:e4:a2:64:6c:4a:
22:83:d2:ab:30:6b:af:dd:3d:3c:c3:e0:0f:29:73:
63:4e:28:e8:6d:2d:07:38:26:b3:b8:a7:00:2c:a1:
ce:11
```

Q:

```
00:95:3c:84:b4:77:ed:a7:61:c9:45:ff:31:fc:1d:
```



```
49:ee:af:82:5c:af:9f:a1:62:00:79:ee:ad:aa:4d:
e8:40:08:be:12:ff:ed:6d:cf:01:66:3b:95:e1:cb:
6c:b7:bc:3f:9a:ab:62:1c:3a:8a:f9:62:36:30:82:
20:94:08:24:a9:f9:51:a3:df:60:2a:ac:b1:a7:81:
55:bc:fb:34:be:35:c0:ac:e5:1e:be:d7:36:b4:f4:
11:08:04:7d:38:a6:bd:1b:80:a7:2a:e7:3d:f2:ff:
73:20:76:7d:38:6e:7a:f1:25:2a:b8:4d:5b:38:57:
31:91:2e:9d:f8:b2:57:d0:eb:c4:c9:ac:8e:12:75:
5b:20:e9:e4:26:25:de:a4:a8:78:f3:12:b2:8e:76:
2b:4b:29:00:3e:4a:08:8b:8c:13:65:05:a0:b4:a6:
b5:22:6a:ba:0b:fb:11:0f:31:cf:e4:a2:64:6c:4a:
22:83:d2:ab:30:6b:af:dd:3d:3c:c3:e0:0f:29:73:
63:4e:28:e8:6d:2d:07:38:26:b3:b8:a7:00:2c:a1:
ce:11
```

```
Q:
00:95:3c:84:b4:77:ed:a7:61:c9:45:ff:31:fc:1d:
5a:b2:61:21:39:8a:57:57:3a:6c:34:52:39:90:1a:
9b:26:91
```

```
G:
00:8c:1a:0e:49:af:28:c1:76:5c:45:01:d1:f8:6c:
c5:c3:86:71:1a:99:0c:e7:bb:7e:bd:c5:d7:ec:56:
d5:b9:27:b0:42:88:6c:31:e1:8d:79:8c:51:7c:d2:
c7:93:67:ed:00:c2:98:8c:23:3a:98:d3:d3:db:29:
f8:ed:93:9f:d5:46:6d:1f:1a:8c:0c:49:3a:73:e4:
de:81:b7:cd:90:b1:67:48:97:ea:fa:47:a5:d5:2e:
49:29:4f:ea:f5:1d:79:fd:0e:4a:01:d3:da:83:a0:
9e:6b:6f:ee:e2:18:e2:00:3c:fd:fe:70:ce:5a:5d:
ee:6d:7d:f3:f9:aa:6b:58:2a:f8:7d:86:c9:31:18:
f3:d7:12:1e:94:e7:3a:93:42:3f:4d:52:a6:aa:8e:
d0:ca:85:5b:e5:9c:08:70:f2:03:ec:d8:9c:d6:2f:
f4:ed:0d:af:65:00:c9:7e:68:ea:b3:f7:bd:7f:7b:
78:42:41:77:51:8c:79:6f:bb:8f:2c:d5:e9:fd:ae:
f0:78:c7:20:b9:34:19:1a:33:60:6e:d3:07:fe:83:
8b:27:95:54:eb:8b:ed:66:03:18:e8:68:d8:c1:df:
4c:d6:b5:b5:45:73:ae:3d:a7:48:83:f7:c8:2c:ee:
e7:a5:86:e9:16:e5:8b:1a:cc:7d:94:d3:71:8e:59:
c4:47
```

X509v3 extensions:

X509v3 Subject Key Identifier:

B7:59:9A:D9:06:88:5A:77:B5:CC:8C:40:DD:B7:45:59:B0:D4:42:AA

X509v3 Authority Key Identifier:

keyid:B7:59:9A:D9:06:88:5A:77:B5:CC:8C:40:DD:B7:45:59:B0:D4:42:AA

X509v3 Basic Constraints: critical

CA:TRUE

Signature Algorithm: dsa\_with\_SHA256

r:

1c:a2:5e:96:cf:a8:d8:01:36:1a:2b:4a:80:af:95:
34:21:d6:e8:69:24:ef:86:92:25:e6:8c:d1:36:c3:
fd:04

s:

65:8f:61:71:16:a5:a4:83:c2:cd:bb:cd:71:12:c6:
69:f4:44:96:92:40:9c:68:e2:37:0d:f1:1c:d8:9c:
26:5e

crypto6 ☐



2.a, b

```
→ Desktop openssl ecparam -name secp160r1 -genkey -out ec_client.key
→ Desktop openssl ecparam -name secp160r1 -genkey -out ec_server.key
→ Desktop
→ Desktop
→ Desktop openssl ec -in ec_client.key -pubout -out client_pub.key
read EC key
writing EC key
→ Desktop openssl ec -in ec_server.key -pubout -out server_pub.key
read EC key
writing EC key
→ Desktop cat client_pub.key
-----BEGIN PUBLIC KEY-----
MD4wEAYHKOZIZj0CAQYFK4EEAAgDKgAEK2MqpH5de7CqRI52SBLrRTctuQ4be8LO
urnyIGtwUPXPl6AGhyYZkg==
-----END PUBLIC KEY-----
→ Desktop cat server_pub.key
-----BEGIN PUBLIC KEY-----
MD4wEAYHKOZIZj0CAQYFK4EEAAgDKgAEp2tFWbxq00oGWJ5gGf3JYlBfk5kdWbsZ
gUj4791yZ759VK2UCK8jtQ==
-----END PUBLIC KEY-----
→ Desktop
```

2.c, d, e

```
→ Desktop openssl pkeyutl -derive -inkey ec_server.key -peerkey client_pub.key
-out secret2
→ Desktop openssl pkeyutl -derive -inkey ec_client.key -peerkey server_pub.key
-out secret1
→ Desktop cat secret1
T+d09l.0("0w<00%
→ Desktop cat secret2
T+d09l.0("0w<00%
→ Desktop
```

Result → secret1 and secret2 files have the same content.

کلید مشترک از طریق زیر محاسبه می شود. که برای هر دو طرف ارتباط یکسان بدست می آید لذا محتوای دو فایل secret1 و secret2 کاملاً یکسان است.

Server :

$K_{pr\_server} = a$  ,  $K_{pub\_client} = bP$

$K_{pr\_server} K_{pub\_client} = abP$

client :

$K_{pr\_client} = b$  ,  $K_{pub\_server} = aP$

$K_{pr\_client} K_{pub\_server} = baP$