Investigating the Decay of Stronium-90 in GEANT4

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***Abstract*—This experiment investigates the beta-minus decay of Strontium 90 ($Sr90$) directly, and indirectly from Yttrium (Y90), using the GEANT4 platform, with Monte-Carlo methods. The maximum energy corresponding to $Sr90 \rightarrow Y90$ (E_{Sr}) and $Y90 \rightarrow Zr90$ (E_Y) were found by analysing the absorption curves of these electrons through low Atomic number (Z) materials (aluminium and copper), of varying thickness. These values were found to be $E_{Sr} = (0.53 \pm 0.08)MeV$ and $E_Y = (2.16 \pm 0.18)MeV$ using aluminium, and $E_{Sr} = (0.61 \pm 0.12)MeV$ and $E_Y = (2.23 \pm 0.13)MeV$ using copper. Discrepancies between these values and the theoretical values; $E_{Sr} = 0.54MeV$ and $E_Y = 2.27MeV$ are within our accepted error margin. As a result, this concluded that the mean rate of energy loss with areal density, due to inelastic collisions in materials with low-Z, is identical, within an error margin.**

I. Introduction

RADIOACTIVE decay is a stochastic process, by which an unstable nucleus of an atom releases energy through the emission of ionising radiation, such as $\beta^-$and γ-decay, which are of particular interest in our investigation. β^- decay occurs due to nucleus-internal processes, converting a neutron into a proton, and emitting an electron and its anti-neutrino. γ-decay is the emission of one or more photons from the nuclei.

This release of ionising radiation can be a potential hazard for exposed organisms and hardware, this is why it's particularly important to study its propagation and absorption. In complex experiments, risk and expenses escalate excessively, as a result, the need for computer simulations has risen, in-turn, circumventing these constraints.

Individual decays of a radioactive isotope, Strontium-90 ($Sr90$) was simulated on the GEANT4 platform. This source decays through the emission of γ-rays and β^- particles. This experiment mainly investigated how these electrons interact with matter as they propagate, thereby losing energy and being absorbed. This gives insight into the maximum range of different electrons from $Sr90$ and the energy associated with these, allowing comparison with the theoretical values.

An analysis into the mean rate of energy loss with areal distance was also carried out, seeking consistencies with low atomic number (Z) materials; where the ratio of electrons to matter is approximately the same, such as in Aluminium and Copper.

II. Theory

A. Radioactive decay of Strontium-90

Strontium-90 undergoes β^- decay into Yttrium-90 ($Y90$) with a half-life of $\tau_{Sr} = 28years$ and decay energy of $E_{Sr} = 0.54MeV$, that gets shared amongst the electron and anti-neutrino - as neither is bound to its final state. This value also corresponds to the maximum energy an electron can obtain.

At a lower energy level, $Y90$ undertakes β^- decay 99.98% of the time, into a stable isotope, Zirconium ($Zr90$), with a $\tau_Y = 64hours$ and $E_Y = 2.27MeV$, in the remaining 0.02% events, $Y90$ undergoes a lower energy β^- decay followed by γ-emission [1]. As $\tau_{Sr} \gg \tau_Y$, $Sr90$ can be considered a pure β^- source, with half of these coming from the $Sr \rightarrow Y$ transition and the other half from $Y \rightarrow Zr$. Illustrated in Fig.1 is the complete decay path of $Sr90$.

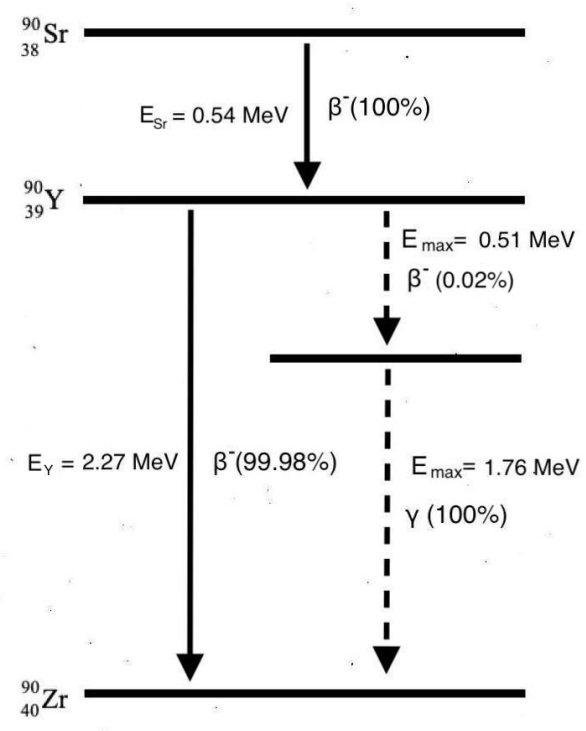

Fig. 1. **The complete radioactive decay scheme of *Sr90***; showing both origins of interest of β^- particle emission, from the $Sr \rightarrow Y$ and $Y \rightarrow Zr$ transition.

B. Electron and matter interactions

As electrons propagate through matter, they can interact with other atomic electrons; through inelastic collisions or without necessarily colliding with them; losing energy. The force between like charges repels electrons away from its path, whenever it's large enough to unbound the atomic electron from its shell, excitation or ionisation results, releasing γ-radiation. Electrons, along its path, are deflected due to elastic collisions with atomic nuclei.

Another form of energy loss is bremsstrahlung radiation, which occurs when an electron experiences a considerable Coulomb force near a nucleus, decelerating and consequently

releasing electromagnetic radiation. However, this mechanism of electron energy loss is negligible for low-Z materials, and at the relatively low energies, we are considering.

C. Maximum range of electrons

The total distance an electron travels in a material before losing all its energy and gets absorbed is referred to as its range (r). This value directly depends on the starting energy of the electron, and the material, as the density will affect the number of electrons in its path. A characteristic we will employ is that all electrons of the same energy have the same range in a specific material [2].

The number of interaction depends on the number of electrons it will encounter along its path, in low-Z elements, they have approximately the same number of electrons per gram of material. This inverse relationship between the density (ρ) and electron range, can be explored using the areal density (R) of a material;

$$R = \rho r. \tag{1}$$

This value can be used to determine the minimum energy electrons must have (in MeV), to penetrate through an areal density [1],

$$R = 0.11\left(\sqrt{1 + 22.4E^2} - 1\right). \tag{2}$$

D. Gradual energy deposition

In every inelastic collision, the energy transferred by the emitted electron is very small, consequently, atomic electron excitation is more probable than their ionisation. Even when ionisation occurs, the arising secondary electrons have a very low mean kinetic energy (few eV's) and are absorbed easily in the material. Therefore, on average, the total energy loss will be the sum of all these gradual losses in its path.

A quantum theory representing the mean rate of energy loss with distance was developed by Bohr, Bethe and Bloch in the early 20th century, which can be written approximately as [3]:

$$\frac{\overline{dE}}{dx} = 0.306\rho\frac{Z}{A}(\frac{v}{c})^{-2}\ln\left(\frac{1.16E}{I}\right). \tag{3}$$

In the above, E is the energy of the incoming electron and I is its excitation energy - dependent on the material. This equation has an inverse relationship to the velocity (v/c), as higher energy electrons interact less. The dependence on Z is quite subtle, as the ratio of Z/A (atomic number to the weight of the material), is equal to the number of electrons per unit mass. This value is ≈ 0.5 for low-Z materials.

Multiplying (3) with r, we can re-write the equation in terms of areal density. The ratio U, of this mean energy loss between the two different (low-Z) materials, using the same source, at the same areal density, can be simplified to;

$$U = ln\left(\frac{1.16E_1}{I_1}\right)/ln\left(\frac{1.16E_2}{I_2}\right). \tag{4}$$

The excitation energy for aluminium and copper are; $I_{Al} = 150eV$ and $I_{Cu} = 279eV$ [1]. U should be ≈ 1 for low-Z absorbers, as their rate of energy loss with areal density should be close to identical.

III. The Simulation

The simulation involved Monte Carlo methods, using random sample generators on GEANT4 platform. For this experiment, visualisations and back-scattering were not recorded, to optimise running time.

A. Experimental apparatus

Ionising radiation was measured using a Silicon detector. The detector consists of $300\mu m$ semiconducting silicon wafer, with two main bands; a valence and a conducting. The energy required to excite an electron from the valence into the conducting band (to produce an electron-hole pair) is $3.6eV$[1]. In our experiment, our β^- and γ-particles, lose on average a much greater amount, consequently, this threshold can be neglected. A potential difference of $50V$ will collect any ionised electrons produced within $100ns$ of its creation, building a count. This detector was placed at a separation of

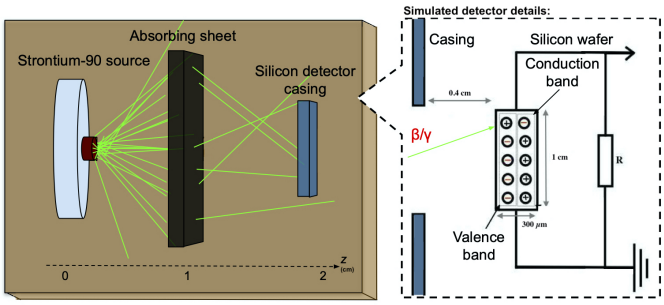

Fig. 2. **Schematic of the simulation setup**, on the lab bench. The electrical circuitry of the semiconducting Silicon wafer is also shown, with its characteristic bands.

$z = 2cm$ from our source, where aluminium or copper sheets, of a specified thickness were placed in between, to investigate their absorbing characteristics.

B. Data recording

The activity of the source was normalised with a value of $1kBq$. This was chosen at random because if used consistently, it wouldn't alter any relationship.

Assuming each simulated event is independent and has a fixed probability. The chance of obtaining a certain number of decays (n) in a given time is given by a Poisson distribution and depends on the mean number of decays ($\bar{n}$). This distribution can be approximated to a Gaussian for a large mean ($\bar{n} > 30$). Therefore, a reasonable estimate for the error associated with each measurement was given by the square root of that number ($\sqrt{\bar{n}}$).

As we increased the thickness of the absorber, the number of counts detected decreased, therefore preliminary data values were needed before each run, to ensure an adequate number of decays at each thickness was obtained (> 50), in turn reducing its fractional error ($\sqrt{\bar{n}}/\bar{n}$). Moreover, we increased data collection near points of interest - where there were deviations from the expected linear trend.

IV. Results

To represent our data, we plotted the logarithmic relationship between the absorbed count rate and the areal density of a material. Three predominant linear regions are visible, corresponding to the different decays of Sr.

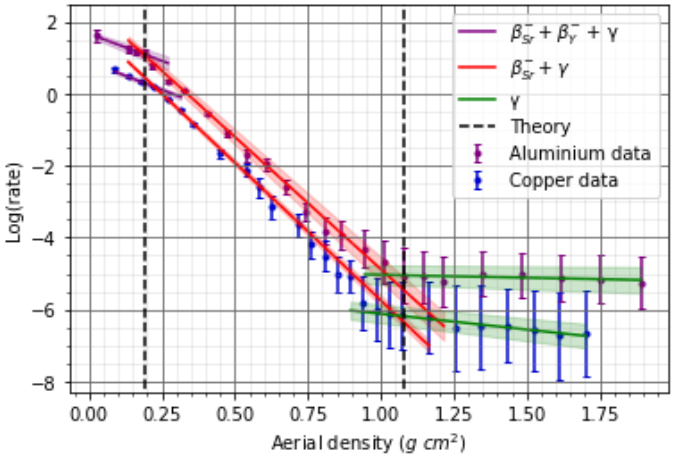

Fig. 3. **Logarithmic absorption graph for Aluminium and Copper as a function of areal density**, showing the best fit lines and the corresponding error of the fit shaded in the same colour. The theoretical-expected values of each transition are also indicated as a black dashed line.

Best fit lines were plotted (subjectively) and their co-variance matrix calculated. The areal density corresponding to the maximum range is the x-intercept between two linear fits. Evaluating the first intercept gives this value for the electrons coming from $Sr90$ (R_{Sr}) meanwhile, the second intercept gives those from $Y90$ (R_Y). These were then used to calculate the maximum energy of electrons from each transition (E_{Sr} and E_Y), using (2), for each material.

The uncertainty was calculated through the propagation of errors, however, we took into account the precision of the graph when this was greater.

The values obtained for aluminium were $R_{Sr} = (0.19 \pm 0.03) gcm^{-2}$and $R_Y = (1.02 \pm 0.08) gcm^{-2}$, which corresponds to energies of $E_{Sr} = (0.53 \pm 0.08) MeV$ and $E_Y = (2.16 \pm 0.18) MeV$respectively.

Similarly for copper, $R_{Sr} = (0.22 \pm 0.04) gcm^{-2}$ and $R_Y = (1.05 \pm 0.06) gcm^{-2}$, giving energy values of $E_{Sr} = (0.61 \pm 0.12) MeV$ and $E_Y = (2.23 \pm 0.13) MeV$.

The theoretical values of each transition are $E_{Sr} = 0.54 MeV$ and $E_Y = 2.27 MeV$. These differ from our experimental values by less than their corresponding error.

In addition, the mean rate of energy loss with areal distance in Aluminium relative to Copper (U) was calculated, for the different decays of Sr; the values were $U_{Sr} = 1.06 \pm 0.25$ and $U_Y = 1.06 \pm 0.10$, as expected, these values are ≈ 1.

V. Analysis

In theory, the corresponding counts at an aerial density should be the same for both materials. However, on average, it is lower for Copper (as seen in Fig. 3). This can be due to its higher density, giving rise to a higher chance of electron back-scattering; appearing to be absorbed faster. Electrons are largely susceptible to this due to their low mass. At larger thicknesses, this scattering can become more predominant, as more interactions occur. However, in future experiments, a detector encircling the source could account for this[4].

Overall, our values are slightly lower than the theoretical values, this can arise from the nature of the source, as this is not a point source, and particles propagate from its surface too.

Moreover, in Copper (with a higher Z), bremsstrahlung energy losses could become more significant and could explain why its gradient for β^-_Y is steeper. Equation (3) doesn't take into account these types of losses, consequently, our values for U are slightly larger than 1.

Nonetheless, deviations of U from 1, are smaller than its errors, therefore, conclusions of the effect of the atomic number on the mean rate of energy exchanged per collision are beyond the precision of this investigation. This experiment could benefit from more tests using various other materials to get an appreciation for the significance of Z on this type of radiative losses. Furthermore, implications of γ interactions could also be explored, as Compton scattering could be a source of error, at these energies.

More repeats would also be favourable to reduce the statistical errors arising from the measurement of these inherently random processes. Furthermore, provided the duration of this experiment was longer; the simulations could have been run for extended periods of times, also reducing our errors. A recent study [5] suggests, that around 2500 replications are required to be within one unit of the true value 95% of the time, in Monte Carlo based simulations.

VI. Conclusion

This experiment was able to deduce the maximum energy from the different decays within Strontium-90 using our knowledge of electron interactions - exploring the fact that for low-Z absorbers, the energy of electrons at an areal density, is the same. These values agreed with their theoretical counterparts, making the areal density an appropriate method to calculate the maximum energy of electron decay in materials such as aluminium and copper, providing evidence of inelastic collisions of electrons in these.

References

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