TT3010 - Audio technology and room acoustics.
Exercise 6 - Music scales
Solutions

Jan Arne Bosnes

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All tasks are based on chapter 9 in Rossings "Science of Sound" **?**. It is recommended that the student will try to do every task, but tasks marked *Mandatory* are to be handed in for approval (online). Deadline is November 23 at 16:00.

1

The ratio of frequencies for a semitone in the scale of equal temperament is given as follows:

$$f_2/f_1 = 2^{1/12} = 1.05946 \tag{1}$$

For a major third, we know that it has a step of four semitones, so the ratio of frequencies can be obtained by multiplying the semitone of equal temperament four times:

$$f_2/f_1 = \left(2^{1/12}\right)^4 = 2^{(4\cdot\frac{1}{12})} = 1.260 \tag{2}$$

This number is very close to $5/4 = 1.25$, and therefore the fifth harmonic of the lower tone is very close to the fourth harmonic of the higher tone, and gives a perception of *consonance*.

A minor third has a step of three semitones. Then the ratio of frequencies, for a minor third in equal temperament, can be obtained by multiplying the ratio of the frequencies for a semitone of equal temperament 3 times:

$$f_2/f_1 = \left(2^{1/12}\right)^3 = 2^{(3\cdot\frac{1}{12})} = 1.189 \tag{3}$$

2

So, the rate will grow by 1.059 which we know is about the same as $2^{\frac{1}{12}}$ = the step for one semi-tone. For twelve years, we will then get a step which is the same as moving twelve sem-tones = one octave. Therefore, the ratio should be 2. Numerically, we can check: $\left(2^{1/12}\right)^{12} = 2^{\left(\frac{1}{12}\cdot 12\right)} = 2^1 = 2$.

3

Using table 9.2, you will find the following.

- Center frequency 31.5 Hz is closest to the frequency $B_0 = 30.868$ Hz on the musical scale.
- Center frequency 63 Hz is closest to the frequency $B_1 = 60.735$ Hz on the musical scale.
- Center frequency 125 Hz is closest to the frequency $B_2 = 123.47$ Hz on the musical scale.
- Center frequency 250 Hz is closest to the frequency $B_3 = 246.94$ Hz on the musical scale.
- Center frequency 500 Hz is closest to the frequency $B_4 = 493.88$ Hz on the musical scale.
- Center frequency 1000 Hz is closest to the frequency $B_5 = 987.77$ Hz on the musical scale.
- Center frequency 2000 Hz is closest to the frequency $B_6 = 1975.5$ Hz on the musical scale.
- Center frequency 4000 Hz is closest to the frequency $B_7 = 3951.5$ Hz on the musical scale.
- Center frequency 8000 Hz is closest to the frequency $B_8 = 7902.1$ Hz on the musical scale.

4

Using table 9.2, you will find the following.

- The touch-tone frequency 697 Hz is closest to the frequency $F_5 = 698.46$ Hz on the musical scale.
- The touch-tone frequency 770 Hz is closest to the frequency $G_5 = 783.99$ Hz on the musical scale.
- The touch-tone frequency 850 Hz is closest to the frequency $G_5^{\#}$ Hz on the musical scale.
- The touch-tone frequency 941 Hz is closest to the frequency $A_5^{\#} = 932.33$ Hz on the musical scale.

- The touch-tone frequency 1209 Hz is closest to the frequency $D_6^{\#} = 1244.5$ Hz on the musical scale.
- The touch-tone frequency 1337 Hz is closest to the frequency $E_6 = 1318.5$ Hz on the musical scale.
- The touch-tone frequency 1477 Hz is closest to the frequency $F_6^{\#}$ Hz on the musical scale.

5

The fifth has the ratio 3:2 and the fourth has the ratio 4:3. We will then get the following ratio:

$$\frac{3}{2} \cdot \frac{4}{3} = \frac{4}{2} = \frac{2}{1} \tag{4}$$

Which is the corresponding ratio to an interval of one whole octave.

We use the same procedure when considering a major sixth (5:3) and a minor third(6:5).

$$\frac{5}{3} \cdot \frac{6}{5} = \frac{6}{3} = \frac{2}{1} \tag{5}$$

So, a major sixth and a minor third correspond to an octave.

6

7

The ratio of the interval C : G correspond to a ratio 3:2, which is a perfect fifth.

The ratio of the interval E : B as follows,

$$\frac{E}{B} = \frac{(\frac{E}{C})}{(\frac{B}{C})} = \frac{3}{2} \tag{6}$$

which is a perfect fifth.

The ratio of the interval F : C as follows,

$$\frac{F}{C} = \frac{(\frac{F}{C})}{(\frac{C}{C})} = \frac{3}{2} \tag{7}$$

which is a perfect fifth.

The ratio of the interval G : D as follows,

$$\frac{G}{D} = \frac{(\frac{G}{C})}{(\frac{D}{C})} = \frac{3}{4} \tag{8}$$

which is not a perfect fifth.

The ratio of the interval A : E as follows,

$$\frac{A}{F} = \frac{(\frac{A}{C})}{(\frac{E}{C})} = \frac{4}{5} \tag{9}$$

which is not a perfect fifth.
The ratio of the interval D : A as follows,

$$\frac{D}{A} = \frac{(\frac{D}{C})}{(\frac{A}{C})} = \frac{40}{27} \tag{10}$$

which is not a perfect fifth.

8

The formula for the conversion of cents to a frequency ratio is

$$R_{cent} = 10^{(cent \cdot \log(2))/1200} \tag{11}$$

So, for 25 cent, $R_{25} = 1.0145$, which is the frequency ratio.

For A_4, the corresponding frequency is 440 Hz. The new added frequency with 25 cents are $f_2 = R_{25} \cdot f_{A4} = 446.38Hz$. So, adding 25 cent to A_4 gives us $A_4 + 25 = 440\ Hz\ +\ 446.38\ Hz = 886.38\ Hz$ and subtracting 25 cents gives us $A_4 - 25?440\ Hz\ - 446.38\ Hz\ = 6.38\ Hz$

9

The formula for the conversion of frequency ratio to cents is,

$$I = \frac{1200}{\log 2} \log R \text{ cent} \tag{12}$$

where R is the ratio of the frequencies. Now the standard C's from table 9.2 that are near the frequencies 128 Hz, 256 Hz and 512 Hz and C_3 with frequency 130.81 Hz, C_4 with frequency 261.63 Hz, and C_5 with frequency 535.25 Hz.

The frequency ratio for 128 Hz will then be $R = \frac{130.81\ Hz}{128\ Hz}$.

Using the conversion formula, we will get,

$$I = \frac{1200}{\log 2} \log \frac{130.81}{128} = 37.67 \text{ cents} \tag{13}$$

So, the tuning fork of frequency 128 Hz is 37.67 cents flat compared to the standard frequency of 130.81 Hz.

The frequency ratio for 256 Hz will then be $R = \frac{261.63\ Hz}{256\ Hz}$.

Using the conversion formula, we will get,

$$I = \frac{1200}{\log 2} \log \frac{261.63}{256} = 37.67 \text{ cents} \tag{14}$$

So, the tuning fork of frequency 256 Hz is 37.67 cents flat compared to the standard frequency of 261.63 Hz.

The frequency ratio for 512 Hz will then be $R = \frac{523.25\ Hz}{512\ Hz}$.

Using the conversion formula, we will get,

$$I(cent) = \frac{1200}{\log 2} \log \frac{532.25}{512} = 37.67 \text{ cents} \tag{15}$$

So, the tuning fork of frequency 512 Hz is 37.67 cents flat compared to the standard frequency of 523.25 Hz.

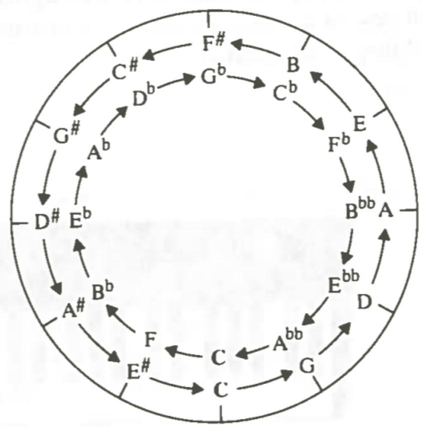

FIGURE 9.2
The circle of fifths. The outer circle visits all 12 notes on the chromatic scale by going up by fifths (or down by fourths). The inner circle goes down by fifths (or up by fourths).

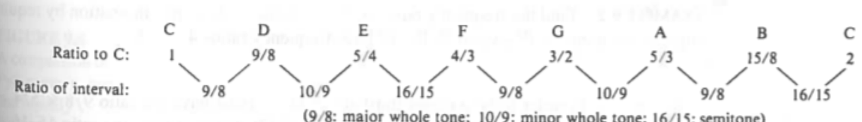

FIGURE 9.5 Frequency ratios of notes in the just diatonic scale. Numbers in the bottom row give intervals between two adjacent notes.

FIGURE 9.6
A comparison of Pythagorean, just, and equally tempered scales on a scale of cents (see Table 9.2).

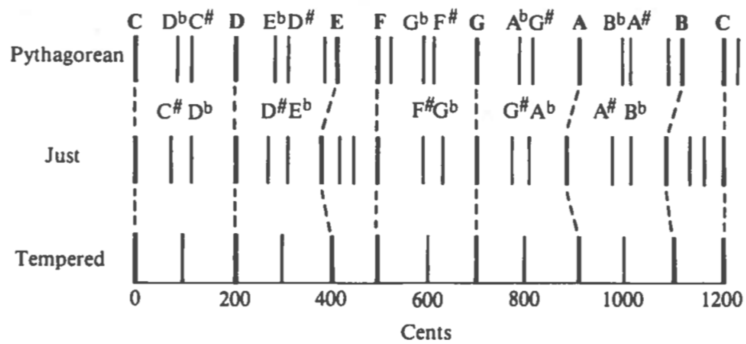

TABLE 9.2 Frequencies of notes in tempered scale

C_0	16.352		C_3	130.81		C_6	1046.5	
		17.324			138.59			1108.7
D_0	18.354		D_3	146.83		D_6	1174.7	
		19.445			155.56			1244.5
E_0	20.602		E_3	164.81		E_6	1318.5	
F_0	21.827		F_3	174.61		F_6	1396.9	
		23.125			185.00			1480.0
G_0	24.500		G_3	196.00		G_6	1568.0	
		25.957			207.65			1661.2
A_0	27.500		A_3	220.00		A_6	1760.0	
		29.135			233.08			1864.7
B_0	30.868		B_3	246.94		B_6	1975.5	
C_1	32.703		C_4	261.63		C_7	2093.0	
		34.648			277.18			2217.5
D_1	36.708		D_4	293.66		D_7	2349.3	
		38.891			311.13			2489.0
E_1	41.203		E_4	329.63		E_7	2637.0	
F_1	43.654		F_4	349.23		F_7	2793.8	
		46.249			369.99			2960.0
G_1	48.999		G_4	392.00		G_7	3136.0	
		51.913			415.30			3322.4
A_1	55.000		A_4	440.00		A_7	3520.0	
		58.270			466.16			3729.3
B_1	61.735		B_4	493.88		B_7	3951.1	
C_2	65.406		C_5	523.25		C_8	4186.0	
		69.296			554.37			4434.9
D_2	73.416		D_5	587.33		D_8	4698.6	
		77.782			622.25			4978.0
E_2	82.407		E_5	659.26		E_8	5274.0	
F_2	87.307		F_5	698.46		F_8	5587.7	
		92.499			739.99			5919.9
G_2	97.999		G_5	783.99		G_8	6271.9	
		103.83			830.61			6644.9
A_2	110.00		A_5	880.00		A_8	7040.0	
		116.54			932.33			7458.6
B_2	123.47		B_5	987.77		B_8	7902.1	