National University of Computer & Emerging Sciences, Karachi
Spring – 2018 CS-Department
Final Examination (Weightage 60%)
22^{nd} May 2018, 9 am – 12 noon

Course Code: CS211	Course Name: Discrete Structures
Instructor Name: Jalaluddin Qureshi	
Student Roll No:	Group:

- Return the question paper. There are **15 questions and 3 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- All the answers must be solved according to the sequence given in the question paper.
- **Marks will be awarded iff justifications has been provided.**

Time: 3 Hours. **Max Marks**: 140 marks (marks awarded out of 120, + 20 marks bonus)

Question 1: When Rehmat was a final student of BSc Computer Science students, he wanted to advance plan his career track. He enquired from the HR director of Filtronic Software the requirement to be hired as a Data Analytic Engineer in their company. The HR director replied, "If you graduate with a CGPA of 3.5 or higher, and you have a strong letter of recommendation from a faculty member of your department, then we will hire you as a Data Analytic Engineer." Rehmat graduated with a CGPA of 3.26 and had a strong letter of recommendation from a faculty member of his department. He then applied to Filtronic Software for the Data Analytic Engineer position, and was successfully hired by the company. Justify using logic theory whether or not the HR director lied to Rehmat? **(5 marks)**

Question 2: Determine using the pigeonhole principle the minimum number of people who must be sampled so that the birthday of at least 3 persons is on the same month. **(5 marks)**

Question 3: For the following recursive equation, determine the value of β, where K=4, and ρ_1=1.5 :

$$\beta = \sum_{t=2}^{K} \delta_t \qquad \delta_t = \binom{K}{t} \rho_t \qquad \rho_t = \sum_{s=1}^{t-1} \binom{t-1}{s} \rho_{t-s}$$

Write a pseudo-code to determine the value of β for any arbitrary value of K. **(10+5 marks)**

Question 4: a) By using proof by induction or proof by strong induction, prove the following Pascal Identity:

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

(b) By using proof by contradiction: show that for every integer n, n^2 is even if and only if n is even.
(7+4 marks)

Question 5: A company has 20 employees, 12 males and 8 females. How many ways are there to form a committee of 5 employees that contains at least one male and at least one female? Show conceptually how the tree diagram can be used to solve this problem (you don't need to solve the problem using tree diagram, just explain how it can be used for the given problem). **(6+3 marks)**

Question 6: Let the functions f, g : **R** → **R** be defined as follows: **(3+5+2=10 marks)**

$$f(x) = \begin{cases} \frac{x}{2(x-1)} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}, \quad g(x) = \begin{cases} \frac{x}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1. \end{cases}$$

(a) Give formal definitions of surjective, injective and bijective functions. (b) Show whether or not the functions *f* and *g* are bijective functions or not. (c) And find the inverse of these two functions.

Question 7: Determine the value of c at the end of the execution of the following program written using C++ pseudocode. Is it possible to shorten the conditions following the if statements? If yes, then what will be those new if conditions.
You need to show your working ***in detail*** for all the steps i.e. the values of x, y, and c, and the truth value for the if statements for each value of j (i.e. ***for all steps***). (Hint: The function pow(a,b) should be interpreted as follow: a^b).

```
int c=7;          int x=5;          int y=2;
for (int j=1; j<5; j++) {
        x = ⌈ 0.89 ∗ pow (x,j) ⌉;
        y = ⌊ 8.25 ∗ pow (y,j) ⌋ + ⌊ j ∗ −0.2 ⌋;
        if ¬(x<y) ∧ (j>3) {
                c = c + x – y;
        }
        else if (x<y) ∨ ¬(j>1) {
        c = c + 2*x + y;
        }
}
```

(8+2=10 marks)

Question 8: Let *q, r, s* denote the following statements:
q: I shall play tennis. r: The sun is shining. s: The humidity is low.

Write the following in symbolic form.
a) If the sun is shining, their either I shall play tennis or the humidity is not low.
b) If humidity is high and after sunset I shall play tennis.
c) With the aid of an example, define tautology. **(2+2+2=6 marks)**

Question 9: Using the algebra of propositions and/or De Morgans' law, simplify the following:
a) $(p \vee q) \wedge \neg(\neg p \wedge q)$
b) $\neg(\neg((p \vee q) \wedge r) \vee \neg q)$ **(3+4=7 marks)**

Question 10: Let *p(x), q(x), r(x), s(x)*, and *t(x)* be the following statements.
p(x): x>0
q(x): x is even
r(x): x is a perfect square
s(x): x is (exactly) divisible by 4
t(x): x is (exactly) divisible by 5 **(2x6=12 marks)**

Write the following statements in symbolic form.
(a) At least one integer is even.
(b) There exists a positive integer that is even.
(c) If x is even, then x is not divisible by 5.
(d) No even integer is divisible by 5.
(e) There exists an even integer divisible by 5.
(f) If x is even and x is a perfect square, then x is divisible by 4.

Question 11: Determine $|A \cup B \cup C|$ when $|A|=50$, $|B|=500$, and $|C|=5000$, and draw Venn diagram for each of the cases if (a) $A \subseteq B \subseteq C$ (b) $A \cap B = A \cap C = B \cap C = \{\ \}$ (c) $|A \cap B| = |A \cap C| = |B \cap C| = 3$, and $|A \cap B \cap C| = 1$.
(3x3=9 marks)

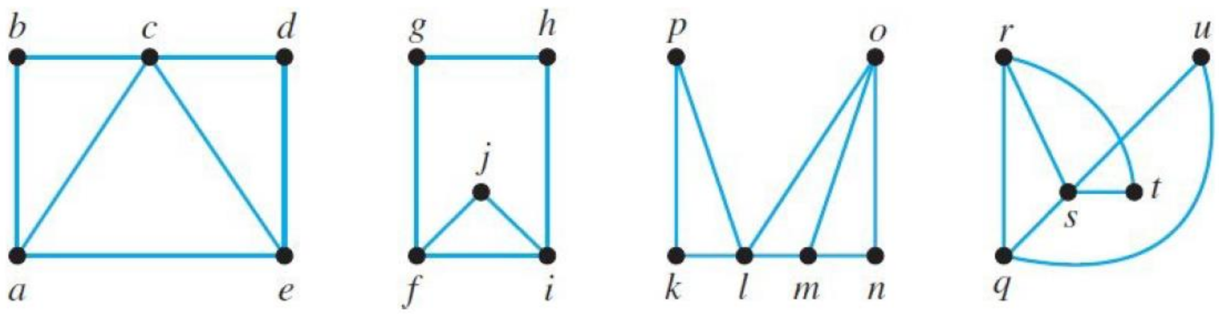
Question 12: Let $A = \{c, z, \varepsilon, f, \Psi\}$, $B = \{f, \Delta\}$, $C = \{\varepsilon, \Psi\}$, and the universal set $\mathcal{U} = (\{21, \pi\} \setminus C) \cup B \cup C$.
Answer each of the following questions. Give reasons for your answers.

(a) Find $B^c \cap \mathcal{U}$ (b) Is $C \subseteq A$? **(2x4+3=11 marks)**

(c) Is $C \subseteq C$? (d) Is C a proper subset of A?

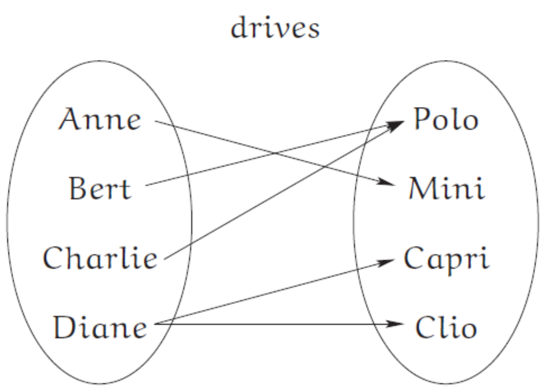
(e) If A={1, f, t, 8, 6}, B={5, pi}, and C={97, J, Y}. Determine, (AUB) x C; (AxC) U (BxC)

Question 13: Of the following four graph, **(4+4+4+3+3=18 marks)**
(a) which of the two graphs are isomorphic. Prove that these two graphs are isomorphic graph.
(b) For the first graph starting from left hand side (graph Q), find the solution of its minimum connector problem. You may assume that all edges have equal weight.
(c) Write a pseudocode of a program which takes the adjacency matrix of two graphs each with 4 vertices as an input, and outputs whether or not the two graphs are isomorphic or not.
(d) Express the third graph from left hand side (graph W) using the G=[V,E] notation.
(e) What is the disadvantage of saving a sparse graph using adjacency matrix?



Q **L** **W** **Z**

Question 14: A small survey yielded the following relationship between people and the cars they drive:



A relationship between these cars and the fuel they use was then compiled:
fuel = {(Polo,Unleaded), (Mini,Unleaded), (Capri, Leaded), (Clio,Diesel)}
(a) Write out the drives relationship as a set of ordered pairs.
(b) Draw an arrow diagram of the fuel relationship. **(3+2+3+4=12 marks)**
(c) Describe how two relations may be composed to create a new relation. Illustrate your answer by drawing the arrow diagram of the composition of the drives and fuel relations above.
(d) With the aid of a suitable example, explain how a relational database containing the list of "employees of a company" and "public/ private company status of a company" can be used to determine the list of people/ employees working in public company using the concept of relation.