Lab 7

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#Rcpp

We will get some experience with speeding up R code using C++ via the `Rcpp` package.

First, clear the workspace and load the `Rcpp` package.

```
pacman::p_load(Rcpp)
```

Create a variable `n` to be 10 and a vaiable `Nvec` to be 100 initially. Create a random vector via `rnorm` `Nvec` times and load it into a `Nvec` x `n` dimensional matrix.

```
n = 10
Nvec = 100
X = matrix(data = rnorm(Nvec*n), nrow = Nvec)
X
```

```
##               [,1]         [,2]        [,3]        [,4]         [,5]
##   [1,]  1.06909802 -1.304278889  0.49374181  2.05845140  0.207386377
##   [2,] -1.33867899 -1.059121057 -0.39028848  0.52840098 -0.900964660
##   [3,] -2.19262509  0.121596851 -0.19038123  1.64071899 -2.112021787
##   [4,] -0.37511485  1.775098728 -0.68384097 -0.42131952 -1.499354524
##   [5,]  0.26736802  1.058828130 -2.79619959  0.69435592 -0.102161765
##   [6,] -0.14765310  0.074026481  0.13669575  1.19099511 -0.717742182
##   [7,]  0.22878403  0.696767883 -0.51957613  0.23519972 -0.712048316
##   [8,] -1.12549989  0.053771686 -0.04983288  1.17744039 -0.946787534
##   [9,] -1.61272348 -0.027380718 -1.94501602  0.31341674  1.863694630
##  [10,] -0.07160224 -0.827677292 -1.00131787  0.44006092  0.549886849
##  [11,]  0.32954145 -0.883855314  0.47336163 -1.78461268 -1.418616854
##  [12,]  0.70396818 -1.671719835  1.11299165 -0.02500380  0.764380741
##  [13,]  0.22432825  0.242317697  0.56611425 -0.82320354  1.787792774
##  [14,] -0.22015949  2.212714621 -0.37145181  0.37203012  0.934941870
##  [15,]  0.10043275  1.718975176  0.70359018  0.28804407 -0.917403826
##  [16,]  1.90057973 -1.653472667  0.29520285  0.74504793 -0.258820399
##  [17,]  0.14221420  0.722212156  0.71057413 -0.57228086 -0.011866487
##  [18,]  1.00987252 -0.552544870  1.35522060  0.11386497 -0.238610912
##  [19,]  0.48054951 -0.444136468  1.45246280 -0.26875304  0.548773310
##  [20,] -0.07630641  0.681859437 -0.30415320  1.04549807 -0.489456652
##  [21,] -0.28948878 -0.288941474  0.29640583 -1.45833928  0.402741296
##  [22,] -1.45182465 -1.152110523  0.64168398 -0.19891412  1.715581472
##  [23,] -0.50416746  1.112955726  2.47886417 -0.33144452  1.202350159
##  [24,]  1.20535411  1.486411493 -0.99601695  0.79948488  1.133602468
##  [25,] -1.49116511 -1.965683944  0.40677929  0.36119221  1.262425990
##  [26,]  1.25588206  0.414034052 -0.68375548 -0.48635310  0.737156928
##  [27,]  0.03797380 -1.463791751  0.80297137 -0.96684362  1.030136953
##  [28,] -0.51546961 -0.702880392  1.07593829  0.25153041 -0.579867048
```

```
##  [29,] -0.18461895  0.284734765 -0.56125382 -1.10254488  0.730647963
##  [30,]  1.37894803  0.899271646  0.37052832 -0.01328218 -0.449610500
##  [31,] -1.19289785 -0.469141424 -0.11272248  0.38564267 -1.882386353
##  [32,] -0.45536721 -0.765824755 -0.11983952 -1.21004482  1.040841732
##  [33,] -0.07851135 -1.471704427  0.93716486  0.08051922  1.307802516
##  [34,]  1.23820115 -0.233933560  0.33341220 -0.51998821  0.825480667
##  [35,] -0.06199043 -2.465256166 -0.23639360 -1.10491380 -1.033775727
##  [36,] -0.89399992  0.014565210  0.24220954  1.14348641 -1.298970220
##  [37,] -0.30718747 -2.322246456  1.57984071  0.50782057  0.768452024
##  [38,]  1.63409845  0.939334221 -0.55789446  1.50237346  0.440552318
##  [39,] -0.26878852 -0.228583588 -1.52870979 -1.10806323 -0.665562027
##  [40,] -1.25316943  0.442447431 -0.11347173  1.41917284  1.278459040
##  [41,] -1.53585778  0.506310902  0.55229479  0.17628309 -0.339828801
##  [42,] -0.99121471 -1.082776762  0.60805089 -0.25050314  0.843896078
##  [43,]  0.14284095  1.551955719  1.94402941 -0.01620043 -0.604763700
##  [44,]  0.32684406 -2.044046244  0.25791158 -0.59163620 -0.462648197
##  [45,] -0.50998687  1.664005803 -2.57653582 -0.64537383 -0.951589385
##  [46,]  1.54218192 -0.235342751  0.24332962  1.27792541 -0.002963238
##  [47,]  0.12365171 -0.696613335  1.03379257  0.28196810 -0.486795882
##  [48,] -0.28082791  0.661476518  1.40002894 -1.37269158  1.922194050
##  [49,]  0.19194026 -0.003765585 -0.81782027 -0.64493872 -0.255313383
##  [50,] -0.62157825 -2.653524029 -1.33957207  0.80840494  0.977680255
##  [51,]  2.22787895 -0.389368671  1.35525718  0.43997305 -0.913364937
##  [52,]  0.66150346  1.393819267 -0.85276016  1.93986824  1.242712512
##  [53,] -0.59454309 -0.938695408  0.68378968 -0.26542844 -1.484822138
##  [54,] -0.99813439 -1.002338003  0.93118104 -0.74213000 -0.670680446
##  [55,]  1.01279096  0.942874646 -0.98400636  1.16673342  0.367200388
##  [56,]  1.91841068  1.711873920 -0.07920480  0.23937069  0.575468340
##  [57,] -0.20342566  0.289978696 -1.27326932 -0.46294952 -1.358707298
##  [58,]  0.39239193 -0.331241590 -1.60694202  0.73959116  0.332915440
##  [59,]  0.59750298 -0.389059137 -1.66786454  1.18284919 -0.120073227
##  [60,] -0.83227560  0.510552568 -1.27292283 -0.31986976 -0.422751629
##  [61,]  0.60957227  0.216070764  1.87047971  1.33034853  0.424767837
##  [62,]  1.26881062  1.522059479  0.35080758 -2.08827096 -1.034537272
##  [63,] -1.09623562  1.153007281  0.07020569  0.52842129  0.498946970
##  [64,] -0.54702661 -0.906102818  0.13951579 -0.13599184  0.503189237
##  [65,] -1.24772481 -0.359207922 -1.18785774 -0.06655669  0.952387189
##  [66,] -0.47580674 -0.380498346  0.80324247  1.07946305  0.283765055
##  [67,] -0.79094312  0.378436560 -0.67496134  0.18199515 -0.988991942
##  [68,]  0.59502529 -1.557657979  0.43605584 -0.35043819  2.727346775
##  [69,]  1.37413553  0.546270070  1.70816709  0.22201072 -2.631426220
##  [70,] -0.34479887 -1.611667222  0.77647672  0.07816061  0.604717073
##  [71,]  1.05866651 -1.072556138  0.89514424 -2.03457143 -1.976691349
##  [72,]  0.71842456 -0.256653551 -0.03666363  1.34293842 -0.622618113
##  [73,] -1.20896080  0.529024286  0.83330152  0.58256433  0.708102993
##  [74,]  0.40644706 -1.791535011  1.40598638 -0.48201446  0.551348550
##  [75,]  0.71464287  0.904177405  0.28182263 -0.17363390  0.702890924
##  [76,] -0.63240465 -0.146630106 -2.86882847 -0.95694495 -0.291434977
##  [77,]  0.89848105  1.772077443  0.16372409  0.19208913  0.727065401
##  [78,] -1.05312347 -1.044073037  1.14214627 -0.35220711 -0.177636197
##  [79,] -0.31796734 -0.356608225  0.67125780  0.31676366  1.268958885
##  [80,]  1.91580793  0.653650235  0.51447179  0.06007512  0.638313771
##  [81,]  0.26736524  0.038015617  0.76677652  0.50143388  0.913218825
##  [82,] -0.50639711 -0.363909016  0.18271854  0.63933049 -2.011387156
```

```
##  [83,]  1.73203673 -0.664491572 -1.16605324 -0.52532837  1.164862971
##  [84,] -0.45226288  0.688615877 -0.70221484 -0.20699960  0.123451813
##  [85,]  1.83641292 -1.848877153  0.58602094  0.44065668 -1.849443265
##  [86,]  0.27891882 -1.276504991  0.83693913  1.06380073  1.050164249
##  [87,] -1.46526286  0.660096241  1.48468285 -0.72915754  0.087355794
##  [88,]  0.74075658 -0.997379524 -0.19933325 -0.79618497  0.011242069
##  [89,]  0.25622044  0.620527287  0.35210607  1.08436119  0.031804871
##  [90,]  0.48677304  2.421084389 -0.82262843 -0.08943306 -1.640532826
##  [91,] -0.89375720 -0.986953657  0.57928961 -0.27193233  1.136388721
##  [92,]  1.65695370  0.932477833  1.02258307 -0.73288609  1.738160941
##  [93,] -1.17420220 -0.689029049  1.47562687  2.19276979 -1.940606532
##  [94,]  0.21535972 -0.105282752 -2.07053175 -0.75752657  0.550426689
##  [95,] -0.69950244 -0.860664674  0.92753951 -0.54516022  0.412057866
##  [96,]  0.36579624 -0.628991016  1.61990751  0.69243031 -2.000354458
##  [97,]  1.06619712  1.163307056  0.30302184  0.95965539  1.550202641
##  [98,] -0.15271017 -2.193107142  2.47856081  0.34783171 -0.649069537
##  [99,]  0.03319772  0.229884180 -0.10204767 -0.30182564 -0.230362911
## [100,]  0.20933603 -0.806508841  0.37132954  0.16681112 -0.393975686
##              [,6]          [,7]         [,8]        [,9]       [,10]
##   [1,]  2.04186259 -0.2342730663 -2.804630994  0.64109322  0.59377459
##   [2,] -1.40722166  0.6005584362  0.824472009 -0.50672646 -0.08120781
##   [3,]  1.41078935  0.8024619365 -1.932903120 -0.74414301 -0.95862457
##   [4,]  1.59352330  1.1648421233 -1.285800820 -1.51393631 -0.62758281
##   [5,]  0.24807597 -0.2180380127  0.752202425 -0.31480504  0.81998947
##   [6,] -0.51199725  2.1185429772  1.542237519  0.01723986  0.20498948
##   [7,]  0.75971207  3.9293018886 -0.400444403  0.34653228  1.23207051
##   [8,] -1.61817598  2.3901961737  0.097013690 -1.43417081 -1.06772854
##   [9,]  0.41682907 -1.3259568753 -0.111371476 -0.34264016 -0.15847498
##  [10,] -2.55283988 -0.7285243720  1.494530919 -0.44062718 -1.22556350
##  [11,]  0.32908117  0.6544500911  0.147203706  2.63822246 -0.20108028
##  [12,]  1.15274790  1.5483241258  1.219974350 -1.82823724  1.05151868
##  [13,]  0.71004642  0.2534842263 -1.883614269  0.48320585 -0.26196976
##  [14,] -0.41546353  0.7007010394 -0.771813435 -0.54321954 -0.71726548
##  [15,] -1.36435122  1.9995454855 -1.321472068 -0.20790295 -0.30556332
##  [16,]  0.10145940  0.6703678048  0.967939307 -0.37752329 -0.99934106
##  [17,]  1.47161005  1.0938482868 -1.242794174  0.99602331  1.41576747
##  [18,] -1.13207251 -0.2585174819 -0.603347151 -0.71018189 -0.44292154
##  [19,]  0.51465236  0.4877493147  0.884878743  0.20794064 -0.50895289
##  [20,]  0.38229668 -1.2720073208  1.404752307 -0.07540626  0.07407732
##  [21,]  0.20421519  0.1837051697 -1.321873715 -0.14714847 -0.16782147
##  [22,]  0.26642943  0.2504797857  0.708134774 -0.24953449  0.33992676
##  [23,]  0.06217772  1.6325842620 -0.064725920  1.61857035  0.39979238
##  [24,] -2.03239912  0.9871143986 -0.076292288 -0.64361886 -0.33187318
##  [25,] -1.71672537 -1.7491697614 -0.241802730  0.20539716 -0.32336584
##  [26,] -0.45213696 -0.2810358384  0.174191559  0.07627880  2.03599664
##  [27,]  0.91529579  1.4890624362 -1.728027936  0.41599784  0.61615392
##  [28,] -1.43594886 -0.2285825123 -1.769526966 -0.45358118 -1.29822105
##  [29,]  0.72833743 -0.3432136624 -0.873656978 -0.91505024 -0.81274301
##  [30,] -0.63958486 -1.7199514315  0.287686116 -0.23909955  1.20125306
##  [31,]  0.15070655  1.1695872190  0.895040346 -0.63222353 -0.55527128
##  [32,]  0.75120781 -0.0006997679  0.412028451 -0.28212790 -0.72341715
##  [33,]  0.50626929 -0.8043146780  0.231472618 -0.34902370 -0.84514901
##  [34,] -0.81350589 -0.1252313295  0.177419210 -1.39175727  0.11533388
##  [35,]  0.82802779 -1.1826542598 -0.127785341 -0.66647480 -0.63586483
```

```
##  [36,]  0.46624037  1.6463678494 -0.008869827 -0.44782176 -0.20947380
##  [37,]  0.02717969 -1.7637164135  0.609193230 -0.57015835  0.41058782
##  [38,]  0.63231885 -0.2680868799  1.075848918 -0.49990587  1.01142988
##  [39,]  1.58006590 -0.6890503999 -0.818175283 -0.81193777 -0.66033757
##  [40,] -1.74448146 -0.6811357109  1.512483971  0.52515183  0.13942560
##  [41,]  0.69407424 -1.4924103176  0.152999145 -1.84967911 -0.70400479
##  [42,] -0.16359003 -0.3955096249 -0.555780686 -1.17001250 -1.06375816
##  [43,] -1.38821483  0.1449295932 -1.072369702  0.30792127  0.40796265
##  [44,] -0.24582881 -0.2223687783 -1.180337229  0.10951104  1.01436166
##  [45,] -0.30618447 -0.5387666199  1.118309958  1.47388188 -1.54454258
##  [46,] -0.73636430  0.2592388489 -0.572808591 -0.47993622  0.01873225
##  [47,] -1.37730554  0.1329294263  0.863037093  0.87064440 -0.54296360
##  [48,]  1.74599364 -0.8258307107 -0.751518078  2.07552718  0.33161386
##  [49,]  0.75427812 -1.5939865170 -0.851086620  0.41480071 -0.52571886
##  [50,]  0.71032436 -0.4762867598  1.516172577  0.84984760  0.47453827
##  [51,]  1.06234330 -0.4247417974 -0.192910525  1.50412252  1.18719797
##  [52,] -2.61881518 -0.1281424043 -1.190475667  0.10360692  2.59868597
##  [53,]  1.16480513  1.6055263529 -0.267572497  1.42142349  0.28769030
##  [54,] -0.23151303  0.4489845300  0.310721051  0.76191294 -0.68323085
##  [55,] -0.17342901 -0.8669270455 -0.271081000 -0.13287899  0.44594785
##  [56,]  2.54946097  0.0613702741 -0.169729964 -1.14753960 -0.23130974
##  [57,]  0.47457549 -0.2801336083 -0.123378621  0.80938166  0.15747913
##  [58,]  0.40615920  0.6584201625 -0.438156550 -1.37612618  0.24018434
##  [59,]  1.79225832 -0.2755777076  0.012057151 -0.53315964  0.80027523
##  [60,] -0.22529519 -0.8151696460 -1.661167471 -0.90636758  0.35305701
##  [61,]  1.46642316  0.8702300930 -1.296932914  1.13684699  1.59222095
##  [62,]  0.84117772  1.4350293532 -2.112113539 -1.01787824  1.66217854
##  [63,]  1.39458229 -0.8114042697  0.483311907 -0.35533927  0.24745855
##  [64,] -0.07072813 -1.9661662310  0.872896465 -0.04136017 -1.53412539
##  [65,]  0.06879811  0.6397406417  0.926542043  0.58607396 -0.24292391
##  [66,] -0.22225391  0.9758111870 -1.012945058  1.39568153 -0.19569207
##  [67,]  0.59237454  2.2401784243  0.320972753 -0.54653518 -1.26621833
##  [68,] -0.18145483 -1.7815260956 -0.564876551  0.09818641 -0.52272670
##  [69,]  1.49141681  1.1548731342  0.541366566  0.13528743  0.43147579
##  [70,] -2.48467747 -0.5421615832  0.625134272  1.27897650  1.35239763
##  [71,] -0.27984073 -1.4636835415 -1.060358644 -0.44666662  1.65199961
##  [72,] -0.89705630  2.2208372928 -0.043189070 -0.80449595 -1.08687303
##  [73,]  1.02384259  0.1037493802 -0.744350907 -0.34911521  1.21053952
##  [74,]  1.04918497  0.0797320337  0.929796885 -1.32931632  0.68321972
##  [75,]  0.96351568 -0.2876983374  0.243708328 -1.95024102  0.39610830
##  [76,]  0.84674545 -0.6013144553 -0.970857237 -2.48165439  0.87475393
##  [77,] -0.59176065  0.8545485355  0.020602193  0.60655657 -1.96963003
##  [78,]  0.42636840 -1.1797638796  0.520560346 -1.00907051 -0.67390788
##  [79,]  0.59740468 -0.7840654969  0.440011612  1.91222531 -0.59380238
##  [80,]  0.66570314  0.6305186978 -0.163529593  1.25916527 -0.97355275
##  [81,] -0.85669262 -2.2567294633  0.022074285  0.01143627 -0.05318336
##  [82,]  0.59875872  0.1490371441  0.679025301  1.04004535  1.02503223
##  [83,] -2.16755871  0.1057386775 -0.709875914 -1.11706713 -2.57469862
##  [84,] -0.39858216  2.0275669083  0.254827118 -1.60076127 -0.28824513
##  [85,]  1.09459870  0.2255875535  0.579827515  1.71929883 -2.14791637
##  [86,] -0.07798745  2.4488685332  0.961907725 -0.44278707  1.66829187
##  [87,]  1.11769063 -0.2551213255  1.087696469  1.29707852 -0.41817348
##  [88,]  2.67236336 -1.8452387730 -1.955722778 -0.66937153 -1.38415043
##  [89,]  1.67929134 -0.5862000035 -0.123011629  0.61152763  0.08602936
```

```
##  [90,] -0.59707806  0.0656066425 -0.637289173 -0.38622441  1.60067900
##  [91,]  0.83501330  0.8694474794 -0.503463111  2.07506544  0.67865718
##  [92,] -1.52004498  0.5995098073  0.969965726  0.76043619 -1.92554123
##  [93,]  0.27080458 -0.7749532217 -0.764065245  0.35222160 -0.28176851
##  [94,]  0.70255223 -0.0598975181 -0.082040480 -0.77145756  0.19810797
##  [95,] -2.80615683 -1.7789462139  0.700101915 -2.03729709 -1.28350898
##  [96,]  2.07223286 -0.2403334340  0.068401752  0.07697870 -0.05766763
##  [97,]  0.67019431  0.2500044154 -0.155643501  2.32291501 -0.23657377
##  [98,]  0.42536897  1.2935720922  1.950909353  0.67844992 -0.72009091
##  [99,]  0.60996012  0.3334279501 -1.007894001  0.26883244  0.77295494
## [100,]  0.11842221  1.1341512481  2.172081277 -0.88519852  0.47439962
```

Write a function `all_angles` that measures the angle between each of the pairs of vectors. You should measure the vector on a scale of 0 to 180 degrees with negative angles coerced to be positive.

```
angle = function(u,v){
  acos(sum(u*v)/sqrt(sum(u^2)*sum(v^2)))*(180/pi)
}
all_angles = function(X){
  A = matrix(NA, nrow = nrow(X), ncol = nrow(X))
  for(i in 1:nrow(X)-1){
    for(j in (i+1): nrow(X)){
      A[i,j] = angle(X[i,],X[j,])
    }
  }
  A
}
all_angles(X)
```

```
##        [,1]     [,2]     [,3]     [,4]     [,5]      [,6]     [,7]      [,8]
##   [1,]   NA 114.2651 68.29577 86.60337 99.02640 105.42689 83.81153 105.81187
##   [2,]   NA       NA 75.66951 99.82513 88.07019  56.28014 87.07716  44.56397
##   [3,]   NA       NA       NA 49.27104 91.55784  83.33639 74.75552  62.64702
##   [4,]   NA       NA       NA       NA 77.64771  89.09316 63.27676  72.65795
##   [5,]   NA       NA       NA       NA       NA  82.59129 80.39059  92.06846
##   [6,]   NA       NA       NA       NA       NA        NA 51.13376  46.80891
##   [7,]   NA       NA       NA       NA       NA        NA       NA  65.45747
##   [8,]   NA       NA       NA       NA       NA        NA       NA        NA
##   [9,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [10,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [11,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [12,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [13,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [14,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [15,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [16,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [17,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [18,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [19,]   NA       NA       NA       NA       NA        NA       NA        NA
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##  [22,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [23,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [24,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [25,]   NA       NA       NA       NA       NA        NA       NA        NA
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##  [37,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [48,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [74,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [78,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [79,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [80,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [82,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [83,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [85,]   NA       NA       NA       NA       NA       NA       NA       NA
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## [100,]   NA       NA       NA       NA       NA       NA       NA       NA
##             [,9]     [,10]     [,11]     [,12]     [,13]     [,14]     [,15]
##   [1,] 91.67734 122.28400  91.88608  84.65139  64.54961  98.80620  94.56852
##   [2,] 89.12049  54.05513  93.52073  85.36998 132.54349 103.70771  84.49968
##   [3,] 87.67495 110.88766  96.63567  99.24240  94.17983  80.87939  71.41007
##   [4,] 95.04499 121.35397  97.75275  93.72531  85.36259  61.81385  62.29859
##   [5,] 62.55756  80.71534 109.98150 101.26845 112.14307  76.34599  98.88706
##   [6,] 113.74322 80.81762  88.51157  69.34606 123.86531  88.22293  67.44572
##   [7,] 112.40492 117.84722 78.56859  72.72161  87.36887  77.16147  56.23712
##   [8,] 101.46113 71.13238 104.91132  83.22304 110.77038  70.22635  50.33557
##   [9,]       NA 74.67003 118.92086 103.37645  80.73562  77.42099 120.59119
##  [10,]       NA       NA 104.32845  98.66965 117.01141  91.38634  99.08308
##  [11,]       NA       NA       NA  99.10665  87.34025 119.71714  90.82940
##  [12,]       NA       NA       NA       NA  92.40211 109.45832 101.89189
##  [13,]       NA       NA       NA       NA       NA  69.87356  84.60501
##  [14,]       NA       NA       NA       NA       NA       NA  52.26524
##  [15,]       NA       NA       NA       NA       NA       NA       NA
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##  [32,]       NA       NA       NA       NA       NA       NA       NA
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##  [43,]       NA       NA       NA       NA       NA       NA       NA
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##  [65,]       NA       NA       NA       NA       NA       NA       NA
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##  [68,]       NA       NA       NA       NA       NA       NA       NA
##  [69,]       NA       NA       NA       NA       NA       NA       NA
##  [70,]       NA       NA       NA       NA       NA       NA       NA
##  [71,]       NA       NA       NA       NA       NA       NA       NA
##  [72,]       NA       NA       NA       NA       NA       NA       NA
##  [73,]       NA       NA       NA       NA       NA       NA       NA
##  [74,]       NA       NA       NA       NA       NA       NA       NA
##  [75,]       NA       NA       NA       NA       NA       NA       NA
##  [76,]       NA       NA       NA       NA       NA       NA       NA
##  [77,]       NA       NA       NA       NA       NA       NA       NA
##  [78,]       NA       NA       NA       NA       NA       NA       NA
##  [79,]       NA       NA       NA       NA       NA       NA       NA
##  [80,]       NA       NA       NA       NA       NA       NA       NA
##  [81,]       NA       NA       NA       NA       NA       NA       NA
##  [82,]       NA       NA       NA       NA       NA       NA       NA
##  [83,]       NA       NA       NA       NA       NA       NA       NA
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##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##           [,16]     [,17]     [,18]     [,19]     [,20]     [,21]     [,22]
##   [1,]  80.43569  63.58311  82.59337  92.06259 100.44056  82.81117  95.74841
##   [2,]  82.86834 123.47281  87.20744 101.38317  86.63334 107.60881  76.81048
##   [3,] 100.88603  81.59180  98.33158 110.07316  90.83518  86.56666  99.15875
##   [4,] 104.16894  70.70258 102.50046 107.45716  95.53809  75.70340 115.12560
##   [5,]  99.08170  98.79573 127.80321 129.25689  60.75144 118.71138 107.91662
##   [6,]  68.98400  93.66426  95.58529  77.26458  82.75419 126.20346  87.94228
##   [7,]  88.09765  52.92136 107.25877  90.06165 112.57533  89.39485  97.12192
##   [8,]  80.33540 107.94957  78.92506  95.63447 100.37054 100.10350  90.46525
##   [9,] 114.80591 107.03659 122.30915 115.51483  77.28237  87.97624  65.41453
##  [10,]  73.38946 156.85810  76.72698  95.60881  77.06842 111.68258  83.36020
##  [11,]  84.60481  68.57712  95.29879  73.75928 107.47452  75.42281  98.97388
##  [12,]  58.67799  83.96754  81.50555  55.34757  99.65477  90.33196  60.04038
##  [13,] 105.11447  57.50495  86.89935  80.06430 130.02660  42.06519  77.99318
##  [14,] 113.64368  85.80572  95.67048 104.50416  94.87275  86.05899 101.15402
##  [15,]  98.98706  74.78470  72.66824  98.23534 112.96209  83.64058 115.57748
##  [16,]        NA 110.14222  64.06621  58.83396  89.66046 108.71038  94.16738
##  [17,]        NA        NA 105.12427  84.62974 113.38191  66.51754  93.20228
##  [18,]        NA        NA        NA  74.19356 106.05361  81.88814 100.16277
##  [19,]        NA        NA        NA        NA  96.43584  88.55389  65.81943
##  [20,]        NA        NA        NA        NA        NA 143.25097  99.60254
##  [21,]        NA        NA        NA        NA        NA        NA  79.40323
##  [22,]        NA        NA        NA        NA        NA        NA        NA
##  [23,]        NA        NA        NA        NA        NA        NA        NA
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##  [66,]       NA       NA       NA       NA       NA       NA       NA
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##  [68,]       NA       NA       NA       NA       NA       NA       NA
##  [69,]       NA       NA       NA       NA       NA       NA       NA
##  [70,]       NA       NA       NA       NA       NA       NA       NA
##  [71,]       NA       NA       NA       NA       NA       NA       NA
##  [72,]       NA       NA       NA       NA       NA       NA       NA
##  [73,]       NA       NA       NA       NA       NA       NA       NA
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##  [79,]       NA       NA       NA       NA       NA       NA       NA
##  [80,]       NA       NA       NA       NA       NA       NA       NA
##  [81,]       NA       NA       NA       NA       NA       NA       NA
##  [82,]       NA       NA       NA       NA       NA       NA       NA
##  [83,]       NA       NA       NA       NA       NA       NA       NA
##  [84,]       NA       NA       NA       NA       NA       NA       NA
##  [85,]       NA       NA       NA       NA       NA       NA       NA
##  [86,]       NA       NA       NA       NA       NA       NA       NA
##  [87,]       NA       NA       NA       NA       NA       NA       NA
##  [88,]       NA       NA       NA       NA       NA       NA       NA
##  [89,]       NA       NA       NA       NA       NA       NA       NA
##  [90,]       NA       NA       NA       NA       NA       NA       NA
##  [91,]       NA       NA       NA       NA       NA       NA       NA
##  [92,]       NA       NA       NA       NA       NA       NA       NA
##  [93,]       NA       NA       NA       NA       NA       NA       NA
```

```
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,23]     [,24]     [,25]     [,26]     [,27]     [,28]     [,29]
##   [1,]  89.99514 104.88808  91.02138  92.19893  60.06846  80.35211  90.08605
##   [2,] 105.23201  87.92597  66.72086 108.52157 104.68195  73.16064 113.94888
##   [3,]  98.01993 106.74122  97.95064 131.95346  87.13993  67.37487  83.74595
##   [4,]  97.15788  89.90106 133.37313 105.85601  89.55462  89.04023  61.35142
##   [5,] 122.53902  68.30996 108.96904  63.74332 120.02091 127.65701  88.45501
##   [6,]  76.64098  74.25088 108.45985  97.18294  99.82156 101.56799 131.20824
##   [7,]  69.10100  78.91520 132.48638  83.71772  67.08761 106.08743 102.24493
##   [8,]  89.91423  65.82968  92.32353 117.10093  97.09784  65.38004 100.80153
##   [9,] 110.50155  87.91375  63.92911  88.00143  97.88915 100.70103  61.82812
##  [10,] 113.33207  63.28176  55.30105  92.69586 121.50718  78.04123  98.85186
##  [11,]  72.95206 117.88934  98.75511  95.46465  71.69622  92.42156 103.85093
##  [12,]  84.33343  98.46769  98.15003  83.08486  65.78279 105.42565  93.86097
##  [13,]  62.38135  89.10070  88.16534  86.47254  44.80804  80.77361  55.53453
##  [14,]  75.94379  46.02468 106.53465  91.78730  99.63556  85.49709  68.49051
##  [15,]  66.96638  59.10871 112.76695  97.68999  87.23606  65.54289  97.30394
##  [16,] 102.65730  85.77396  94.21065  98.02989  87.87037  88.22791 105.28618
##  [17,]  54.28414 105.59118 121.27705  75.87354  50.97179 104.89488  88.97447
##  [18,]  89.17620  79.15397  74.37588  95.26018  85.47271  45.25370  96.46124
##  [19,]  56.19109 103.39509  92.87209 102.70505  72.92447  95.35361  98.11398
##  [20,] 110.12148  94.21533  92.56810  89.12100 146.95189 114.47448 107.76010
##  [21,]  78.38269 103.90857  84.39153  94.06410  44.51679  68.58447  47.24373
##  [22,]  71.93564 105.71158  57.75129  95.86893  67.25848  95.51753  85.45569
##  [23,]        NA  90.03721  97.06469  92.08630  65.17186  91.15391 103.40752
##  [24,]        NA        NA  97.19363  70.66581 106.08253  88.09835  91.36075
##  [25,]        NA        NA        NA  99.09740  88.97761  61.27497  92.37710
##  [26,]        NA        NA        NA        NA  88.28915 122.37030  97.13127
##  [27,]        NA        NA        NA        NA        NA  81.97889  76.87453
##  [28,]        NA        NA        NA        NA        NA        NA  84.71499
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##           [,30]     [,31]     [,32]     [,33]     [,34]     [,35]     [,36]
##  [1,]  93.54377 105.85008 102.17280  77.04221 101.45963  82.17514  83.21129
##  [2,] 108.25647  44.04310  96.07917  94.90031  95.90490  80.34958  61.67450
##  [3,] 117.75507  53.49681 102.50019 100.72554 126.23595  83.03160  39.86166
##  [4,] 102.25093  67.86587  94.25137 118.14321 101.61654  92.96009  59.27170
##  [5,]  78.82141  89.04179 102.50380 121.16027  96.04224 103.32259  91.66567
##  [6,] 106.36012  49.92723 108.67107 109.58769  97.76546 113.37517  45.24519
##  [7,] 113.27605  69.82516 103.32813 125.24103 103.08084 114.44738  51.12031
##  [8,] 117.65350  46.37687 103.27964 104.53452  89.42044 102.86514  42.29385
##  [9,] 101.38598 104.61351  66.24373  74.92351  97.16696  87.00785 108.33335
## [10,]  87.30959  85.70103  86.09144  78.39877  69.69745  85.98624 104.14619
## [11,]  99.11966  82.79825  83.86980  98.17877 113.98769  74.60708  91.74452
## [12,]  99.59690  79.46157  70.53567  67.19709  62.95018  78.57037  77.13492
## [13,] 103.27498 131.05351  68.81822  73.08791  84.37212  97.55901 110.33264
## [14,]  95.46673 100.83276  99.15954 109.90357  86.07359 136.47020  84.61834
## [15,]  95.30309  77.97383 122.38911 126.68840  90.50771 125.21447  61.16665
## [16,]  94.58235  78.92268  85.73906  67.81915  68.18002  71.07847  82.60076
## [17,]  93.96545 101.96845  96.39920 108.19762 108.66041 105.77563  81.65344
## [18,]  71.46235  97.77622 103.06338  72.61285  52.15500  80.66029  96.19667
## [19,]  99.75307  90.69682  65.52056  55.15882  76.93338  87.01399  90.56893
## [20,]  62.95119  81.88034 105.68004  93.13665 101.50616  94.35851  90.67672
## [21,] 106.66018 105.61224  60.04870  79.53308  82.16088  72.08230 104.06699
## [22,] 116.57795  91.22777  50.50282  50.16594  85.77400  83.78858  93.50920
## [23,] 101.10657 100.35552  89.96032  89.58454  98.91294 124.86756  84.14177
## [24,]  81.59518 104.29001 109.22240 109.60994  62.59790 131.16919  95.07554
## [25,]  93.47742 100.94088  77.25265  53.38251  84.36075  72.19519 110.68135
## [26,]  51.81639 125.99364 101.83257 107.75751  65.05931 106.20674 122.89218
## [27,] 117.59622 102.11595  68.64406  74.05852  88.04845  80.03697  88.24523
## [28,]  98.83309  83.81078  98.67276  77.92105  84.81886  77.37185  81.55397
## [29,] 102.71818 105.16158  51.21111  76.13034  77.00007  73.56783 107.66340
## [30,]        NA 116.57362 119.04995 101.78156  67.30796  96.30255 120.79411
## [31,]        NA        NA  94.13576 103.51951 111.76312  77.00180  34.06569
## [32,]        NA        NA        NA  50.88472  80.37011  62.05723 107.69738
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## [34,]        NA        NA        NA        NA        NA  86.69278 116.07230
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##           [,37]     [,38]     [,39]     [,40]     [,41]     [,42]     [,43]
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##   [2,]  81.21767 108.00860 101.60487  65.31200  83.57739  76.84369  97.28418
##   [3,] 101.66137 106.43853  68.16590 105.35449  65.00776  78.91421  88.50461
##   [4,] 129.48947  92.10517  54.19319 126.03995  70.52958  94.17590  85.69327
##   [5,] 116.68710  53.32145  72.88426  79.18128  93.57993 120.78347 116.47116
##   [6,] 103.86299  76.19566 121.50935  74.19990 106.40927 110.07231  88.63454
```

```
##   [7,] 128.61197  83.83979  96.54860 108.63370 120.39840 117.82718  85.49483
##   [8,] 105.89889 101.40165 103.60690  79.77700  85.32673  78.08236  80.64252
##   [9,]  83.11004  88.76983  67.99836  65.61455  69.39285  69.64862 123.31717
##  [10,]  77.17646  91.36677 104.47976  49.65340  88.73106  74.66904 102.01701
##  [11,]  98.36800 117.34193  88.70445 111.55624 121.05654 109.37537  86.61571
##  [12,]  66.35809  75.80358  97.24229 102.57495  91.24183  78.13888 107.33632
##  [13,]  94.01372 105.01434  83.40426 108.40090  99.66092  73.38550  80.39133
##  [14,] 129.12456  82.30212  93.41174  79.81190  83.17289  90.71431  69.49075
##  [15,] 129.01466  98.76315 108.47240  96.51807 100.97413 100.84953  43.11326
##  [16,]  73.66918  74.60094  98.50788 100.56656 106.57396  87.30009 109.02328
##  [17,] 108.90009  90.72116  88.75424 120.44664 109.68023 113.21407  72.59478
##  [18,]  67.38850  97.54828 110.10556  97.14973  88.82356  69.20008  59.09955
##  [19,]  68.12005  90.67898 112.28242  93.95709  95.69628  79.74346  88.80702
##  [20,]  81.22363  53.92550  91.29255  63.60295  65.15974 108.11841 101.39459
##  [21,]  92.37466 133.12765  65.99029 123.48287  88.91771  62.12777  82.07376
##  [22,]  55.30724 103.95416 101.21811  68.09742  79.81010  55.09142 108.83748
##  [23,]  96.00864  99.92396 127.21740  83.38096 106.75361  96.21528  56.63509
##  [24,] 117.26950  67.99005 111.82272  69.17042 107.98945 100.26989  76.57725
##  [25,]  45.43151 114.36410 101.70031  56.99937  78.28534  50.27005  94.45352
##  [26,]  94.25951  57.83516  99.97005  86.86659 117.07488 121.02800  87.94292
##  [27,]  83.72968 113.13648  86.73590 120.21830 111.95458  74.74275  90.72709
##  [28,]  78.55033 128.54913  94.23658  93.33148  77.35430  53.43730  62.58150
##  [29,]  98.61047 105.30343  42.41554 114.39112  66.32257  59.59943 104.97204
##  [30,]  81.23696  59.37804 100.17938  85.22873  87.03478 113.81173  68.05449
##  [31,]  97.56933 103.78749  83.24721  93.96765  75.13609  87.75328  98.93738
##  [32,]  74.66083 111.66675  65.39677  98.42384  78.94285  54.73641 125.48331
##  [33,]  36.23399  99.11608  90.21447  84.44928  73.32735  42.47474 110.18502
##  [34,]  74.47284  75.74650  98.82385  92.32778  90.52386  73.46294  85.96866
##  [35,]  59.28951 113.03858  53.01422 116.72056  74.13766  62.92420 119.90430
##  [36,] 105.41375  94.13862  93.08189  97.35227  83.06067  92.93733  86.49763
##  [37,]        NA  93.25819 100.40202  77.08554  71.67808  57.02684 100.23545
##  [38,]        NA        NA 100.95033  80.43711  96.13248 122.27906  98.99336
##  [39,]        NA        NA        NA 126.82277  68.18047  79.03007 121.32690
##  [40,]        NA        NA        NA        NA  85.28595  87.51645  87.80032
##  [41,]        NA        NA        NA        NA        NA  56.77901  92.08791
##  [42,]        NA        NA        NA        NA        NA        NA  98.54496
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##  [89,]        NA        NA        NA        NA        NA        NA        NA
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##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
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##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,44]     [,45]     [,46]     [,47]     [,48]     [,49]     [,50]
##   [1,]  64.11407 119.29652  64.51860 106.36329  75.87098  75.23559  82.90280
##   [2,]  83.58615  85.68810  92.47410  61.40715 135.58897 117.36560  72.21229
##   [3,]  91.43866  89.62792  94.12703 104.80451 104.25034  85.37185  99.53789
##   [4,] 106.35150  79.78962  97.34283 127.79170  99.42069  81.73521 124.72518
##   [5,] 111.17514  56.19600  91.00266 118.95565 110.62179  80.74079  77.42055
##   [6,] 107.55988  89.04268  78.81086  65.41076 121.27836 146.53323  84.82092
##   [7,]  92.84282  94.05032  84.81698 100.10441  98.05103 121.53178  99.77216
##   [8,] 102.17387  90.76685  75.94457  74.04933 135.55304 129.99738 102.42386
##   [9,] 101.53229  73.79477 111.94691 119.70447  81.88237  67.79478  62.58742
##  [10,]  97.94895  72.50779  80.17381  59.32581 121.55601  98.06586  71.98248
##  [11,]  70.27298  76.69195 109.58124  66.59598  70.69869  80.26725  86.12458
##  [12,]  76.81852 132.97184  80.45578  92.95347  96.82387 122.99690  73.86515
##  [13,]  81.70635 107.51089  92.07140 112.42145  43.18264  75.52468 103.45431
```

```
##  [14,] 132.50875  76.24892  83.89702 110.67727  89.00807  97.28301 127.03977
##  [15,] 101.69252  91.14477  70.56314  82.61209 104.78069 115.69707 141.42635
##  [16,]  80.58027 103.68987  55.34949  65.60506 112.98772 103.83309  72.49440
##  [17,]  79.57946 105.94941  98.60608 112.53617  53.99514  90.73538 106.21221
##  [18,]  68.55009 122.72556  48.69754  61.46392 103.65795  99.47833 112.69601
##  [19,]  94.23975 112.84390  90.17319  65.20741  68.93412 111.88791  85.33758
##  [20,] 120.48159  68.17119  94.12240  88.52132 100.65621  82.42607  77.42526
##  [21,]  63.82707 104.19593 107.43983 108.40575  62.91539  71.56475 107.16565
##  [22,]  84.44909 113.94886 112.96366  87.62670  72.33280 109.81861  57.54347
##  [23,] 100.36006 106.14423  97.95186  75.19435  53.07334 118.42676 107.05435
##  [24,] 114.35054  80.46624  56.96381  88.32325 112.25681 110.77275 108.81988
##  [25,]  69.34743 101.05168  92.88124  67.77959  87.89434  84.87120  63.83343
##  [26,]  79.37010  95.70241  78.88960 104.69407  83.87361  91.39842  87.42649
##  [27,]  53.64718 126.07655  90.93323 100.43834  63.38923  94.70555  85.68195
##  [28,]  70.58701 105.32731  72.21805  68.87722 102.07673  87.67874 110.86893
##  [29,]  94.36534  84.34241 107.95769 132.79427  74.84351  57.49897 102.45999
##  [30,]  87.65244  92.38895  72.22261  88.43829  90.64352  78.08901 106.12021
##  [31,]  95.87854  80.76240 101.21964  76.73391 127.45936 110.80754  86.29072
##  [32,]  88.21347  90.06500 123.73494 101.55376  68.21405  78.51772  68.23820
##  [33,]  77.97622 116.83005  90.09607  81.25852  72.61688  84.20619  63.23734
##  [34,]  81.98459 115.04880  61.69841  89.00313 102.45929 101.91297 100.30336
##  [35,]  52.70133  96.25865 101.27279  89.89935  96.19477  60.83857  66.94727
##  [36,]  98.47446  95.33719  85.84859  86.55038 117.45717 120.18051  96.18755
##  [37,]  63.12577 125.85158  86.10729  72.71010  83.96425  90.03384  59.97192
##  [38,] 111.33674  91.66991  62.83417 104.36656  99.34796  98.63093  84.36368
##  [39,]  83.50249  71.87746 112.68782 132.55463  89.69811  43.15187  87.59125
##  [40,] 114.06952  78.99083  91.92524  66.54289  96.14950 110.37959  71.95734
##  [41,] 105.51714  92.05723 108.18335 105.79105  94.93888  77.32325  98.57548
##  [42,]  78.37187 112.29167  95.67352  89.70201  89.68239  86.90605  83.88843
##  [43,]  92.08033 103.01345  75.27667  72.12868  82.85759 104.58111 142.64623
##  [44,]        NA 121.65039  80.29772  84.45628  89.76854  80.58432  75.33553
##  [45,]        NA        NA 114.25504  90.97851  95.71318  67.16799  89.44254
##  [46,]        NA        NA        NA  78.40465 115.57304 105.25225 100.04133
##  [47,]        NA        NA        NA        NA 101.35196 113.60105  84.40364
##  [48,]        NA        NA        NA        NA        NA  70.77945  90.98595
##  [49,]        NA        NA        NA        NA        NA        NA  87.94414
##  [50,]        NA        NA        NA        NA        NA        NA        NA
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##  [66,]        NA        NA        NA        NA        NA        NA        NA
##  [67,]        NA        NA        NA        NA        NA        NA        NA
```

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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,51]     [,52]     [,53]     [,54]     [,55]     [,56]     [,57]
##   [1,]  57.31884  83.80714  74.12439 103.06625  75.47094  72.62338  92.70954
##   [2,] 119.28951  88.96074  82.82835  63.76625 109.76522 138.04532  89.22438
##   [3,] 101.73026 102.37172  65.47067  80.94612  98.42913  89.18750  73.56497
##   [4,] 101.99667 104.00056  79.65793 101.20433  90.59317  58.57584  69.18384
##   [5,] 102.07825  68.64335 109.86634 132.18755  48.26623  74.95137  60.33262
##   [6,]  92.44329  85.79260  69.67417  79.87958  99.49475  98.86307  95.87864
##   [7,]  84.59515  83.61921  54.90846  91.26219  98.51029  80.18589  81.25815
##   [8,] 121.84980  85.81639  84.14403  77.70314 103.50032 107.17205 101.71399
##   [9,] 126.59499  82.33921 118.99225 108.56432  72.86539  91.74957  86.98562
##  [10,] 118.84687  80.30968 123.26142  86.53582  83.02379 120.31049 101.91867
##  [11,]  63.44227 116.36499  43.18531  48.31511 122.50968 107.35218  61.57812
##  [12,]  82.33443 102.52460  80.62165  88.62795 108.63678  76.29662 121.66234
##  [13,]  86.75137  88.60665  90.36815  95.21809  93.48129  73.26368 107.02831
##  [14,] 117.80096  68.22230 114.88167 117.22424  68.61536  69.15281 100.43055
##  [15,]  95.69848  71.49227  81.23774  89.37749  91.53574  90.85855  94.24930
##  [16,]  70.51637 106.71417  84.62244  84.92163  93.22661  82.20749 106.75334
##  [17,]  61.28973  87.52331  55.21444  91.76783  97.38360  70.79574  80.24637
##  [18,]  76.50227  84.22560 101.77750  83.64683  93.15878  96.93830 123.23607
##  [19,]  71.15644 120.03735  75.59566  65.35090 120.05097  80.37791 124.15263
##  [20,]  85.25003  88.73871 108.64323 107.82673  62.00448  79.05883  77.84558
```

```
##  [21,]  98.85913 104.86232  80.82575  74.44157 117.98710  91.60186  92.62936
##  [22,] 110.57368  98.94579  88.98842  71.17980 118.30873 106.80727 122.70065
##  [23,]  78.06547  87.32055  68.88090  71.45173 111.61279  91.26367 112.85080
##  [24,] 107.80464  49.32234 125.59754 122.82192  57.31398  83.67381 107.16554
##  [25,] 107.61087  82.49000 107.03552  72.24013  97.65013 133.70291 110.16479
##  [26,]  72.50683  51.29339 109.51999 126.61376  59.76680  80.60278  88.24113
##  [27,]  78.58459  97.67163  60.25408  74.88066 117.55626  90.16234 103.62879
##  [28,] 100.85096  89.98143  90.51708  67.99092 104.14946 115.46674 105.20812
##  [29,] 116.96949 108.72032 109.03691 101.49311  92.36251  65.90188  89.61294
##  [30,]  62.61070  64.42725 118.72433 119.11891  54.96671  78.88619  87.56025
##  [31,] 106.02513 115.80510  60.28849  60.97321 118.77115 105.24414  74.67900
##  [32,] 112.00377 129.58213  91.35025  71.91308 121.21934  86.77519 100.52246
##  [33,]  91.14416 113.01691  99.88945  76.27835 105.50908  91.00972 127.22627
##  [34,]  93.33260  78.02359 128.11664 110.02018  81.02015  78.60621 127.94440
##  [35,]  87.13036 129.03015  79.30886  67.27585 113.19956  99.76055  77.38129
##  [36,]  96.63509  99.80075  55.23017  75.17766 107.03159  91.84535  85.27202
##  [37,]  80.30490  97.98884  97.52438  76.70182 100.77871 105.57044 119.52803
##  [38,]  69.66128  65.91584 112.49472 142.21493  40.95759  52.39579  95.57582
##  [39,] 102.57445 119.23015  88.66449  96.33577  91.33653  70.83248  56.16753
##  [40,] 111.88974  62.86757 116.24617  92.47299  76.42092 116.97158 107.54688
##  [41,] 115.05255 108.09877 108.77958  90.59776  93.75729  82.06921  96.24406
##  [42,] 121.79624 108.54711 103.13963  71.95095 114.22007 102.74041 122.19581
##  [43,]  77.22854  67.66558  89.15561  85.72808  89.99310  96.46920 102.35788
##  [44,]  69.39155  89.35307  70.71569  74.33267 105.62098 112.88366  87.56364
##  [45,] 108.00698  97.37249  95.69586  90.51914  79.17380  93.45801  46.31909
##  [46,]  69.23764  60.22284 104.49798 115.73222  59.56953  79.47516 113.56104
##  [47,]  79.90408  92.93560  79.30281  52.10074 109.75151 129.19304 103.27174
##  [48,]  72.16878  99.36126  81.29582  83.91189  98.69418  77.84154  93.99287
##  [49,]  83.78714 104.46609  98.78230  98.34495  75.77640  78.88681  56.77629
##  [50,]  90.81897  97.60436  83.98981  82.23577  93.40566 107.35970  87.24136
##  [51,]        NA  89.22329  67.97598  93.09744  81.11903  74.18272  84.21309
##  [52,]        NA        NA 115.76266 126.68161  47.94799  97.06356  99.74722
##  [53,]        NA        NA        NA  50.68474 128.31327 100.82845  69.39509
##  [54,]        NA        NA        NA        NA 154.18397 129.19037  87.53188
##  [55,]        NA        NA        NA        NA        NA  65.58872  85.09183
##  [56,]        NA        NA        NA        NA        NA        NA  92.96118
##  [57,]        NA        NA        NA        NA        NA        NA        NA
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```

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##  [75,]        NA        NA        NA        NA        NA        NA        NA
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##  [82,]        NA        NA        NA        NA        NA        NA        NA
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,58]     [,59]     [,60]     [,61]     [,62]     [,63]     [,64]
##   [1,]  75.49961  60.77375  84.85283  44.01838  82.46374  89.21875 101.41386
##   [2,]  84.24164 100.44085  88.67149 117.56073 115.41358 105.88859  83.49056
##   [3,]  76.19229  76.66829  65.03752  81.39105  85.22473  75.49918  99.29833
##   [4,]  68.59744  76.05897  61.82983  90.09545  51.76601  74.66821 115.25647
##   [5,]  52.93184  48.22568  69.36452 107.74546  91.46817  74.01521 100.93229
##   [6,]  83.34222  91.12955 121.04257  85.48573  98.80117  98.59083 114.16215
##   [7,]  71.23673  80.44501  97.46819  65.98570  58.94353  99.88526 152.42915
##   [8,]  73.43896 103.71993  88.74696 103.19902  94.21812 104.88023 104.81817
##   [9,]  68.82657  69.64208  62.35977 109.87274 114.76989  59.22680  64.88719
##  [10,]  86.81216 104.42208  94.73888 137.09025 132.62552 108.44360  57.29391
##  [11,] 124.78232 107.50800 110.21912  84.05104  82.09777 115.70662  94.07320
##  [12,]  71.80953  77.09462 116.90196  76.09010  82.65924  92.42991  99.95251
##  [13,]  92.81807  99.00346  83.10832  64.81876  69.27096  88.67997  97.13189
##  [14,]  77.29854  98.60455  72.51845  91.15097  77.22560  74.16183 106.96075
##  [15,]  91.30944 115.50337  82.98443  79.17216  61.37629 107.26160 130.28826
##  [16,]  77.90469  79.98003 127.78316  91.51196 104.71843 119.40572  81.43690
##  [17,]  96.94651  84.53386  90.19327  39.25806  46.42279  82.51647 133.83625
##  [18,] 100.56278 116.98738  97.42253  87.02475  85.40159 123.38357  83.55986
##  [19,] 115.47847 109.03611 150.36929  73.72364  98.12128  92.79610  78.70453
##  [20,]  93.21444  68.11869  95.52468 100.40096 118.37741  53.99714  67.97090
##  [21,]  95.39058 109.40182  68.40267  87.93857  58.13831 101.76152  93.79803
##  [22,]  93.37954  97.71806 105.18079  87.37654 112.65941  75.20089  72.46783
##  [23,] 124.35679 123.37928 119.81855  52.96378  80.29682  85.81424 109.42032
##  [24,]  68.41058  96.74276  86.74811 101.10683  87.93011 104.88326 107.29640
##  [25,] 102.28849 108.10258  84.43995 104.49531 130.11708  96.34484  49.59117
##  [26,]  79.19971  76.35736  85.54940  81.99796  69.20346  95.77475 113.13665
##  [27,]  86.44874  93.03790  94.01020  58.33170  66.00299 106.99053 110.32608
```

```
##  [28,] 100.12990 121.45469  74.13372  94.72700  93.15548 114.60341  78.61014
##  [29,]  72.66717  85.31674  58.64004 108.38495  72.36649  76.81978  77.38695
##  [30,] 101.52626  87.86736  84.40964  88.11107  79.34848  87.16850  89.27872
##  [31,]  84.38075  88.72217  93.90820 105.22681  96.57778  91.58148  94.05598
##  [32,]  89.38885  93.61203  97.52518 109.55807  98.73597  82.94796  60.99781
##  [33,]  96.30007  93.43047 109.29284  89.70678 119.99644  85.13975  45.39015
##  [34,]  76.23636  98.57903  94.28308 102.89669  79.32272 109.42090  86.51039
##  [35,]  87.35595  79.77156  82.07924 109.90955  96.18754 102.19958  59.07845
##  [36,]  76.80874  82.61249  93.80057  77.51303  86.13106  86.60504 114.92526
##  [37,] 101.56160  91.24365 104.07097  85.80888 118.59839  86.82249  53.27924
##  [38,]  66.51048  52.15782 102.25329  76.48231  90.92052  72.01174 102.57062
##  [39,]  67.14180  62.08138  53.69156 110.95578  74.62980  78.23345  79.77458
##  [40,] 100.11552 103.09209  99.88877 102.34802 136.69947  76.44649  70.60438
##  [41,]  90.23608  88.19388  67.96059 106.80821  98.84015  49.78809  60.57055
##  [42,]  86.71209 105.49578  79.87950 105.68317 106.10380  88.64642  56.13271
##  [43,] 121.53443 130.29021  87.97321  69.04033  67.09628  98.29007 111.02171
##  [44,]  88.63052  88.06093  80.64768  78.98218  77.62889 123.92480  94.19792
##  [45,]  94.28147  88.66953  80.05091 125.89532 102.76096  83.68140  78.54233
##  [46,]  69.39391  81.69945  97.15673  73.86944  87.24740 117.36805 105.10928
##  [47,] 126.46220 128.32011 127.34570  95.59071 119.06576 121.09645  74.68754
##  [48,] 120.77432  99.92753  99.48460  61.92119  82.78485  69.80074  84.42411
##  [49,]  92.33939  74.32363  59.77771 102.56382  87.77893  80.86573  64.94166
##  [50,]  78.20356  63.74502 105.06147  96.25491 130.68943  87.77833  66.84218
##  [51,] 107.44901  79.18930 114.60922  48.40943  76.28971  99.88436 103.77659
##  [52,]  76.27898  86.93937  74.02559  77.17145  85.64308  95.03970 115.18657
##  [53,] 104.23187  90.91734 106.65817  64.03030  77.84854 101.07365 111.76279
##  [54,] 127.29287 126.16902 109.90014  96.14706 103.00923 107.46558  74.57630
##  [55,]  65.50673  59.21656  73.04415  86.16300  92.07211  78.02124  96.09153
##  [56,]  69.65092  59.77814  91.79212  72.89040  65.35454  64.82381 102.39201
##  [57,]  89.35321  71.05225  67.01547 102.92268  79.52632  88.66239  97.37347
##  [58,]        NA  44.25020  67.27408  97.04204  82.11288  89.99194 106.76396
##  [59,]        NA        NA  78.06154  81.21376  89.17574  70.48952  98.02635
##  [60,]        NA        NA        NA 105.57191  70.69337  81.48471  95.06112
##  [61,]        NA        NA        NA        NA  72.13051  81.80528 121.18585
##  [62,]        NA        NA        NA        NA        NA  97.55989 136.33055
##  [63,]        NA        NA        NA        NA        NA        NA  79.54583
##  [64,]        NA        NA        NA        NA        NA        NA        NA
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##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
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##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
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##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,65]     [,66]     [,67]     [,68]     [,69]     [,70]     [,71]
##   [1,] 110.57756  60.17966 101.67364  76.03015  80.08354  99.13819  84.44727
##   [2,]  68.38849  86.53113  68.75052 108.32014  98.70912  65.15699  96.01457
##   [3,]  93.16314  71.22751  54.96549 118.60276  76.69044 119.18372  97.90736
##   [4,] 101.65311 101.10124  49.79541 127.16161  68.10403 150.57478  90.86908
##   [5,]  70.33411 116.39405  82.50731 107.52042 101.15283 105.44467 105.20283
##   [6,]  74.37983  75.97401  52.96759 128.78994  66.56245  84.50838 115.24116
##   [7,]  81.81620  70.99094  50.94697 130.27191  63.80584 104.75575 101.68579
##   [8,]  81.36434  76.46117  44.03777 120.07284  86.71104  92.80077 112.07752
##   [9,]  52.80395 100.86197  98.44221  64.74577 145.39182  93.51959 114.08898
##  [10,]  70.74244 101.78623  94.67297  74.51001 123.47131  61.89412 104.14667
##  [11,]  88.16385  75.29346  86.60061  99.83521  67.55309  78.82144  69.50158
##  [12,]  89.98137  98.07942  82.72247  85.20896  73.78134  90.32323  87.33370
##  [13,]  93.82235  69.44498 102.19714  58.54946 105.62598  99.51190  92.77233
##  [14,]  87.07817  85.42102  72.34568  99.22958 103.69496 115.94134 123.50073
##  [15,] 107.94700  67.94036  63.57980 123.83562  72.71567  98.78185  95.67354
##  [16,]  98.33228  89.90728  81.57017  82.18905  71.11662  88.53880  92.09727
##  [17,] 100.83344  69.12145  88.93788 103.63094  64.64682 102.92792  79.11534
##  [18,] 135.17345  84.41266 107.05646  73.66506  81.24098  73.48882  65.96703
##  [19,]  92.40302  81.03915  89.19687  75.38881  69.28654  85.83741  96.37723
##  [20,]  88.70754 113.22027  98.01346 100.14244  83.06914  95.81689 100.60007
##  [21,]  96.24654  86.44831  93.01808  71.90255 101.78502  96.73967  67.85491
##  [22,]  55.67752  81.95044  95.36636  62.32461 116.20766  69.27145 106.28172
##  [23,]  88.16514  54.13884  89.13122  91.93983  77.80713  79.80098 103.07872
##  [24,]  87.96359  90.62538  84.38162  91.98003 107.41482  88.43024 114.88471
##  [25,]  77.13461  78.99207 119.42962  48.66095 133.90806  46.66224  88.77962
##  [26,]  95.89216 109.30555 123.34056  82.15053  96.69747  72.86748  72.16816
##  [27,]  89.43949  62.57257  90.02712  73.26989  89.28030  86.72926  79.81098
##  [28,] 112.34298  67.13086  90.02507  80.66270  97.71418  79.66593  78.86461
##  [29,]  87.37597 112.73418  84.90768  70.23549 112.59771 125.58809  89.47203
##  [30,] 128.75222 117.20065 133.97506  86.11336  80.36909  78.50401  58.14569
##  [31,]  78.84408  90.27763  40.00761 134.96691  66.03273 100.92119  96.05019
##  [32,]  61.03684 105.34609  85.32350  59.89205 112.03170  97.90051  96.85693
##  [33,]  85.40591  86.83246 108.90756  36.36083 108.09249  80.04670  94.64392
##  [34,] 109.69680 117.27534 108.73840  64.52252  99.60771  81.71148  76.02149
```

```
##  [35,]  95.14805 108.79864  95.12240  73.79903  89.68392  90.49432  57.58380
##  [36,]  86.59924  68.73732  37.56096 136.98975  60.10593 108.42383 106.72410
##  [37,]  96.44048  90.99240 124.71095  50.80997  99.88582  60.11552  74.89861
##  [38,]  98.45287 107.00111  99.85527  94.39834  77.85996 100.91170 102.47760
##  [39,]  87.83963 118.82392  77.33435  91.16315  94.20489 130.29870  76.64854
##  [40,]  62.12974  82.37481 102.67386  80.32678 121.69292  56.43387 121.71382
##  [41,]  97.23026 113.65970  87.50226  89.11423  95.11694 111.02690  89.42888
##  [42,]  85.03758  88.81747  90.67260  56.51549 118.51579  87.63107  92.86511
##  [43,] 127.12416  70.77369 100.43360 103.13372  73.06919  79.06636  75.43687
##  [44,] 106.19693  81.19736 112.37592  72.99718  89.74299  63.50547  44.24192
##  [45,]  65.19811 104.83004  73.18777 108.77555  99.58609 103.35501 106.66804
##  [46,] 122.13794  77.44316  98.37011  84.25088  80.45856  84.15571  88.87719
##  [47,]  91.85264  69.39616  94.34630  89.50198  81.83985  48.41145  88.94382
##  [48,]  87.63328  76.07140 114.35284  64.21323  94.63321  90.31523  88.73340
##  [49,]  98.79556 106.81241 109.53585  71.62155 102.00590 107.42952  70.45637
##  [50,]  51.35599  88.13172  97.96508  68.83734 110.19166  68.52082 101.84181
##  [51,] 125.02805  78.47669 114.02531  89.67322  49.68015  84.07511  63.46082
##  [52,]  95.93585  79.48188 112.98182  88.16365 108.54360  66.45229  94.34789
##  [53,]  86.26797  59.91964  65.17678 117.11367  54.27020  91.90237  81.69715
##  [54,]  78.59783  69.80647  75.95254  94.60045  81.13110  71.01196  82.21430
##  [55,] 102.08595 100.57057 110.67015  84.93607 100.52803  97.59391  98.97285
##  [56,] 108.63515 108.67494  84.38240  92.12342  69.22461 141.86859  97.11111
##  [57,]  83.69290 101.30512  78.44089 120.32497  79.05817 104.82579  75.63560
##  [58,]  80.48977 103.20488  71.35233  93.40645 101.79613 113.06519 103.49322
##  [59,]  84.89334 104.02865  86.49312  94.15583  85.34909 115.14594  95.35927
##  [60,]  96.31523 104.42006  92.56293  94.08992 112.49191 104.47151  74.93394
##  [61,] 109.50650  52.23340 100.20803  91.08265  63.98244  90.25925  88.70847
##  [62,] 119.85492  99.34382  84.00799 110.61358  64.67467 113.09303  60.07090
##  [63,]  79.55387 101.99215  90.72544  92.98636  92.82827 115.14907 110.14054
##  [64,]  79.78671 104.61091 107.67600  54.41065 115.07632  80.92805  93.53000
##  [65,]        NA  85.09578  70.57955  87.27702 122.48755  82.66115 129.67376
##  [66,]        NA        NA  83.93748  88.97998  87.97930  74.11254 108.83159
##  [67,]        NA        NA        NA 131.55443  73.03205 123.12834 117.12984
##  [68,]        NA        NA        NA        NA 128.11714  70.43219  88.68386
##  [69,]        NA        NA        NA        NA        NA 108.77101  72.20911
##  [70,]        NA        NA        NA        NA        NA        NA  78.19819
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
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##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
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##  [83,]        NA        NA        NA        NA        NA        NA        NA
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##  [88,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [89,]        NA        NA        NA        NA        NA        NA        NA
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,72]     [,73]     [,74]     [,75]     [,76]     [,77]     [,78]
##   [1,]  88.70023  67.26026  83.34024  90.73208  91.21275 102.13945  96.39103
##   [2,]  65.40271 102.56957  89.93836 115.69580  86.53085 106.83364  76.10278
##   [3,]  73.12514  68.89904 103.27577  97.58091  76.28094  97.21638  81.16530
##   [4,]  76.83777  78.09376 103.24775  67.99568  61.22012  80.98435  97.09102
##   [5,]  91.92939  96.73040 113.48567  79.49587  55.06355  92.23729 119.37660
##   [6,]  45.83990  95.16503  88.73246 100.03394 109.28479  81.04187 105.44670
##   [7,]  59.22110  78.90095  95.24713  92.86703  89.65644  83.46431 128.60469
##   [8,]  33.48317  95.27920  99.10636  98.35068  90.95927  76.50016  92.60775
##   [9,] 114.82604  76.13394  99.86728  84.41284  58.89725  98.85696  82.47322
##  [10,]  76.41750 127.89029  97.28368 103.79850  92.54638  80.63454  81.82499
##  [11,]  98.73656 110.92377  93.14480 128.94906 113.08152  90.99454  96.33172
##  [12,]  75.64787  77.82840  26.85579  61.81136  87.49124 109.90227  75.25367
##  [13,] 103.62387  68.25428  89.82911  82.29699  93.08552  74.80221  98.53871
##  [14,]  74.96349  77.69935 124.22283  72.10612  85.58392  45.18963 114.86944
##  [15,]  52.71410  87.48213 117.60920  94.98199 102.17939  59.18145 117.50816
##  [16,]  55.18630 123.06529  62.60987  89.18456 105.80584  83.78738  85.39923
##  [17,] 102.06639  55.66871  89.10469  86.68628  94.89870  94.43996 113.11040
##  [18,]  69.79108 104.47670  74.70248  85.76749 108.95062  83.25434  75.35218
##  [19,]  84.34141  91.52217  50.98054  82.32365 130.87604  77.08781  70.70193
##  [20,] 103.30477  91.97312  95.30114  78.92961  90.33720  94.15698  79.33922
##  [21,] 105.19345  79.87754  82.92390  87.99561  75.83766  94.00166  81.39025
##  [22,] 102.92240  64.31344  55.77620  87.48170  93.65181 108.68232  61.89812
##  [23,]  90.44872  64.96003  88.27555  96.69047 134.27917  69.10905  99.86956
##  [24,]  59.01432 105.19407 115.78348  80.98131  90.57690  51.46395 127.65863
##  [25,] 104.48835  90.09634  83.52307 111.49560  95.68430 107.16211  60.14841
##  [26,] 108.52846  88.46770  87.33815  73.66510  77.41813 101.70141 120.99353
##  [27,]  90.05329  69.59222  67.26846  94.59459  91.67044 101.60231  93.46232
##  [28,]  71.71446  95.17722  97.84081 108.17496  99.11204  84.09291  71.41029
##  [29,] 105.01160  85.23210  88.83594  62.78225  53.70643  82.95034  77.92577
##  [30,] 111.70092  95.72836  91.10485  70.59166  90.30827  96.67429  94.18692
##  [31,]  60.88852  96.68040  86.74512 102.64291  85.53241 100.06864  74.88143
##  [32,] 107.36559  92.46654  62.03314  80.14712  78.24954  93.74470  61.94123
##  [33,] 100.28224  86.20338  50.52288  82.31222 100.78501  97.77145  46.74176
##  [34,]  80.96764 103.94912  62.96850  54.93588  82.36240  85.56685  84.55555
##  [35,] 102.67429 107.06382  61.40374  94.63963  69.59355 125.03196  50.01225
##  [36,]  49.67517  76.32963  91.71421  97.43675  92.53619  91.31299  91.01636
##  [37,] 107.36169  81.24202  45.78766  86.75467  99.10112 123.11305  42.71360
##  [38,]  85.35611  86.57697  85.75721  58.10420  87.19483  85.34573 111.79667
##  [39,] 106.33510  94.14111  91.28191  77.22069  40.07276 104.39825  78.19454
##  [40,]  92.55737  87.41554 103.52646 102.64598 107.47450  82.63306  88.56486
##  [41,] 104.41653  70.23707  80.10268  63.18627  70.19747 101.92757  41.19554
```

```
##  [42,]  89.49223  81.80693  67.21345  83.04229  81.24277  97.37479  43.42029
##  [43,]  84.84792  78.42894 106.75912  93.09211 117.76654  71.99409  99.29125
##  [44,]  97.65137  91.15676  67.20464 106.41577  81.49889 131.96194  81.03564
##  [45,]  96.24638 120.07139 135.31752 108.64327  82.49334  67.73883 106.48008
##  [46,]  52.50266 101.36778  88.90872  82.86947 100.90249  77.46113 109.17731
##  [47,]  72.44760 118.10835  88.57349 126.45052 139.76648  80.00442  81.18599
##  [48,] 132.29033  65.29496  85.92070  88.93038 107.59610  84.94103  88.11455
##  [49,] 126.56586  99.79799 105.07664  91.88944  67.55152  96.38173  82.23069
##  [50,] 100.23717  94.55805  70.51188 106.90442  85.54660 118.82952  79.14963
##  [51,]  97.82255  90.38263  77.36236  92.77606 115.71805  97.44547 101.19484
##  [52,]  85.62519  78.11949 112.55622  89.64096  88.25376  87.88775 125.72099
##  [53,]  81.93557  82.91965  83.63218 119.61618 105.91879 102.99534  92.87375
##  [54,]  88.07460  99.55567  79.81487 128.05075 115.38876  96.49712  62.84529
##  [55,]  92.10582  90.82336 114.21897  72.94770  77.50583  79.72602 119.21488
##  [56,]  90.29463  78.66804  82.06829  40.01326  78.57493  73.18634 100.42048
##  [57,] 104.91438 104.92279 120.41346 111.55215  68.94055 100.84878 105.96502
##  [58,]  66.73332  87.66272  87.21862  67.64956  45.12401  98.11281 106.07128
##  [59,]  92.43298  80.62165  83.44086  71.64476  56.74320 111.78474 100.43890
##  [60,] 103.60029  78.27569 114.26298  83.65492  39.28857 105.48979  91.97155
##  [61,]  90.64082  50.64552  80.11117  87.20815 114.00656  93.84501 104.93013
##  [62,]  90.45777  77.14454  89.04360  67.65145  71.65105  91.87290 110.21102
##  [63,] 118.15331  49.69132  89.28979  62.32785  79.33362  95.01495  74.41176
##  [64,] 110.74716 107.30086  80.45009  94.57777  94.14028  92.25600  44.50413
##  [65,]  92.47786  90.82488  97.10571 108.55175  83.61826  89.33740  94.40996
##  [66,]  71.92582  73.82028 103.31085 127.09308 126.91997  78.12106 104.57019
##  [67,]  49.56902  95.39890  99.47618  94.00443  83.18345  72.68541  97.29653
##  [68,] 112.38430  90.41706  70.71791  84.72253  93.83853  92.77613  71.82289
##  [69,]  74.98419  89.79788  77.79038  83.42863 107.50637  90.18408  92.96800
##  [70,]  97.80584  97.17433  83.77505 121.20349 113.35550 107.07028  87.09330
##  [71,] 111.37664  96.38135  73.20969  87.80513  78.22882 123.66703  76.94634
##  [72,]        NA 106.93609  94.30957  95.24104 100.06015  66.23822 106.22775
##  [73,]        NA        NA  79.13847  71.04495  84.85063 108.32961  83.12391
##  [74,]        NA        NA        NA  64.44935  89.55252 119.77878  54.78892
##  [75,]        NA        NA        NA        NA  66.66801  90.04091  80.58893
##  [76,]        NA        NA        NA        NA        NA 116.08478  87.93218
##  [77,]        NA        NA        NA        NA        NA        NA 113.74264
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,79]     [,80]     [,81]     [,82]     [,83]     [,84]     [,85]
##   [1,]  78.74153  73.71563  87.54732  83.24974  99.85758 114.43375  73.87934
##   [2,] 107.88560 132.34190 100.65499  72.18412  86.11461  69.37881  92.13498
##   [3,] 104.77354 106.93682 113.31051  70.22681 105.55632  78.02810  85.23804
##   [4,] 125.45636  90.96220 126.00043  88.91086  95.70573  57.92754  96.02307
##   [5,] 107.17890  99.41572  99.66069  85.31264  91.24640  75.75187 106.74876
##   [6,] 102.48323  89.90643 121.16591  65.14707  99.62170  58.78177  81.79778
##   [7,] 111.27612  78.61672 149.91043  73.38074 104.72732  54.67535  90.92905
##   [8,] 120.54302 105.84545 115.33910  89.65495  75.16880  41.47851  94.21472
##   [9,]  75.68576 112.00320  74.37505 111.85334  85.04175  90.27245 119.04102
##  [10,]  89.58738 104.40462  67.45958 107.20318  51.20202  83.80378  90.80825
##  [11,]  72.52834  71.32302 109.89011  62.08098 101.52124 111.01982  53.36318
##  [12,] 101.05643  92.43363 105.72721  90.91746  96.52505  71.67707  88.23665
##  [13,]  70.53713  63.44204  86.32760 125.47762  80.99740  96.02389  99.77865
##  [14,] 100.36777  77.57014  93.13186 123.10751  74.13581  57.58651 120.21900
##  [15,] 117.52409  80.08700 110.90682  92.72625  82.09177  58.90435  98.39227
##  [16,]  90.51486  67.27221  94.84920  89.39893  66.24936  89.57280  44.54435
##  [17,]  86.46532  71.12935 114.73506  75.47551 125.02822  94.18463  94.55620
##  [18,] 101.61171  82.85972  65.78117 105.94389  61.63927  96.78333  77.77300
##  [19,]  63.46655  60.62780  89.02782  94.01053  92.44457  95.63314  64.60318
##  [20,]  81.70512  98.90018  68.29188  69.06539 108.99440 105.62312  87.59840
##  [21,]  93.39457  89.76612  98.30974 115.22470  79.50286  87.01522  99.85775
##  [22,]  66.65778 108.79787  84.50856 101.46401  99.63506  86.02445 108.04721
##  [23,]  65.08275  65.63089  95.45654  89.98549 109.63457  89.82509  95.30213
##  [24,] 107.84570  76.48795  86.58353 122.30635  52.97317  59.58058 109.88230
##  [25,]  67.35444 116.09792  50.43052 102.77677  76.65821 112.00005  98.23667
##  [26,]  99.66195  87.25915  81.01242  95.49781  93.27275  94.87586 114.93143
##  [27,]  83.20039  78.73934 110.05854  98.32903  93.23420  89.35519  87.55099
##  [28,]  97.29038  98.94222  76.13636 102.45022  65.16675  92.41901  83.61283
##  [29,]  97.30652  90.05160  91.75761 132.62237  68.23758  76.74267 105.24171
##  [30,]  98.59566  89.25875  53.42738  86.55151  94.51351 115.11575  99.11991
##  [31,] 113.17213 114.40430 126.25969  56.57493 102.38057  63.31963  75.00302
##  [32,]  72.45890  90.57025  94.77757 116.25685  78.15434  84.39724  88.92659
##  [33,]  57.99557  86.98832  62.05484 112.28407  77.50043 109.60554  77.54225
##  [34,] 110.72985  86.76146  71.31219 130.44231  53.70052  76.29369 102.98777
##  [35,]  92.79797 107.94953  88.80706  83.03456  83.31394 107.54180  64.33970
##  [36,] 110.83388 100.25502 130.87715  60.35414 108.64866  60.19944  80.57652
##  [37,]  69.44770 107.54169  52.91924  89.63848  94.71864 120.58175  84.31811
##  [38,]  94.73539  74.69671  81.54964  87.37817  96.96934  89.80105  94.20013
##  [39,] 104.79201 100.62983 105.61946  94.55902  85.79035  87.04744  86.12485
##  [40,]  69.19520 105.42838  60.93917  93.86654  89.61244  92.15674 111.33177
##  [41,]  97.80170 121.68763  72.33777  95.63564  98.43817  86.93735 106.21386
##  [42,]  86.15064 110.19097  74.29675 121.68826  67.94779  84.14736  97.44696
##  [43,]  97.52196  82.12642  77.70225  89.19016  95.58514  92.97642 101.53596
##  [44,]  96.58312 103.84494  89.07052  79.84234  89.95859 110.90539  79.92608
##  [45,]  85.50019  86.26477  98.26167  84.42280  82.50519  87.09274  83.80423
##  [46,] 105.26541  70.96864  78.57339  99.81533  64.43736  88.77695  78.57524
##  [47,]  73.80748  84.31310  77.28048  75.41590  81.37066 103.46658  62.76103
##  [48,]  45.24525  65.71944  77.45603  98.39919 109.73194 119.95644  92.92689
```

```
##  [49,]  78.39065  86.45714  73.46113  95.38851  84.33056 123.25879  79.60196
##  [50,]  64.54033 102.64826  87.35805  77.46538  96.32945 105.85099  78.17333
##  [51,]  76.14373  58.64464  84.18870  62.89436 110.98675 129.09940  58.18286
##  [52,] 100.46977  95.71600  72.30611  97.96672  86.40076  87.61762 124.66968
##  [53,]  84.21069  83.82529 131.43782  46.61347 121.28347  93.91585  60.47973
##  [54,]  74.16159  97.51619 102.23271  70.50377  97.38327  96.66051  65.26261
##  [55,]  94.87177  80.20425  66.65768  99.91734  81.26410  98.48877 103.06118
##  [56,]  96.07762  59.03885  96.28973 104.03436  92.34985  82.95496  88.06162
##  [57,]  98.21141  96.64271 112.08580  56.79806 106.06228  99.30549  77.65487
##  [58,] 122.26377 100.02646 109.86033 104.13294  73.19895  57.89405 102.64799
##  [59,]  98.05767  92.72797 101.31288  77.30958 102.74311  91.83426  86.32660
##  [60,] 118.84450 121.95960  88.13833 100.02960  84.87861  82.57593 121.50084
##  [61,]  74.39246  66.89248  93.63760  74.15093 122.64338 106.59302  85.91620
##  [62,] 125.15028  82.31832 115.83283  92.06878  97.59783  73.34553 103.11947
##  [63,]  76.00996 100.70788  80.20879  86.60871 124.24224  93.14345 113.06289
##  [64,]  60.69395  98.63137  51.96040 102.98767  74.27168 115.78048  76.71520
##  [65,]  71.06949 101.51217 106.68766  92.45761  90.87782  73.67489  99.89547
##  [66,]  64.46808  73.11932  96.10269  79.98204  97.76521  99.36485  77.28097
##  [67,] 111.23810  89.59045 145.37187  81.49074  87.85098  42.39266  79.66355
##  [68,]  57.81922  81.78615  48.24514 128.16103  66.17036 117.01685  92.05827
##  [69,] 103.22772  71.81012 113.94470  52.39338 116.47960  91.35151  59.88230
##  [70,]  73.89433 104.76767  65.07849  82.83089  89.28802 110.90483  95.04194
##  [71,] 109.49242 103.10171  82.34171  76.05546  97.11065 112.41364  84.93283
##  [72,] 114.83980  79.16530 116.40619  92.24171  65.31738  52.41919  73.97325
##  [73,]  84.63392 104.35545  89.35755  85.81052 130.51147  89.09450 123.40279
##  [74,]  89.82874  96.88176  88.22632  90.80387  98.39485  92.04954  83.50563
##  [75,] 110.82193  89.07179  83.81760 114.67898  89.58168  72.07771 113.23119
##  [76,] 126.53168 122.69518 102.57886 100.02026  84.51707  69.74454 115.79316
##  [77,]  80.06418  45.79192  90.76845 117.10706  61.20539  75.06683  81.63022
##  [78,]  81.95792 116.81584  69.47857  90.41237  95.11309 102.31002  84.86802
##  [79,]        NA  66.38883  66.20262  90.12379  99.86540 130.21063  73.14918
##  [80,]        NA        NA  92.81180 102.00567  77.02786 100.37148  59.41913
##  [81,]        NA        NA        NA 108.14863  80.19663 128.00063  98.89157
##  [82,]        NA        NA        NA        NA 137.25532 106.30699  68.79819
##  [83,]        NA        NA        NA        NA        NA  75.08762  83.62328
##  [84,]        NA        NA        NA        NA        NA        NA 112.04944
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,86]     [,87]     [,88]     [,89]     [,90]     [,91]     [,92]
##   [1,]  83.10270 103.65310  57.63123  53.03946  98.21367  69.61443 111.81480
```

```
##   [2,]  74.78760  98.35740 122.14171 126.35495  96.10478  98.86927 103.13375
##   [3,]  99.04716  88.61586  75.49964  74.42371  83.20691  91.88687 127.32575
##   [4,] 105.41763  93.55974  72.71362  80.58564  60.64594 109.40337 108.65738
##   [5,]  95.94924 112.61774  98.72261  83.82339  60.34976 110.56815 106.22588
##   [6,]  49.21227  89.76606 137.79900  97.73497  85.52651  91.13237  85.24624
##   [7,]  56.08607  96.46724 111.50970  90.68099  70.46770  74.28650  97.21698
##   [8,]  71.26140 104.65031 119.43741 117.43445  85.99218 106.57172  87.87684
##   [9,] 104.21932  91.61122  77.69803  86.04348 102.59271  85.77128 103.14798
##  [10,]  93.11758 107.00476 111.46888 125.09875 105.20022 112.92398  67.23307
##  [11,]  96.29221  66.17473  88.82484  93.54486  96.29706  60.22513  84.33299
##  [12,]  38.48876  94.73966  91.36496  93.99576 110.50859  84.53151  93.57780
##  [13,]  93.40301  84.72611  63.98124  80.53040 103.28242  59.47913  75.31831
##  [14,]  99.34316  95.45522  99.80837  88.15853  68.09680 104.15740  69.89881
##  [15,]  85.33742  99.95066 114.17721 103.18616  58.81201  99.82324  77.17062
##  [16,]  67.75056 108.64175  84.98324  92.35998 116.93922  99.86628  72.50923
##  [17,]  78.17651  74.99651  83.86395  66.24622  73.18882  54.72967 102.85674
##  [18,]  87.20336 109.99591  86.36459 107.41464  96.52017 109.61041  69.50682
##  [19,]  66.15160  60.01045  89.75436  82.50305 127.49370  72.23333  57.90190
##  [20,] 103.94939  78.63890  95.23915  61.77387  79.80048 115.46436  99.65578
##  [21,] 101.83907  86.53720  61.94486 107.10028  98.31515  71.10557  89.64944
##  [22,]  60.80224  68.49869  96.27374  98.41101 132.90883  58.10619  90.55893
##  [23,]  68.69207  53.90368 111.31751  81.57261  95.86251  52.31542  65.57609
##  [24,]  81.89316 123.80649 118.36523 108.98953  70.88983 113.59403  57.65614
##  [25,]  93.46690  89.97861  90.26416 108.51461 123.36289  79.23491  89.44315
##  [26,]  76.77440 111.82537 103.86900  95.10483  63.53908  90.84326  88.79336
##  [27,]  65.37624  89.94147  73.96683  93.29894 110.80188  45.02247  94.34779
##  [28,] 103.82407 101.22405  81.21797 109.65826  99.58062  97.57798  85.89946
##  [29,] 117.54026  93.28935  47.75212  93.73080  96.27202  98.17556  88.90826
##  [30,] 104.57462 101.01850  92.53479  82.33897  58.07342 118.58314  87.69954
##  [31,]  81.21967  83.05164 104.05164 100.93151  88.88254 101.31255 111.12526
##  [32,]  91.31642  71.11546  66.11428 100.81308 133.19601  73.43317  79.79610
##  [33,]  83.78156  79.04809  63.58005  84.82358 153.44424  76.84136  78.75643
##  [34,]  77.02968 119.77210  90.06036 116.91199  93.17308 116.91968  64.08088
##  [35,] 104.30415  93.72864  50.70420  99.04957 115.68109  93.28618 111.69089
##  [36,]  69.00279  87.70612 104.22252  84.44846  83.36317  90.74258 111.98497
##  [37,]  78.37680  83.85824  77.66979  87.89542 128.51293  82.38907  97.30740
##  [38,]  74.53245 107.15132  97.78953  61.70916  71.85499 109.62762  88.72266
##  [39,] 123.56346  96.20811  41.16446  84.70867  87.03888 101.51881 117.50402
##  [40,]  82.15920  81.67622 129.36093  96.53411  97.60458  89.68675  76.22747
##  [41,] 113.27885  75.27887  70.07254  80.98753  92.33324 116.27691 109.02998
##  [42,]  93.07130  88.99004  68.12562 107.49625 129.90209  90.11973  88.57062
##  [43,]  94.72654  83.35836 109.03156  93.26358  62.27502  94.57865  75.17233
##  [44,]  80.13985 109.51150  73.98838 105.98187 100.10232  72.28476 112.78234
##  [45,] 127.19220  81.18238  95.98179  91.81679  74.79769 102.64506  82.35802
##  [46,]  72.25071 135.75322  94.11902  88.80643  83.70237 109.64984  77.74246
##  [47,]  80.11424  79.29488 119.64436 108.80296 107.97393  86.04129  63.23342
##  [48,]  98.98635  51.14182  71.08505  63.50414 104.16321  48.14797  79.51338
##  [49,] 146.30516  89.74895  39.18941  73.11414  88.50672  94.50973 102.88032
##  [50,]  71.60782  89.35516  88.60894  86.92190 126.77442  66.75049 104.73499
##  [51,]  82.61504  86.77788  79.87973  58.56740  84.40016  76.97629  91.56203
##  [52,]  77.19701 120.99624 119.42223  95.20072  60.37340  94.86699  89.74610
##  [53,]  74.69783  68.29077  90.50560  81.25629  95.89043  54.00522 108.70216
##  [54,]  88.52153  56.85431  95.51982 108.27778 118.41133  65.35557  86.51996
##  [55,] 100.90840 121.24155  88.76405  68.38224  62.62919 114.39256  90.88332
```

```
##  [56,]  93.68674  93.17470  62.86890  53.05708  78.20299 104.90234  85.97324
##  [57,] 121.33995  87.73451  81.35885  82.96136  62.54845  92.14578 117.67719
##  [58,]  74.83956 137.85690  81.15815  93.27585  83.01038 107.25369 109.94249
##  [59,]  84.11556 111.60593  68.59830  59.27838  83.57209  95.40229 125.59964
##  [60,] 121.13382 113.23597  71.90686  98.55692  61.34553 107.74339 122.58509
##  [61,]  64.33878  80.65919  87.58474  52.11413  87.54390  56.83951  98.63933
##  [62,]  89.87633 100.79862  80.91129  93.03464  52.64194  94.06087  98.95233
##  [63,] 100.36505  59.84322  80.44552  50.59173  85.64871  90.35299 108.07695
##  [64,] 116.92657  73.81476  69.26182  90.13392 128.34333  97.93708  79.78320
##  [65,]  78.36804  76.41320 107.55435 100.89708 112.33900  65.81404  88.93295
##  [66,]  71.95705  81.37264 100.92077  78.81098 105.34310  48.54114  85.67318
##  [67,]  81.41844  87.33879  98.71436  96.75723  87.60795  95.53849  93.33440
##  [68,]  93.16226  93.77460  65.70814  90.85121 130.25209  76.01818  71.92045
##  [69,]  80.68168  78.70566  89.88773  68.91488  72.43773  95.79006  97.05704
##  [70,]  71.68801  91.35487 121.22417 115.71933 102.81005  71.60663  81.81221
##  [71,] 102.54518  98.78698  72.98658 101.77043  73.86721  98.04256 108.37100
##  [72,]  64.49416 116.82494 109.39291 105.17487  90.29885 105.35775  76.64809
##  [73,]  73.97042  72.78085  87.34438  63.61216  85.39461  67.24695 117.17718
##  [74,]  58.23590  84.99348  77.55753  91.33825 120.27386  83.35468  95.48611
##  [75,]  85.36374  97.86688  74.32433  77.89191  79.04140 117.69794  89.90264
##  [76,] 102.74358 117.99383  68.99433 100.77783  72.39907 111.14306 125.62635
##  [77,] 101.07818  85.46306  99.07728  87.44162  88.01140  98.84112  36.65280
##  [78,]  99.51461  67.78106  68.47717  92.20897 121.80396  96.58863 100.44020
##  [79,]  94.27955  52.73052  82.93300  63.33623 127.73657  50.29273  71.80932
##  [80,]  89.90586  84.54431  80.45439  66.59201  97.01806  78.56150  49.75493
##  [81,] 108.73287  88.89467  82.73228  82.58389  99.83278 100.83443  76.58199
##  [82,]  83.20248  73.87463 102.77892  72.91809  76.09542  79.57174 124.06960
##  [83,] 100.37632 124.65101  83.06768 122.22875 103.10878 113.36613  54.71895
##  [84,]  69.73335 105.64979 111.39266 117.50010  77.64713 107.39175  85.27620
##  [85,]  94.82202  84.20846  72.27511  76.21620 112.63789  84.98667  80.81886
##  [86,]        NA  97.89537 119.36639  96.86677 102.73208  69.60376  89.46119
##  [87,]        NA        NA  91.76499  70.28585 105.33849  63.44298  83.62554
##  [88,]        NA        NA        NA  67.00706 105.79490  91.17050 103.94777
##  [89,]        NA        NA        NA        NA  89.12237  79.01042 103.12014
##  [90,]        NA        NA        NA        NA        NA 116.00548 104.46969
##  [91,]        NA        NA        NA        NA        NA        NA  92.71405
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,93]     [,94]     [,95]     [,96]     [,97]     [,98]     [,99]
##   [1,]  63.90264  94.03632 118.43696  62.75041  70.54935  91.18532  58.31697
##   [2,]  72.94394  98.20956  63.53483  98.06630 127.64925  70.76261 116.67392
##   [3,]  48.14284  95.15885 103.10591  63.98415 105.12016  91.23582  72.19182
##   [4,]  88.59610  72.74268 110.18170  75.92144 103.60136 110.25901  62.51507
##   [5,] 109.43394  47.06460 106.96358 109.71068  88.76038 127.06466  87.51466
##   [6,]  83.32868 106.80628  99.94317  85.03935  90.33815  59.84800 104.82249
##   [7,]  98.31727  85.90082 133.12948  86.41351  82.06153  84.87946  58.42253
##   [8,]  77.87179 101.48690  75.58795  97.63014 112.24099  78.34888 106.04797
```

```
##   [9,] 102.14167  54.47222  82.73369 120.36424  89.73227 120.42530  97.25249
##  [10,]  98.46757  87.87722  43.18970 123.28796 103.52734  87.73375 147.56781
##  [11,]  89.11532 103.17220 110.23425  72.89451  80.62586  65.54479  75.38123
##  [12,]  99.30322  85.86762  91.51347  74.92054 104.06512  57.96645  93.21094
##  [13,] 106.74067  84.82468 103.45950 101.07715  62.57647 104.43754  61.31738
##  [14,] 105.52263  84.20162  92.27689 118.80567  72.53221 124.21008  86.49575
##  [15,]  84.50206 111.43489  94.69136  96.02747  87.35234  97.11727  75.70885
##  [16,]  88.04526  95.06588  85.40359  71.57610  90.26543  54.00358 115.77355
##  [17,]  91.98458  93.78664 140.94178  73.06810  67.19545  93.37069  27.89336
##  [18,]  76.12455 120.24804  54.87673  82.21914 101.90566  75.96539 104.43343
##  [19,]  93.36737 116.09570  88.16304  69.45005  74.44906  37.87395 108.40272
##  [20,]  74.49589  92.08030  86.51053  76.38435  87.26390  92.28152 115.67858
##  [21,] 105.33552  79.10253  87.69466  97.67628  99.88073 100.22345  60.61537
##  [22,]  96.46070  90.41980  76.90858 101.04619  94.53734  67.22166 103.37095
##  [23,]  91.27147 125.93317 105.51392  88.57350  57.04708  69.51618  78.25760
##  [24,] 116.00192  82.60899  80.33658 133.31990  76.97194 115.04576 103.76655
##  [25,]  76.54771 101.16352  48.21843 108.05611 100.85397  83.70247 115.75950
##  [26,] 124.82602  69.19927 100.30121 112.94619  80.31588 115.83692  72.97909
##  [27,]  97.18850  88.00663 109.96468  85.47278  83.45870  78.06484  54.47820
##  [28,]  60.37392 117.09616  57.45831  88.23097 107.50445  85.53802  96.35605
##  [29,] 115.22462  52.30507  83.56115 103.86258 100.03832 119.40215  79.77857
##  [30,]  91.79054  96.03407  78.89961  87.37245  88.96679 109.08149  90.88087
##  [31,]  65.65250  97.63500  88.78491  67.64190 123.61349  62.86359 100.91001
##  [32,] 117.83851  65.23346  80.22512  99.15439  97.73476  78.70765 101.14835
##  [33,]  85.06408  95.83667  65.95382  82.65118  89.45360  64.25993 115.52836
##  [34,] 120.02390  80.41967  57.07460 109.22470 104.08957  94.41283 107.72477
##  [35,]  80.03631  76.01965  74.62756  67.74744 125.69600  75.09997  94.45735
##  [36,]  58.56984 105.11641 106.63360  64.21658 102.92357  68.69581  83.49453
##  [37,]  70.38195 105.80772  60.57131  75.15509 103.16390  62.80502 111.88686
##  [38,]  99.37046  78.92568 106.87754  86.65457  70.00926 101.15212  94.51466
##  [39,]  97.37532  44.91203  98.58624  81.68049 111.87588 113.65187  72.08683
##  [40,]  86.95744 104.47503  66.39162 119.62041  79.66360  89.25878 127.94782
##  [41,]  69.52156  89.97273  62.57978  76.49384 119.54368  95.65943 104.89938
##  [42,]  82.70416  90.52499  49.95881  95.73796 114.97484  80.82168 109.18422
##  [43,]  74.82493 134.09859  82.38194  87.69537  82.14633  93.41543  78.27120
##  [44,]  80.07205  90.00814  87.00783  79.32831 110.23866  81.02519  67.98063
##  [45,] 102.94922  71.61340  95.97865 106.90834  82.79913 110.19521 101.26626
##  [46,]  80.78890 102.22319  85.47499  87.61498  81.24973  90.60965  95.05657
##  [47,]  71.96372 136.77390  66.10624  85.81336  88.05171  48.07282 124.87169
##  [48,]  99.27644  96.27519 109.66364  86.52677  53.60106  90.93659  68.31729
##  [49,]  89.76662  66.47416  93.79564  84.60231  88.11617 119.11065  76.62712
##  [50,]  89.89882  73.92942  93.29646  91.70253  89.48735  71.80771 105.86031
##  [51,]  75.66005 111.00470 117.93421  51.79243  65.33661  73.70542  69.50161
##  [52,]  94.89179  91.51830  89.47480 121.53688  76.57041 121.56535  81.11348
##  [53,]  68.58751 107.11238 125.31266  54.62419  87.99984  56.22698  62.81975
##  [54,]  71.55319 118.73495  79.03918  74.43700 103.58883  44.85299  99.89394
##  [55,]  95.14741  74.05210  97.61697 103.94280  69.17528 131.78623  88.38578
##  [56,] 104.83362  70.98087 115.93283  73.97217  71.27793 105.19760  75.20941
##  [57,]  86.77638  69.54613 116.53098  81.11079  95.35687 108.76708  67.33877
##  [58,] 103.86383  44.94628  99.65164 102.48370 103.25727 113.29911  82.16118
##  [59,]  88.93333  53.25223 120.68943  75.93106  88.16016 105.45656  74.43203
##  [60,]  88.37242  61.85882  83.91498 105.76157 114.96809 143.54477  65.75422
##  [61,]  70.83382 114.91596 127.92394  63.09219  57.15450  78.90139  54.21257
##  [62,] 106.02894  78.93414 112.14387  83.49199  96.93029 109.71955  39.90373
```

```
##  [63,]  82.36796  83.69508 100.30419  80.35813  81.57408 102.41249  87.95465
##  [64,]  82.31736  91.62175  52.12684  90.19361  97.78248  79.50856 133.76621
##  [65,] 109.22833  68.15553  95.69971 118.92727  84.52045  86.44559 104.34792
##  [66,]  62.58041 125.11477 108.14972  84.04314  60.07350  71.60503  77.16198
##  [67,]  88.35520  83.15636 105.04943  83.44626 100.94011  78.47104  90.46015
##  [68,] 102.59970  83.97603  67.73712 106.57989  77.98428  91.87087 104.05861
##  [69,]  70.02822 113.25691 115.61101  36.54117  89.30433  63.72565  74.49985
##  [70,]  85.92359 113.21831  66.75543 107.80512  91.07316  73.60839 105.69364
##  [71,]  84.72744  92.23073  81.51541  71.32100 117.67998  91.89984  68.72189
##  [72,]  82.74045 100.34808  88.93724  89.25252  94.50453  73.56375 102.96972
##  [73,]  74.88143  97.24758 106.81917  80.13194  83.67912  96.13065  60.89569
##  [74,]  92.13467  91.06890  80.05001  67.60342 108.64958  54.84084  97.30924
##  [75,] 109.42875  70.23040  86.04751  86.66076  97.99612 105.50316  88.29093
##  [76,] 106.54569  32.40240  89.23653 102.66241 124.79867 128.58764  73.41864
##  [77,] 105.22389 100.20937  91.47190 106.61759  55.92881  95.56948 105.09232
##  [78,]  68.19110 101.60789  53.52948  68.20623 122.38974  65.06054 114.58700
##  [79,]  83.79063 107.83759  97.51997  86.90570  47.24504  72.08901 101.34804
##  [80,] 103.55567  99.06959 116.13180  82.70478  37.80361  82.97150  85.27286
##  [81,]  80.87218 105.87801  54.81025  98.38828  81.78816  99.00853 114.46537
##  [82,]  55.01337 108.85669 114.82062  51.22906  95.38052  67.36056  77.24674
##  [83,] 113.10633  75.56788  59.42354 121.66107  94.81654 101.36432 115.63821
##  [84,] 112.02240  71.73879  87.50050 110.82470 109.82442  95.93182  92.95899
##  [85,]  71.07415 105.56520 101.49604  53.01069  79.73316  53.96969  99.79626
##  [86,]  94.80404  98.21541 100.74326  89.64461  86.94148  60.39723  89.63631
##  [87,]  80.46956 114.31558  97.28531  71.89203  75.32103  65.04313  93.85557
##  [88,]  84.52507  70.43106  95.47251  67.06281  92.94306 100.01820  74.53839
##  [89,]  67.54646  97.64144 126.24776  54.68267  54.72754  90.59380  74.25405
##  [90,]  93.30761  83.91628 104.32016  94.03057  95.09930 126.93842  63.28086
##  [91,]  88.74331  99.75067 118.50847  88.01560  62.09673  71.24323  67.08138
##  [92,] 117.13187 104.28630  74.10430 112.69883  62.67806  80.65706 119.02304
##  [93,]        NA 129.31931  88.23245  48.94559  96.74361  70.12166  90.08890
##  [94,]        NA        NA  99.96347 113.19537 101.70373 124.97240  79.13302
##  [95,]        NA        NA        NA 108.24447 122.28211  86.23036 134.23441
##  [96,]        NA        NA        NA        NA  93.72791  58.66904  78.71655
##  [97,]        NA        NA        NA        NA        NA  92.27845  82.26400
##  [98,]        NA        NA        NA        NA        NA        NA 111.11997
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##           [,100]
##   [1,] 111.53420
##   [2,]  62.37073
##   [3,] 101.13031
##   [4,]  97.92243
##   [5,]  87.53637
##   [6,]  43.56881
##   [7,]  73.31099
##   [8,]  70.01774
##   [9,] 108.90126
##  [10,]  78.43017
##  [11,]  90.60211
##  [12,]  40.71036
##  [13,] 127.09814
##  [14,] 112.22804
##  [15,]  98.26508
```

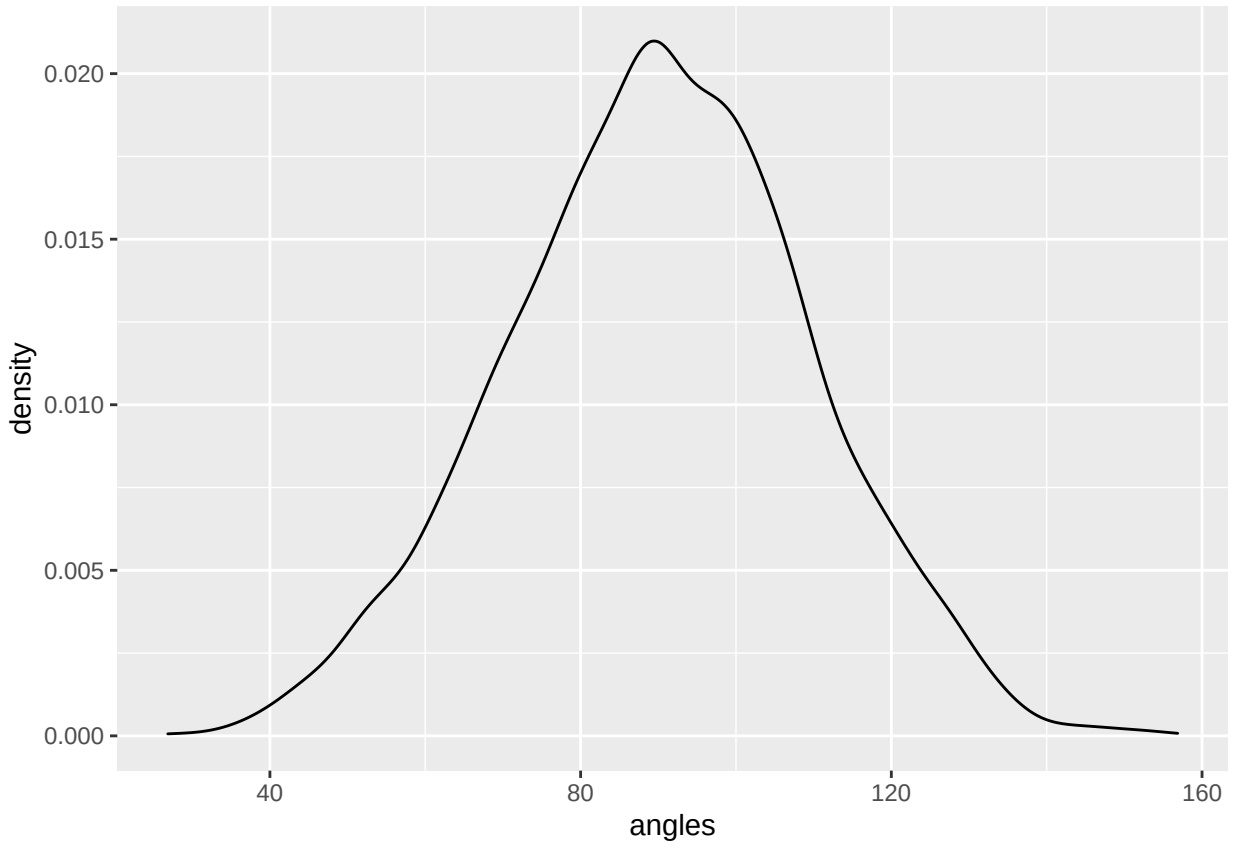
```
##  [16,]  56.85758
##  [17,] 102.53199
##  [18,]  90.39101
##  [19,]  61.74354
##  [20,]  77.62295
##  [21,] 117.01912
##  [22,]  72.24783
##  [23,]  90.72003
##  [24,]  92.91579
##  [25,] 101.12117
##  [26,]  88.46158
##  [27,]  96.67901
##  [28,] 112.65364
##  [29,] 115.20815
##  [30,]  95.32306
##  [31,]  56.64930
##  [32,]  82.93512
##  [33,]  85.45769
##  [34,]  75.88710
##  [35,]  84.73337
##  [36,]  68.18069
##  [37,]  77.46428
##  [38,]  73.22829
##  [39,] 106.00321
##  [40,]  84.16062
##  [41,]  92.17985
##  [42,]  95.05509
##  [43,] 107.39899
##  [44,]  94.17966
##  [45,] 101.75064
##  [46,]  88.12331
##  [47,]  72.26051
##  [48,] 116.05571
##  [49,] 132.15620
##  [50,]  70.89286
##  [51,]  89.02645
##  [52,] 104.19251
##  [53,]  79.22936
##  [54,]  78.98309
##  [55,] 108.74128
##  [56,]  91.49847
##  [57,] 103.60561
##  [58,]  82.54112
##  [59,]  84.29601
##  [60,] 126.36136
##  [61,]  96.45482
##  [62,]  98.70397
##  [63,]  94.22165
##  [64,]  94.09099
##  [65,]  78.79858
##  [66,] 105.29688
##  [67,]  70.80969
##  [68,] 107.30311
##  [69,]  67.80827
```

```
##  [70,]  83.24399
##  [71,]  94.65029
##  [72,]  68.59481
##  [73,]  98.33865
##  [74,]  52.46115
##  [75,]  78.82256
##  [76,]  95.64509
##  [77,] 102.72435
##  [78,]  78.65362
##  [79,] 103.44889
##  [80,]  99.32878
##  [81,] 109.56854
##  [82,]  72.15561
##  [83,]  99.14868
##  [84,]  67.63305
##  [85,]  80.90755
##  [86,]  48.24765
##  [87,]  87.30305
##  [88,] 115.62979
##  [89,] 102.20195
##  [90,] 100.18115
##  [91,]  97.66245
##  [92,]  89.05562
##  [93,]  95.19878
##  [94,]  92.95064
##  [95,]  85.44563
##  [96,]  77.56801
##  [97,] 109.07873
##  [98,]  49.89962
##  [99,] 114.10905
## [100,]        NA
```

Plot the density of these angles.

```
pacman::p_load(ggplot2)
ggplot(data.frame(angles = c(all_angles(X)))) +
  aes(x = angles) +
  geom_density()
```

```
## Warning: Removed 5050 rows containing non-finite values (stat_density).
```



Write an Rcpp function `all_angles_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
cppFunction("
  NumericMatrix all_angles_cpp(NumericMatrix X){
    int n = X.nrow();
    int p = X.ncol();
    NumericMatrix A(n,n);
    std::fill(A.begin(),A.end(),NA_REAL);
    for(int i_1=0; i_1<n-1;i_1++){
      for(int i_2=i_1+1; i_2<n; i_2++){
        double sum_sqd_u = 0;
        double sum_sqd_v = 0;
        double sum_u_v = 0;
        for(int j=0; j<p;j++){
          sum_sqd_u += pow(X(i_1,j),2);
          sum_sqd_v += pow(X(i_2,j),2);
          sum_u_v += X(i_1,j)*X(i_2,j);
        }
        A(i_1, i_2) = acos(sum_u_v/sqrt(sum_sqd_u*sum_sqd_v))*(180/M_PI);
      }
    }
    return A;
  }
")

all_angles_cpp(X)
```

```
##        [,1]     [,2]     [,3]     [,4]     [,5]      [,6]     [,7]      [,8]
##   [1,]   NA 114.2651 68.29577 86.60337 99.02640 105.42689 83.81153 105.81187
##   [2,]   NA       NA 75.66951 99.82513 88.07019  56.28014 87.07716  44.56397
##   [3,]   NA       NA       NA 49.27104 91.55784  83.33639 74.75552  62.64702
##   [4,]   NA       NA       NA       NA 77.64771  89.09316 63.27676  72.65795
##   [5,]   NA       NA       NA       NA       NA  82.59129 80.39059  92.06846
##   [6,]   NA       NA       NA       NA       NA        NA 51.13376  46.80891
##   [7,]   NA       NA       NA       NA       NA        NA       NA  65.45747
##   [8,]   NA       NA       NA       NA       NA        NA       NA        NA
##   [9,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [10,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [11,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [12,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [13,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [14,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [15,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [16,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [17,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [18,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [19,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [20,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [21,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [22,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [23,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [24,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [25,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [26,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [27,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [28,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [29,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [30,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [31,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [32,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [33,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [34,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [35,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [36,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [37,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [38,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [39,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [40,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [41,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [42,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [43,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [44,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [45,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [46,]   NA       NA       NA       NA       NA        NA       NA        NA
##  [47,]   NA       NA       NA       NA       NA        NA       NA        NA
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## [100,]   NA       NA       NA       NA       NA       NA       NA       NA
##             [,9]     [,10]     [,11]     [,12]     [,13]     [,14]     [,15]
##   [1,] 91.67734 122.28400  91.88608  84.65139  64.54961  98.80620  94.56852
##   [2,] 89.12049  54.05513  93.52073  85.36998 132.54349 103.70771  84.49968
##   [3,] 87.67495 110.88766  96.63567  99.24240  94.17983  80.87939  71.41007
##   [4,] 95.04499 121.35397  97.75275  93.72531  85.36259  61.81385  62.29859
##   [5,] 62.55756  80.71534 109.98150 101.26845 112.14307  76.34599  98.88706
##   [6,] 113.74322 80.81762  88.51157  69.34606 123.86531  88.22293  67.44572
```

```
##   [7,] 112.40492 117.84722  78.56859  72.72161  87.36887  77.16147  56.23712
##   [8,] 101.46113  71.13238 104.91132  83.22304 110.77038  70.22635  50.33557
##   [9,]        NA  74.67003 118.92086 103.37645  80.73562  77.42099 120.59119
##  [10,]        NA        NA 104.32845  98.66965 117.01141  91.38634  99.08308
##  [11,]        NA        NA        NA  99.10665  87.34025 119.71714  90.82940
##  [12,]        NA        NA        NA        NA  92.40211 109.45832 101.89189
##  [13,]        NA        NA        NA        NA        NA  69.87356  84.60501
##  [14,]        NA        NA        NA        NA        NA        NA  52.26524
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##           [,16]     [,17]     [,18]     [,19]     [,20]     [,21]     [,22]
##   [1,]  80.43569  63.58311  82.59337  92.06259 100.44056  82.81117  95.74841
##   [2,]  82.86834 123.47281  87.20744 101.38317  86.63334 107.60881  76.81048
##   [3,] 100.88603  81.59180  98.33158 110.07316  90.83518  86.56666  99.15875
##   [4,] 104.16894  70.70258 102.50046 107.45716  95.53809  75.70340 115.12560
##   [5,]  99.08170  98.79573 127.80321 129.25689  60.75144 118.71138 107.91662
##   [6,]  68.98400  93.66426  95.58529  77.26458  82.75419 126.20346  87.94228
##   [7,]  88.09765  52.92136 107.25877  90.06165 112.57533  89.39485  97.12192
##   [8,]  80.33540 107.94957  78.92506  95.63447 100.37054 100.10350  90.46525
##   [9,] 114.80591 107.03659 122.30915 115.51483  77.28237  87.97624  65.41453
##  [10,]  73.38946 156.85810  76.72698  95.60881  77.06842 111.68258  83.36020
##  [11,]  84.60481  68.57712  95.29879  73.75928 107.47452  75.42281  98.97388
##  [12,]  58.67799  83.96754  81.50555  55.34757  99.65477  90.33196  60.04038
##  [13,] 105.11447  57.50495  86.89935  80.06430 130.02660  42.06519  77.99318
```

```
##  [14,] 113.64368  85.80572  95.67048 104.50416  94.87275  86.05899 101.15402
##  [15,]  98.98706  74.78470  72.66824  98.23534 112.96209  83.64058 115.57748
##  [16,]        NA 110.14222  64.06621  58.83396  89.66046 108.71038  94.16738
##  [17,]        NA        NA 105.12427  84.62974 113.38191  66.51754  93.20228
##  [18,]        NA        NA        NA  74.19356 106.05361  81.88814 100.16277
##  [19,]        NA        NA        NA        NA  96.43584  88.55389  65.81943
##  [20,]        NA        NA        NA        NA        NA 143.25097  99.60254
##  [21,]        NA        NA        NA        NA        NA        NA  79.40323
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##  [66,]        NA        NA        NA        NA        NA        NA        NA
##  [67,]        NA        NA        NA        NA        NA        NA        NA
```

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##  [74,]        NA        NA        NA        NA        NA        NA        NA
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,23]     [,24]     [,25]     [,26]     [,27]     [,28]     [,29]
##   [1,]  89.99514 104.88808  91.02138  92.19893  60.06846  80.35211  90.08605
##   [2,] 105.23201  87.92597  66.72086 108.52157 104.68195  73.16064 113.94888
##   [3,]  98.01993 106.74122  97.95064 131.95346  87.13993  67.37487  83.74595
##   [4,]  97.15788  89.90106 133.37313 105.85601  89.55462  89.04023  61.35142
##   [5,] 122.53902  68.30996 108.96904  63.74332 120.02091 127.65701  88.45501
##   [6,]  76.64098  74.25088 108.45985  97.18294  99.82156 101.56799 131.20824
##   [7,]  69.10100  78.91520 132.48638  83.71772  67.08761 106.08743 102.24493
##   [8,]  89.91423  65.82968  92.32353 117.10093  97.09784  65.38004 100.80153
##   [9,] 110.50155  87.91375  63.92911  88.00143  97.88915 100.70103  61.82812
##  [10,] 113.33207  63.28176  55.30105  92.69586 121.50718  78.04123  98.85186
##  [11,]  72.95206 117.88934  98.75511  95.46465  71.69622  92.42156 103.85093
##  [12,]  84.33343  98.46769  98.15003  83.08486  65.78279 105.42565  93.86097
##  [13,]  62.38135  89.10070  88.16534  86.47254  44.80804  80.77361  55.53453
##  [14,]  75.94379  46.02468 106.53465  91.78730  99.63556  85.49709  68.49051
##  [15,]  66.96638  59.10871 112.76695  97.68999  87.23606  65.54289  97.30394
##  [16,] 102.65730  85.77396  94.21065  98.02989  87.87037  88.22791 105.28618
##  [17,]  54.28414 105.59118 121.27705  75.87354  50.97179 104.89488  88.97447
##  [18,]  89.17620  79.15397  74.37588  95.26018  85.47271  45.25370  96.46124
##  [19,]  56.19109 103.39509  92.87209 102.70505  72.92447  95.35361  98.11398
##  [20,] 110.12148  94.21533  92.56810  89.12100 146.95189 114.47448 107.76010
```

```
##  [21,]  78.38269 103.90857  84.39153  94.06410  44.51679  68.58447  47.24373
##  [22,]  71.93564 105.71158  57.75129  95.86893  67.25848  95.51753  85.45569
##  [23,]        NA  90.03721  97.06469  92.08630  65.17186  91.15391 103.40752
##  [24,]        NA        NA  97.19363  70.66581 106.08253  88.09835  91.36075
##  [25,]        NA        NA        NA  99.09740  88.97761  61.27497  92.37710
##  [26,]        NA        NA        NA        NA  88.28915 122.37030  97.13127
##  [27,]        NA        NA        NA        NA        NA  81.97889  76.87453
##  [28,]        NA        NA        NA        NA        NA        NA  84.71499
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##  [61,]        NA        NA        NA        NA        NA        NA        NA
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##  [67,]        NA        NA        NA        NA        NA        NA        NA
##  [68,]        NA        NA        NA        NA        NA        NA        NA
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##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
```

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## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,30]     [,31]     [,32]     [,33]     [,34]     [,35]     [,36]
##   [1,]  93.54377 105.85008 102.17280  77.04221 101.45963  82.17514  83.21129
##   [2,] 108.25647  44.04310  96.07917  94.90031  95.90490  80.34958  61.67450
##   [3,] 117.75507  53.49681 102.50019 100.72554 126.23595  83.03160  39.86166
##   [4,] 102.25093  67.86587  94.25137 118.14321 101.61654  92.96009  59.27170
##   [5,]  78.82141  89.04179 102.50380 121.16027  96.04224 103.32259  91.66567
##   [6,] 106.36012  49.92723 108.67107 109.58769  97.76546 113.37517  45.24519
##   [7,] 113.27605  69.82516 103.32813 125.24103 103.08084 114.44738  51.12031
##   [8,] 117.65350  46.37687 103.27964 104.53452  89.42044 102.86514  42.29385
##   [9,] 101.38598 104.61351  66.24373  74.92351  97.16696  87.00785 108.33335
##  [10,]  87.30959  85.70103  86.09144  78.39877  69.69745  85.98624 104.14619
##  [11,]  99.11966  82.79825  83.86980  98.17877 113.98769  74.60708  91.74452
##  [12,]  99.59690  79.46157  70.53567  67.19709  62.95018  78.57037  77.13492
##  [13,] 103.27498 131.05351  68.81822  73.08791  84.37212  97.55901 110.33264
##  [14,]  95.46673 100.83276  99.15954 109.90357  86.07359 136.47020  84.61834
##  [15,]  95.30309  77.97383 122.38911 126.68840  90.50771 125.21447  61.16665
##  [16,]  94.58235  78.92268  85.73906  67.81915  68.18002  71.07847  82.60076
##  [17,]  93.96545 101.96845  96.39920 108.19762 108.66041 105.77563  81.65344
##  [18,]  71.46235  97.77622 103.06338  72.61285  52.15500  80.66029  96.19667
##  [19,]  99.75307  90.69682  65.52056  55.15882  76.93338  87.01399  90.56893
##  [20,]  62.95119  81.88034 105.68004  93.13665 101.50616  94.35851  90.67672
##  [21,] 106.66018 105.61224  60.04870  79.53308  82.16088  72.08230 104.06699
##  [22,] 116.57795  91.22777  50.50282  50.16594  85.77400  83.78858  93.50920
##  [23,] 101.10657 100.35552  89.96032  89.58454  98.91294 124.86756  84.14177
##  [24,]  81.59518 104.29001 109.22240 109.60994  62.59790 131.16919  95.07554
##  [25,]  93.47742 100.94088  77.25265  53.38251  84.36075  72.19519 110.68135
##  [26,]  51.81639 125.99364 101.83257 107.75751  65.05931 106.20674 122.89218
##  [27,] 117.59622 102.11595  68.64406  74.05852  88.04845  80.03697  88.24523
```

```
##  [28,]  98.83309  83.81078  98.67276  77.92105  84.81886  77.37185  81.55397
##  [29,] 102.71818 105.16158  51.21111  76.13034  77.00007  73.56783 107.66340
##  [30,]        NA 116.57362 119.04995 101.78156  67.30796  96.30255 120.79411
##  [31,]        NA        NA  94.13576 103.51951 111.76312  77.00180  34.06569
##  [32,]        NA        NA        NA  50.88472  80.37011  62.05723 107.69738
##  [33,]        NA        NA        NA        NA  73.10304  61.80627 107.91135
##  [34,]        NA        NA        NA        NA        NA  86.69278 116.07230
##  [35,]        NA        NA        NA        NA        NA        NA  96.98126
##  [36,]        NA        NA        NA        NA        NA        NA        NA
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##  [46,]        NA        NA        NA        NA        NA        NA        NA
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##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
```

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##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
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##  [89,]        NA        NA        NA        NA        NA        NA        NA
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##  [92,]        NA        NA        NA        NA        NA        NA        NA
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##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,37]     [,38]     [,39]     [,40]     [,41]     [,42]     [,43]
##   [1,]  78.08158  81.37686  83.10856 113.19718  98.21709  88.74127  91.98791
##   [2,]  81.21767 108.00860 101.60487  65.31200  83.57739  76.84369  97.28418
##   [3,] 101.66137 106.43853  68.16590 105.35449  65.00776  78.91421  88.50461
##   [4,] 129.48947  92.10517  54.19319 126.03995  70.52958  94.17590  85.69327
##   [5,] 116.68710  53.32145  72.88426  79.18128  93.57993 120.78347 116.47116
##   [6,] 103.86299  76.19566 121.50935  74.19990 106.40927 110.07231  88.63454
##   [7,] 128.61197  83.83979  96.54860 108.63370 120.39840 117.82718  85.49483
##   [8,] 105.89889 101.40165 103.60690  79.77700  85.32673  78.08236  80.64252
##   [9,]  83.11004  88.76983  67.99836  65.61455  69.39285  69.64862 123.31717
##  [10,]  77.17646  91.36677 104.47976  49.65340  88.73106  74.66904 102.01701
##  [11,]  98.36800 117.34193  88.70445 111.55624 121.05654 109.37537  86.61571
##  [12,]  66.35809  75.80358  97.24229 102.57495  91.24183  78.13888 107.33632
##  [13,]  94.01372 105.01434  83.40426 108.40090  99.66092  73.38550  80.39133
##  [14,] 129.12456  82.30212  93.41174  79.81190  83.17289  90.71431  69.49075
##  [15,] 129.01466  98.76315 108.47240  96.51807 100.97413 100.84953  43.11326
##  [16,]  73.66918  74.60094  98.50788 100.56656 106.57396  87.30009 109.02328
##  [17,] 108.90009  90.72116  88.75424 120.44664 109.68023 113.21407  72.59478
##  [18,]  67.38850  97.54828 110.10556  97.14973  88.82356  69.20008  59.09955
##  [19,]  68.12005  90.67898 112.28242  93.95709  95.69628  79.74346  88.80702
##  [20,]  81.22363  53.92550  91.29255  63.60295  65.15974 108.11841 101.39459
##  [21,]  92.37466 133.12765  65.99029 123.48287  88.91771  62.12777  82.07376
##  [22,]  55.30724 103.95416 101.21811  68.09742  79.81010  55.09142 108.83748
##  [23,]  96.00864  99.92396 127.21740  83.38096 106.75361  96.21528  56.63509
##  [24,] 117.26950  67.99005 111.82272  69.17042 107.98945 100.26989  76.57725
##  [25,]  45.43151 114.36410 101.70031  56.99937  78.28534  50.27005  94.45352
##  [26,]  94.25951  57.83516  99.97005  86.86659 117.07488 121.02800  87.94292
##  [27,]  83.72968 113.13648  86.73590 120.21830 111.95458  74.74275  90.72709
##  [28,]  78.55033 128.54913  94.23658  93.33148  77.35430  53.43730  62.58150
##  [29,]  98.61047 105.30343  42.41554 114.39112  66.32257  59.59943 104.97204
##  [30,]  81.23696  59.37804 100.17938  85.22873  87.03478 113.81173  68.05449
##  [31,]  97.56933 103.78749  83.24721  93.96765  75.13609  87.75328  98.93738
##  [32,]  74.66083 111.66675  65.39677  98.42384  78.94285  54.73641 125.48331
##  [33,]  36.23399  99.11608  90.21447  84.44928  73.32735  42.47474 110.18502
##  [34,]  74.47284  75.74650  98.82385  92.32778  90.52386  73.46294  85.96866
```

```
##  [35,]  59.28951 113.03858  53.01422 116.72056  74.13766  62.92420 119.90430
##  [36,] 105.41375  94.13862  93.08189  97.35227  83.06067  92.93733  86.49763
##  [37,]        NA  93.25819 100.40202  77.08554  71.67808  57.02684 100.23545
##  [38,]        NA        NA 100.95033  80.43711  96.13248 122.27906  98.99336
##  [39,]        NA        NA        NA 126.82277  68.18047  79.03007 121.32690
##  [40,]        NA        NA        NA        NA  85.28595  87.51645  87.80032
##  [41,]        NA        NA        NA        NA        NA  56.77901  92.08791
##  [42,]        NA        NA        NA        NA        NA        NA  98.54496
##  [43,]        NA        NA        NA        NA        NA        NA        NA
##  [44,]        NA        NA        NA        NA        NA        NA        NA
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##  [46,]        NA        NA        NA        NA        NA        NA        NA
##  [47,]        NA        NA        NA        NA        NA        NA        NA
##  [48,]        NA        NA        NA        NA        NA        NA        NA
##  [49,]        NA        NA        NA        NA        NA        NA        NA
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##  [53,]        NA        NA        NA        NA        NA        NA        NA
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##  [55,]        NA        NA        NA        NA        NA        NA        NA
##  [56,]        NA        NA        NA        NA        NA        NA        NA
##  [57,]        NA        NA        NA        NA        NA        NA        NA
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##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,44]     [,45]     [,46]     [,47]     [,48]     [,49]     [,50]
##   [1,]  64.11407 119.29652  64.51860 106.36329  75.87098  75.23559  82.90280
##   [2,]  83.58615  85.68810  92.47410  61.40715 135.58897 117.36560  72.21229
##   [3,]  91.43866  89.62792  94.12703 104.80451 104.25034  85.37185  99.53789
##   [4,] 106.35150  79.78962  97.34283 127.79170  99.42069  81.73521 124.72518
##   [5,] 111.17514  56.19600  91.00266 118.95565 110.62179  80.74079  77.42055
##   [6,] 107.55988  89.04268  78.81086  65.41076 121.27836 146.53323  84.82092
##   [7,]  92.84282  94.05032  84.81698 100.10441  98.05103 121.53178  99.77216
##   [8,] 102.17387  90.76685  75.94457  74.04933 135.55304 129.99738 102.42386
##   [9,] 101.53229  73.79477 111.94691 119.70447  81.88237  67.79478  62.58742
##  [10,]  97.94895  72.50779  80.17381  59.32581 121.55601  98.06586  71.98248
##  [11,]  70.27298  76.69195 109.58124  66.59598  70.69869  80.26725  86.12458
##  [12,]  76.81852 132.97184  80.45578  92.95347  96.82387 122.99690  73.86515
##  [13,]  81.70635 107.51089  92.07140 112.42145  43.18264  75.52468 103.45431
##  [14,] 132.50875  76.24892  83.89702 110.67727  89.00807  97.28301 127.03977
##  [15,] 101.69252  91.14477  70.56314  82.61209 104.78069 115.69707 141.42635
##  [16,]  80.58027 103.68987  55.34949  65.60506 112.98772 103.83309  72.49440
##  [17,]  79.57946 105.94941  98.60608 112.53617  53.99514  90.73538 106.21221
##  [18,]  68.55009 122.72556  48.69754  61.46392 103.65795  99.47833 112.69601
##  [19,]  94.23975 112.84390  90.17319  65.20741  68.93412 111.88791  85.33758
##  [20,] 120.48159  68.17119  94.12240  88.52132 100.65621  82.42607  77.42526
##  [21,]  63.82707 104.19593 107.43983 108.40575  62.91539  71.56475 107.16565
##  [22,]  84.44909 113.94886 112.96366  87.62670  72.33280 109.81861  57.54347
##  [23,] 100.36006 106.14423  97.95186  75.19435  53.07334 118.42676 107.05435
##  [24,] 114.35054  80.46624  56.96381  88.32325 112.25681 110.77275 108.81988
##  [25,]  69.34743 101.05168  92.88124  67.77959  87.89434  84.87120  63.83343
##  [26,]  79.37010  95.70241  78.88960 104.69407  83.87361  91.39842  87.42649
##  [27,]  53.64718 126.07655  90.93323 100.43834  63.38923  94.70555  85.68195
##  [28,]  70.58701 105.32731  72.21805  68.87722 102.07673  87.67874 110.86893
##  [29,]  94.36534  84.34241 107.95769 132.79427  74.84351  57.49897 102.45999
##  [30,]  87.65244  92.38895  72.22261  88.43829  90.64352  78.08901 106.12021
##  [31,]  95.87854  80.76240 101.21964  76.73391 127.45936 110.80754  86.29072
##  [32,]  88.21347  90.06500 123.73494 101.55376  68.21405  78.51772  68.23820
##  [33,]  77.97622 116.83005  90.09607  81.25852  72.61688  84.20619  63.23734
##  [34,]  81.98459 115.04880  61.69841  89.00313 102.45929 101.91297 100.30336
##  [35,]  52.70133  96.25865 101.27279  89.89935  96.19477  60.83857  66.94727
##  [36,]  98.47446  95.33719  85.84859  86.55038 117.45717 120.18051  96.18755
##  [37,]  63.12577 125.85158  86.10729  72.71010  83.96425  90.03384  59.97192
##  [38,] 111.33674  91.66991  62.83417 104.36656  99.34796  98.63093  84.36368
##  [39,]  83.50249  71.87746 112.68782 132.55463  89.69811  43.15187  87.59125
##  [40,] 114.06952  78.99083  91.92524  66.54289  96.14950 110.37959  71.95734
##  [41,] 105.51714  92.05723 108.18335 105.79105  94.93888  77.32325  98.57548
```

```
##  [42,]  78.37187 112.29167  95.67352  89.70201  89.68239  86.90605  83.88843
##  [43,]  92.08033 103.01345  75.27667  72.12868  82.85759 104.58111 142.64623
##  [44,]        NA 121.65039  80.29772  84.45628  89.76854  80.58432  75.33553
##  [45,]        NA        NA 114.25504  90.97851  95.71318  67.16799  89.44254
##  [46,]        NA        NA        NA  78.40465 115.57304 105.25225 100.04133
##  [47,]        NA        NA        NA        NA 101.35196 113.60105  84.40364
##  [48,]        NA        NA        NA        NA        NA  70.77945  90.98595
##  [49,]        NA        NA        NA        NA        NA        NA  87.94414
##  [50,]        NA        NA        NA        NA        NA        NA        NA
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##  [67,]        NA        NA        NA        NA        NA        NA        NA
##  [68,]        NA        NA        NA        NA        NA        NA        NA
##  [69,]        NA        NA        NA        NA        NA        NA        NA
##  [70,]        NA        NA        NA        NA        NA        NA        NA
##  [71,]        NA        NA        NA        NA        NA        NA        NA
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##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
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##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
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##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
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##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
```

```
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##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,51]     [,52]     [,53]     [,54]     [,55]     [,56]     [,57]
##   [1,]  57.31884  83.80714  74.12439 103.06625  75.47094  72.62338  92.70954
##   [2,] 119.28951  88.96074  82.82835  63.76625 109.76522 138.04532  89.22438
##   [3,] 101.73026 102.37172  65.47067  80.94612  98.42913  89.18750  73.56497
##   [4,] 101.99667 104.00056  79.65793 101.20433  90.59317  58.57584  69.18384
##   [5,] 102.07825  68.64335 109.86634 132.18755  48.26623  74.95137  60.33262
##   [6,]  92.44329  85.79260  69.67417  79.87958  99.49475  98.86307  95.87864
##   [7,]  84.59515  83.61921  54.90846  91.26219  98.51029  80.18589  81.25815
##   [8,] 121.84980  85.81639  84.14403  77.70314 103.50032 107.17205 101.71399
##   [9,] 126.59499  82.33921 118.99225 108.56432  72.86539  91.74957  86.98562
##  [10,] 118.84687  80.30968 123.26142  86.53582  83.02379 120.31049 101.91867
##  [11,]  63.44227 116.36499  43.18531  48.31511 122.50968 107.35218  61.57812
##  [12,]  82.33443 102.52460  80.62165  88.62795 108.63678  76.29662 121.66234
##  [13,]  86.75137  88.60665  90.36815  95.21809  93.48129  73.26368 107.02831
##  [14,] 117.80096  68.22230 114.88167 117.22424  68.61536  69.15281 100.43055
##  [15,]  95.69848  71.49227  81.23774  89.37749  91.53574  90.85855  94.24930
##  [16,]  70.51637 106.71417  84.62244  84.92163  93.22661  82.20749 106.75334
##  [17,]  61.28973  87.52331  55.21444  91.76783  97.38360  70.79574  80.24637
##  [18,]  76.50227  84.22560 101.77750  83.64683  93.15878  96.93830 123.23607
##  [19,]  71.15644 120.03735  75.59566  65.35090 120.05097  80.37791 124.15263
##  [20,]  85.25003  88.73871 108.64323 107.82673  62.00448  79.05883  77.84558
##  [21,]  98.85913 104.86232  80.82575  74.44157 117.98710  91.60186  92.62936
##  [22,] 110.57368  98.94579  88.98842  71.17980 118.30873 106.80727 122.70065
##  [23,]  78.06547  87.32055  68.88090  71.45173 111.61279  91.26367 112.85080
##  [24,] 107.80464  49.32234 125.59754 122.82192  57.31398  83.67381 107.16554
##  [25,] 107.61087  82.49000 107.03552  72.24013  97.65013 133.70291 110.16479
##  [26,]  72.50683  51.29339 109.51999 126.61376  59.76680  80.60278  88.24113
##  [27,]  78.58459  97.67163  60.25408  74.88066 117.55626  90.16234 103.62879
##  [28,] 100.85096  89.98143  90.51708  67.99092 104.14946 115.46674 105.20812
##  [29,] 116.96949 108.72032 109.03691 101.49311  92.36251  65.90188  89.61294
##  [30,]  62.61070  64.42725 118.72433 119.11891  54.96671  78.88619  87.56025
##  [31,] 106.02513 115.80510  60.28849  60.97321 118.77115 105.24414  74.67900
##  [32,] 112.00377 129.58213  91.35025  71.91308 121.21934  86.77519 100.52246
##  [33,]  91.14416 113.01691  99.88945  76.27835 105.50908  91.00972 127.22627
##  [34,]  93.33260  78.02359 128.11664 110.02018  81.02015  78.60621 127.94440
##  [35,]  87.13036 129.03015  79.30886  67.27585 113.19956  99.76055  77.38129
##  [36,]  96.63509  99.80075  55.23017  75.17766 107.03159  91.84535  85.27202
##  [37,]  80.30490  97.98884  97.52438  76.70182 100.77871 105.57044 119.52803
##  [38,]  69.66128  65.91584 112.49472 142.21493  40.95759  52.39579  95.57582
##  [39,] 102.57445 119.23015  88.66449  96.33577  91.33653  70.83248  56.16753
##  [40,] 111.88974  62.86757 116.24617  92.47299  76.42092 116.97158 107.54688
##  [41,] 115.05255 108.09877 108.77958  90.59776  93.75729  82.06921  96.24406
##  [42,] 121.79624 108.54711 103.13963  71.95095 114.22007 102.74041 122.19581
##  [43,]  77.22854  67.66558  89.15561  85.72808  89.99310  96.46920 102.35788
##  [44,]  69.39155  89.35307  70.71569  74.33267 105.62098 112.88366  87.56364
##  [45,] 108.00698  97.37249  95.69586  90.51914  79.17380  93.45801  46.31909
##  [46,]  69.23764  60.22284 104.49798 115.73222  59.56953  79.47516 113.56104
##  [47,]  79.90408  92.93560  79.30281  52.10074 109.75151 129.19304 103.27174
##  [48,]  72.16878  99.36126  81.29582  83.91189  98.69418  77.84154  93.99287
```

```
##  [49,]  83.78714 104.46609  98.78230  98.34495  75.77640  78.88681  56.77629
##  [50,]  90.81897  97.60436  83.98981  82.23577  93.40566 107.35970  87.24136
##  [51,]        NA  89.22329  67.97598  93.09744  81.11903  74.18272  84.21309
##  [52,]        NA        NA 115.76266 126.68161  47.94799  97.06356  99.74722
##  [53,]        NA        NA        NA  50.68474 128.31327 100.82845  69.39509
##  [54,]        NA        NA        NA        NA 154.18397 129.19037  87.53188
##  [55,]        NA        NA        NA        NA        NA  65.58872  85.09183
##  [56,]        NA        NA        NA        NA        NA        NA  92.96118
##  [57,]        NA        NA        NA        NA        NA        NA        NA
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##  [67,]        NA        NA        NA        NA        NA        NA        NA
##  [68,]        NA        NA        NA        NA        NA        NA        NA
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##  [70,]        NA        NA        NA        NA        NA        NA        NA
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##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
##  [76,]        NA        NA        NA        NA        NA        NA        NA
##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,58]     [,59]     [,60]     [,61]     [,62]     [,63]     [,64]
##   [1,]  75.49961  60.77375  84.85283  44.01838  82.46374  89.21875 101.41386
```

```
##   [2,]  84.24164 100.44085  88.67149 117.56073 115.41358 105.88859  83.49056
##   [3,]  76.19229  76.66829  65.03752  81.39105  85.22473  75.49918  99.29833
##   [4,]  68.59744  76.05897  61.82983  90.09545  51.76601  74.66821 115.25647
##   [5,]  52.93184  48.22568  69.36452 107.74546  91.46817  74.01521 100.93229
##   [6,]  83.34222  91.12955 121.04257  85.48573  98.80117  98.59083 114.16215
##   [7,]  71.23673  80.44501  97.46819  65.98570  58.94353  99.88526 152.42915
##   [8,]  73.43896 103.71993  88.74696 103.19902  94.21812 104.88023 104.81817
##   [9,]  68.82657  69.64208  62.35977 109.87274 114.76989  59.22680  64.88719
##  [10,]  86.81216 104.42208  94.73888 137.09025 132.62552 108.44360  57.29391
##  [11,] 124.78232 107.50800 110.21912  84.05104  82.09777 115.70662  94.07320
##  [12,]  71.80953  77.09462 116.90196  76.09010  82.65924  92.42991  99.95251
##  [13,]  92.81807  99.00346  83.10832  64.81876  69.27096  88.67997  97.13189
##  [14,]  77.29854  98.60455  72.51845  91.15097  77.22560  74.16183 106.96075
##  [15,]  91.30944 115.50337  82.98443  79.17216  61.37629 107.26160 130.28826
##  [16,]  77.90469  79.98003 127.78316  91.51196 104.71843 119.40572  81.43690
##  [17,]  96.94651  84.53386  90.19327  39.25806  46.42279  82.51647 133.83625
##  [18,] 100.56278 116.98738  97.42253  87.02475  85.40159 123.38357  83.55986
##  [19,] 115.47847 109.03611 150.36929  73.72364  98.12128  92.79610  78.70453
##  [20,]  93.21444  68.11869  95.52468 100.40096 118.37741  53.99714  67.97090
##  [21,]  95.39058 109.40182  68.40267  87.93857  58.13831 101.76152  93.79803
##  [22,]  93.37954  97.71806 105.18079  87.37654 112.65941  75.20089  72.46783
##  [23,] 124.35679 123.37928 119.81855  52.96378  80.29682  85.81424 109.42032
##  [24,]  68.41058  96.74276  86.74811 101.10683  87.93011 104.88326 107.29640
##  [25,] 102.28849 108.10258  84.43995 104.49531 130.11708  96.34484  49.59117
##  [26,]  79.19971  76.35736  85.54940  81.99796  69.20346  95.77475 113.13665
##  [27,]  86.44874  93.03790  94.01020  58.33170  66.00299 106.99053 110.32608
##  [28,] 100.12990 121.45469  74.13372  94.72700  93.15548 114.60341  78.61014
##  [29,]  72.66717  85.31674  58.64004 108.38495  72.36649  76.81978  77.38695
##  [30,] 101.52626  87.86736  84.40964  88.11107  79.34848  87.16850  89.27872
##  [31,]  84.38075  88.72217  93.90820 105.22681  96.57778  91.58148  94.05598
##  [32,]  89.38885  93.61203  97.52518 109.55807  98.73597  82.94796  60.99781
##  [33,]  96.30007  93.43047 109.29284  89.70678 119.99644  85.13975  45.39015
##  [34,]  76.23636  98.57903  94.28308 102.89669  79.32272 109.42090  86.51039
##  [35,]  87.35595  79.77156  82.07924 109.90955  96.18754 102.19958  59.07845
##  [36,]  76.80874  82.61249  93.80057  77.51303  86.13106  86.60504 114.92526
##  [37,] 101.56160  91.24365 104.07097  85.80888 118.59839  86.82249  53.27924
##  [38,]  66.51048  52.15782 102.25329  76.48231  90.92052  72.01174 102.57062
##  [39,]  67.14180  62.08138  53.69156 110.95578  74.62980  78.23345  79.77458
##  [40,] 100.11552 103.09209  99.88877 102.34802 136.69947  76.44649  70.60438
##  [41,]  90.23608  88.19388  67.96059 106.80821  98.84015  49.78809  60.57055
##  [42,]  86.71209 105.49578  79.87950 105.68317 106.10380  88.64642  56.13271
##  [43,] 121.53443 130.29021  87.97321  69.04033  67.09628  98.29007 111.02171
##  [44,]  88.63052  88.06093  80.64768  78.98218  77.62889 123.92480  94.19792
##  [45,]  94.28147  88.66953  80.05091 125.89532 102.76096  83.68140  78.54233
##  [46,]  69.39391  81.69945  97.15673  73.86944  87.24740 117.36805 105.10928
##  [47,] 126.46220 128.32011 127.34570  95.59071 119.06576 121.09645  74.68754
##  [48,] 120.77432  99.92753  99.48460  61.92119  82.78485  69.80074  84.42411
##  [49,]  92.33939  74.32363  59.77771 102.56382  87.77893  80.86573  64.94166
##  [50,]  78.20356  63.74502 105.06147  96.25491 130.68943  87.77833  66.84218
##  [51,] 107.44901  79.18930 114.60922  48.40943  76.28971  99.88436 103.77659
##  [52,]  76.27898  86.93937  74.02559  77.17145  85.64308  95.03970 115.18657
##  [53,] 104.23187  90.91734 106.65817  64.03030  77.84854 101.07365 111.76279
##  [54,] 127.29287 126.16902 109.90014  96.14706 103.00923 107.46558  74.57630
##  [55,]  65.50673  59.21656  73.04415  86.16300  92.07211  78.02124  96.09153
```

```
##  [56,]  69.65092  59.77814  91.79212  72.89040  65.35454  64.82381 102.39201
##  [57,]  89.35321  71.05225  67.01547 102.92268  79.52632  88.66239  97.37347
##  [58,]       NA  44.25020  67.27408  97.04204  82.11288  89.99194 106.76396
##  [59,]       NA        NA  78.06154  81.21376  89.17574  70.48952  98.02635
##  [60,]       NA        NA        NA 105.57191  70.69337  81.48471  95.06112
##  [61,]       NA        NA        NA        NA  72.13051  81.80528 121.18585
##  [62,]       NA        NA        NA        NA        NA  97.55989 136.33055
##  [63,]       NA        NA        NA        NA        NA        NA  79.54583
##  [64,]       NA        NA        NA        NA        NA        NA        NA
##  [65,]       NA        NA        NA        NA        NA        NA        NA
##  [66,]       NA        NA        NA        NA        NA        NA        NA
##  [67,]       NA        NA        NA        NA        NA        NA        NA
##  [68,]       NA        NA        NA        NA        NA        NA        NA
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##  [70,]       NA        NA        NA        NA        NA        NA        NA
##  [71,]       NA        NA        NA        NA        NA        NA        NA
##  [72,]       NA        NA        NA        NA        NA        NA        NA
##  [73,]       NA        NA        NA        NA        NA        NA        NA
##  [74,]       NA        NA        NA        NA        NA        NA        NA
##  [75,]       NA        NA        NA        NA        NA        NA        NA
##  [76,]       NA        NA        NA        NA        NA        NA        NA
##  [77,]       NA        NA        NA        NA        NA        NA        NA
##  [78,]       NA        NA        NA        NA        NA        NA        NA
##  [79,]       NA        NA        NA        NA        NA        NA        NA
##  [80,]       NA        NA        NA        NA        NA        NA        NA
##  [81,]       NA        NA        NA        NA        NA        NA        NA
##  [82,]       NA        NA        NA        NA        NA        NA        NA
##  [83,]       NA        NA        NA        NA        NA        NA        NA
##  [84,]       NA        NA        NA        NA        NA        NA        NA
##  [85,]       NA        NA        NA        NA        NA        NA        NA
##  [86,]       NA        NA        NA        NA        NA        NA        NA
##  [87,]       NA        NA        NA        NA        NA        NA        NA
##  [88,]       NA        NA        NA        NA        NA        NA        NA
##  [89,]       NA        NA        NA        NA        NA        NA        NA
##  [90,]       NA        NA        NA        NA        NA        NA        NA
##  [91,]       NA        NA        NA        NA        NA        NA        NA
##  [92,]       NA        NA        NA        NA        NA        NA        NA
##  [93,]       NA        NA        NA        NA        NA        NA        NA
##  [94,]       NA        NA        NA        NA        NA        NA        NA
##  [95,]       NA        NA        NA        NA        NA        NA        NA
##  [96,]       NA        NA        NA        NA        NA        NA        NA
##  [97,]       NA        NA        NA        NA        NA        NA        NA
##  [98,]       NA        NA        NA        NA        NA        NA        NA
##  [99,]       NA        NA        NA        NA        NA        NA        NA
## [100,]       NA        NA        NA        NA        NA        NA        NA
##            [,65]     [,66]     [,67]     [,68]     [,69]     [,70]     [,71]
##   [1,] 110.57756  60.17966 101.67364  76.03015  80.08354  99.13819  84.44727
##   [2,]  68.38849  86.53113  68.75052 108.32014  98.70912  65.15699  96.01457
##   [3,]  93.16314  71.22751  54.96549 118.60276  76.69044 119.18372  97.90736
##   [4,] 101.65311 101.10124  49.79541 127.16161  68.10403 150.57478  90.86908
##   [5,]  70.33411 116.39405  82.50731 107.52042 101.15283 105.44467 105.20283
##   [6,]  74.37983  75.97401  52.96759 128.78994  66.56245  84.50838 115.24116
##   [7,]  81.81620  70.99094  50.94697 130.27191  63.80584 104.75575 101.68579
##   [8,]  81.36434  76.46117  44.03777 120.07284  86.71104  92.80077 112.07752
```

```
##   [9,]  52.80395 100.86197  98.44221  64.74577 145.39182  93.51959 114.08898
##  [10,]  70.74244 101.78623  94.67297  74.51001 123.47131  61.89412 104.14667
##  [11,]  88.16385  75.29346  86.60061  99.83521  67.55309  78.82144  69.50158
##  [12,]  89.98137  98.07942  82.72247  85.20896  73.78134  90.32323  87.33370
##  [13,]  93.82235  69.44498 102.19714  58.54946 105.62598  99.51190  92.77233
##  [14,]  87.07817  85.42102  72.34568  99.22958 103.69496 115.94134 123.50073
##  [15,] 107.94700  67.94036  63.57980 123.83562  72.71567  98.78185  95.67354
##  [16,]  98.33228  89.90728  81.57017  82.18905  71.11662  88.53880  92.09727
##  [17,] 100.83344  69.12145  88.93788 103.63094  64.64682 102.92792  79.11534
##  [18,] 135.17345  84.41266 107.05646  73.66506  81.24098  73.48882  65.96703
##  [19,]  92.40302  81.03915  89.19687  75.38881  69.28654  85.83741  96.37723
##  [20,]  88.70754 113.22027  98.01346 100.14244  83.06914  95.81689 100.60007
##  [21,]  96.24654  86.44831  93.01808  71.90255 101.78502  96.73967  67.85491
##  [22,]  55.67752  81.95044  95.36636  62.32461 116.20766  69.27145 106.28172
##  [23,]  88.16514  54.13884  89.13122  91.93983  77.80713  79.80098 103.07872
##  [24,]  87.96359  90.62538  84.38162  91.98003 107.41482  88.43024 114.88471
##  [25,]  77.13461  78.99207 119.42962  48.66095 133.90806  46.66224  88.77962
##  [26,]  95.89216 109.30555 123.34056  82.15053  96.69747  72.86748  72.16816
##  [27,]  89.43949  62.57257  90.02712  73.26989  89.28030  86.72926  79.81098
##  [28,] 112.34298  67.13086  90.02507  80.66270  97.71418  79.66593  78.86461
##  [29,]  87.37597 112.73418  84.90768  70.23549 112.59771 125.58809  89.47203
##  [30,] 128.75222 117.20065 133.97506  86.11336  80.36909  78.50401  58.14569
##  [31,]  78.84408  90.27763  40.00761 134.96691  66.03273 100.92119  96.05019
##  [32,]  61.03684 105.34609  85.32350  59.89205 112.03170  97.90051  96.85693
##  [33,]  85.40591  86.83246 108.90756  36.36083 108.09249  80.04670  94.64392
##  [34,] 109.69680 117.27534 108.73840  64.52252  99.60771  81.71148  76.02149
##  [35,]  95.14805 108.79864  95.12240  73.79903  89.68392  90.49432  57.58380
##  [36,]  86.59924  68.73732  37.56096 136.98975  60.10593 108.42383 106.72410
##  [37,]  96.44048  90.99240 124.71095  50.80997  99.88582  60.11552  74.89861
##  [38,]  98.45287 107.00111  99.85527  94.39834  77.85996 100.91170 102.47760
##  [39,]  87.83963 118.82392  77.33435  91.16315  94.20489 130.29870  76.64854
##  [40,]  62.12974  82.37481 102.67386  80.32678 121.69292  56.43387 121.71382
##  [41,]  97.23026 113.65970  87.50226  89.11423  95.11694 111.02690  89.42888
##  [42,]  85.03758  88.81747  90.67260  56.51549 118.51579  87.63107  92.86511
##  [43,] 127.12416  70.77369 100.43360 103.13372  73.06919  79.06636  75.43687
##  [44,] 106.19693  81.19736 112.37592  72.99718  89.74299  63.50547  44.24192
##  [45,]  65.19811 104.83004  73.18777 108.77555  99.58609 103.35501 106.66804
##  [46,] 122.13794  77.44316  98.37011  84.25088  80.45856  84.15571  88.87719
##  [47,]  91.85264  69.39616  94.34630  89.50198  81.83985  48.41145  88.94382
##  [48,]  87.63328  76.07140 114.35284  64.21323  94.63321  90.31523  88.73340
##  [49,]  98.79556 106.81241 109.53585  71.62155 102.00590 107.42952  70.45637
##  [50,]  51.35599  88.13172  97.96508  68.83734 110.19166  68.52082 101.84181
##  [51,] 125.02805  78.47669 114.02531  89.67322  49.68015  84.07511  63.46082
##  [52,]  95.93585  79.48188 112.98182  88.16365 108.54360  66.45229  94.34789
##  [53,]  86.26797  59.91964  65.17678 117.11367  54.27020  91.90237  81.69715
##  [54,]  78.59783  69.80647  75.95254  94.60045  81.13110  71.01196  82.21430
##  [55,] 102.08595 100.57057 110.67015  84.93607 100.52803  97.59391  98.97285
##  [56,] 108.63515 108.67494  84.38240  92.12342  69.22461 141.86859  97.11111
##  [57,]  83.69290 101.30512  78.44089 120.32497  79.05817 104.82579  75.63560
##  [58,]  80.48977 103.20488  71.35233  93.40645 101.79613 113.06519 103.49322
##  [59,]  84.89334 104.02865  86.49312  94.15583  85.34909 115.14594  95.35927
##  [60,]  96.31523 104.42006  92.56293  94.08992 112.49191 104.47151  74.93394
##  [61,] 109.50650  52.23340 100.20803  91.08265  63.98244  90.25925  88.70847
##  [62,] 119.85492  99.34382  84.00799 110.61358  64.67467 113.09303  60.07090
```

```
##  [63,]  79.55387 101.99215  90.72544  92.98636  92.82827 115.14907 110.14054
##  [64,]  79.78671 104.61091 107.67600  54.41065 115.07632  80.92805  93.53000
##  [65,]        NA  85.09578  70.57955  87.27702 122.48755  82.66115 129.67376
##  [66,]        NA        NA  83.93748  88.97998  87.97930  74.11254 108.83159
##  [67,]        NA        NA        NA 131.55443  73.03205 123.12834 117.12984
##  [68,]        NA        NA        NA        NA 128.11714  70.43219  88.68386
##  [69,]        NA        NA        NA        NA        NA 108.77101  72.20911
##  [70,]        NA        NA        NA        NA        NA        NA  78.19819
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
##  [76,]        NA        NA        NA        NA        NA        NA        NA
##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,72]     [,73]     [,74]     [,75]     [,76]     [,77]     [,78]
##   [1,]  88.70023  67.26026  83.34024  90.73208  91.21275 102.13945  96.39103
##   [2,]  65.40271 102.56957  89.93836 115.69580  86.53085 106.83364  76.10278
##   [3,]  73.12514  68.89904 103.27577  97.58091  76.28094  97.21638  81.16530
##   [4,]  76.83777  78.09376 103.24775  67.99568  61.22012  80.98435  97.09102
##   [5,]  91.92939  96.73040 113.48567  79.49587  55.06355  92.23729 119.37660
##   [6,]  45.83990  95.16503  88.73246 100.03394 109.28479  81.04187 105.44670
##   [7,]  59.22110  78.90095  95.24713  92.86703  89.65644  83.46431 128.60469
##   [8,]  33.48317  95.27920  99.10636  98.35068  90.95927  76.50016  92.60775
##   [9,] 114.82604  76.13394  99.86728  84.41284  58.89725  98.85696  82.47322
##  [10,]  76.41750 127.89029  97.28368 103.79850  92.54638  80.63454  81.82499
##  [11,]  98.73656 110.92377  93.14480 128.94906 113.08152  90.99454  96.33172
##  [12,]  75.64787  77.82840  26.85579  61.81136  87.49124 109.90227  75.25367
##  [13,] 103.62387  68.25428  89.82911  82.29699  93.08552  74.80221  98.53871
##  [14,]  74.96349  77.69935 124.22283  72.10612  85.58392  45.18963 114.86944
##  [15,]  52.71410  87.48213 117.60920  94.98199 102.17939  59.18145 117.50816
```

```
##  [16,]  55.18630 123.06529  62.60987  89.18456 105.80584  83.78738  85.39923
##  [17,] 102.06639  55.66871  89.10469  86.68628  94.89870  94.43996 113.11040
##  [18,]  69.79108 104.47670  74.70248  85.76749 108.95062  83.25434  75.35218
##  [19,]  84.34141  91.52217  50.98054  82.32365 130.87604  77.08781  70.70193
##  [20,] 103.30477  91.97312  95.30114  78.92961  90.33720  94.15698  79.33922
##  [21,] 105.19345  79.87754  82.92390  87.99561  75.83766  94.00166  81.39025
##  [22,] 102.92240  64.31344  55.77620  87.48170  93.65181 108.68232  61.89812
##  [23,]  90.44872  64.96003  88.27555  96.69047 134.27917  69.10905  99.86956
##  [24,]  59.01432 105.19407 115.78348  80.98131  90.57690  51.46395 127.65863
##  [25,] 104.48835  90.09634  83.52307 111.49560  95.68430 107.16211  60.14841
##  [26,] 108.52846  88.46770  87.33815  73.66510  77.41813 101.70141 120.99353
##  [27,]  90.05329  69.59222  67.26846  94.59459  91.67044 101.60231  93.46232
##  [28,]  71.71446  95.17722  97.84081 108.17496  99.11204  84.09291  71.41029
##  [29,] 105.01160  85.23210  88.83594  62.78225  53.70643  82.95034  77.92577
##  [30,] 111.70092  95.72836  91.10485  70.59166  90.30827  96.67429  94.18692
##  [31,]  60.88852  96.68040  86.74512 102.64291  85.53241 100.06864  74.88143
##  [32,] 107.36559  92.46654  62.03314  80.14712  78.24954  93.74470  61.94123
##  [33,] 100.28224  86.20338  50.52288  82.31222 100.78501  97.77145  46.74176
##  [34,]  80.96764 103.94912  62.96850  54.93588  82.36240  85.56685  84.55555
##  [35,] 102.67429 107.06382  61.40374  94.63963  69.59355 125.03196  50.01225
##  [36,]  49.67517  76.32963  91.71421  97.43675  92.53619  91.31299  91.01636
##  [37,] 107.36169  81.24202  45.78766  86.75467  99.10112 123.11305  42.71360
##  [38,]  85.35611  86.57697  85.75721  58.10420  87.19483  85.34573 111.79667
##  [39,] 106.33510  94.14111  91.28191  77.22069  40.07276 104.39825  78.19454
##  [40,]  92.55737  87.41554 103.52646 102.64598 107.47450  82.63306  88.56486
##  [41,] 104.41653  70.23707  80.10268  63.18627  70.19747 101.92757  41.19554
##  [42,]  89.49223  81.80693  67.21345  83.04229  81.24277  97.37479  43.42029
##  [43,]  84.84792  78.42894 106.75912  93.09211 117.76654  71.99409  99.29125
##  [44,]  97.65137  91.15676  67.20464 106.41577  81.49889 131.96194  81.03564
##  [45,]  96.24638 120.07139 135.31752 108.64327  82.49334  67.73883 106.48008
##  [46,]  52.50266 101.36778  88.90872  82.86947 100.90249  77.46113 109.17731
##  [47,]  72.44760 118.10835  88.57349 126.45052 139.76648  80.00442  81.18599
##  [48,] 132.29033  65.29496  85.92070  88.93038 107.59610  84.94103  88.11455
##  [49,] 126.56586  99.79799 105.07664  91.88944  67.55152  96.38173  82.23069
##  [50,] 100.23717  94.55805  70.51188 106.90442  85.54660 118.82952  79.14963
##  [51,]  97.82255  90.38263  77.36236  92.77606 115.71805  97.44547 101.19484
##  [52,]  85.62519  78.11949 112.55622  89.64096  88.25376  87.88775 125.72099
##  [53,]  81.93557  82.91965  83.63218 119.61618 105.91879 102.99534  92.87375
##  [54,]  88.07460  99.55567  79.81487 128.05075 115.38876  96.49712  62.84529
##  [55,]  92.10582  90.82336 114.21897  72.94770  77.50583  79.72602 119.21488
##  [56,]  90.29463  78.66804  82.06829  40.01326  78.57493  73.18634 100.42048
##  [57,] 104.91438 104.92279 120.41346 111.55215  68.94055 100.84878 105.96502
##  [58,]  66.73332  87.66272  87.21862  67.64956  45.12401  98.11281 106.07128
##  [59,]  92.43298  80.62165  83.44086  71.64476  56.74320 111.78474 100.43890
##  [60,] 103.60029  78.27569 114.26298  83.65492  39.28857 105.48979  91.97155
##  [61,]  90.64082  50.64552  80.11117  87.20815 114.00656  93.84501 104.93013
##  [62,]  90.45777  77.14454  89.04360  67.65145  71.65105  91.87290 110.21102
##  [63,] 118.15331  49.69132  89.28979  62.32785  79.33362  95.01495  74.41176
##  [64,] 110.74716 107.30086  80.45009  94.57777  94.14028  92.25600  44.50413
##  [65,]  92.47786  90.82488  97.10571 108.55175  83.61826  89.33740  94.40996
##  [66,]  71.92582  73.82028 103.31085 127.09308 126.91997  78.12106 104.57019
##  [67,]  49.56902  95.39890  99.47618  94.00443  83.18345  72.68541  97.29653
##  [68,] 112.38430  90.41706  70.71791  84.72253  93.83853  92.77613  71.82289
##  [69,]  74.98419  89.79788  77.79038  83.42863 107.50637  90.18408  92.96800
```

```
##  [70,]  97.80584  97.17433  83.77505 121.20349 113.35550 107.07028  87.09330
##  [71,] 111.37664  96.38135  73.20969  87.80513  78.22882 123.66703  76.94634
##  [72,]        NA 106.93609  94.30957  95.24104 100.06015  66.23822 106.22775
##  [73,]        NA        NA  79.13847  71.04495  84.85063 108.32961  83.12391
##  [74,]        NA        NA        NA  64.44935  89.55252 119.77878  54.78892
##  [75,]        NA        NA        NA        NA  66.66801  90.04091  80.58893
##  [76,]        NA        NA        NA        NA        NA 116.08478  87.93218
##  [77,]        NA        NA        NA        NA        NA        NA 113.74264
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,79]     [,80]     [,81]     [,82]     [,83]     [,84]     [,85]
##   [1,]  78.74153  73.71563  87.54732  83.24974  99.85758 114.43375  73.87934
##   [2,] 107.88560 132.34190 100.65499  72.18412  86.11461  69.37881  92.13498
##   [3,] 104.77354 106.93682 113.31051  70.22681 105.55632  78.02810  85.23804
##   [4,] 125.45636  90.96220 126.00043  88.91086  95.70573  57.92754  96.02307
##   [5,] 107.17890  99.41572  99.66069  85.31264  91.24640  75.75187 106.74876
##   [6,] 102.48323  89.90643 121.16591  65.14707  99.62170  58.78177  81.79778
##   [7,] 111.27612  78.61672 149.91043  73.38074 104.72732  54.67535  90.92905
##   [8,] 120.54302 105.84545 115.33910  89.65495  75.16880  41.47851  94.21472
##   [9,]  75.68576 112.00320  74.37505 111.85334  85.04175  90.27245 119.04102
##  [10,]  89.58738 104.40462  67.45958 107.20318  51.20202  83.80378  90.80825
##  [11,]  72.52834  71.32302 109.89011  62.08098 101.52124 111.01982  53.36318
##  [12,] 101.05643  92.43363 105.72721  90.91746  96.52505  71.67707  88.23665
##  [13,]  70.53713  63.44204  86.32760 125.47762  80.99740  96.02389  99.77865
##  [14,] 100.36777  77.57014  93.13186 123.10751  74.13581  57.58651 120.21900
##  [15,] 117.52409  80.08700 110.90682  92.72625  82.09177  58.90435  98.39227
##  [16,]  90.51486  67.27221  94.84920  89.39893  66.24936  89.57280  44.54435
##  [17,]  86.46532  71.12935 114.73506  75.47551 125.02822  94.18463  94.55620
##  [18,] 101.61171  82.85972  65.78117 105.94389  61.63927  96.78333  77.77300
##  [19,]  63.46655  60.62780  89.02782  94.01053  92.44457  95.63314  64.60318
##  [20,]  81.70512  98.90018  68.29188  69.06539 108.99440 105.62312  87.59840
##  [21,]  93.39457  89.76612  98.30974 115.22470  79.50286  87.01522  99.85775
##  [22,]  66.65778 108.79787  84.50856 101.46401  99.63506  86.02445 108.04721
```

```
##  [23,]  65.08275  65.63089  95.45654  89.98549 109.63457  89.82509  95.30213
##  [24,] 107.84570  76.48795  86.58353 122.30635  52.97317  59.58058 109.88230
##  [25,]  67.35444 116.09792  50.43052 102.77677  76.65821 112.00005  98.23667
##  [26,]  99.66195  87.25915  81.01242  95.49781  93.27275  94.87586 114.93143
##  [27,]  83.20039  78.73934 110.05854  98.32903  93.23420  89.35519  87.55099
##  [28,]  97.29038  98.94222  76.13636 102.45022  65.16675  92.41901  83.61283
##  [29,]  97.30652  90.05160  91.75761 132.62237  68.23758  76.74267 105.24171
##  [30,]  98.59566  89.25875  53.42738  86.55151  94.51351 115.11575  99.11991
##  [31,] 113.17213 114.40430 126.25969  56.57493 102.38057  63.31963  75.00302
##  [32,]  72.45890  90.57025  94.77757 116.25685  78.15434  84.39724  88.92659
##  [33,]  57.99557  86.98832  62.05484 112.28407  77.50043 109.60554  77.54225
##  [34,] 110.72985  86.76146  71.31219 130.44231  53.70052  76.29369 102.98777
##  [35,]  92.79797 107.94953  88.80706  83.03456  83.31394 107.54180  64.33970
##  [36,] 110.83388 100.25502 130.87715  60.35414 108.64866  60.19944  80.57652
##  [37,]  69.44770 107.54169  52.91924  89.63848  94.71864 120.58175  84.31811
##  [38,]  94.73539  74.69671  81.54964  87.37817  96.96934  89.80105  94.20013
##  [39,] 104.79201 100.62983 105.61946  94.55902  85.79035  87.04744  86.12485
##  [40,]  69.19520 105.42838  60.93917  93.86654  89.61244  92.15674 111.33177
##  [41,]  97.80170 121.68763  72.33777  95.63564  98.43817  86.93735 106.21386
##  [42,]  86.15064 110.19097  74.29675 121.68826  67.94779  84.14736  97.44696
##  [43,]  97.52196  82.12642  77.70225  89.19016  95.58514  92.97642 101.53596
##  [44,]  96.58312 103.84494  89.07052  79.84234  89.95859 110.90539  79.92608
##  [45,]  85.50019  86.26477  98.26167  84.42280  82.50519  87.09274  83.80423
##  [46,] 105.26541  70.96864  78.57339  99.81533  64.43736  88.77695  78.57524
##  [47,]  73.80748  84.31310  77.28048  75.41590  81.37066 103.46658  62.76103
##  [48,]  45.24525  65.71944  77.45603  98.39919 109.73194 119.95644  92.92689
##  [49,]  78.39065  86.45714  73.46113  95.38851  84.33056 123.25879  79.60196
##  [50,]  64.54033 102.64826  87.35805  77.46538  96.32945 105.85099  78.17333
##  [51,]  76.14373  58.64464  84.18870  62.89436 110.98675 129.09940  58.18286
##  [52,] 100.46977  95.71600  72.30611  97.96672  86.40076  87.61762 124.66968
##  [53,]  84.21069  83.82529 131.43782  46.61347 121.28347  93.91585  60.47973
##  [54,]  74.16159  97.51619 102.23271  70.50377  97.38327  96.66051  65.26261
##  [55,]  94.87177  80.20425  66.65768  99.91734  81.26410  98.48877 103.06118
##  [56,]  96.07762  59.03885  96.28973 104.03436  92.34985  82.95496  88.06162
##  [57,]  98.21141  96.64271 112.08580  56.79806 106.06228  99.30549  77.65487
##  [58,] 122.26377 100.02646 109.86033 104.13294  73.19895  57.89405 102.64799
##  [59,]  98.05767  92.72797 101.31288  77.30958 102.74311  91.83426  86.32660
##  [60,] 118.84450 121.95960  88.13833 100.02960  84.87861  82.57593 121.50084
##  [61,]  74.39246  66.89248  93.63760  74.15093 122.64338 106.59302  85.91620
##  [62,] 125.15028  82.31832 115.83283  92.06878  97.59783  73.34553 103.11947
##  [63,]  76.00996 100.70788  80.20879  86.60871 124.24224  93.14345 113.06289
##  [64,]  60.69395  98.63137  51.96040 102.98767  74.27168 115.78048  76.71520
##  [65,]  71.06949 101.51217 106.68766  92.45761  90.87782  73.67489  99.89547
##  [66,]  64.46808  73.11932  96.10269  79.98204  97.76521  99.36485  77.28097
##  [67,] 111.23810  89.59045 145.37187  81.49074  87.85098  42.39266  79.66355
##  [68,]  57.81922  81.78615  48.24514 128.16103  66.17036 117.01685  92.05827
##  [69,] 103.22772  71.81012 113.94470  52.39338 116.47960  91.35151  59.88230
##  [70,]  73.89433 104.76767  65.07849  82.83089  89.28802 110.90483  95.04194
##  [71,] 109.49242 103.10171  82.34171  76.05546  97.11065 112.41364  84.93283
##  [72,] 114.83980  79.16530 116.40619  92.24171  65.31738  52.41919  73.97325
##  [73,]  84.63392 104.35545  89.35755  85.81052 130.51147  89.09450 123.40279
##  [74,]  89.82874  96.88176  88.22632  90.80387  98.39485  92.04954  83.50563
##  [75,] 110.82193  89.07179  83.81760 114.67898  89.58168  72.07771 113.23119
##  [76,] 126.53168 122.69518 102.57886 100.02026  84.51707  69.74454 115.79316
```

```
##  [77,]  80.06418  45.79192  90.76845 117.10706  61.20539  75.06683  81.63022
##  [78,]  81.95792 116.81584  69.47857  90.41237  95.11309 102.31002  84.86802
##  [79,]        NA  66.38883  66.20262  90.12379  99.86540 130.21063  73.14918
##  [80,]        NA        NA  92.81180 102.00567  77.02786 100.37148  59.41913
##  [81,]        NA        NA        NA 108.14863  80.19663 128.00063  98.89157
##  [82,]        NA        NA        NA        NA 137.25532 106.30699  68.79819
##  [83,]        NA        NA        NA        NA        NA  75.08762  83.62328
##  [84,]        NA        NA        NA        NA        NA        NA 112.04944
##  [85,]        NA        NA        NA        NA        NA        NA        NA
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##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
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##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,86]     [,87]     [,88]     [,89]     [,90]     [,91]     [,92]
##   [1,]  83.10270 103.65310  57.63123  53.03946  98.21367  69.61443 111.81480
##   [2,]  74.78760  98.35740 122.14171 126.35495  96.10478  98.86927 103.13375
##   [3,]  99.04716  88.61586  75.49964  74.42371  83.20691  91.88687 127.32575
##   [4,] 105.41763  93.55974  72.71362  80.58564  60.64594 109.40337 108.65738
##   [5,]  95.94924 112.61774  98.72261  83.82339  60.34976 110.56815 106.22588
##   [6,]  49.21227  89.76606 137.79900  97.73497  85.52651  91.13237  85.24624
##   [7,]  56.08607  96.46724 111.50970  90.68099  70.46770  74.28650  97.21698
##   [8,]  71.26140 104.65031 119.43741 117.43445  85.99218 106.57172  87.87684
##   [9,] 104.21932  91.61122  77.69803  86.04348 102.59271  85.77128 103.14798
##  [10,]  93.11758 107.00476 111.46888 125.09875 105.20022 112.92398  67.23307
##  [11,]  96.29221  66.17473  88.82484  93.54486  96.29706  60.22513  84.33299
##  [12,]  38.48876  94.73966  91.36496  93.99576 110.50859  84.53151  93.57780
##  [13,]  93.40301  84.72611  63.98124  80.53040 103.28242  59.47913  75.31831
##  [14,]  99.34316  95.45522  99.80837  88.15853  68.09680 104.15740  69.89881
##  [15,]  85.33742  99.95066 114.17721 103.18616  58.81201  99.82324  77.17062
##  [16,]  67.75056 108.64175  84.98324  92.35998 116.93922  99.86628  72.50923
##  [17,]  78.17651  74.99651  83.86395  66.24622  73.18882  54.72967 102.85674
##  [18,]  87.20336 109.99591  86.36459 107.41464  96.52017 109.61041  69.50682
##  [19,]  66.15160  60.01045  89.75436  82.50305 127.49370  72.23333  57.90190
##  [20,] 103.94939  78.63890  95.23915  61.77387  79.80048 115.46436  99.65578
##  [21,] 101.83907  86.53720  61.94486 107.10028  98.31515  71.10557  89.64944
##  [22,]  60.80224  68.49869  96.27374  98.41101 132.90883  58.10619  90.55893
##  [23,]  68.69207  53.90368 111.31751  81.57261  95.86251  52.31542  65.57609
##  [24,]  81.89316 123.80649 118.36523 108.98953  70.88983 113.59403  57.65614
##  [25,]  93.46690  89.97861  90.26416 108.51461 123.36289  79.23491  89.44315
##  [26,]  76.77440 111.82537 103.86900  95.10483  63.53908  90.84326  88.79336
##  [27,]  65.37624  89.94147  73.96683  93.29894 110.80188  45.02247  94.34779
##  [28,] 103.82407 101.22405  81.21797 109.65826  99.58062  97.57798  85.89946
##  [29,] 117.54026  93.28935  47.75212  93.73080  96.27202  98.17556  88.90826
```

```
##  [30,] 104.57462 101.01850  92.53479  82.33897  58.07342 118.58314  87.69954
##  [31,]  81.21967  83.05164 104.05164 100.93151  88.88254 101.31255 111.12526
##  [32,]  91.31642  71.11546  66.11428 100.81308 133.19601  73.43317  79.79610
##  [33,]  83.78156  79.04809  63.58005  84.82358 153.44424  76.84136  78.75643
##  [34,]  77.02968 119.77210  90.06036 116.91199  93.17308 116.91968  64.08088
##  [35,] 104.30415  93.72864  50.70420  99.04957 115.68109  93.28618 111.69089
##  [36,]  69.00279  87.70612 104.22252  84.44846  83.36317  90.74258 111.98497
##  [37,]  78.37680  83.85824  77.66979  87.89542 128.51293  82.38907  97.30740
##  [38,]  74.53245 107.15132  97.78953  61.70916  71.85499 109.62762  88.72266
##  [39,] 123.56346  96.20811  41.16446  84.70867  87.03888 101.51881 117.50402
##  [40,]  82.15920  81.67622 129.36093  96.53411  97.60458  89.68675  76.22747
##  [41,] 113.27885  75.27887  70.07254  80.98753  92.33324 116.27691 109.02998
##  [42,]  93.07130  88.99004  68.12562 107.49625 129.90209  90.11973  88.57062
##  [43,]  94.72654  83.35836 109.03156  93.26358  62.27502  94.57865  75.17233
##  [44,]  80.13985 109.51150  73.98838 105.98187 100.10232  72.28476 112.78234
##  [45,] 127.19220  81.18238  95.98179  91.81679  74.79769 102.64506  82.35802
##  [46,]  72.25071 135.75322  94.11902  88.80643  83.70237 109.64984  77.74246
##  [47,]  80.11424  79.29488 119.64436 108.80296 107.97393  86.04129  63.23342
##  [48,]  98.98635  51.14182  71.08505  63.50414 104.16321  48.14797  79.51338
##  [49,] 146.30516  89.74895  39.18941  73.11414  88.50672  94.50973 102.88032
##  [50,]  71.60782  89.35516  88.60894  86.92190 126.77442  66.75049 104.73499
##  [51,]  82.61504  86.77788  79.87973  58.56740  84.40016  76.97629  91.56203
##  [52,]  77.19701 120.99624 119.42223  95.20072  60.37340  94.86699  89.74610
##  [53,]  74.69783  68.29077  90.50560  81.25629  95.89043  54.00522 108.70216
##  [54,]  88.52153  56.85431  95.51982 108.27778 118.41133  65.35557  86.51996
##  [55,] 100.90840 121.24155  88.76405  68.38224  62.62919 114.39256  90.88332
##  [56,]  93.68674  93.17470  62.86890  53.05708  78.20299 104.90234  85.97324
##  [57,] 121.33995  87.73451  81.35885  82.96136  62.54845  92.14578 117.67719
##  [58,]  74.83956 137.85690  81.15815  93.27585  83.01038 107.25369 109.94249
##  [59,]  84.11556 111.60593  68.59830  59.27838  83.57209  95.40229 125.59964
##  [60,] 121.13382 113.23597  71.90686  98.55692  61.34553 107.74339 122.58509
##  [61,]  64.33878  80.65919  87.58474  52.11413  87.54390  56.83951  98.63933
##  [62,]  89.87633 100.79862  80.91129  93.03464  52.64194  94.06087  98.95233
##  [63,] 100.36505  59.84322  80.44552  50.59173  85.64871  90.35299 108.07695
##  [64,] 116.92657  73.81476  69.26182  90.13392 128.34333  97.93708  79.78320
##  [65,]  78.36804  76.41320 107.55435 100.89708 112.33900  65.81404  88.93295
##  [66,]  71.95705  81.37264 100.92077  78.81098 105.34310  48.54114  85.67318
##  [67,]  81.41844  87.33879  98.71436  96.75723  87.60795  95.53849  93.33440
##  [68,]  93.16226  93.77460  65.70814  90.85121 130.25209  76.01818  71.92045
##  [69,]  80.68168  78.70566  89.88773  68.91488  72.43773  95.79006  97.05704
##  [70,]  71.68801  91.35487 121.22417 115.71933 102.81005  71.60663  81.81221
##  [71,] 102.54518  98.78698  72.98658 101.77043  73.86721  98.04256 108.37100
##  [72,]  64.49416 116.82494 109.39291 105.17487  90.29885 105.35775  76.64809
##  [73,]  73.97042  72.78085  87.34438  63.61216  85.39461  67.24695 117.17718
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##  [76,] 102.74358 117.99383  68.99433 100.77783  72.39907 111.14306 125.62635
##  [77,] 101.07818  85.46306  99.07728  87.44162  88.01140  98.84112  36.65280
##  [78,]  99.51461  67.78106  68.47717  92.20897 121.80396  96.58863 100.44020
##  [79,]  94.27955  52.73052  82.93300  63.33623 127.73657  50.29273  71.80932
##  [80,]  89.90586  84.54431  80.45439  66.59201  97.01806  78.56150  49.75493
##  [81,] 108.73287  88.89467  82.73228  82.58389  99.83278 100.83443  76.58199
##  [82,]  83.20248  73.87463 102.77892  72.91809  76.09542  79.57174 124.06960
##  [83,] 100.37632 124.65101  83.06768 122.22875 103.10878 113.36613  54.71895
```

```
##  [84,]  69.73335 105.64979 111.39266 117.50010  77.64713 107.39175  85.27620
##  [85,]  94.82202  84.20846  72.27511  76.21620 112.63789  84.98667  80.81886
##  [86,]        NA  97.89537 119.36639  96.86677 102.73208  69.60376  89.46119
##  [87,]        NA        NA  91.76499  70.28585 105.33849  63.44298  83.62554
##  [88,]        NA        NA        NA  67.00706 105.79490  91.17050 103.94777
##  [89,]        NA        NA        NA        NA  89.12237  79.01042 103.12014
##  [90,]        NA        NA        NA        NA        NA 116.00548 104.46969
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##  [92,]        NA        NA        NA        NA        NA        NA        NA
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##           [,93]     [,94]     [,95]     [,96]     [,97]     [,98]     [,99]
##   [1,]  63.90264  94.03632 118.43696  62.75041  70.54935  91.18532  58.31697
##   [2,]  72.94394  98.20956  63.53483  98.06630 127.64925  70.76261 116.67392
##   [3,]  48.14284  95.15885 103.10591  63.98415 105.12016  91.23582  72.19182
##   [4,]  88.59610  72.74268 110.18170  75.92144 103.60136 110.25901  62.51507
##   [5,] 109.43394  47.06460 106.96358 109.71068  88.76038 127.06466  87.51466
##   [6,]  83.32868 106.80628  99.94317  85.03935  90.33815  59.84800 104.82249
##   [7,]  98.31727  85.90082 133.12948  86.41351  82.06153  84.87946  58.42253
##   [8,]  77.87179 101.48690  75.58795  97.63014 112.24099  78.34888 106.04797
##   [9,] 102.14167  54.47222  82.73369 120.36424  89.73227 120.42530  97.25249
##  [10,]  98.46757  87.87722  43.18970 123.28796 103.52734  87.73375 147.56781
##  [11,]  89.11532 103.17220 110.23425  72.89451  80.62586  65.54479  75.38123
##  [12,]  99.30322  85.86762  91.51347  74.92054 104.06512  57.96645  93.21094
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##  [14,] 105.52263  84.20162  92.27689 118.80567  72.53221 124.21008  86.49575
##  [15,]  84.50206 111.43489  94.69136  96.02747  87.35234  97.11727  75.70885
##  [16,]  88.04526  95.06588  85.40359  71.57610  90.26543  54.00358 115.77355
##  [17,]  91.98458  93.78664 140.94178  73.06810  67.19545  93.37069  27.89336
##  [18,]  76.12455 120.24804  54.87673  82.21914 101.90566  75.96539 104.43343
##  [19,]  93.36737 116.09570  88.16304  69.45005  74.44906  37.87395 108.40272
##  [20,]  74.49589  92.08030  86.51053  76.38435  87.26390  92.28152 115.67858
##  [21,] 105.33552  79.10253  87.69466  97.67628  99.88073 100.22345  60.61537
##  [22,]  96.46070  90.41980  76.90858 101.04619  94.53734  67.22166 103.37095
##  [23,]  91.27147 125.93317 105.51392  88.57350  57.04708  69.51618  78.25760
##  [24,] 116.00192  82.60899  80.33658 133.31990  76.97194 115.04576 103.76655
##  [25,]  76.54771 101.16352  48.21843 108.05611 100.85397  83.70247 115.75950
##  [26,] 124.82602  69.19927 100.30121 112.94619  80.31588 115.83692  72.97909
##  [27,]  97.18850  88.00663 109.96468  85.47278  83.45870  78.06484  54.47820
##  [28,]  60.37392 117.09616  57.45831  88.23097 107.50445  85.53802  96.35605
##  [29,] 115.22462  52.30507  83.56115 103.86258 100.03832 119.40215  79.77857
##  [30,]  91.79054  96.03407  78.89961  87.37245  88.96679 109.08149  90.88087
##  [31,]  65.65250  97.63500  88.78491  67.64190 123.61349  62.86359 100.91001
##  [32,] 117.83851  65.23346  80.22512  99.15439  97.73476  78.70765 101.14835
##  [33,]  85.06408  95.83667  65.95382  82.65118  89.45360  64.25993 115.52836
##  [34,] 120.02390  80.41967  57.07460 109.22470 104.08957  94.41283 107.72477
##  [35,]  80.03631  76.01965  74.62756  67.74744 125.69600  75.09997  94.45735
##  [36,]  58.56984 105.11641 106.63360  64.21658 102.92357  68.69581  83.49453
```

```
##  [37,]  70.38195 105.80772  60.57131  75.15509 103.16390  62.80502 111.88686
##  [38,]  99.37046  78.92568 106.87754  86.65457  70.00926 101.15212  94.51466
##  [39,]  97.37532  44.91203  98.58624  81.68049 111.87588 113.65187  72.08683
##  [40,]  86.95744 104.47503  66.39162 119.62041  79.66360  89.25878 127.94782
##  [41,]  69.52156  89.97273  62.57978  76.49384 119.54368  95.65943 104.89938
##  [42,]  82.70416  90.52499  49.95881  95.73796 114.97484  80.82168 109.18422
##  [43,]  74.82493 134.09859  82.38194  87.69537  82.14633  93.41543  78.27120
##  [44,]  80.07205  90.00814  87.00783  79.32831 110.23866  81.02519  67.98063
##  [45,] 102.94922  71.61340  95.97865 106.90834  82.79913 110.19521 101.26626
##  [46,]  80.78890 102.22319  85.47499  87.61498  81.24973  90.60965  95.05657
##  [47,]  71.96372 136.77390  66.10624  85.81336  88.05171  48.07282 124.87169
##  [48,]  99.27644  96.27519 109.66364  86.52677  53.60106  90.93659  68.31729
##  [49,]  89.76662  66.47416  93.79564  84.60231  88.11617 119.11065  76.62712
##  [50,]  89.89882  73.92942  93.29646  91.70253  89.48735  71.80771 105.86031
##  [51,]  75.66005 111.00470 117.93421  51.79243  65.33661  73.70542  69.50161
##  [52,]  94.89179  91.51830  89.47480 121.53688  76.57041 121.56535  81.11348
##  [53,]  68.58751 107.11238 125.31266  54.62419  87.99984  56.22698  62.81975
##  [54,]  71.55319 118.73495  79.03918  74.43700 103.58883  44.85299  99.89394
##  [55,]  95.14741  74.05210  97.61697 103.94280  69.17528 131.78623  88.38578
##  [56,] 104.83362  70.98087 115.93283  73.97217  71.27793 105.19760  75.20941
##  [57,]  86.77638  69.54613 116.53098  81.11079  95.35687 108.76708  67.33877
##  [58,] 103.86383  44.94628  99.65164 102.48370 103.25727 113.29911  82.16118
##  [59,]  88.93333  53.25223 120.68943  75.93106  88.16016 105.45656  74.43203
##  [60,]  88.37242  61.85882  83.91498 105.76157 114.96809 143.54477  65.75422
##  [61,]  70.83382 114.91596 127.92394  63.09219  57.15450  78.90139  54.21257
##  [62,] 106.02894  78.93414 112.14387  83.49199  96.93029 109.71955  39.90373
##  [63,]  82.36796  83.69508 100.30419  80.35813  81.57408 102.41249  87.95465
##  [64,]  82.31736  91.62175  52.12684  90.19361  97.78248  79.50856 133.76621
##  [65,] 109.22833  68.15553  95.69971 118.92727  84.52045  86.44559 104.34792
##  [66,]  62.58041 125.11477 108.14972  84.04314  60.07350  71.60503  77.16198
##  [67,]  88.35520  83.15636 105.04943  83.44626 100.94011  78.47104  90.46015
##  [68,] 102.59970  83.97603  67.73712 106.57989  77.98428  91.87087 104.05861
##  [69,]  70.02822 113.25691 115.61101  36.54117  89.30433  63.72565  74.49985
##  [70,]  85.92359 113.21831  66.75543 107.80512  91.07316  73.60839 105.69364
##  [71,]  84.72744  92.23073  81.51541  71.32100 117.67998  91.89984  68.72189
##  [72,]  82.74045 100.34808  88.93724  89.25252  94.50453  73.56375 102.96972
##  [73,]  74.88143  97.24758 106.81917  80.13194  83.67912  96.13065  60.89569
##  [74,]  92.13467  91.06890  80.05001  67.60342 108.64958  54.84084  97.30924
##  [75,] 109.42875  70.23040  86.04751  86.66076  97.99612 105.50316  88.29093
##  [76,] 106.54569  32.40240  89.23653 102.66241 124.79867 128.58764  73.41864
##  [77,] 105.22389 100.20937  91.47190 106.61759  55.92881  95.56948 105.09232
##  [78,]  68.19110 101.60789  53.52948  68.20623 122.38974  65.06054 114.58700
##  [79,]  83.79063 107.83759  97.51997  86.90570  47.24504  72.08901 101.34804
##  [80,] 103.55567  99.06959 116.13180  82.70478  37.80361  82.97150  85.27286
##  [81,]  80.87218 105.87801  54.81025  98.38828  81.78816  99.00853 114.46537
##  [82,]  55.01337 108.85669 114.82062  51.22906  95.38052  67.36056  77.24674
##  [83,] 113.10633  75.56788  59.42354 121.66107  94.81654 101.36432 115.63821
##  [84,] 112.02240  71.73879  87.50050 110.82470 109.82442  95.93182  92.95899
##  [85,]  71.07415 105.56520 101.49604  53.01069  79.73316  53.96969  99.79626
##  [86,]  94.80404  98.21541 100.74326  89.64461  86.94148  60.39723  89.63631
##  [87,]  80.46956 114.31558  97.28531  71.89203  75.32103  65.04313  93.85557
##  [88,]  84.52507  70.43106  95.47251  67.06281  92.94306 100.01820  74.53839
##  [89,]  67.54646  97.64144 126.24776  54.68267  54.72754  90.59380  74.25405
##  [90,]  93.30761  83.91628 104.32016  94.03057  95.09930 126.93842  63.28086
```

```
##  [91,]  88.74331  99.75067 118.50847  88.01560  62.09673  71.24323  67.08138
##  [92,] 117.13187 104.28630  74.10430 112.69883  62.67806  80.65706 119.02304
##  [93,]        NA 129.31931  88.23245  48.94559  96.74361  70.12166  90.08890
##  [94,]        NA        NA  99.96347 113.19537 101.70373 124.97240  79.13302
##  [95,]        NA        NA        NA 108.24447 122.28211  86.23036 134.23441
##  [96,]        NA        NA        NA        NA  93.72791  58.66904  78.71655
##  [97,]        NA        NA        NA        NA        NA  92.27845  82.26400
##  [98,]        NA        NA        NA        NA        NA        NA 111.11997
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##          [,100]
##   [1,] 111.53420
##   [2,]  62.37073
##   [3,] 101.13031
##   [4,]  97.92243
##   [5,]  87.53637
##   [6,]  43.56881
##   [7,]  73.31099
##   [8,]  70.01774
##   [9,] 108.90126
##  [10,]  78.43017
##  [11,]  90.60211
##  [12,]  40.71036
##  [13,] 127.09814
##  [14,] 112.22804
##  [15,]  98.26508
##  [16,]  56.85758
##  [17,] 102.53199
##  [18,]  90.39101
##  [19,]  61.74354
##  [20,]  77.62295
##  [21,] 117.01912
##  [22,]  72.24783
##  [23,]  90.72003
##  [24,]  92.91579
##  [25,] 101.12117
##  [26,]  88.46158
##  [27,]  96.67901
##  [28,] 112.65364
##  [29,] 115.20815
##  [30,]  95.32306
##  [31,]  56.64930
##  [32,]  82.93512
##  [33,]  85.45769
##  [34,]  75.88710
##  [35,]  84.73337
##  [36,]  68.18069
##  [37,]  77.46428
##  [38,]  73.22829
##  [39,] 106.00321
##  [40,]  84.16062
##  [41,]  92.17985
##  [42,]  95.05509
##  [43,] 107.39899
```

```
##  [44,]  94.17966
##  [45,] 101.75064
##  [46,]  88.12331
##  [47,]  72.26051
##  [48,] 116.05571
##  [49,] 132.15620
##  [50,]  70.89286
##  [51,]  89.02645
##  [52,] 104.19251
##  [53,]  79.22936
##  [54,]  78.98309
##  [55,] 108.74128
##  [56,]  91.49847
##  [57,] 103.60561
##  [58,]  82.54112
##  [59,]  84.29601
##  [60,] 126.36136
##  [61,]  96.45482
##  [62,]  98.70397
##  [63,]  94.22165
##  [64,]  94.09099
##  [65,]  78.79858
##  [66,] 105.29688
##  [67,]  70.80969
##  [68,] 107.30311
##  [69,]  67.80827
##  [70,]  83.24399
##  [71,]  94.65029
##  [72,]  68.59481
##  [73,]  98.33865
##  [74,]  52.46115
##  [75,]  78.82256
##  [76,]  95.64509
##  [77,] 102.72435
##  [78,]  78.65362
##  [79,] 103.44889
##  [80,]  99.32878
##  [81,] 109.56854
##  [82,]  72.15561
##  [83,]  99.14868
##  [84,]  67.63305
##  [85,]  80.90755
##  [86,]  48.24765
##  [87,]  87.30305
##  [88,] 115.62979
##  [89,] 102.20195
##  [90,] 100.18115
##  [91,]  97.66245
##  [92,]  89.05562
##  [93,]  95.19878
##  [94,]  92.95064
##  [95,]  85.44563
##  [96,]  77.56801
##  [97,] 109.07873
```

```
##  [98,]  49.89962
##  [99,] 114.10905
## [100,]        NA
```

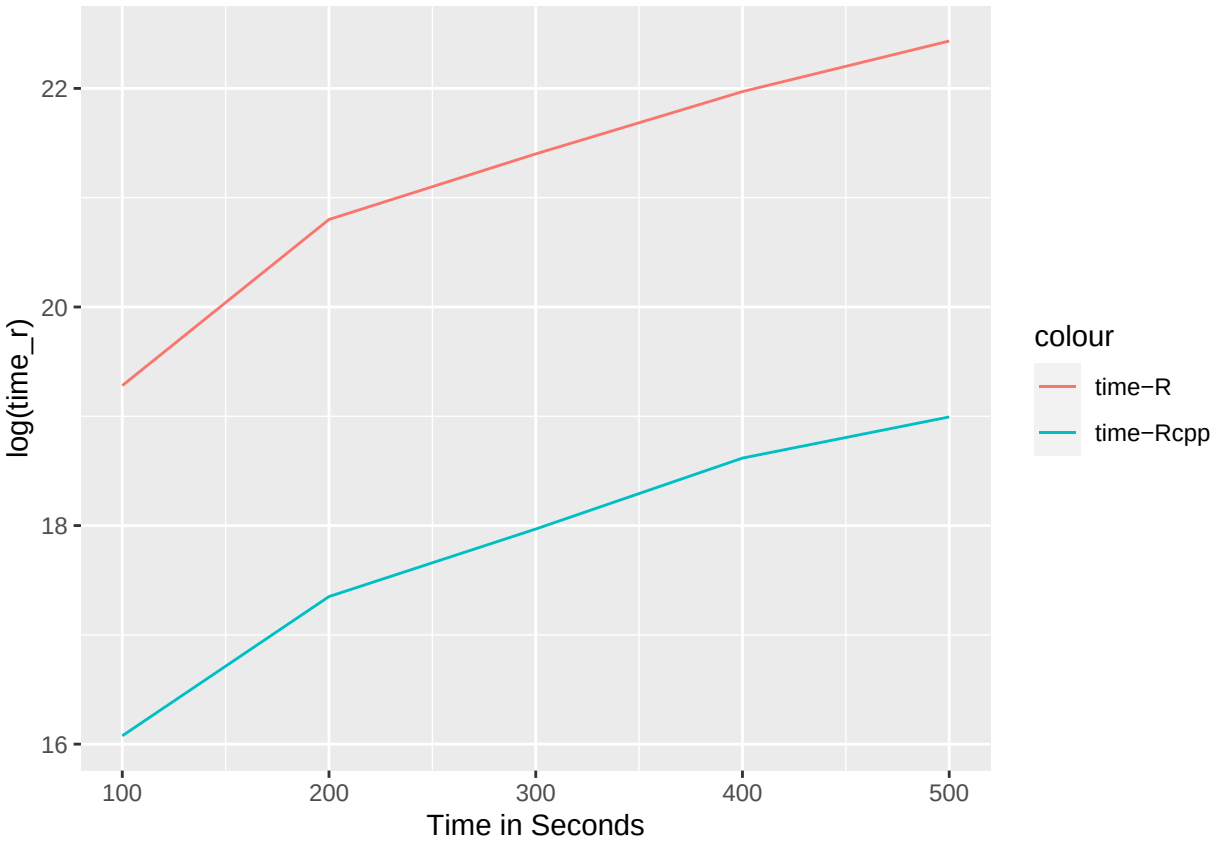
Test the time difference between these functions for `n = 1000` and `Nvec = 100, 500, 1000, 5000` using the package `microbenchmark`. Store the results in a matrix with rows representing `Nvec` and two columns for base R and Rcpp.

```
pacman::p_load(microbenchmark)

n = 1000
Nvec = c(100, 200, 300, 400, 500)
time_r = c()
time_cpp = c()
for (i in 1:length(Nvec)){
  X = c()
  for (j in 1:n){
    x = rnorm(Nvec[i])
    X = cbind(X, x)
  }
  time_r = c(time_r, mean(microbenchmark(angles_r = all_angles(X), times = 3, unit = "s")$time))
  time_cpp = c(time_cpp, mean(microbenchmark(angles_cpp = all_angles_cpp(X), times = 3, unit = "s")$time
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot. We will see later how to create "long" matrices that make such plots easier.

```
pacman::p_load(ggplot2)
ggplot() +
  geom_line(aes(x = Nvec, y = log(time_r), col = "time-R")) +
  geom_line(aes(x = Nvec, y = log(time_cpp), col = "time-Rcpp")) +
  xlab("Time in Seconds")
```



Let `Nvec = 10000` and vary `n` to be 10, 100, 1000. Plot the density of angles for all three values of `n` on one plot using color to signify `n`. Make sure you have a color legend. This is not easy.

```
Nvec = 10000
X = c()
for (i in 1:10){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_1 = all_angles(X)
X = c()
for (i in 1:100){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_2 = all_angles(X)
X = c()
for (i in 1:1000){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_3 = all_angles(X)

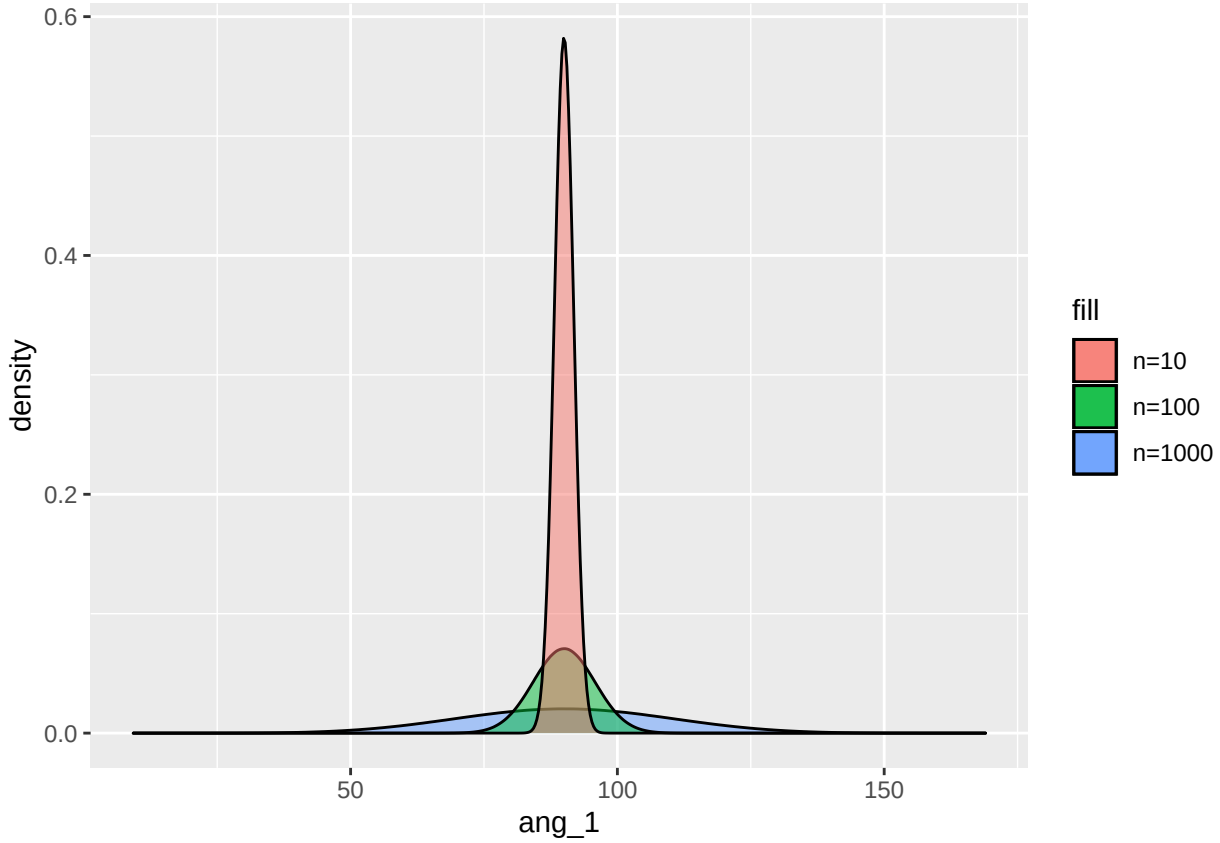
ggplot() +
  geom_density(aes(x = ang_1, fill = "yellow"), alpha = .5) +
  geom_density(aes(x = ang_2, fill = "red"), alpha = .5) +
  geom_density(aes(x = ang_3, fill = "blue"), alpha = .5) +
```

```
  scale_fill_discrete(labels = c("n=10", "n=100", "n=1000"))
```

```
## Warning: Removed 50005000 rows containing non-finite values (stat_density).
```

```
## Warning: Removed 50005000 rows containing non-finite values (stat_density).
```

```
## Warning: Removed 50005000 rows containing non-finite values (stat_density).
```



Write an R function `nth_fibonnaci` that finds the nth Fibonnaci number via recursion but allows you to specify the starting number. For instance, if the sequency started at 1, you get the familiar 1, 1, 2, 3, 5, etc. But if it started at 0.01, you would get 0.01, 0.01, 0.02, 0.03, 0.05, etc.

```
nth_fibonnaci = function(n,start){
   if (n-1 <= 1){
        return(start)
   }
  return(nth_fibonnaci(n-1, start) + nth_fibonnaci(n-2, start))
}

nth_fibonnaci(9,0.1)
```

```
## [1] 3.4
```

Write an Rcpp function `nth_fibonnaci_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
cppFunction("
  double nth_fibonnaci_cpp(int n, double start){
     if (n-1 <= 1)
```

```
        return start;
    return nth_fibonnaci_cpp(n-1, start) + nth_fibonnaci_cpp(n-2, start);
  }
")
```

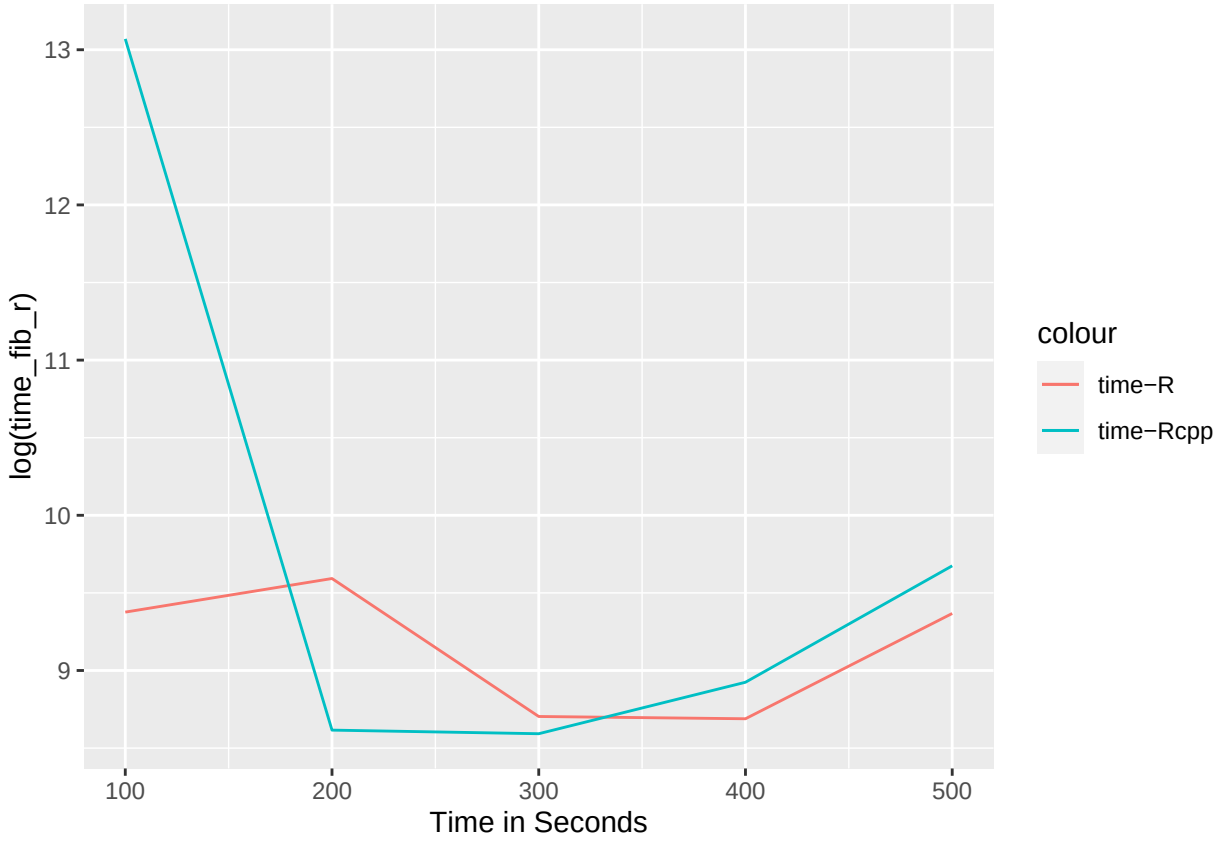
Time the difference in these functions for n = 100, 200, ..., 1500 while starting the sequence at the smallest possible floating point value in R. Store the results in a matrix.

```
pacman::p_load(microbenchmark)

n = 1000
Nvec = c(100, 200, 300, 400, 500)
time_fib_r = c()
time_fib_cpp = c()
for (i in 1:length(Nvec)){
 time_fib_r  = c(time_fib_r, mean(microbenchmark(angles_r = nth_fibonnaci(i, .Machine$double.min), times
  time_fib_cpp = c(time_fib_cpp, mean(microbenchmark(angles_cpp = nth_fibonnaci_cpp(i, .Machine$double.m
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
pacman::p_load(ggplot2)
ggplot() +
  geom_line(aes(x = Nvec, y = log(time_fib_r), col = "time-R")) +
  geom_line(aes(x = Nvec, y = log(time_fib_cpp), col = "time-Rcpp")) +
  xlab("Time in Seconds")
```



Data Wrangling / Munging / Carpentry

Throughout this assignment you can use either the `tidyverse` package suite or `data.table` to answer but not base R. You can mix `data.table` with `magrittr` piping if you wish but don't go back and forth between `tbl_df`'s and `data.table` objects.

```
pacman::p_load(tidyverse, magrittr, data.table)
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below.

```
data(storms)
str(storms)
```

```
## tibble[,13] [10,010 x 13] (S3: tbl_df/tbl/data.frame)
##  $ name       : chr [1:10010] "Amy" "Amy" "Amy" "Amy" ...
##  $ year       : num [1:10010] 1975 1975 1975 1975 1975 ...
##  $ month      : num [1:10010] 6 6 6 6 6 6 6 6 6 6 ...
##  $ day        : int [1:10010] 27 27 27 27 28 28 28 28 29 29 ...
##  $ hour       : num [1:10010] 0 6 12 18 0 6 12 18 0 6 ...
##  $ lat        : num [1:10010] 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4 34 ...
##  $ long       : num [1:10010] -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -74.8 ...
##  $ status     : chr [1:10010] "tropical depression" "tropical depression" "tropical depression" "trop
##  $ category   : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<..: 1 1 1 1 1 1 1 1 2 2 ...
##  $ wind       : int [1:10010] 25 25 25 25 25 25 25 30 35 40 ...
##  $ pressure   : int [1:10010] 1013 1013 1013 1013 1012 1012 1011 1006 1004 1002 ...
##  $ ts_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
##  $ hu_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
```

```
head(storms)
```

```
## # A tibble: 6 x 13
##   name   year month   day  hour   lat  long status       category  wind pressure
##   <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>        <ord>    <int>    <int>
## 1 Amy    1975     6    27     0  27.5 -79   tropical de~ -1          25     1013
## 2 Amy    1975     6    27     6  28.5 -79   tropical de~ -1          25     1013
## 3 Amy    1975     6    27    12  29.5 -79   tropical de~ -1          25     1013
## 4 Amy    1975     6    27    18  30.5 -79   tropical de~ -1          25     1013
## 5 Amy    1975     6    28     0  31.5 -78.8 tropical de~ -1          25     1012
## 6 Amy    1975     6    28     6  32.4 -78.7 tropical de~ -1          25     1012
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

Reorder the columns so name is first, status is second, category is third and the rest are the same.

```
storms %>%
  select(name, status, category, everything())
```

```
## # A tibble: 10,010 x 13
##    name  status      category  year month   day  hour   lat  long  wind pressure
##    <chr> <chr>       <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>    <int>
##  1 Amy   tropical d~ -1        1975     6    27     0  27.5 -79      25     1013
##  2 Amy   tropical d~ -1        1975     6    27     6  28.5 -79      25     1013
##  3 Amy   tropical d~ -1        1975     6    27    12  29.5 -79      25     1013
##  4 Amy   tropical d~ -1        1975     6    27    18  30.5 -79      25     1013
##  5 Amy   tropical d~ -1        1975     6    28     0  31.5 -78.8    25     1012
##  6 Amy   tropical d~ -1        1975     6    28     6  32.4 -78.7    25     1012
##  7 Amy   tropical d~ -1        1975     6    28    12  33.3 -78      25     1011
##  8 Amy   tropical d~ -1        1975     6    28    18  34   -77      30     1006
```

```
##  9 Amy   tropical s~ 0         1975     6    29     0  34.4 -75.8    35     1004
## 10 Amy   tropical s~ 0         1975     6    29     6  34   -74.8    40     1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storms only in the 1970's.

```
storms %>%
  filter(year>=1970 & year<=1979 )
```

```
## # A tibble: 546 x 13
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 536 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storm observations only with category 4 and above and wind speed 100MPH and above.

```
storms %>%
  filter(category >= 4 & wind >= 100)
```

```
## # A tibble: 416 x 13
##    name   year month   day  hour   lat  long status    category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>    <int>
##  1 Anita  1977     9     2     0  24.6 -96.2 hurricane 5          140      931
##  2 Anita  1977     9     2     6  24.2 -97.1 hurricane 5          150      926
##  3 Anita  1977     9     2    12  23.7 -98   hurricane 4          120      940
##  4 David  1979     8    28     0  12.2 -52.9 hurricane 4          115      947
##  5 David  1979     8    28     6  12.5 -54.4 hurricane 4          125      941
##  6 David  1979     8    28    12  12.8 -55.7 hurricane 4          130      938
##  7 David  1979     8    28    18  13.2 -56.9 hurricane 4          125      941
##  8 David  1979     8    29     0  13.7 -58   hurricane 4          120      944
##  9 David  1979     8    29     6  14.2 -59.2 hurricane 4          120      942
## 10 David  1979     8    29    12  14.8 -60.3 hurricane 4          125      938
## # ... with 406 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Create a new feature `wind_speed_per_unit_pressure`.

```
storms %>%
  mutate(wind_speed_per_unit_pressure = wind/pressure)
```

```
## # A tibble: 10,010 x 14
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
```

```
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Create a new feature: `average_diameter` which averages the two diameter metrics. If one is missing, then use the value of the one that is present. If both are missing, leave missing.

```
storms %<>%
  rowwise() %>%
  arrange(desc(year)) %>%
  mutate(average_diameter = ifelse(!is.na(ts_diameter) & !is.na(hu_diameter), mean(c(ts_diameter,hu_dia
storms
```

```
## # A tibble: 10,010 x 14
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>
```

For each storm, summarize the maximum wind speed. "Summarize" means create a new dataframe with only the summary metrics you care about.

```
storms %>%
  group_by(name) %>%
  summarise(max_wind_speed = max(wind, na.rm = TRUE))
```

```
## # A tibble: 198 x 2
##    name     max_wind_speed
##  * <chr>             <int>
##  1 AL011993             30
##  2 AL012000             25
##  3 AL021992             30
##  4 AL021994             30
##  5 AL021999             30
##  6 AL022000             30
##  7 AL022001             25
##  8 AL022003             30
##  9 AL022006             45
## 10 AL031987             40
```

```
## # ... with 188 more rows
```

Order your dataset by maximum wind speed storm but within the rows of storm show the observations in time order from early to late.

```
storms %>%
  group_by(name) %>%
  mutate(max_wind_by_storm = max(wind, na.rm = TRUE)) %>%
  select(name, max_wind_by_storm, everything()) %>%
  arrange(desc(max_wind_by_storm), year, month, day, hour)
```

```
## # A tibble: 10,010 x 15
## # Groups:   name [198]
##    name   max_wind_by_sto~  year month   day  hour   lat  long status   category
##    <chr>             <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>    <ord>
##  1 Gilbe~              160  1988     9     8    18  12   -54   tropica~ -1
##  2 Gilbe~              160  1988     9     9     0  12.7 -55.6 tropica~ -1
##  3 Gilbe~              160  1988     9     9     6  13.3 -57.1 tropica~ -1
##  4 Gilbe~              160  1988     9     9    12  14   -58.6 tropica~ -1
##  5 Gilbe~              160  1988     9     9    18  14.5 -60.1 tropica~ 0
##  6 Gilbe~              160  1988     9    10     0  14.8 -61.5 tropica~ 0
##  7 Gilbe~              160  1988     9    10     6  15   -62.8 tropica~ 0
##  8 Gilbe~              160  1988     9    10    12  15.3 -64.1 tropica~ 0
##  9 Gilbe~              160  1988     9    10    18  15.7 -65.4 tropica~ 0
## 10 Gilbe~              160  1988     9    11     0  15.9 -66.8 hurrica~ 1
## # ... with 10,000 more rows, and 5 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, average_diameter <dbl>
```

Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
  arrange(year, desc(wind)) %>%
  slice(1) %>%
  select(name, year, wind)
```

```
## # A tibble: 41 x 3
## # Groups:   year [41]
##    name      year  wind
##    <chr>    <dbl> <int>
##  1 Caroline  1975   100
##  2 Belle     1976   105
##  3 Anita     1977   150
##  4 Cora      1978    80
##  5 David     1979   150
##  6 Ivan      1980    90
##  7 Harvey    1981   115
##  8 Debby     1982   115
##  9 Alicia    1983   100
## 10 Diana     1984   115
## # ... with 31 more rows
```

For each named storm, find its maximum category, wind speed, pressure and diameters. Do not allow the max to be NA (unless all the measurements for that storm were NA).

```
storms %>%
  group_by(name) %>%
```

```
  mutate(max_wind_by_storm = max(wind, na.rm = TRUE), max_category_by_storm = max(category, na.rm = TRU
select(max_wind_by_storm, max_category_by_storm, max_pressure_by_storm, max_hu_diameter_by_storm, max_t
  arrange(name) %>%
  distinct()
```

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## Adding missing grouping variables: `name`

## # A tibble: 198 x 6
## # Groups:   name [198]
##    name   max_wind_by_storm max_category_by_~ max_pressure_by~ max_hu_diameter_~
##    <chr>              <int> <ord>                        <int>             <dbl>
##  1 AL011~                30 -1                            1003              -Inf
##  2 AL012~                25 -1                            1010              -Inf
##  3 AL021~                30 -1                            1009              -Inf
##  4 AL021~                30 -1                            1017              -Inf
##  5 AL021~                30 -1                            1006              -Inf
##  6 AL022~                30 -1                            1010              -Inf
##  7 AL022~                25 -1                            1012              -Inf
##  8 AL022~                30 -1                            1010              -Inf
##  9 AL022~                45 0                             1008                 0
## 10 AL031~                40 0                             1015              -Inf
## # ... with 188 more rows, and 1 more variable: max_ts_diameter_by_storm <dbl>
```
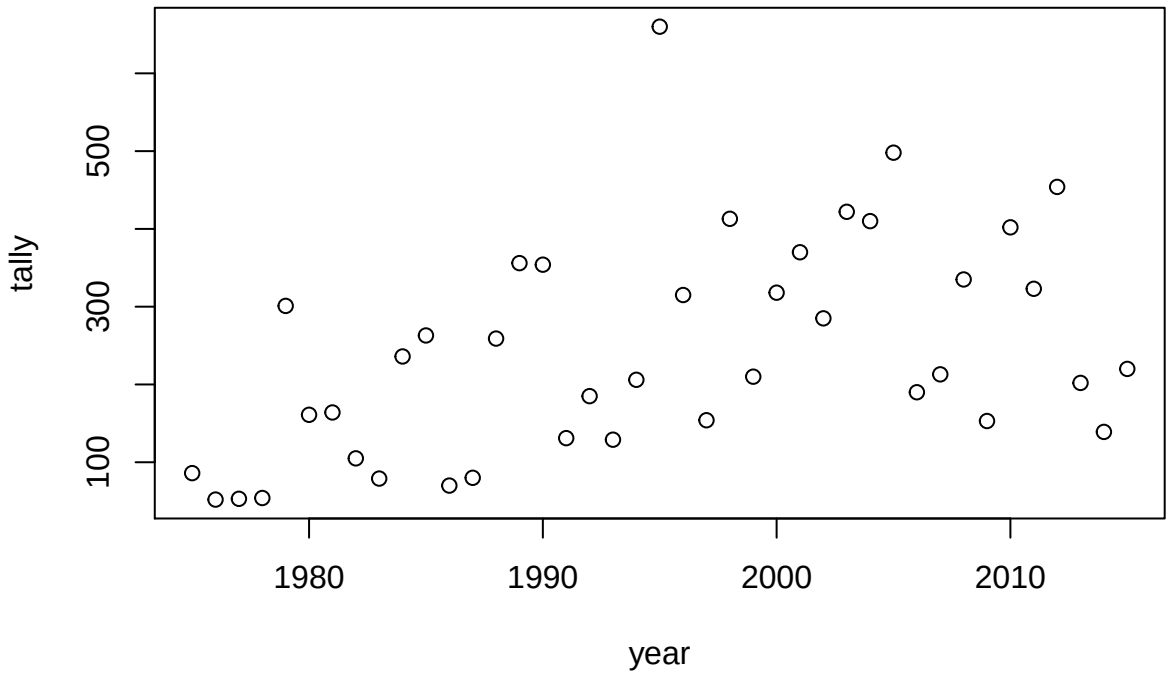

For each year in the dataset, tally the number of storms. “Tally” is a fancy word for “count the number of”. Plot the number of storms by year. Any pattern?

```
storms %>%
  group_by(year) %>%
  summarise(tally = n()) %>%
  plot()
```



For each year in the dataset, tally the storms by category.

```
storms %>%
  group_by(year, category) %>%
  summarise(tally = n())
```

```
## `summarise()` has grouped output by 'year'. You can override using the `.groups` argument.
```

```
## # A tibble: 233 x 3
## # Groups:   year [41]
##     year category tally
##    <dbl> <ord>    <int>
##  1  1975 -1          30
##  2  1975 0           33
##  3  1975 1           12
##  4  1975 2            9
##  5  1975 3            2
##  6  1976 -1          10
##  7  1976 0           20
##  8  1976 1           10
##  9  1976 2            9
## 10  1976 3            3
## # ... with 223 more rows
```

For each year in the dataset, find the maximum wind speed per status level.

```
storms %>%
  group_by(year, status) %>%
  mutate(max_wind_by_storm = max(wind, na.rm = TRUE)) %>%
  arrange(year, status, desc(max_wind_by_storm)) %>%
  select(year, status, max_wind_by_storm) %>%
  distinct
```

```
## # A tibble: 123 x 3
## # Groups:   year, status [123]
```

```
##     year status              max_wind_by_storm
##    <dbl> <chr>                           <int>
##  1  1975 hurricane                         100
##  2  1975 tropical depression                30
##  3  1975 tropical storm                     60
##  4  1976 hurricane                         105
##  5  1976 tropical depression                30
##  6  1976 tropical storm                     60
##  7  1977 hurricane                         150
##  8  1977 tropical depression                30
##  9  1977 tropical storm                     60
## 10  1978 hurricane                          80
## # ... with 113 more rows
```

For each storm, summarize its average location in latitude / longitude coordinates.

```
storms %>%
  group_by(name) %>%
  summarise(avg_lat = mean(lat), avg_long = mean(long))
```

```
## # A tibble: 198 x 3
##    name     avg_lat avg_long
##  * <chr>      <dbl>    <dbl>
##  1 AL011993   24.7     -78.0
##  2 AL012000   20.8     -93.1
##  3 AL021992   26.7     -84.5
##  4 AL021994   33.6     -79.7
##  5 AL021999   20.4     -96.4
##  6 AL022000    9.9     -28.5
##  7 AL022001   11.9     -45.3
##  8 AL022003    9.62    -43.4
##  9 AL022006   41.3     -63.5
## 10 AL031987   30.8     -88.7
## # ... with 188 more rows
```

For each storm, summarize its duration in number of hours (to the nearest 6hr increment).

```
storms %>%
  group_by(name) %>%
  mutate(duration = (n()-1)*6) %>%
  select(name, duration) %>%
  distinct
```

```
## # A tibble: 198 x 2
## # Groups:   name [198]
##    name      duration
##    <chr>        <dbl>
##  1 Ana            594
##  2 Bill           450
##  3 Claudette     1074
##  4 Danny          870
##  5 Erika          618
##  6 Fred           300
##  7 Henri          570
##  8 Nine            72
##  9 Ida            372
## 10 Joaquin        246
```

```
## # ... with 188 more rows
```

For storm in a category, create a variable `storm_number` that enumerates the storms 1, 2, ... (in date order).

```
storms %>%
  group_by(category) %>%
  mutate(storm_number = dense_rank(paste(year, month, day))) %>%
  select(category, storm_number, year, month, day, name) %>%
  distinct %>%
  arrange(category, storm_number)
```

```
## # A tibble: 3,945 x 6
## # Groups:   category [7]
##    category storm_number  year month   day name
##    <ord>           <int> <dbl> <dbl> <int> <chr>
##  1 -1                  1  1975     6    27 Amy
##  2 -1                  2  1975     6    28 Amy
##  3 -1                  3  1975     8    24 Caroline
##  4 -1                  4  1975     8    25 Caroline
##  5 -1                  5  1975     8    26 Caroline
##  6 -1                  6  1975     8    27 Caroline
##  7 -1                  7  1975     8    28 Caroline
##  8 -1                  8  1975     8    29 Caroline
##  9 -1                  9  1975     9     1 Caroline
## 10 -1                 10  1976    10     4 Gloria
## # ... with 3,935 more rows
```

Convert year, month, day, hour into the variable `timestamp` using the `lubridate` package. Although the new package `clock` just came out, `lubridate` still seems to be standard. Next year I'll probably switch the class to be using `clock`.

```
pacman::p_load(lubridate)
storms %>%
  unite(timestamp, year, month, day, hour, sep = "-", remove = FALSE)
```

```
## # A tibble: 10,010 x 15
##    name  timestamp   year month   day  hour   lat  long status    category  wind
##    <chr> <chr>      <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>
##  1 Ana   2015-5-9-6  2015     5     9     6  32.2 -77.5 tropical~ 0           50
##  2 Ana   2015-5-9-~  2015     5     9    12  32.5 -77.8 tropical~ 0           50
##  3 Ana   2015-5-9-~  2015     5     9    18  32.7 -78   tropical~ 0           45
##  4 Ana   2015-5-10~  2015     5    10     0  33.1 -78.3 tropical~ 0           45
##  5 Ana   2015-5-10~  2015     5    10     6  33.5 -78.6 tropical~ 0           40
##  6 Ana   2015-5-10~  2015     5    10    10  33.8 -78.8 tropical~ 0           40
##  7 Ana   2015-5-10~  2015     5    10    12  33.9 -78.8 tropical~ 0           35
##  8 Ana   2015-5-10~  2015     5    10    18  34.3 -78.7 tropical~ -1          30
##  9 Ana   2015-5-11~  2015     5    11     0  34.7 -78.5 tropical~ -1          30
## 10 Ana   2015-5-11~  2015     5    11     6  35.5 -78   tropical~ -1          30
## # ... with 10,000 more rows, and 4 more variables: pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, average_diameter <dbl>
```

Using the `lubridate` package, create new variables `day_of_week` which is a factor with levels "Sunday", "Monday", ... "Saturday" and `week_of_year` which is integer 1, 2, ..., 52.

```
storms %<>%
  mutate(timestamp = make_datetime(year, month, day),
         day_of_week = wday(ymd(timestamp), label = TRUE, abbr = FALSE),
```

```
         week_of_year = week(ymd(timestamp)))
```

For each storm, summarize the day in which is started in the following format "Friday, June 27, 1975".

```
storms %>%
  group_by(name) %>%
  summarise(start_date = min(timestamp)) %>%
  mutate(start_date = paste(weekdays(start_date), paste(months(start_date), day(start_date), sep = " ")
```

```
## # A tibble: 198 x 2
##    name     start_date
##  * <chr>    <chr>
##  1 AL011993 Monday, May 31, 1993
##  2 AL012000 Wednesday, June 7, 2000
##  3 AL021992 Thursday, June 25, 1992
##  4 AL021994 Wednesday, July 20, 1994
##  5 AL021999 Friday, July 2, 1999
##  6 AL022000 Friday, June 23, 2000
##  7 AL022001 Wednesday, July 11, 2001
##  8 AL022003 Wednesday, June 11, 2003
##  9 AL022006 Monday, July 17, 2006
## 10 AL031987 Sunday, August 9, 1987
## # ... with 188 more rows
```

Create a new factor variable `decile_windspeed` by binning wind speed into 10 bins.

```
storms %>%
  mutate(decile_windspeed = ntile(wind, 10))
```

```
## # A tibble: 10,010 x 18
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 7 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>, decile_windspeed <int>
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
serious_storms = copy(storms)
serious_storms %>%
  filter(category>=3)
```

```
## # A tibble: 779 x 17
## # Rowwise:
##    name     year month   day  hour   lat  long status    category  wind pressure
##    <chr>   <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>    <int>
```

```
##  1 Danny    2015     8    21    12  13.8 -47.8 hurricane 3          110      960
##  2 Danny    2015     8    21    18  14.3 -48.6 hurricane 3          105      966
##  3 Joaquin  2015    10     1     0  23.9 -72.9 hurricane 3          100      951
##  4 Joaquin  2015    10     1     6  23.5 -73.3 hurricane 3          110      947
##  5 Joaquin  2015    10     1    12  23.1 -73.7 hurricane 4          115      942
##  6 Joaquin  2015    10     1    18  23   -74.2 hurricane 4          115      936
##  7 Joaquin  2015    10     2     0  22.9 -74.4 hurricane 4          120      931
##  8 Joaquin  2015    10     2     6  23   -74.7 hurricane 4          120      935
##  9 Joaquin  2015    10     2    12  23.4 -74.8 hurricane 4          115      937
## 10 Joaquin  2015    10     2    16  23.6 -74.8 hurricane 3          110      940
## # ... with 769 more rows, and 6 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>
```

In `serious_storms`, merge the variables lat and long together into `lat_long` with values `lat / long` as a string.

```
serious_storms %>%
  unite(lat_long, lat, long, sep = " / ")
```

```
## # A tibble: 10,010 x 16
##    name   year month   day  hour lat_long   status       category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <chr>      <chr>        <ord>    <int>    <int>
##  1 Ana    2015     5     9     6 32.2 / -7~ tropical st~ 0           50      998
##  2 Ana    2015     5     9    12 32.5 / -7~ tropical st~ 0           50     1001
##  3 Ana    2015     5     9    18 32.7 / -78 tropical st~ 0           45     1001
##  4 Ana    2015     5    10     0 33.1 / -7~ tropical st~ 0           45     1001
##  5 Ana    2015     5    10     6 33.5 / -7~ tropical st~ 0           40     1002
##  6 Ana    2015     5    10    10 33.8 / -7~ tropical st~ 0           40     1002
##  7 Ana    2015     5    10    12 33.9 / -7~ tropical st~ 0           35     1002
##  8 Ana    2015     5    10    18 34.3 / -7~ tropical de~ -1          30     1006
##  9 Ana    2015     5    11     0 34.7 / -7~ tropical de~ -1          30     1009
## 10 Ana    2015     5    11     6 35.5 / -78 tropical de~ -1          30     1010
## # ... with 10,000 more rows, and 6 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>
```

Let's return now to the original storms data frame. For each category, find the average wind speed, pressure and diameters (do not count the NA's in your averaging).

```
storms %>%
  group_by(category) %>%
  mutate(avg_wind_speed = mean(wind), avg_pressure = mean(pressure), avg_ts_diameter = mean(ts_diameter
```

```
## # A tibble: 10,010 x 21
## # Groups:   category [7]
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
```

```
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 10 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>, avg_wind_speed <dbl>,
## #   avg_pressure <dbl>, avg_ts_diameter <dbl>, avg_hu_diameter <dbl>
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
storms %>%
  group_by(name) %>%
  summarise(max_category = max(category, na.rm = TRUE), max_wind_speed = max(wind, na.rm = TRUE), max_p
```

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## # A tibble: 198 x 6
##    name     max_category max_wind_speed max_pressure max_ts_diameter
##  * <chr>    <ord>                 <int>        <int>           <dbl>
##  1 AL011993 -1                       30         1003            -Inf
##  2 AL012000 -1                       25         1010            -Inf
##  3 AL021992 -1                       30         1009            -Inf
##  4 AL021994 -1                       30         1017            -Inf
##  5 AL021999 -1                       30         1006            -Inf
##  6 AL022000 -1                       30         1010            -Inf
##  7 AL022001 -1                       25         1012            -Inf
##  8 AL022003 -1                       30         1010            -Inf
##  9 AL022006 0                        45         1008              69.0
## 10 AL031987 0                        40         1015            -Inf
## # ... with 188 more rows, and 1 more variable: max_hu_diameter <dbl>
```

Calculate the distance from each storm observation to Miami in a new variable `distance_to_miami`. This is very challenging. You will need a function that computes distances from two sets of latitude / longitude coordinates.

```
MIAMI_LAT_LONG_COORDS = c(25.7617, -80.1918)
distance_between_coords = function(lat_1, long_1, lat_2, long_2){
  lat_1 = lat_1*180/pi
  long_1 = long_1*180/pi
  lat_2 = lat_2*180/pi
```

```
  long_2 = long_2*180/pi

  a = sin(lat_2 - lat_1 / 2)^2 + (cos(lat_2) * cos(lat_1)) * sin(long_2 - long_1 / 2)^2
  c = 2 * atan2(sqrt(a), sqrt(1-a))
  distance = 6373.0 * c #multiply by radius of earth in km
  distance
}

storms %>%
  mutate(distance_to_miami = distance_between_coords(lat,long, MIAMI_LAT_LONG_COORDS[1], MIAMI_LAT_LONG
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## # A tibble: 10,010 x 18
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
```

```
## # ... with 10,000 more rows, and 7 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>, distance_to_miami <dbl>
```

For each storm observation, use the function from the previous question to calculate the distance it moved since the previous observation.

```
storms %<>%
  group_by(name) %>%
  mutate(distance_from_previous = ifelse(name != lag(name), 0, distance_between_coords(lat, long, lag(la
   mutate(distance_from_previous = ifelse(is.na(distance_from_previous), 0, distance_from_previous))
```

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```

For each storm, find the total distance it moved over its observations and its total displacement. "Distance" is a scalar quantity that refers to "how much ground an object has covered" during its motion. "Displacement" is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

```
storms %>%
  group_by(name) %>%
  mutate(distance = sum(distance_from_previous), displacement = paste(round(last(lat) - first(lat), 2),
```

```
## # A tibble: 10,010 x 20
## # Groups:   name [198]
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 9 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>, timestamp <dttm>,
## #   day_of_week <ord>, week_of_year <dbl>, distance_from_previous <dbl>,
## #   distance <dbl>, displacement <chr>
```

For each storm observation, calculate the average speed the storm moved in location.

```
storms %<>%
  mutate(average_speed_in_location = distance_from_previous / 6) #the data is based on 6-hour increment
```

For each storm, calculate its average ground speed (how fast its eye is moving which is different from windspeed around the eye).

```
storms %<>%
  group_by(name) %>%
  mutate(avg_ground_speed = mean(average_speed_in_location, na.rm = TRUE))
```

Is there a relationship between average ground speed and maximum category attained? Use a dataframe summary (not a regression).

```
speed_and_category = storms %>%
  group_by(name) %>%
  summarize(avg_ground_speed, maximum_category = as.numeric(max(category)))
```

```
## `summarise()` has grouped output by 'name'. You can override using the `.groups` argument.
```

```
cor(speed_and_category$avg_ground_speed, speed_and_category$maximum_category)
```

```
## [1] 0.0546193
```

Now we want to transition to building real design matrices for prediction. This is more in tune with what happens in the real world. Large data dump and you convert it into X and y how you see fit.

Suppose we wish to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the `y` and identify which features you need $x_1, ...x_p$ and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to "featurize" as creatively as

you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

```
new_storms = storms %>%
  group_by(name) %>%
  summarise(
    y = max(wind),
    max_category = max(category),
    ts_diameter = if_else(is.na(ts_diameter), 0, ts_diameter),
    hu_diameter = if_else(is.na(hu_diameter), 0, hu_diameter)
      ) %>%
  ungroup() %>%
  select(-name)
```

```
## `summarise()` has grouped output by 'name'. You can override using the `.groups` argument.
```

```
new_storms
```

```
## # A tibble: 10,010 x 4
##        y max_category ts_diameter hu_diameter
##    <int> <ord>              <dbl>       <dbl>
##  1    30 -1                     0           0
##  2    30 -1                     0           0
##  3    30 -1                     0           0
##  4    30 -1                     0           0
##  5    30 -1                     0           0
##  6    30 -1                     0           0
##  7    30 -1                     0           0
##  8    30 -1                     0           0
##  9    25 -1                     0           0
## 10    25 -1                     0           0
## # ... with 10,000 more rows
```

Fit your model. Validate it.

```
n = nrow(new_storms)
K = 5

test_indices = sample(1 : n, (1 / K)*n)
train_indices = setdiff(1 : n, test_indices)

X = select(new_storms, -y)
y = new_storms$y

Xtrain =  X[train_indices, ]
ytrain = y[train_indices]
Xtest =  X[test_indices, ]
ytest = y[test_indices]

mod = lm(ytrain ~ ., data.frame(Xtrain))
summary(mod)$r.squared #in-sample R^2
```

```
## [1] 0.9694382
```

```
sd(mod$residuals) #in sample standard error
```

```
## [1] 5.461958
```

```
y_hat_oos = predict(mod, data.frame(Xtest))
oos_residuals = ytest - y_hat_oos #oos e's
1 - sum(oos_residuals^2) / sum((ytest - mean(ytest))^2) #oos R^2
```

```
## [1] 0.9695684
```

```
sd(oos_residuals) #oos standard error
```

```
## [1] 5.383377
```

Assess your level of success at this endeavor.

I think that the model I build was a success! Obviously the in-sample metrics are unreliable, and even the high oosR^2 could sometimes be misleading, but the pretty low out of sample standard error value (relative to the units of y) shows that this model can predict with respectable accuracy.

The Forward Stepwise Procedure for Probability Estimation Models

Set a seed and load the `adult` dataset and remove missingness and randomize the order.

```
set.seed(1)
pacman::p_load_gh("coatless/ucidata")
data(adult)
adult = na.omit(adult)
adult = adult[sample(1 : nrow(adult)), ]
```

Copy from the previous lab all cleanups you did to this dataset.

```
adult$fnlwgt = NULL

adult$marital_status = as.character(adult$marital_status)
adult$marital_status = ifelse(adult$marital_status == "Married-AF-spouse" | adult$marital_status == "Ma
adult$marital_status = as.factor(adult$marital_status)

adult$education = as.character(adult$education)
adult$education = ifelse(adult$education == "1st-4th" | adult$education == "Preschool", "<=4th", adult$
adult$education = as.factor(adult$education)
adult$education = NULL

tab = sort(table(adult$native_country))
adult$native_country = as.character(adult$native_country)
adult$native_country= ifelse(adult$native_country %in% names(tab[tab<50]), "Other", adult$native_countr
adult$native_country= as.factor(adult$native_country)

adult$worktype = paste(adult$occupation, adult$workclass, sep = ":")
tab_worktype = sort(table(adult$worktype))
adult$occupation = NULL
adult$workclass = NULL

adult$worktype = as.character(adult$worktype)
adult$worktype = ifelse(adult$worktype %in% names(tab_worktype[tab_worktype<100]), "Other", adult$workt
adult$worktype = as.factor(adult$worktype)

adult$status = paste(as.character(adult$relationship), as.character(adult$marital_status), sep = ":")
adult$status = as.character(adult$status)
```

```
tab_status = sort(table(adult$status))
adult$relationship = NULL
adult$marital_status = NULL
adult$status = as.factor(adult$status)
```

We will be doing model selection. We will split the dataset into 3 distinct subsets. Set the size of our splits here. For simplicitiy, all three splits will be identically sized. We are making it small so the stepwise algorithm can compute quickly. If you have a faster machine, feel free to increase this.

```
Nsplitsize = 1000
```

Now create the following variables: `Xtrain`, `ytrain`, `Xselect`, `yselect`, `Xtest`, `ytest` with `Nsplitsize` observations. Binarize the y values.

```
Xtrain = adult[1 : Nsplitsize, ]
Xtrain$income = NULL
ytrain = ifelse(adult[1 : Nsplitsize, "income"] == ">50K", 1, 0)
Xselect = adult[(Nsplitsize + 1) : (2 * Nsplitsize), ]
Xselect$income = NULL
yselect = ifelse(adult[(Nsplitsize + 1) : (2 * Nsplitsize), "income"] ==">50K", 1, 0)
Xtest = adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), ]
Xtest$income = NULL
ytest = ifelse(adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), "income"] == ">50K", 1, 0)
```

Fit a vanilla logistic regression on the training set.

```
logistic_mod = glm(ytrain ~ ., Xtrain, family = binomial(link = logit))
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

and report the log scoring rule, the Brier scoring rule.

```
p_hat_train = predict(logistic_mod, Xtrain, type = 'response')
```

```
## Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
## prediction from a rank-deficient fit may be misleading
```

```
#in sample log scoring rule
mean(ytrain * log(p_hat_train) + (1 - ytrain) * log(1 - p_hat_train))
```

```
## [1] -0.2671121
```

```
#in sample Brier scoring rule
mean(-(ytrain - p_hat_train)^2)
```

```
## [1] -0.08715781
```

We will be doing model selection using a basis of linear features consisting of all first-order interactions of the 14 raw features (this will include square terms as squares are interactions with oneself).

Create a model matrix from the training data containing all these features. Make sure it has an intercept column too (the one vector is usually an important feature). Cast it as a data frame so we can use it more easily for modeling later on. We're going to need those model matrices (as data frames) for both the select and test sets. So make them here too (copy-paste). Make sure their dimensions are sensible.

```
Xmm_train = data.frame(model.matrix( ~ . , Xtrain))
Xmm_select = data.frame(model.matrix( ~ . , Xselect))
Xmm_test = data.frame(model.matrix( ~ . , Xtest))

dim(Xmm_train)
```

```
## [1] 1000   93
```

```
dim(Xmm_select)
```

```
## [1] 1000   93
```

```
dim(Xmm_test)
```

```
## [1] 1000   93
```

Write code that will fit a model stepwise. You can refer to the chunk in the practice lecture. Use the negative Brier score to do the selection. The negative of the Brier score is always positive and lower means better making this metric kind of like s_e so the picture will be the same as the canonical U-shape for oos performance.

Run the code and hit "stop" when you begin to the see the Brier score degrade appreciably oos. Be patient as it will wobble.

```
pacman::p_load(Matrix)
p_plus_one = ncol(Xmm_train)
predictor_by_iteration = c() #keep a growing list of predictors by iteration
in_sample_brier_by_iteration = c() #keep a growing list of briers by iteration
oos_brier_by_iteration = c() #keep a growing list of briers by iteration
i = 1

repeat {

  #get all predictors left to try
  all_briers = array(NA, p_plus_one) #record all possibilities
  for (j_try in 1 : p_plus_one){
    if (j_try %in% predictor_by_iteration){
      next
    }
    Xmm_sub = Xmm_train[, c(predictor_by_iteration, j_try), drop = FALSE]
    logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family = "binomial"))
    phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type = 'response'))
    all_briers[j_try] = -mean(-(ytrain - phat_train)^2)
  }
  j_star = which.max(all_briers)
  predictor_by_iteration = c(predictor_by_iteration, j_star)
  in_sample_brier_by_iteration = c(in_sample_brier_by_iteration, all_briers[j_star])

  #now let's look at oos
  Xmm_sub = Xmm_train[, predictor_by_iteration, drop = FALSE]

    logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family = "binomial"))
    phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type = 'response'))
    all_briers[j_try] = -mean(-(ytrain - phat_train)^2)

    phat_select = suppressWarnings(predict(logistic_mod, Xmm_select[, predictor_by_iteration, drop = FA

    oos_brier = -mean(-(yselect - phat_select)^2)
    oos_brier_by_iteration = c(oos_brier_by_iteration, oos_brier)

  cat("i =", i, "in-sample_brier =", all_briers[j_star], "oos_brier =", oos_brier, "\n   predictor adde

  i = i + 1
```

```
  if (i > Nsplitsize || i > p_plus_one){
    break
  }
}
```

```
## i = 1 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: X.Intercept.
## i = 2 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: native_countryPoland
## i = 3 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusNot.in.family.Married
## i = 4 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOther.relative.Separated
## i = 5 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOther.relative.Widowed
## i = 6 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOwn.child.Widowed
## i = 7 in-sample_brier = 0.1813554 oos_brier = 0.1855417
##    predictor added: worktypeTransport.moving.Self.emp.not.inc
## i = 8 in-sample_brier = 0.1813548 oos_brier = 0.1855661
##    predictor added: statusUnmarried.Married.spouse.absent
## i = 9 in-sample_brier = 0.1813542 oos_brier = 0.1855927
##    predictor added: worktypeSales.Self.emp.not.inc
## i = 10 in-sample_brier = 0.181353 oos_brier = 0.1856649
##    predictor added: statusUnmarried.Widowed
## i = 11 in-sample_brier = 0.1813499 oos_brier = 0.1856563
##    predictor added: worktypeCraft.repair.Private
## i = 12 in-sample_brier = 0.1813447 oos_brier = 0.1856134
##    predictor added: native_countryIndia
## i = 13 in-sample_brier = 0.1813373 oos_brier = 0.1856355
##    predictor added: native_countryPuerto.Rico
## i = 14 in-sample_brier = 0.1813246 oos_brier = 0.1859607
##    predictor added: worktypeFarming.fishing.Private
## i = 15 in-sample_brier = 0.1813123 oos_brier = 0.1857883
##    predictor added: worktypeFarming.fishing.Self.emp.not.inc
## i = 16 in-sample_brier = 0.1812982 oos_brier = 0.1856838
##    predictor added: statusNot.in.family.Separated
## i = 17 in-sample_brier = 0.1812717 oos_brier = 0.1852927
##    predictor added: worktypeProf.specialty.Federal.gov
## i = 18 in-sample_brier = 0.1812449 oos_brier = 0.1853558
##    predictor added: native_countryGuatemala
## i = 19 in-sample_brier = 0.181218 oos_brier = 0.1857469
##    predictor added: worktypeCraft.repair.Local.gov
## i = 20 in-sample_brier = 0.1811902 oos_brier = 0.1856173
##    predictor added: raceOther
## i = 21 in-sample_brier = 0.1811586 oos_brier = 0.1855962
##    predictor added: worktypeExec.managerial.State.gov
## i = 22 in-sample_brier = 0.1811215 oos_brier = 0.1859505
##    predictor added: worktypeAdm.clerical.Local.gov
## i = 23 in-sample_brier = 0.1810644 oos_brier = 0.185881
##    predictor added: native_countryDominican.Republic
## i = 24 in-sample_brier = 0.1810644 oos_brier = 0.185881
##    predictor added: statusOwn.child.Married.spouse.absent
```

```
## i = 25 in-sample_brier = 0.1810073 oos_brier = 0.1858114
##    predictor added: native_countryVietnam
## i = 26 in-sample_brier = 0.1809499 oos_brier = 0.1860419
##    predictor added: statusOwn.child.Married
## i = 27 in-sample_brier = 0.1808553 oos_brier = 0.1860526
##    predictor added: native_countryOther
## i = 28 in-sample_brier = 0.1807887 oos_brier = 0.1862179
##    predictor added: native_countryUnited.States
## i = 29 in-sample_brier = 0.180699 oos_brier = 0.1868485
##    predictor added: worktypeTech.support.Private
## i = 30 in-sample_brier = 0.1805934 oos_brier = 0.1864382
##    predictor added: worktypeOther.service.Local.gov
## i = 31 in-sample_brier = 0.1804642 oos_brier = 0.1848996
##    predictor added: worktypeExec.managerial.Self.emp.inc
## i = 32 in-sample_brier = 0.1803137 oos_brier = 0.1846994
##    predictor added: native_countryJapan
## i = 33 in-sample_brier = 0.1801419 oos_brier = 0.1849772
##    predictor added: worktypeProtective.serv.State.gov
## i = 34 in-sample_brier = 0.1799592 oos_brier = 0.1847671
##    predictor added: statusOther.relative.Divorced
## i = 35 in-sample_brier = 0.179768 oos_brier = 0.1846089
##    predictor added: worktypeProtective.serv.Private
## i = 36 in-sample_brier = 0.1795723 oos_brier = 0.1842935
##    predictor added: worktypeProf.specialty.Local.gov
## i = 37 in-sample_brier = 0.179356 oos_brier = 0.1841564
##    predictor added: native_countryChina
## i = 38 in-sample_brier = 0.1791469 oos_brier = 0.1840683
##    predictor added: native_countryColumbia
## i = 39 in-sample_brier = 0.1789191 oos_brier = 0.1840311
##    predictor added: worktypeOther.service.State.gov
## i = 40 in-sample_brier = 0.1786884 oos_brier = 0.1838212
##    predictor added: statusOwn.child.Divorced
## i = 41 in-sample_brier = 0.1784501 oos_brier = 0.1838435
##    predictor added: native_countryEl.Salvador
## i = 42 in-sample_brier = 0.1782627 oos_brier = 0.1844303
##    predictor added: statusOther.relative.Married.spouse.absent
## i = 43 in-sample_brier = 0.1780273 oos_brier = 0.1841625
##    predictor added: worktypeTransport.moving.Local.gov
## i = 44 in-sample_brier = 0.1777802 oos_brier = 0.1838986
##    predictor added: worktypeCraft.repair.Self.emp.not.inc
## i = 45 in-sample_brier = 0.1775394 oos_brier = 0.1839145
##    predictor added: worktypeSales.Self.emp.inc
## i = 46 in-sample_brier = 0.1772784 oos_brier = 0.184464
##    predictor added: worktypeAdm.clerical.State.gov
## i = 47 in-sample_brier = 0.1770012 oos_brier = 0.1848479
##    predictor added: native_countryEngland
## i = 48 in-sample_brier = 0.1766289 oos_brier = 0.1852858
##    predictor added: native_countryItaly
## i = 49 in-sample_brier = 0.1762576 oos_brier = 0.1850986
##    predictor added: worktypeTransport.moving.Private
## i = 50 in-sample_brier = 0.1759073 oos_brier = 0.185645
##    predictor added: statusOther.relative.Married
## i = 51 in-sample_brier = 0.1755777 oos_brier = 0.1855656
##    predictor added: worktypePriv.house.serv.Private
```

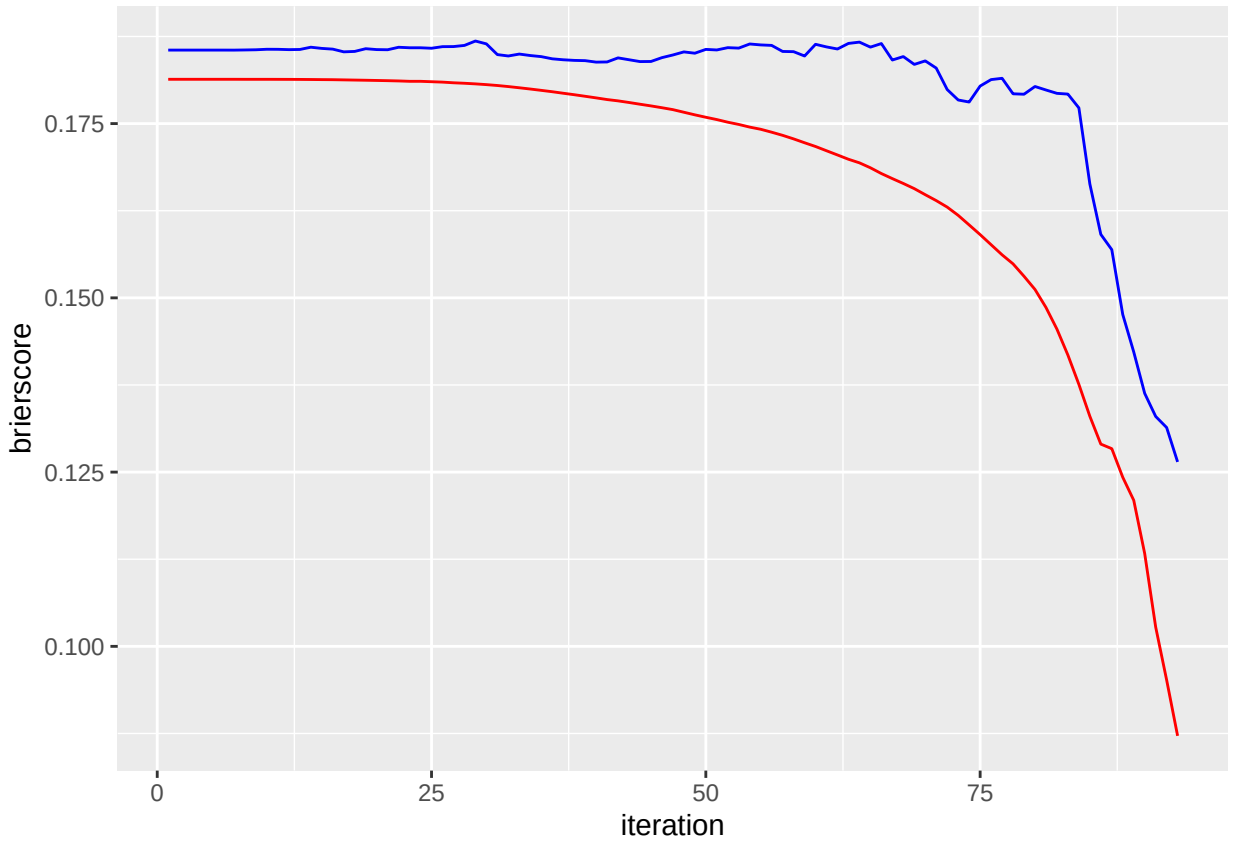
```
## i = 52 in-sample_brier = 0.1752024 oos_brier = 0.1858937
##    predictor added: worktypeOther
## i = 53 in-sample_brier = 0.1748781 oos_brier = 0.1858285
##    predictor added: native_countryGermany
## i = 54 in-sample_brier = 0.1744952 oos_brier = 0.1864225
##    predictor added: native_countryCuba
## i = 55 in-sample_brier = 0.1741871 oos_brier = 0.186287
##    predictor added: statusOwn.child.Separated
## i = 56 in-sample_brier = 0.1737656 oos_brier = 0.1862193
##    predictor added: native_countrySouth
## i = 57 in-sample_brier = 0.1733164 oos_brier = 0.1853527
##    predictor added: worktypeOther.service.Self.emp.not.inc
## i = 58 in-sample_brier = 0.1728051 oos_brier = 0.1853208
##    predictor added: worktypeProf.specialty.Self.emp.inc
## i = 59 in-sample_brier = 0.1722497 oos_brier = 0.1846987
##    predictor added: worktypeSales.Private
## i = 60 in-sample_brier = 0.1717164 oos_brier = 0.1863781
##    predictor added: worktypeProtective.serv.Local.gov
## i = 61 in-sample_brier = 0.1711044 oos_brier = 0.1860013
##    predictor added: statusNot.in.family.Widowed
## i = 62 in-sample_brier = 0.1705002 oos_brier = 0.1857051
##    predictor added: worktypeExec.managerial.Self.emp.not.inc
## i = 63 in-sample_brier = 0.1698833 oos_brier = 0.1865027
##    predictor added: native_countryJamaica
## i = 64 in-sample_brier = 0.1693691 oos_brier = 0.1866908
##    predictor added: raceWhite
## i = 65 in-sample_brier = 0.1686613 oos_brier = 0.1859704
##    predictor added: statusUnmarried.Separated
## i = 66 in-sample_brier = 0.1678313 oos_brier = 0.1864843
##    predictor added: raceBlack
## i = 67 in-sample_brier = 0.1671104 oos_brier = 0.1841216
##    predictor added: worktypeMachine.op.inspct.Private
## i = 68 in-sample_brier = 0.1664096 oos_brier = 0.1846154
##    predictor added: raceAsian.Pac.Islander
## i = 69 in-sample_brier = 0.165671 oos_brier = 0.1834925
##    predictor added: worktypeProf.specialty.Self.emp.not.inc
## i = 70 in-sample_brier = 0.164799 oos_brier = 0.1839977
##    predictor added: native_countryPhilippines
## i = 71 in-sample_brier = 0.1639532 oos_brier = 0.1829634
##    predictor added: statusOther.relative.Never.married
## i = 72 in-sample_brier = 0.1630177 oos_brier = 0.1798843
##    predictor added: worktypeProf.specialty.Private
## i = 73 in-sample_brier = 0.161836 oos_brier = 0.178388
##    predictor added: worktypeHandlers.cleaners.Private
## i = 74 in-sample_brier = 0.1604635 oos_brier = 0.1780931
##    predictor added: worktypeExec.managerial.Local.gov
## i = 75 in-sample_brier = 0.1590754 oos_brier = 0.1803847
##    predictor added: native_countryMexico
## i = 76 in-sample_brier = 0.1576239 oos_brier = 0.18131
##    predictor added: statusNot.in.family.Married.spouse.absent
## i = 77 in-sample_brier = 0.1561724 oos_brier = 0.1814974
##    predictor added: worktypeExec.managerial.Federal.gov
## i = 78 in-sample_brier = 0.154877 oos_brier = 0.1792748
##    predictor added: worktypeAdm.clerical.Private
```

```
## i = 79 in-sample_brier = 0.1530984 oos_brier = 0.1792153
##    predictor added: worktypeProf.specialty.State.gov
## i = 80 in-sample_brier = 0.1512046 oos_brier = 0.1803241
##    predictor added: statusUnmarried.Divorced
## i = 81 in-sample_brier = 0.1486265 oos_brier = 0.1798221
##    predictor added: statusUnmarried.Never.married
## i = 82 in-sample_brier = 0.1455114 oos_brier = 0.1793399
##    predictor added: statusWife.Married
## i = 83 in-sample_brier = 0.141789 oos_brier = 0.179233
##    predictor added: statusNot.in.family.Divorced
## i = 84 in-sample_brier = 0.1375809 oos_brier = 0.1772499
##    predictor added: capital_loss
## i = 85 in-sample_brier = 0.1330105 oos_brier = 0.1663411
##    predictor added: hours_per_week
## i = 86 in-sample_brier = 0.1290151 oos_brier = 0.1591097
##    predictor added: worktypeExec.managerial.Private
## i = 87 in-sample_brier = 0.1283621 oos_brier = 0.1569123
##    predictor added: worktypeOther.service.Private
## i = 88 in-sample_brier = 0.1242607 oos_brier = 0.1476126
##    predictor added: education_num
## i = 89 in-sample_brier = 0.1209538 oos_brier = 0.1422338
##    predictor added: statusOwn.child.Never.married
## i = 90 in-sample_brier = 0.1133092 oos_brier = 0.1362918
##    predictor added: sexMale
## i = 91 in-sample_brier = 0.1027663 oos_brier = 0.1329848
##    predictor added: statusNot.in.family.Never.married
## i = 92 in-sample_brier = 0.09516563 oos_brier = 0.1313902
##    predictor added: age
## i = 93 in-sample_brier = 0.08715781 oos_brier = 0.1264595
##    predictor added: capital_gain
```

Plot the in-sample and oos (select set) Brier score by p. Does this look like what's expected?

```
simulation_results = data.frame(
  iteration = 1 : length(in_sample_brier_by_iteration),
  in_sample_brier_by_iteration = in_sample_brier_by_iteration,
  oos_brier_by_iteration = oos_brier_by_iteration
)

pacman::p_load(ggplot2)
pacman::p_load(latex2exp)
ggplot(simulation_results) +
  geom_line(aes(x = iteration, y = in_sample_brier_by_iteration), color = "red") +
  geom_line(aes(x = iteration, y = oos_brier_by_iteration), color = "blue") +
  #ylim(0, max(c(simulation_results$in_sample_brier_by_iteration, simulation_results$oos_brier_by_itera
  ylab(TeX("$brier score$"))
```


0.175
0.150
0.125
0.100
brierscore
0
25
50
75
iteration