Ray-Cylinder Intersection Derivations

Implicit formula of a cylinder with the axis through point C in direction of $\vec{a}$, where $|\vec{a}|=1$ with radius r. A point X is on the surface of the cylinder if it satisfies:

$$|X-C|^2 = r^2 + [(X-C)\cdot\vec{a}]^2$$

So for the ray-cylinder intersection we want $X = O + t\vec{d}$:

$$\begin{aligned}
|(O+t\vec{d})-C|^2 &= r^2 + [((O+t\vec{d})-C)\cdot\vec{a}]^2\\
|t\vec{d}+(O-C)|^2 &= r^2 + [(t\vec{d}+(O-C))\cdot\vec{a}]^2\\
(t\vec{d}+(O-C))^T(t\vec{d}+(O-C)) &= r^2 + [(t\vec{d}\cdot\vec{a}+(O-C)\cdot\vec{a})(t\vec{d}\cdot\vec{a}+(O-C)\cdot\vec{a})]\\
(t\vec{d}+(O-C))^T(t\vec{d}+(O-C)) &= r^2 + [(t\vec{d}\cdot\vec{a})^2+2(t\vec{d}\cdot\vec{a})((O-C)\cdot\vec{a})+((O-C)\cdot\vec{a})^2]
\end{aligned}$$

For $(t\vec{d}\cdot\vec{a})^2$ we get:

$$\begin{aligned}
(td_1\cdot a_1+td_2\cdot a_2+td_3\cdot a_3)^2 &= (t\cdot(d_1a_1+d_2a_2+d_3a_3))^2\\
&= t^2\cdot(d_1a_1+d_2a_2+d_3a_3)^2\\
&= t^2\cdot(\vec{d}\cdot\vec{a})^2
\end{aligned}$$

So if we substitute that in the formula:

$$(t\vec{d}+(O-C))^T(t\vec{d}+(O-C)) = r^2 + [t^2\cdot(\vec{d}\cdot\vec{a})^2+2(t\vec{d}\cdot\vec{a})((O-C)\cdot\vec{a})+((O-C)\cdot\vec{a})^2]$$

<=>

$$t^2(\vec{d}^T\vec{d})+t2(\vec{d}^T(O-C))+(O-C)^T(O-C) = t^2(\vec{d}\cdot\vec{a})^2+t2(\vec{d}\cdot\vec{a})((O-C)\vec{a})+((O-C)\vec{a})^2+r^2$$

<=>

$$t^2((\vec{d}^T\vec{d})-(\vec{d}\cdot\vec{a})^2)+t2(\vec{d}^T(O-C)-(\vec{d}\cdot\vec{a})((O-C)\vec{a}))+(O-C)^T(O-C)-((O-C)\vec{a})^2-r^2 = 0$$

And therefore we get the parameters for the quadratic formula $at^2+bt+c = 0$:

$$\begin{aligned}
a &= \vec{d}^T\vec{d}-(\vec{d}\cdot\vec{a})^2\\
b &= 2(\vec{d}^T(O-C)-(\vec{d}\cdot\vec{a})((O-C)\cdot\vec{a}))\\
c &= (O-C)^T(O-C)-((O-C)\cdot\vec{a})^2-r^2
\end{aligned}$$

For the normal in the intersection point we first project the intersection point P to the axis of the cylinder to the point X, so that the connecting vector between P and X is orthogonal to the axis:

$$X = C+((P-C)\cdot\vec{d})\vec{d}$$

Finally to get the normal we take the connecting vector and normalize it.

If the intersection is on the outside of the cylinder: $\vec{n} = \dfrac{X-P}{|X-P|}$

If the intersection is on the inside of the cylinder: $\vec{n} = \dfrac{P-X}{|X-P|}$