# Bibliography


[1] F. Farshidian, M. Neunert, A. W. Winkler, G. Rey, and J. Buchli, "An efficient optimal planning and control framework for quadrupedal locomotion," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017, pp. 93–100.

[2] D. Jacobson and D. Mayne, *Differential Dynamic Programming*, ser. Modern analytic and computational methods in science and mathematics. American Elsevier Publishing Company, 1970.

[3] B. Zhou, F. Gao, L. Wang, C. Liu, and S. Shen, "Robust and efficient quadrotor trajectory generation for fast autonomous flight," *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 3529–3536, 2019.

[4] P.-B. Wieber, R. Tedrake, and S. Kuindersma, "Modeling and control of legged robots," in *Springer handbook of robotics*. Springer, 2016, pp. 1203–1234.

[5] R. Grandia, F. Farshidian, A. Dosovitskiy, R. Ranftl, and M. Hutter, "Frequency-aware model predictive control," *IEEE Robotics and Automation Letters*, vol. 4, no. 2, pp. 1517–1524, 2019.

[6] R. Grandia, F. Farshidian, R. Ranftl, and M. Hutter, "Feedback mpc for torque-controlled legged robots," *arXiv preprint arXiv:1905.06144*, 2019.

[7] J.-P. Sleiman, F. Farshidian, and M. Hutter, "Constraint handling in continuous-time ddp-based model predictive control," *arXiv preprint arXiv:2101.06067*, 2021.

[8] M. Diehl, H. G. Bock, and J. P. Schlöder, "A real-time iteration scheme for nonlinear optimization in optimal feedback control," *SIAM Journal on control and optimization*, vol. 43, no. 5, pp. 1714–1736, 2005.

[9] G. Frison and M. Diehl, "Hpipm: a high-performance quadratic programming framework for model predictive control," *arXiv preprint arXiv:2003.02547*, 2020.

[10] M. Diehl, "Lecture notes on optimal control and estimation," 2014.

[11] M. Giftthaler, "Towards a unified framework of efficient algorithms for numerical optimal robot control," Ph.D. dissertation, ETH Zurich, 2018.

[12] R. Sargent and G. Sullivan, "The development of an efficient optimal control package," in *Optimization Techniques*. Springer, 1978, pp. 158–168.

[13] H. G. Bock and K.-J. Plitt, "A multiple shooting algorithm for direct solution of optimal control problems," *IFAC Proceedings Volumes*, vol. 17, no. 2, pp. 1603–1608, 1984.

[14] M. Hutter, C. Gehring, D. Jud, A. Lauber, C. D. Bellicoso, V. Tsounis, J. Hwangbo, K. Bodie, P. Fankhauser, M. Bloesch *et al.*, "Anymal-a highly mobile and dynamic quadrupedal robot," in *2016 IEEE/RSJ international conference on intelligent robots and systems (IROS).* IEEE, 2016, pp. 38–44.

[15] E. Todorov and W. Li, "A generalized iterative lqg method for locally-optimal feedback control of constrained nonlinear stochastic systems," in *Proceedings of the 2005, American Control Conference, 2005.* IEEE, 2005, pp. 300–306.

[16] A. Sideris and J. E. Bobrow, "An efficient sequential linear quadratic algorithm for solving nonlinear optimal control problems," in *Proceedings of the 2005, American Control Conference, 2005.* IEEE, 2005, pp. 2275–2280.

[17] M. Giftthaler, M. Neunert, M. Stäuble, J. Buchli, and M. Diehl, "A family of iterative gauss-newton shooting methods for nonlinear optimal control," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS).* IEEE, 2018, pp. 1–9.

[18] T. A. Howell, B. E. Jackson, and Z. Manchester, "Altro: A fast solver for constrained trajectory optimization," in *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS).* IEEE, 2019, pp. 7674–7679.

[19] G. Frison and J. B. Jørgensen, "Efficient implementation of the riccati recursion for solving linear-quadratic control problems," in *2013 IEEE International Conference on Control Applications (CCA).* IEEE, 2013, pp. 1117–1122.

[20] G. Frison, H. H. B. Sørensen, B. Dammann, and J. B. Jørgensen, "High-performance small-scale solvers for linear model predictive control," in *2014 European Control Conference (ECC).* IEEE, 2014, pp. 128–133.

[21] G. Lantoine and R. P. Russell, "A hybrid differential dynamic programming algorithm for constrained optimal control problems. part 1: Theory," *Journal of Optimization Theory and Applications*, vol. 154, no. 2, pp. 382–417, 2012.

[22] E. Pellegrini and R. P. Russell, "A multiple-shooting differential dynamic programming algorithm. part 1: Theory," *Acta Astronautica*, vol. 170, pp. 686–700, 2020.

[23] "Documentation for optimal control for switched systems algorithm (ocs2)," https://leggedrobotics.github.io/ocs2/, accessed: 2021-01-27.

[24] I. Pólik and T. Terlaky, *Interior Point Methods for Nonlinear Optimization.* Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 215–276.

[25] P. T. Boggs and J. W. Tolle, "Sequential quadratic programming," *Acta numerica*, vol. 4, pp. 1–51, 1995.

[26] P. E. Gill, W. Murray, and M. A. Saunders, "Snopt: An sqp algorithm for large-scale constrained optimization," *SIAM review*, vol. 47, no. 1, pp. 99–131, 2005.

[27] R. Verschueren, G. Frison, D. Kouzoupis, N. van Duijkeren, A. Zanelli, B. Novoselnik, J. Frey, T. Albin, R. Quirynen, and M. Diehl, "Acados: A modular open-source framework for fast embedded optimal control," *arXiv preprint arXiv:1910.13753*, 2019.

[28] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl, "CasADi – A software framework for nonlinear optimization and optimal control," *Mathematical Programming Computation*, vol. 11, no. 1, pp. 1–36, 2019.

[29] D. Kouzoupis, G. Frison, A. Zanelli, and M. Diehl, "Recent advances in quadratic programming algorithms for nonlinear model predictive control," *Vietnam Journal of Mathematics*, vol. 46, no. 4, pp. 863–882, 2018.

[30] J. Nocedal and S. Wright, *Numerical optimization.* Springer Science & Business Media, 2006.

[31] *7. The Trust-Region Subproblem*, pp. 169–248.

[32] G. Frison, "Numerical methods for model predictive control," 2012.

[33] G. Guennebaud, B. Jacob *et al.*, "Eigen v3," http://eigen.tuxfamily.org, 2010.

[34] E. M. Gertz and S. J. Wright, "Object-oriented software for quadratic programming," *ACM Transactions on Mathematical Software (TOMS)*, vol. 29, no. 1, pp. 58–81, 2003.

[35] G. Frison, D. Kouzoupis, T. Sartor, A. Zanelli, and M. Diehl, "Blasfeo: Basic linear algebra subroutines for embedded optimization," *ACM Transactions on Mathematical Software (TOMS)*, vol. 44, no. 4, pp. 1–30, 2018.

[36] H. J. Ferreau, C. Kirches, A. Potschka, H. G. Bock, and M. Diehl, "qpoases: A parametric active-set algorithm for quadratic programming," *Mathematical Programming Computation*, vol. 6, no. 4, pp. 327–363, 2014.

[37] "Documentation for hpipm," https://github.com/giaf/hpipm/blob/master/doc/guide.pdf, accessed: 2021-01-27.

[38] D. Boiroux and J. B. Jørgensen, "Sequential l1 quadratic programming for nonlinear model predictive control," *IFAC-PapersOnLine*, vol. 52, no. 1, pp. 474–479, 2019.

[39] M. Rijnen, A. Saccon, and H. Nijmeijer, "Reference spreading: Tracking performance for impact trajectories of a 1dof setup," *IEEE Transactions on Control Systems Technology*, vol. 28, no. 3, pp. 1124–1131, 2019.

[40] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical programming*, vol. 106, no. 1, pp. 25–57, 2006.

[41] P. Fankhauser and C. Gwerder, "Modeling and control of a ballbot," Master's thesis, ETH Zurich, Zürich, 2010, bachelor Thesis ETH Zurich, 2010.

[42] R. Grandia, A. J. Taylor, A. D. Ames, and M. Hutter, "Multi-layered safety for legged robots via control barrier functions and model predictive control," *arXiv preprint arXiv:2011.00032*, 2020.

[43] R. Verschueren, M. Zanon, R. Quirynen, and M. Diehl, "A sparsity preserving convexification procedure for indefinite quadratic programs arising in direct optimal control," *SIAM Journal on Optimization*, vol. 27, no. 3, pp. 2085–2109, 2017.

[44] R. Verschueren, N. van Duijkeren, R. Quirynen, and M. Diehl, "Exploiting convexity in direct optimal control: a sequential convex quadratic programming method," in *2016 IEEE 55th Conference on Decision and Control (CDC).* IEEE, 2016, pp. 1099–1104.

[45] J. Hall, A. Nurkanović, F. Messerer, and M. Diehl, “A sequential convex programming approach to solving quadratic programs and optimal control problems with linear complementarity constraints,” *IEEE Control Systems Letters*, 2021.

[46] R. Fletcher and S. Leyffer, “Nonlinear programming without a penalty function,” *Mathematical programming*, vol. 91, no. 2, pp. 239–269, 2002.

[47] H. Li, R. J. Frei, and P. M. Wensing, “Model hierarchy predictive control of robotic systems,” *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 3373–3380, 2021.

[48] V. Sindhwani, R. Roelofs, and M. Kalakrishnan, “Sequential operator splitting for constrained nonlinear optimal control,” in *2017 American Control Conference (ACC)*. IEEE, 2017, pp. 4864–4871.

# Appendix A

# Proof of Necessary Conditions for a Nonsingular KKT Matrix

We claim that the KKT matrix $\begin{bmatrix} Q & A^{\mathrm{T}} \\ A & 0 \end{bmatrix}$ is invertible if the following two conditions are fulfilled:

- Matrix $A$ has full row rank
- $Z^{\mathrm{T}}QZ$ is positive definite, where $Z = \begin{bmatrix} z_1 & z_2 & \dots & z_{n-m} \end{bmatrix}$ such that $\{z_1, z_2, \dots, z_{n-m}\}$ form a basis of the null space of matrix $A$.

Now we show why the above two conditions lead to nonsingularity of KKT matrix in 2.8. Suppose there exists $\begin{bmatrix} p \\ q \end{bmatrix}$ such that

$$\begin{bmatrix} Q & A^{\mathrm{T}} \\ A & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = 0 \tag{A.1}$$

then we will have $Ap = 0 \Rightarrow p \in \mathrm{span}\{z_1, z_2, \dots, z_{n-m}\} \Rightarrow p = Zr$ for certain $r \in \mathbb{R}^{n-m}$ and $Qp + A^{\mathrm{T}}q = 0$. If we multiple the transpose of $\begin{bmatrix} p \\ q \end{bmatrix}$ beforehand in A.1,

$$\begin{aligned} 0 = \begin{bmatrix} p \\ q \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q & A^{\mathrm{T}} \\ A & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} &= p^{\mathrm{T}}Qp + p^{\mathrm{T}}A^{\mathrm{T}}q + q^{\mathrm{T}}Ap \\ &= p^{\mathrm{T}}Qp \\ &= r^{\mathrm{T}}Z^{\mathrm{T}}QZr \end{aligned} \tag{A.2}$$

Since $Z^{\mathrm{T}}QZ$ is positive definite, $r^{\mathrm{T}}Z^{\mathrm{T}}QZr = 0$ only holds when $r = 0 \Rightarrow p = Zr = 0 \Rightarrow A^{\mathrm{T}}q = 0$. Since $A$ has full row rank, $q = 0$ must hold. So we can see that A.1 holds only if $p = q = 0$, so the KKT matrix in nonsingular.