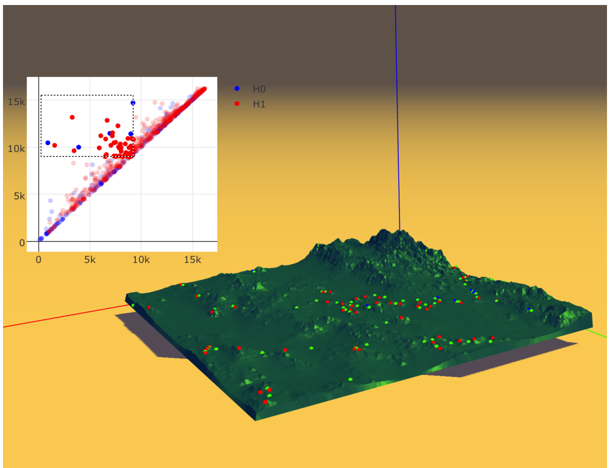
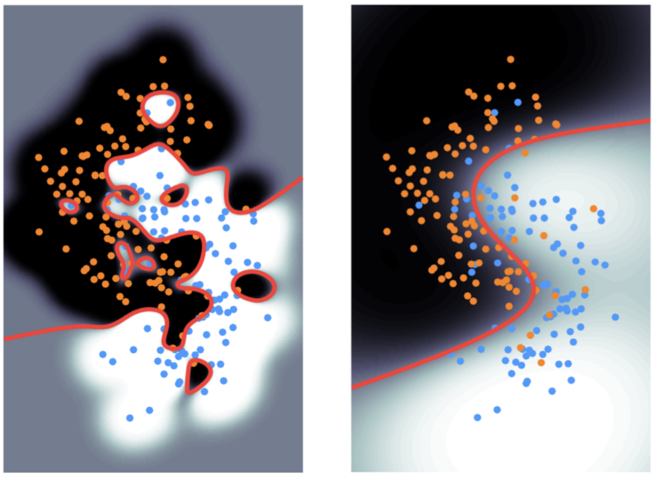
Computational Topology and Topological Data Analysis

This is a course for PhD students. Master students are also allowed and encouraged to participate. The main topic of the course is to introduce computational aspects in applied topology, computational topology, in order to provide the main mathematical ingredients to the the rather new field of research called Topological Data Analysis - TDA.

TDA aims at capturing the shape of data as a robust and multiscale descriptor to be integrated in data handling: classification, comparison, multidimensional reduction, etc ...

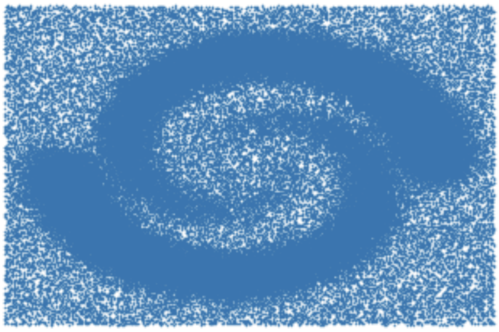
First, we deal with the constructions from data to simplicial complexes according to the kind of data: filtrations of data, point clouds, networks, and topological spaces. For each construction, we underline the possible dependence on a fixed scale parameter. Secondly, we introduce the necessary algebraic structures capturing topological informations out of a simplicial complex at a fixed scale, namely the simplicial homology groups. The so-obtained linear structures are then integrated into the multiscale framework of persistent homology where the entire persistence information is encoded in algebraic terms and the most advantageous persistence summaries available in the literature are discussed. Finally, we introduce the necessary metrics in order to state properties of stability of the introduced multiscale summaries under perturbations of input data. At the end, we give an overview of applications of persistent homology as well as a review of the existing tools in the broader area of Topological Data Analysis (TDA).

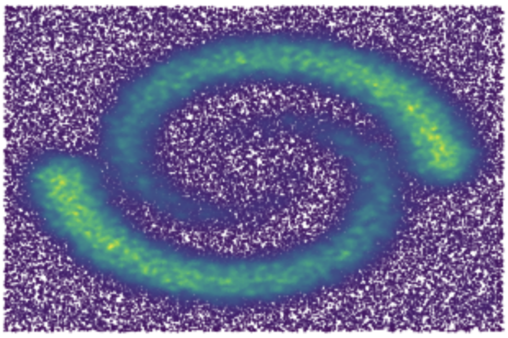




Left: a terrain rendering with simplified critical points. Fugacci et al., STAG, 2016 .

Right: a decision boundary simplified by minimizing connected components. Chao, et al. PMLR, 2019.





Left: a big point cloud with a noisy shaped 2D spiral.

Right: a fuzzy clustering detecting the 2D-spiral with ToMATo algorithm (Topological Data Analysis-based clustering algorithm)

References

Plan

This is a tentative outline of the course, subject to change.

Lecture	Topic
1	Introduction: what is Computational Topology? How to use topology to analyze data? Examples and goals. Lecture 1: Video/Board, Lecture 2: Video/Board
2	Topological Invariants and Discrete Shapes: Lecture 1: Video/Board, Lecture 2: Video/Board
3	Simplicial and Cellular Homology: Lecture 1: Video/Board, Lecture 2: Persistence Pairs Video/Board
4	Persistent Homology: Lecture 1: Video/Board, Lecture 2: Video/Board
5	Level set and zig-zag Persistence: Lecture 1: Video/Board, Lecture 2: Video/Board
6	Discrete Morse Theory and Persistent Homology: Lecture 1: Video/Board, Lecture 2: Video/Board
7	Multiparameter Persistent Homology: Lecture 1: Video, Lecture 2: overview on other TDA open perspectivesVideo/Board
8	Python Lab: Lecture 1: Classifying via TDAVideo/Board, Lecture 2: Video/
9	Student Seminars: Lecture 1: Video/Board, Lecture 2: Video/Board
10	Student Seminars: Lecture 1: Video/Board, Lecture 2: Video/Board
	Q&A session 1: Video, Q&A session 2 Video/Board

Pre-requisites

Geometry 1 and 2, Algebra 1 and 2.

Computational Topology and TDA are deeply rooted into many areas, so some background in algebraic topology, computational algebra, numerical analysis, and machine learning would be helpful, but not required.

Assessment

The final mark is based on:

1. A final presentation by the student of a research paper in the field of computational topology or TDA

The topic will be proposed by the student and approved by the teacher.

Classwork

The format of our classwork will be as follows. There will be two meetings per week from January to February. Seven meetings will be frontal lectures. One meeting will be a Python Lab. The last two meetings will be left for the students' presentations.

Collaboration

Students are encouraged to cohoperate with others, even to share a topic to be presented in the final seminar. Nonetheless, each student has to present its own part. All the names of students involved should be explicitly mentioned in the presentation. This is a matter of academic honesty.

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