In the name of God

Shiraz University

Pattren Recognition

#Mini Project1

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Problem 1

Consider Cauchy distributions in a two-class one-dimensional classification problem:

$$P(x|w_i) = \frac{1}{\pi b}\frac{1}{1+(\frac{x-a_i}{b})^2} \quad \text{i} = 1,2 \qquad a_2 > a_1$$

a) By explicit integration, show that the distributions are normalized.

b) Assume $P(w_1) = P(w_2)$, show that $P(w_1|x) = P(w_2|x)$ if x=$(a_2 + a_1)/2$. plot $P(w_1|x)$ and $P(w_2|x)$ on one axis for the case $a_1 = 3, a_2 = 5$ and b=1.

c) Show that the minimum probability of error is given by:

$$\text{P(error)} = \frac{1}{2} - \frac{1}{\pi} tan^{-1} \left|\frac{a_2 - a_1}{2b}\right|$$

d) What is the maximum value of P(error) and under which conditions can this occur?

e) Design the Bayes minimum error classifier in terms of a_i and b if $P(w_1) = P(w_2)$. Plot the decision boundaries in this case. What is the probability of error?

f) Design the Bayes minimum risk classifier with the following error weights

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

Plot the decision boundaries in this case. What is the probability of error? Compare the result with that of (e).