Climate Change Processes and Climate Modeling

Exercise 1 - Solutions of 0-dimensional Energy-Balance Model

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We have derived the 0-D EBM for the variable of the global mean surface temperature T_s of the following form:

$$C\frac{dT_s}{dt} = \frac{S_0}{4}\left[1 - \alpha_p(x_s(T_s))\right] - (A + BT_s) \tag{1}$$

Note that the heat capcity on the left hand side, C, is the averaged total heat capacity of Earth, taking into account the heat capacity of the atmosphere and of the oceans. The first term on the right-hand side is the incoming solar radiation, the second term is the linearized, empirically estimated outgoing long-wave radiation emitted by the surface.

The planetary albedo α_p shall be parameterized as a function of the edge of the polar ice cap ($x_s = sin(\phi_s)$), which is in turn a function of the mean global surface temperature. In the parameterization, it is assumed that above a certain global mean temperature T_{NoIce}, the Earth is ice-free, while the planet is completely covered by ice for temperatures below T_{Ice}. Inbetween, the ice edge linearly moves from the pole towards the equator:

$$x_s(T_s) = \begin{cases} 1, & T > T_{NoIce} \\ 1 + \frac{T_s - T_{NoIce}}{T_{NoIce} - T_{Ice}}, & T_{Ice} \leq T \leq T_{NoIce} \\ 0, & T < T_{Ice} \end{cases} \tag{2}$$

Further, we set the albedo for ice-covered surface to α_{Ice}, and to α_{NoIce} otherwise. To calculate the planetary albedo, the albedo $\alpha(x)$ is weighted by the distribution of the solar incoming radiation. The latter is approximated by the polynomial $S(x) = (1 + S_2 P_2(x))$, where $P_2(x) = \frac{1}{2}(3x^2 - 1)$. The mean albedo is therewith given by:

$$\alpha_p[x_s(T_s)] = \alpha_{Ice} + (\alpha_{NoIce} - \alpha_{Ice})\left[\left(1 - \frac{S_2}{2}\right)x_s + \frac{S_2}{2}x_s^3\right] \tag{3}$$

The values that shall be used in the following are given in the Table and follow North (1981).

Variable	Value	Unit
C	$2.0 * 10^8$	$[J/K/m^2]$
S0	1340	$[W/m^2]$
S2	-0.477	[]
α_{Ice}	0.62	[]
α_{NoIce}	0.3	[]
T_{Ice}	258	[K]
T_{NoIce}	288	[K]
A	-367.3	$[W/m^2]$
B	2.09	$[W/m^2/K]$

1 Solution of steady-state equation

Assume the steady state solution. Calculate and plot the short-wave as well as the long-wave radiative fluxes for a range of temperatures between 230 and 300 K. To calculate the short-wave flux, implement a function for the albedo as a function of the global mean temperature T_s.

The solutions of the 0-D EBM are then the value(s) of T_s, for which the SW and LW components are equal. Estimate the solution(s): How many solutions do you find for T_s, and how far does the ice cap extend for each of them?

2 Time-dependent EBM

After having estimated the solutions of the EBM by comparing the two flux components, we now will implement the model equation for the time-dependent model. You can use the function for the albedo implemented above. Write a python function for running the EBM, i.e. calculating the tendency dT/dt according to equation 1. As the model is rather linear, we get away with directly calculating the tendency for each time step (i.e. $\delta T = (rhs) * \delta t$) without having to use a more advanced numerical scheme. Implement the EBM-function as a function of the initial temperatre $T_s(t = 0)$ and of the number of timesteps you want to calculate foreward. A time-step of 100 days works well. The return values from the EBM function will be the temperature time-series.

Call the EBM function for different initial value of temperature, ranging from about 230 to 300 K. How many different solutions do you find? How long do you have to run the model to reach convergence? Are the solutions the same as found above, and if not, what could be the possible reason?

[Hint for the last question: consider how the temperature changes for a small deviation to the temperature of a certain solution T_0, i.e. for $T_0 + \delta T$. To derive an expression for the temperature change, insert $T_0 + \delta T$ in equ. 1, and introduce a taylor expansion for the Albedo $\alpha(T_0 + \delta T) = \alpha(T_0) + \frac{\partial \alpha}{\partial T}(T_0)\delta T$.]

3 Solutions for varying the solar constant & the "Snowball Earth paradoxon"

One application of the EBM is to explore the possible climate states for varying the solar constant. For example, the early sun is assumed to have had a solar radiation of only 70% of todays.

Calculate the possible solutions for varying the solar constant between about 0.85 and 1.4 times its current value, using the explicit calculation of solutions as in (1). [Hint: you can simply multiply the SW radiation by the factor S/S0]. Plot the solutions as a function of S/S0. For which range of the solar constant are there more than one solution? Which climate state would the EBM suggest for a faint young sun?

Opposing to the prediction of the EBM, geological evidence suggests that liquid ice was present during times of the "faint young sun". Likewise, another famous paradoxon appears to arise from the EBM predictions, namely the "Snowball Earth Paradox". Geological evidence suggests that glaciation extended to low equatorial latitudes about 600 millions of years ago. From the results obtained so far, a completely ice-covered Earth is indeed one possible climate state for the current solar constant. Given the solutions of the EBM for varying solar constants, how far would the solar constant have to be increased for Earth to leave the "ice-covered" climate state?

As such a strong increase in the solar constant clearly did not occur, researchers searched for other explanations of how Earth could get back to a warm climate state. One common hypothesis to explain the transition from "Snowball Earth" to a warm, partially ice-covered Earth is a drastic increase in the CO_2 concentrations, resulting from constant emission of CO_2 from volcanoes and a shut-down of the carbon sinks on an ice-covered planet. Using the EBM, test how far the radiative forcing through CO_2 would have had to change to allow such a transition (assuming the current solar constant S0). Note that the "Snowball Earth paradoxon" is still not completely solved, and is still a topic of ongoing research. [For further reading, see e.g.: https://www.sciencedaily.com/releases/2011/10/111012083450.htm, or Review Article Hoffman and Schrag, 2002, Section "Snowball Earth" (https://websites.pmc.ucsc.edu/r̃coe/eart206/Hoffman&Schrag_TerraNova02.pdf)]

General hints for the implementation It is recommended to implement the functions of the EBM itself (forward calculation of EBM, LW and SW parameterization) in one python program (e.g. EBM.py), in which you can also define all constants and parameter values (at the top, so they are accessible by all functions). In a separate python program, or a jupyternotebook, you can then call the functions and conduct the analysis. (Note that a restart of the kernel might be necessary for the jupyternotebook to recognize changes in EBM.py).