German University in Cairo
Media Engineering and Technology
DMET 502 Computer Graphics, Winter 2023
Assoc. Prof. Dr. Rimon Elias

Practice assignment 6 solution

Projections

Q 1: Consider a unit cube with two of its corners at $[0,0,0]^T$ and $[1,1,1]^T$. Given a view plane with a normal vector $[-1,1,1]^T$, determine the isometric projection of its corners onto the xy-plane.

Solution:

Using the isometric projection matrix

$$\begin{bmatrix} x' \\ y' \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} cos(-45°) & 0 & -sin(-45°) & 0 \\ -sin(35.2644°)sin(-45°) & cos(35.2644°) & -sin(35.2644°)cos(-45°) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7071 \\ 0.4082 \\ 0 \\ 1 \end{bmatrix}$$

$P = [x,y,z]^T$	$P' = [x',y']^T$
$[0,0,0]^T$	$[0,0]^T$
$[1,0,0]^T$	$[0.7071, 0.4082]^T$
$[0,1,0]^T$	$[0, 0.8165]^T$
$[1,1,0]^T$	$[0.7071, 1.2247]^T$
$[0,0,1]^T$	$[0.7071, -0.4082]^T$
$[1,0,1]^T$	$[1.4142, 0]^T$
$[0,1,1]^T$	$[0.7071, 0.4082]^T$
$[1,1,1]^T$	$[1.4142, 0.8165]^T$

Q 2: Consider a unit cube with two of its corners at $[0,0,0]^T$ and $[1,1,1]^T$. Given a view plane with a normal vector $[-1,-1,1]^T$, determine the isometric projection of its corners onto the yz-plane.

Solution:

The azimuth angle θ with the x-axis $= 45°$

The elevation angle $\varphi = 35.2644°$

The first rotation of the normal is by the angle -θ about the y-axis. The second rotation of the normal is by the angle -φ about the z-axis.

The overall axonometric projection onto the yz-plane for a point $[x,y,z]^T$ is estimated as

$$\begin{bmatrix} 0 \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(-\varphi) & -sin(-\varphi) & 0 \\ sin(-\varphi) & cos(-\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(-\theta) & 0 & sin(-\theta) \\ 0 & 1 & 0 \\ -sin(-\theta) & 0 & cos(-\theta) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ -0.4082 & 0.8165 & 0.4082 \\ 0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$P = [x,y,z]^T$	$P' = [y',z']^T$
$[0,0,0]^T$	$[0,0]^T$
$[1,0,0]^T$	$[-0.4082, 0.7071]^T$
$[0,1,0]^T$	$[0.8165, 0]^T$
$[1,1,0]^T$	$[0.4082, 0.7071]^T$
$[0,0,1]^T$	$[0.4082, 0.7071]^T$
$[1,0,1]^T$	$[0, 1.4142]^T$
$[0,1,1]^T$	$[1.2247, 0.7071]^T$
$[1,1,1]^T$	$[0.8165, 1.4142]^T$

Q 3: Given a view plane with a normal vector $[3,3,4]^T$, determine the homogeneous diametric projection matrix (onto the xy-plane) that can be used with it.

Solution:
$\theta = tan^{-1}(\frac{3}{4}) = 36.8699°$
$\varphi = tan^{-1}(\frac{3}{5}) = 30.9638°$

$$\begin{bmatrix} x' \\ y' \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 & -sin(\theta) & 0 \\ -sin(\varphi)sin(\theta) & cos(\varphi) & -sin(\varphi)cos(\theta) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & 0 & -\frac{3}{5} & 0 \\ -\frac{9}{5\sqrt{34}} & \frac{5}{\sqrt{34}} & -\frac{12}{5\sqrt{34}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Q 4: If the center of projection is placed at $[0.5, 0.5, -d]^T$. Determine the one-point perspective projection matrix onto the xy-plane.

Solution:
Translate using a translation vector $[-0.5, -0.5, 0]^T$

$$M1 = T([-\tfrac{1}{2}, -\tfrac{1}{2}, 0]^T) = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Use the projection matrix where the view plane is the xy-plane

$$M2 = P'_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix}$$

Translate using a translation vector $[0.5, 0.5, 0]^T$

$$M3 = T([\tfrac{1}{2}, \tfrac{1}{2}, 0]^T) = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = M3M2M1$$

$$= \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{2d} & 0 \\ 0 & 1 & \frac{1}{2d} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix}$$

Q 5: If the center of projection is placed at $[0, 0, 0]^T$ and the view plane is placed at $x = d$, derive the projection matrix.

Solution:

Consider the similar triangles

$\frac{y'}{d} = \frac{y}{x}$

$\frac{z'}{d} = \frac{z}{x}$

$y' = \frac{y}{x/d}$

$z' = \frac{z}{x/d}$

$$P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{d} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{x}{d} \end{bmatrix}$$

$$P' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} d \\ \frac{y}{x/d} \\ \frac{z}{x/d} \end{bmatrix}$$