

# ***Embedded System Architecture - CSEN 701***

**Module 6:** *Multi-tasking and Real-Time Systems*

**Lecture 14:** *RTOS Scheduling Algorithms - RM*

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- Task Scheduling
- Rate-Monotonic Scheduling Algorithm



## ES Modeling & Design (2 Modules)

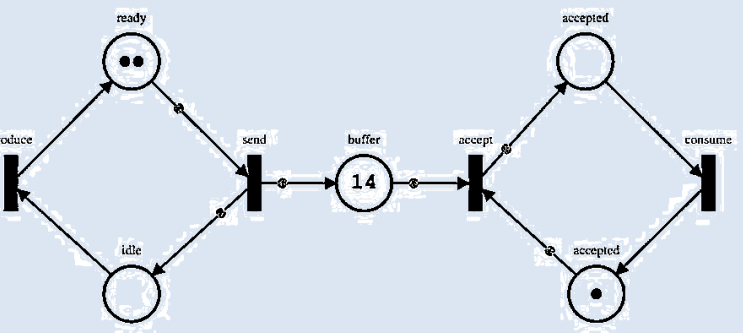
System Modeling



Design Considerations

Power management and optimization

Reliability and fault tolerance



## The Real-time Embedded System

### ES Hardware Components (4 Modules)

Microcontroller  
Fundamentals



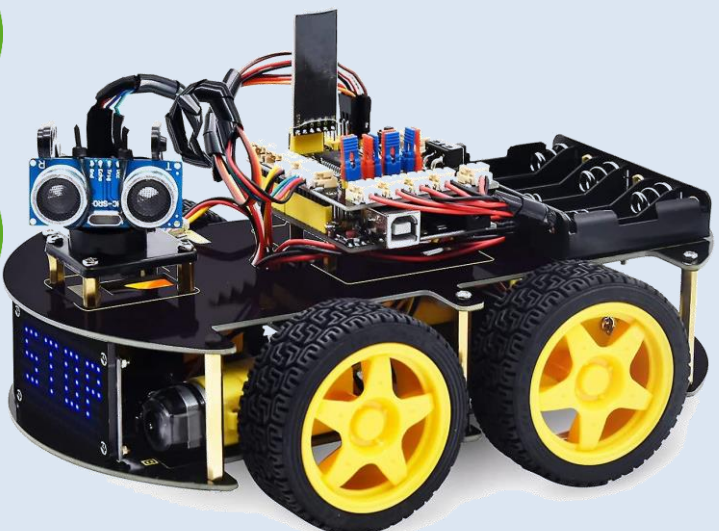
Embedded programming  
languages



Embedded Hardware



Communication and  
Networking



### ES Software Components (1 Module)

Real-Time Systems

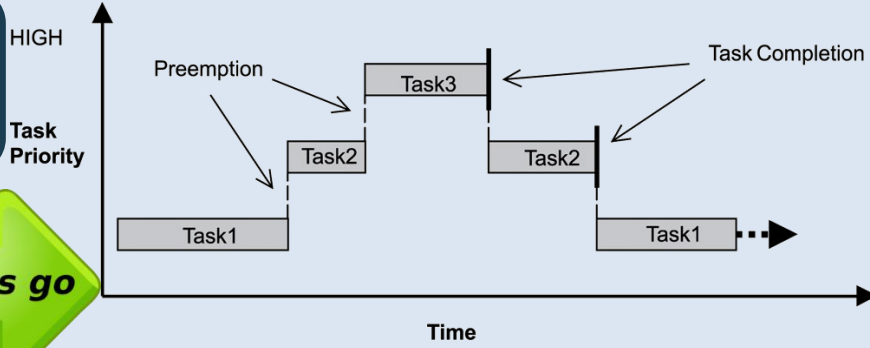


Multi-tasking

Scheduling



Resource Management



### Embedded System Tools & Software Development (2 Modules)

Debugging techniques

Interrupts and exception  
handling

Memory management

Let's go





## Real-Time Constraints

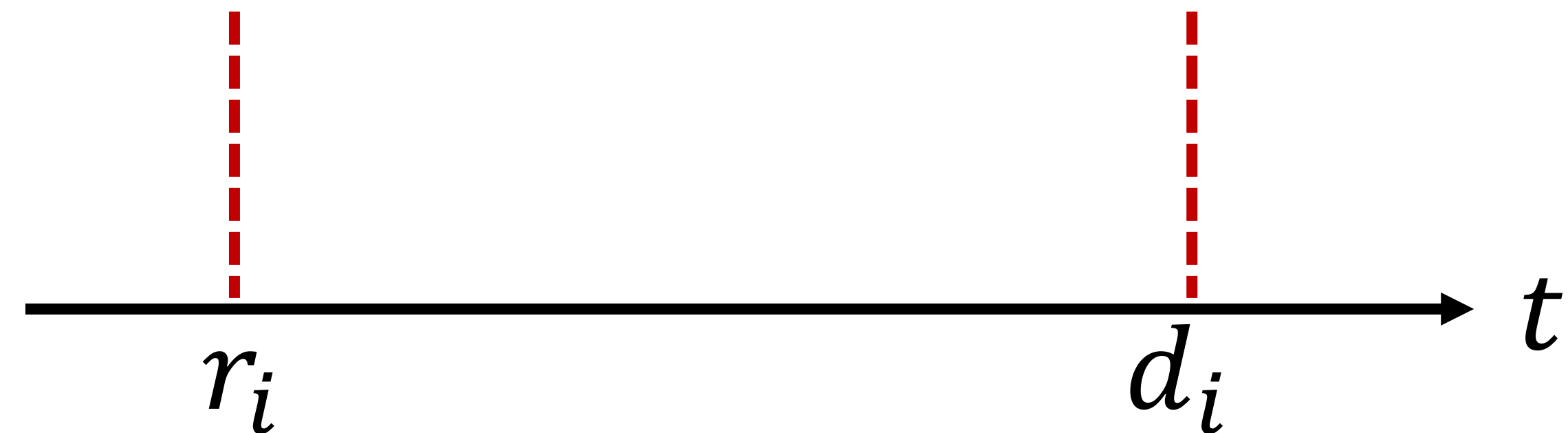
- The **RTOS** is responsible for running concurrent tasks, while **maintaining their timing constraints** which are classified to:
  - **Hard timing constraints**: Missing its deadline is considered a **fatal** error (Example: inflating airbags after an accident)
  - **Soft timing constraints**: Missing the deadline or execute it a little bit late is **not an issue** (Example: opening mobile application)
- **Task scheduling** is used to run the concurrent tasks **while ensuring meeting the deadlines** (Feasible Schedule).

## Tasks

- A Task is the unit of work executed by the CPU
- A Task can be either:
  - **Periodic:**
    - Tasks **repeated** after a **period time** ( $p$ )
    - They have **hard deadlines** as the task must be completed before the end of the period (Ex: Reading sensory data)
  - **Aperiodic:**
    - unpredictable **one shot** tasks, having **soft or no deadlines**. (Ex: pushing a button in a vending machine)
  - **Sporadic:**
    - unpredictable **one shot** tasks, however they introduce a **hard deadline**. (Ex: inflating airbags after an accident).

## Tasks Specification

- In real-time systems, a task ( $i$ ) can be defined by the following timing parameters:
  - Release time ( $r_i$ ):
    - Time instant at which the task is ready to be executed
  - Deadline ( $d_i$ ):
    - Time instant where the execution of the task must be completed



## Tasks Specification

- In real-time systems, a task ( $i$ ) can be defined by the following timing parameters:

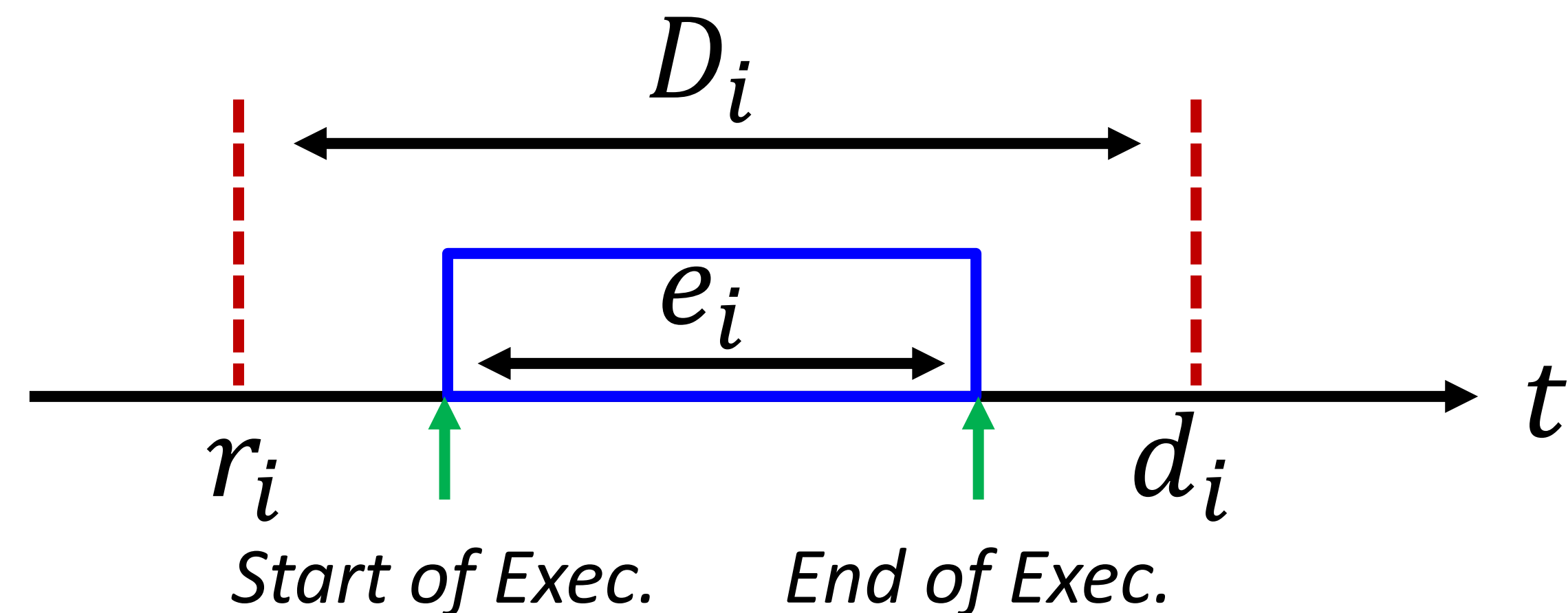
- **Relative deadline ( $D_i$ ):**

- Time difference between the release time and the deadline

$$D_i = d_i - r_i$$

- **Execution time ( $e_i$ ):**

- The time required for the processor to execute the task



## Tasks Specification

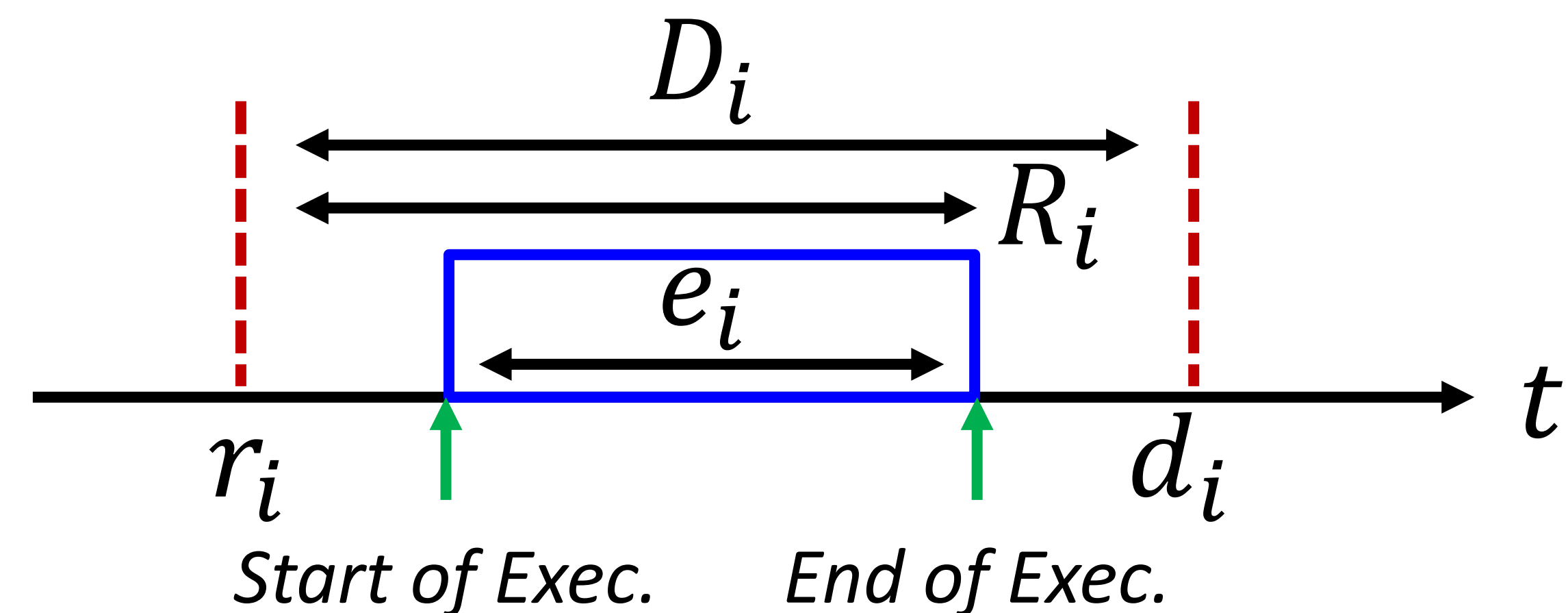
- In real-time systems, a task ( $i$ ) can be defined by the following timing parameters:

- **Response time ( $R_i$ ):**

- Time difference between the release time and the end of execution.

$$R_i = f_i - r_i$$

- The execution of task can start at any time after the release time, but the execution must end before the deadline





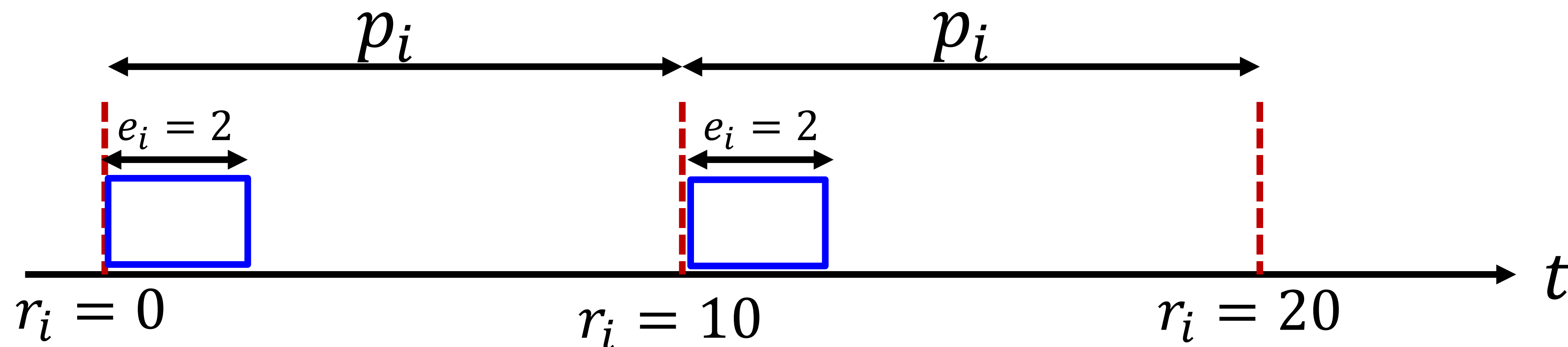
## Periodic Tasks Specification

- For periodic tasks:

- The **relative deadline** is defined by the period of the task. Thus to schedule periodic task. A task is defined as:

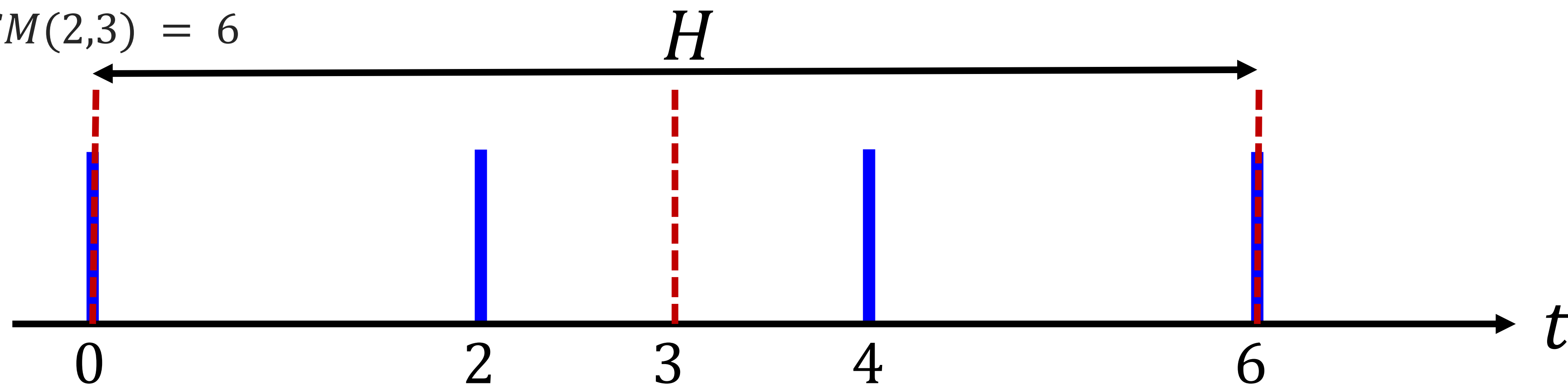
$$T_i = (p_i, e_i)$$

- Ex: A task  $T = (10, 2)$ , is released every 10 time units, have a relative deadline of 10 time units. And requires 2 time units on the processor to be completed.



## Periodic Tasks

- For a set of periodic tasks having different periods ( $p_i$ ).
  - A **hyperperiod** ( $H$ ) defines the **repetition period of the whole set**.
  - A hyperperiod is defined as the **Least common multiple** of the different period values.
- Example:
  - $p_1 = 3$ ,
  - $p_2 = 2$ ,
  - $\therefore H = LCM(2,3) = 6$



- The most well-known static priority scheduling algorithm is Rate-Monotonic Scheduling (RM).
- It is used in scheduling periodic tasks.
- The **priority** is assigned based on the **period**.
  - Tasks having the shortest period has the highest priority.

## Example

- 3 periodic tasks are scheduled using RM algorithm. The tasks are defined as:

$$T_1 = (4, 1),$$

$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$

- Construct the schedule

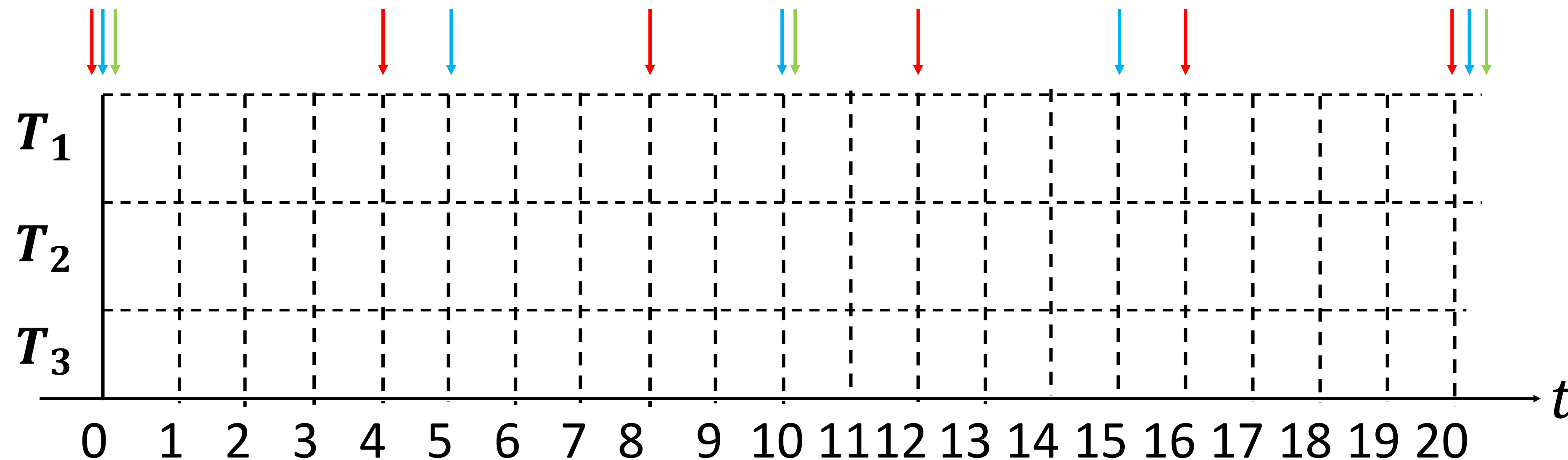
Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3



## Example

- 3 periodic tasks are scheduled using RM algorithm.
  - HyperPeriod:  $H = \text{LCM}(4, 5, 10) = 20$

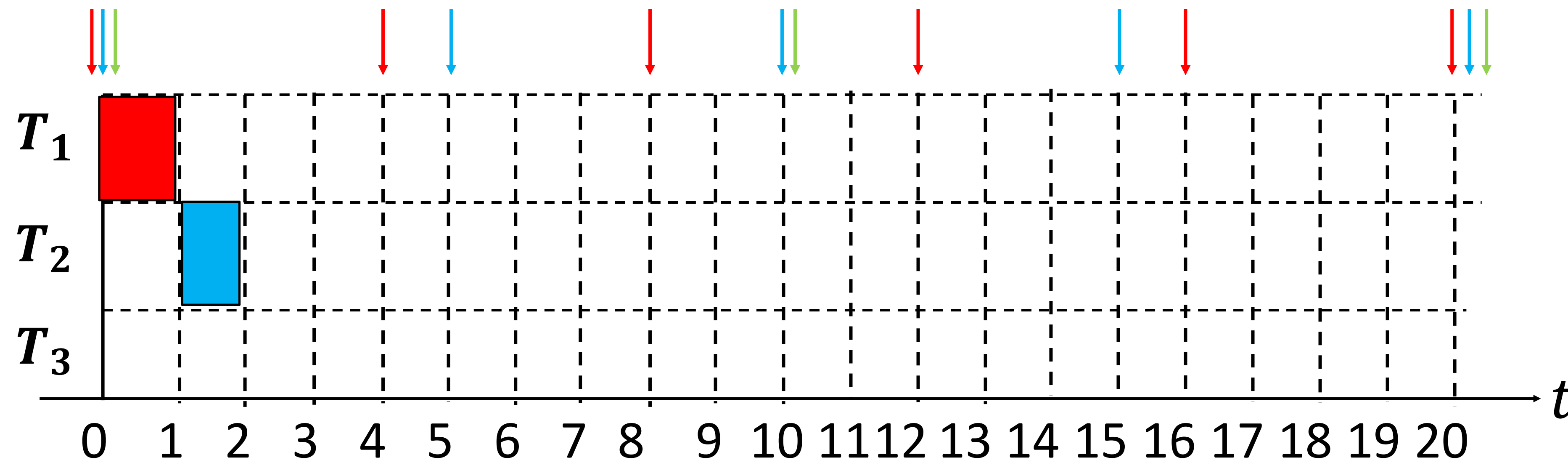
Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3



## Example

- 3 periodic tasks are scheduled using RM algorithm.
  - At  $t=0$ , all tasks are ready.  $T_1$  has the shortest period, thus the highest priority. Thus  $T_1$  is executed.
  - At  $t=1$ , the  $T_1$  instance is finished and the processor is idle. Thus the highest priority is chosen which is  $T_2$

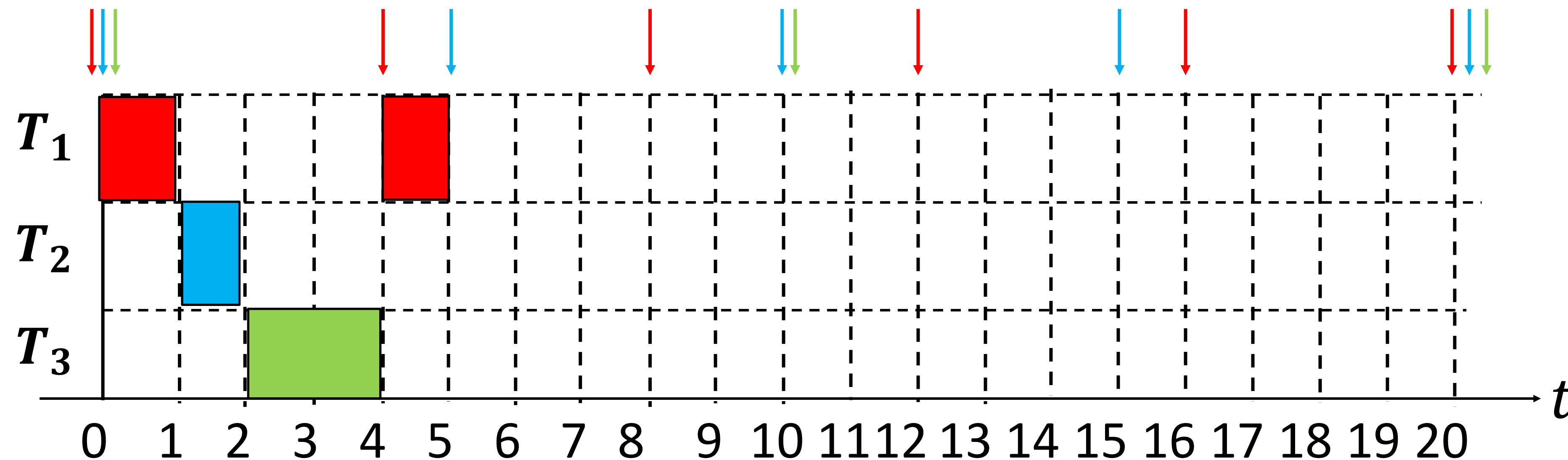
Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3



## Example

- 3 periodic tasks are scheduled using RM algorithm.
  - At  $t=2$  and  $t=3$ , Task 3 is the only task ready to run, thus it is executed.
  - At  $t=4$ ,  $T_1$  is ready, and it has a higher priority, thus  $T_3$  is preempted to execute  $T_1$

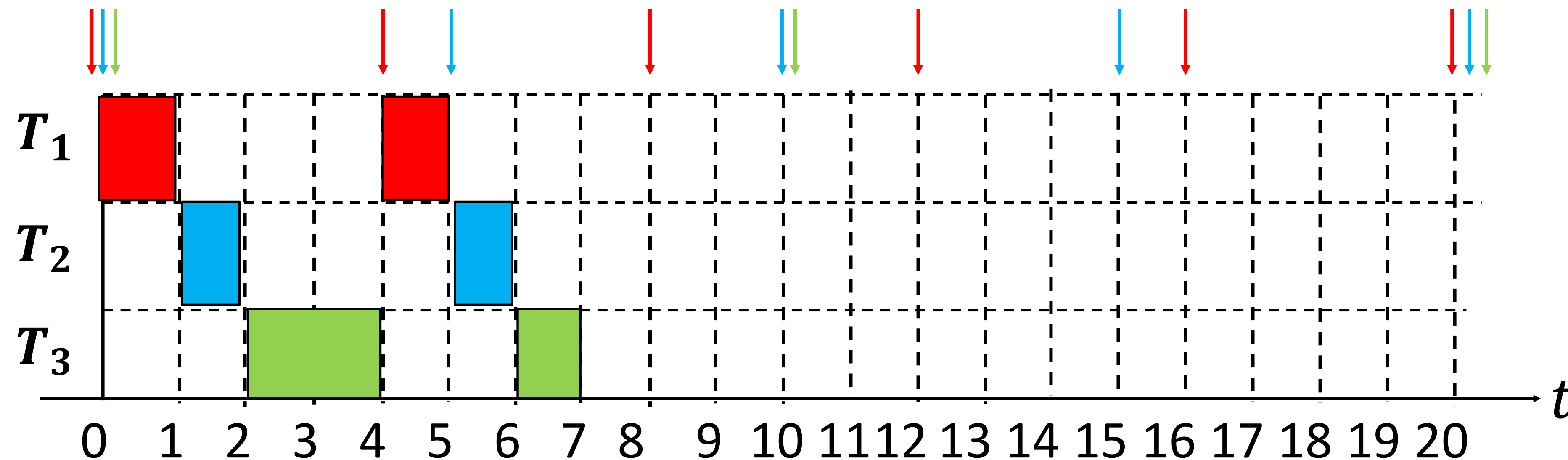
Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3



## Example

- 3 periodic tasks are scheduled using RM algorithm.
  - At  $t=5$ ,  $T_2$  is ready and has higher priority than  $T_3$ , thus  $T_2$  is executed.
  - At  $t=6$ ,  $T_3$  is resumed to finish the remaining 1 time unit

Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3

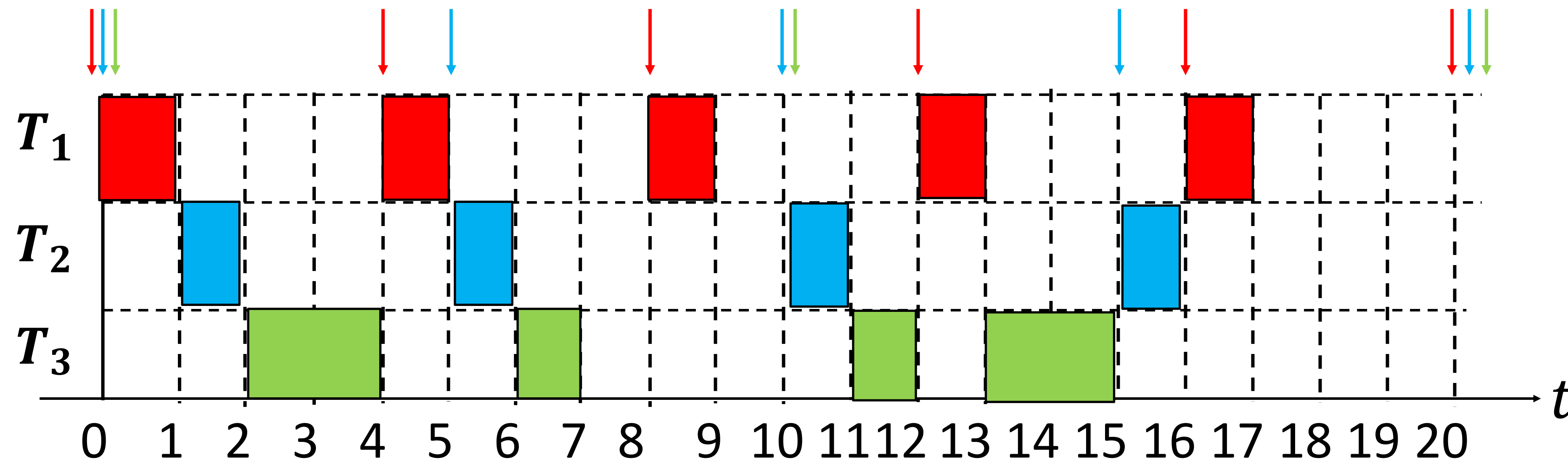




## Example

- 3 periodic tasks are scheduled using RM algorithm.
  - The final schedule can be computed as shown

Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3



## RM Schedulability Check

- To check whether a set of tasks can be scheduled using RM. Two checks are used:
  - Utilization Bound
  - Response Time / Time Demand Analysis

## RM Schedulability Check: *Utilization Bound*

- A **Task utilization** is defined as:

$$U_i = \frac{e_i}{p_i}$$

➤ For a task to be schedulable, its utilization value must be **less than or equal to 1**

- The **total utilization** of a set of tasks is defined as:

$$U_{total} = \sum \frac{e_i}{p_i}$$

## RM Schedulability Check: *Utilization Bound*

- A **sufficient condition for the RM scheduling** to schedule a set of  $n$  tasks is:

$$U_{total} \leq U_{RM}$$
$$U_{RM} = n(2^{\frac{1}{n}} - 1)$$

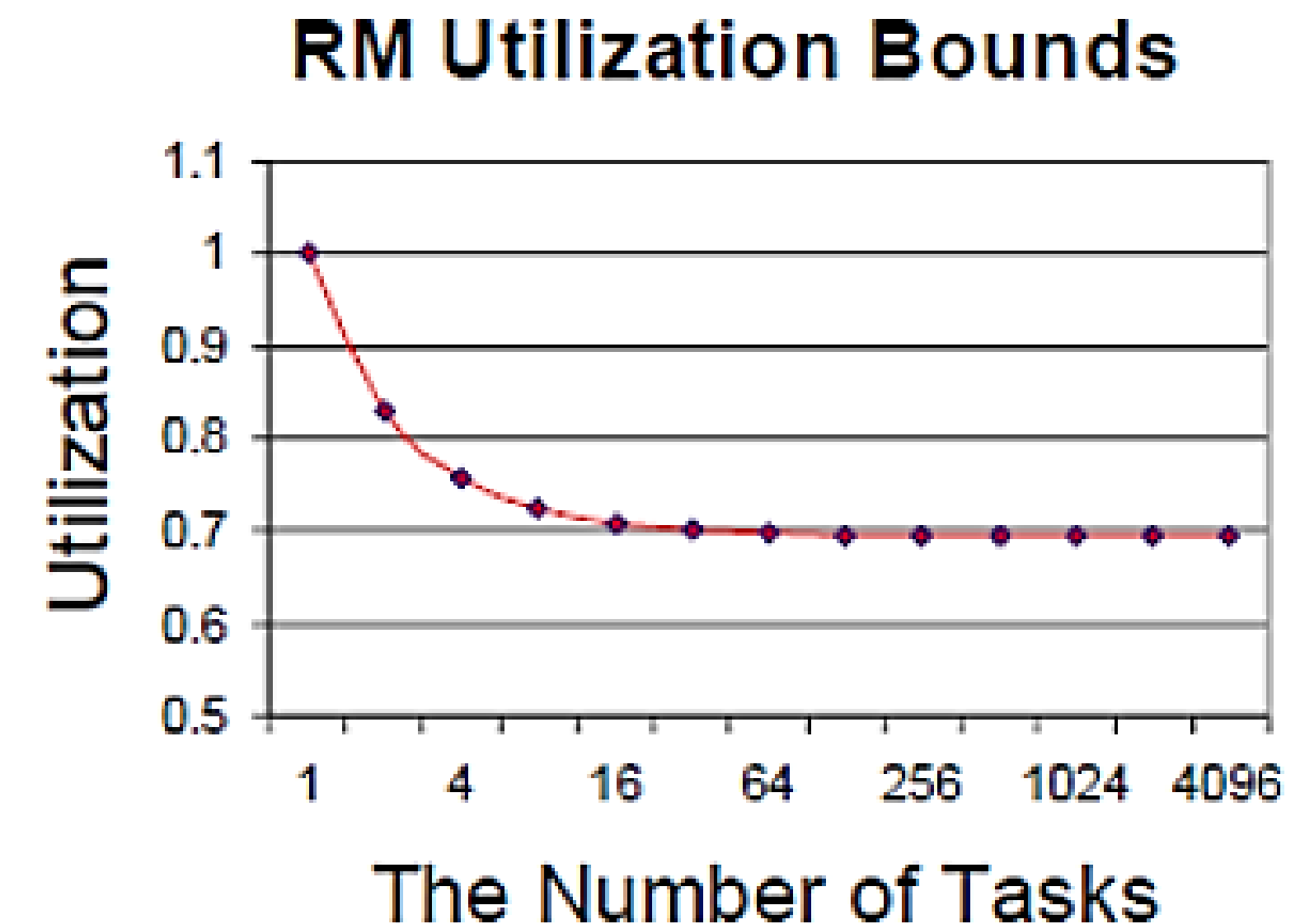
- If the condition is **satisfied**, then this set of tasks **can be scheduled** using **RM**.
- If the condition is **not satisfied**, this set of tasks may or **may not be schedulable** by **RM**



## RM Schedulability Check: *Utilization Bound*

$B(1)=1.0$	$B(4)=0.756$	$B(7)=0.728$
$B(2)=0.828$	$B(5)=0.743$	$B(8)=0.724$
$B(3)=0.779$	$B(6)=0.734$	$U(\infty)=0.693$

Note that  $U(\infty)=0.693$  !



## RM Schedulability Check: *Utilization Bound Example*

- The total utilization is:

$$U_{total} = \frac{1}{4} + \frac{1}{5} + \frac{3}{10} = 0.75$$

- The RM utilization upper bound

$$U_{RM} = 3 * (2^{1/3} - 1) = 0.779$$

$$\therefore U_{total} \leq U_{RM}$$

∴ These tasks are schedulable using RM

Task	p	e	Priority
$T_1$	4	1	1
$T_2$	5	1	2
$T_3$	10	3	3

## RM Schedulability Check

- To check whether a set of tasks can be scheduled using RM. Two checks are used:
  - Utilization Bound
  - Response Time / Time Demand Analysis

## RM Schedulability Check: *Response Time / Time Demand Analysis*

- TDA is a technique to check each Task schedulability using RM scheduling
- The TDA provides a mathematical way to prove that a task is schedulable at its **critical instant**. Thus proving the **schedulability** of a set of tasks using **RM**.
- For each task ( $i$ ), the TDA analysis is performed at its **critical instants**.
- A critical instant of a task ( $i$ ) is defined where **tasks of higher priorities are released** and will **preempt** it, causing the **maximum response time** for the task ( $i$ ).



## RM Schedulability Check: *Response Time / Time Demand Analysis*

- Generally, The time demand of a Task ( $i$ ) to be finished at time  $t$  is the **sum of all the execution times of higher priority tasks and the execution time task( $i$ ):**

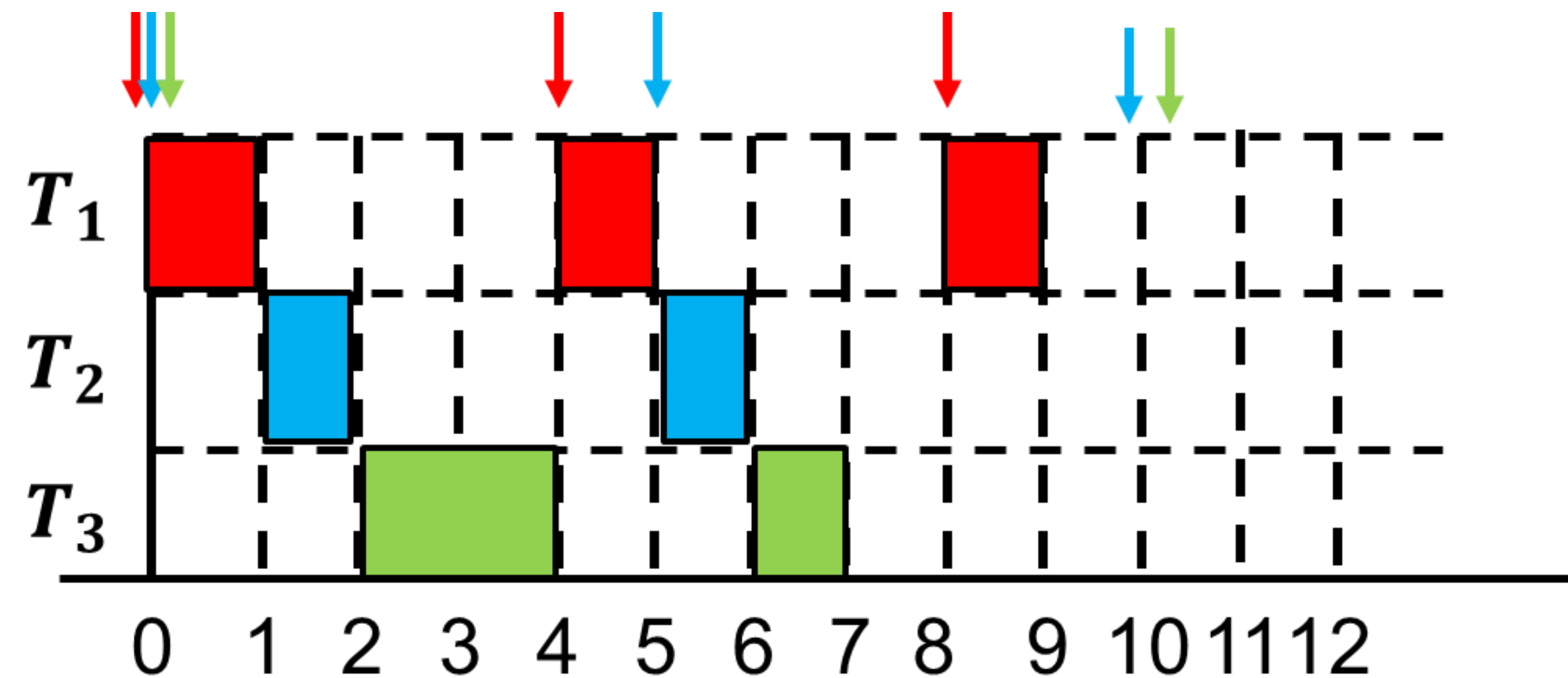
$$w_i(t) = \sum_{j=1}^{i-1} \text{ceil} \left( \frac{t}{p_j} \right) * e_j + e_i$$

- A task is schedulable if:
  - Its utilization  $U = \frac{e_i}{p_i}$  is less than or equal 1
  - There exists a time instant ( $t < p_i$ ) where:

$$w_i(t) \leq t$$

## RM Schedulability Check: *Response Time / Time Demand Analysis*

- The time required to finish T3 in the shown system is :



$$w_3(t) = \underbrace{\text{ceil}\left(\frac{t}{p_1}\right) * e_1}_{\text{Num. of instances of T1 before instance (t)}} + \underbrace{\text{ceil}\left(\frac{t}{p_2}\right) * e_2}_{\text{Num. of instances of T2 before instance (t)}} + e_3$$

Num. of instances of  
T1 before instance (t)

Num. of instances of  
T2 before instance (t)

## RM Schedulability Check: *Response Time / Time Demand Analysis Example*

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

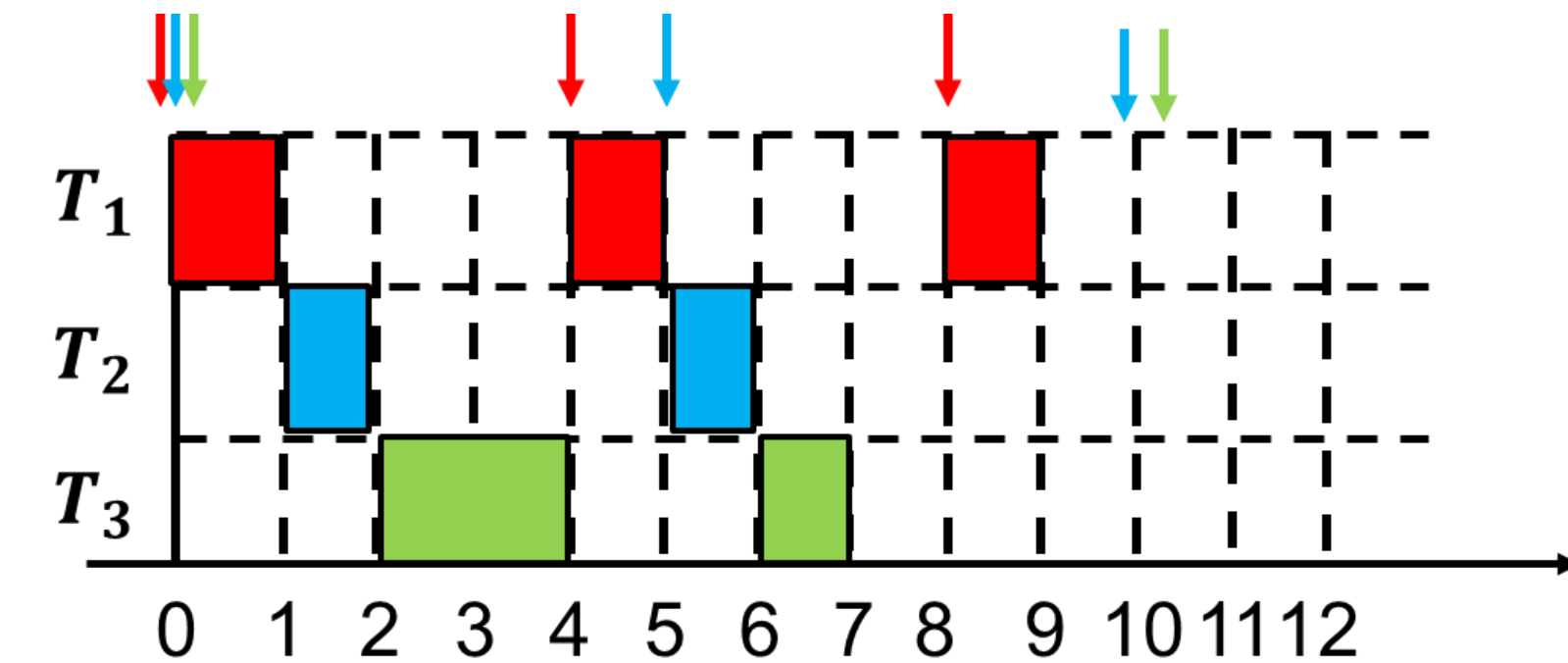
$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$

- Schedulability of Task1:**

$$\text{➤ } U = \frac{1}{4} = 0.25 \leq 1$$

- Task 1 has the highest priority, thus it is certainly schedulable (It is never preempted)



## RM Schedulability Check: *Response Time / Time Demand Analysis Example*

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$

- Schedulability of Task2:

$$\text{➤ } U = \frac{1}{5} = 0.2 \leq 1$$

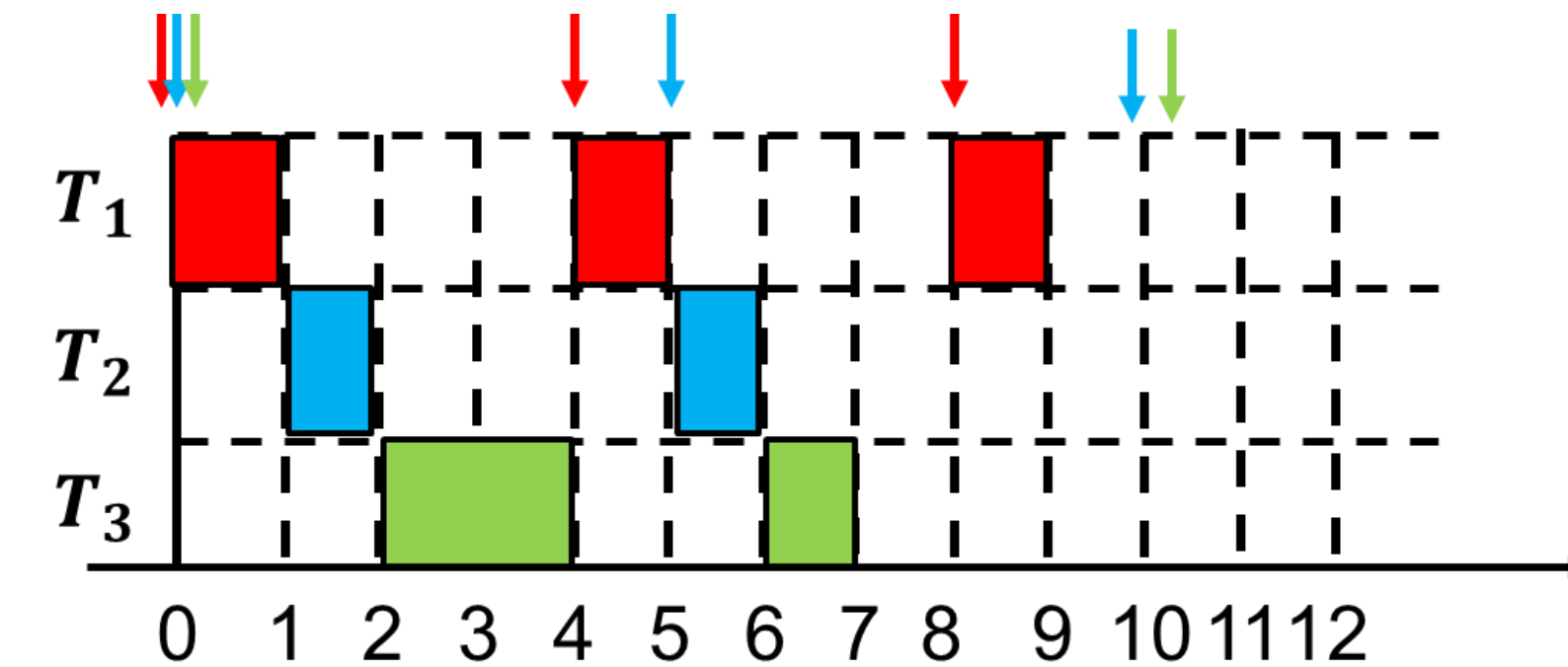
- To find the instance (t), the condition is checked at the **release times of  $T_1$  during the period of  $T_2$**  as well as **the end of the period**. Thus, we will check the condition at  $t = 4, 5$

- At  $t=4$  :

$$w_2(4) = \text{ceil}\left(\frac{4}{p_1}\right) * e_1 + e_2$$

$$= (1 * 1) + 1 = 2 \leq 4 \text{ (Satisfied)}$$

- Task 2 is schedulable before  $t = 4$



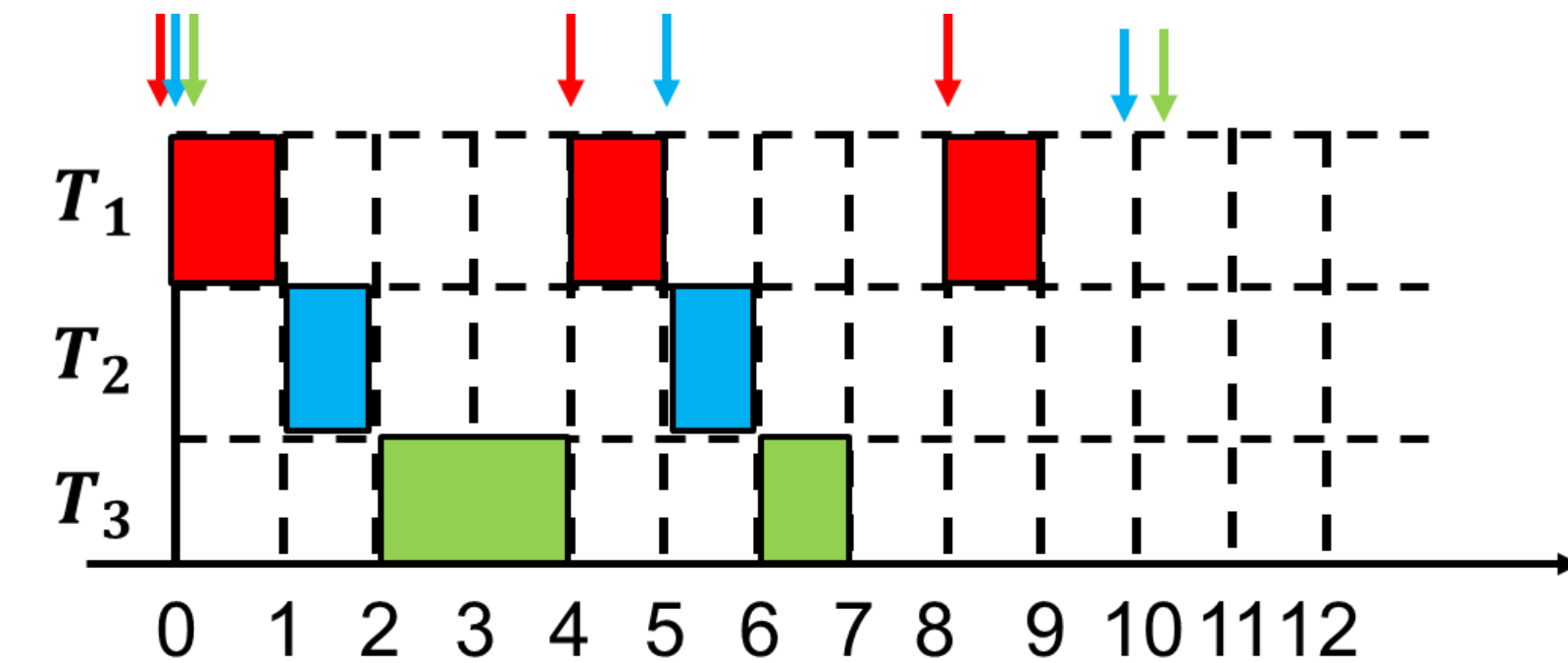
## RM Schedulability Check: *Response Time / Time Demand Analysis Example*

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$



- Schedulability of Task3:**

$$\text{➤ } U = \frac{3}{10} = 0.3 \leq 1$$

- To find the instance (t), the condition is checked at the **release times of  $T_1, T_2$  during the period of  $T_3$**  as well as **the end of the period**. Thus, we will check the condition at  $t = 4, 5, 8, 10$

- At  $t=4$ :

$$\begin{aligned}
 w_3(4) &= \text{ceil}\left(\frac{4}{p_1}\right) * e_1 + \text{ceil}\left(\frac{4}{p_2}\right) * e_2 + e_3 \\
 &= (1 * 1) + (1 * 1) + 3 = 5 \geq 4 \text{ (Not Satisfied)}
 \end{aligned}$$



## RM Schedulability Check: *Response Time / Time Demand Analysis Example*

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$

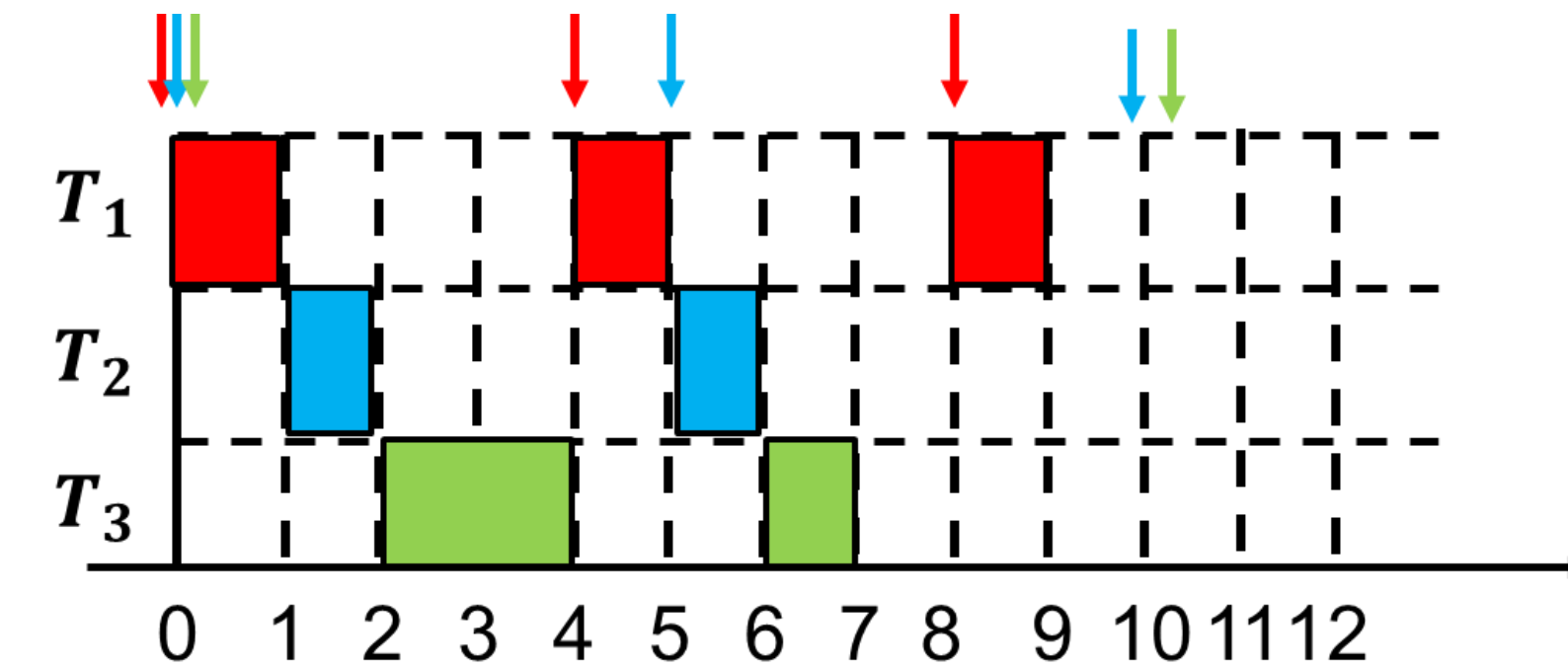
- Schedulability of Task3:**

$$\text{➤ } U = \frac{3}{10} = 0.3 \leq 1$$

- To find the instance (t), the condition is checked at the **release times of  $T_1, T_2$  during the period of  $T_3$**  as well as **the end of the period**. Thus, we will check the condition at **t = 4, 5, 8, 10**

- At t=5 :

$$\begin{aligned}
 w_3(5) &= \text{ceil}\left(\frac{5}{p_1}\right) * e_1 + \text{ceil}\left(\frac{5}{p_2}\right) * e_2 + e_3 \\
 &= (2 * 1) + (1 * 1) + 3 = 6 \geq 5 \text{ (Not Satisfied)}
 \end{aligned}$$



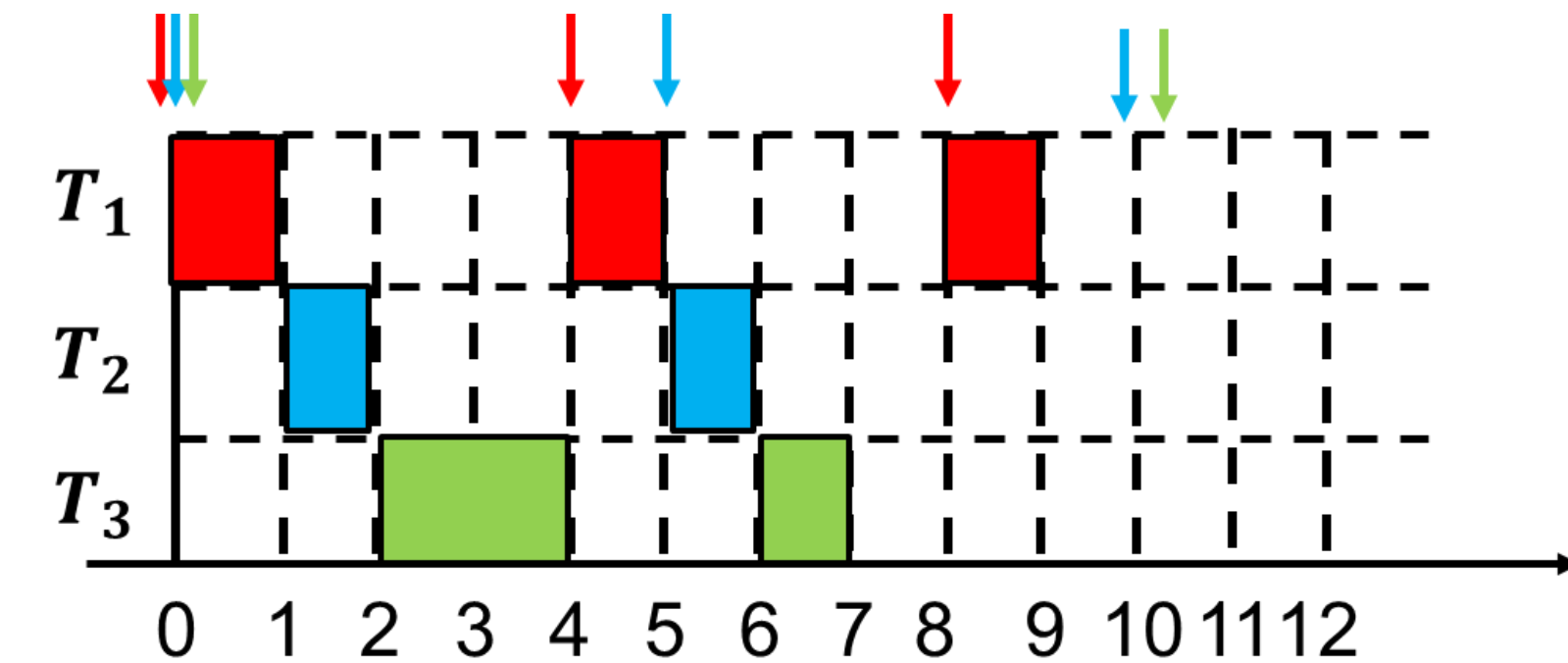
## RM Schedulability Check: *Response Time / Time Demand Analysis Example*

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

$$T_2 = (5, 1),$$

$$T_3 = (10, 3)$$



- Schedulability of Task3:**

$$\text{➤ } U = \frac{3}{10} = 0.3 \leq 1$$

- To find the instance (t), the condition is checked at the **release times of  $T_1, T_2$  during the period of  $T_3$**  as well as **the end of the period**. Thus, we will check the condition at  $t = 4, 5, 8, 10$

- At  $t = 8$ :

$$w_3(8) = \text{ceil}\left(\frac{8}{p_1}\right) * e_1 + \text{ceil}\left(\frac{8}{p_2}\right) * e_2 + e_3$$

$$= (2 * 1) + (2 * 1) + 3 = 7 \leq 8 \text{ (Satisfied)}$$

- Task 3 is schedulable before  $t = 8$

## RM Schedulability Check: *Response Time / Time Demand Analysis* Example

- Check the RM schedulability of the following set of tasks using TDA:

$$T_1 = (4, 1),$$

$$T_2 = (5, 2),$$

$$T_3 = (10, 3.1)$$


- In this example, Task 3 misses the deadline by 0.1 time units
- Using TDA analysis for task 3 at  $t = 4, 5, 8, 10$

$$\text{➤ } w_3(4) = \text{ceil}\left(\frac{4}{p_1}\right) * e_1 + \text{ceil}\left(\frac{4}{p_2}\right) * e_2 + e_3 = 6.1 \geq 4 \text{ (Not Satisfied)}$$

$$\text{➤ } w_3(5) = \text{ceil}\left(\frac{5}{p_1}\right) * e_1 + \text{ceil}\left(\frac{5}{p_2}\right) * e_2 + e_3 = 7.1 \geq 5 \text{ (Not Satisfied)}$$

$$\text{➤ } w_3(8) = \text{ceil}\left(\frac{8}{p_1}\right) * e_1 + \text{ceil}\left(\frac{8}{p_2}\right) * e_2 + e_3 = 9.1 \geq 8 \text{ (Not Satisfied)}$$

$$\text{➤ } w_3(10) = \text{ceil}\left(\frac{10}{p_1}\right) * e_1 + \text{ceil}\left(\frac{10}{p_2}\right) * e_2 + e_3 = 10.1 \geq 10 \text{ (Not Satisfied)}$$

*For Further Inquiries, Please*  
 *send an email*

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***Thank you for your attention!***

***See you next time*** 😊