Assignment 2: Neural Networks

Jan Scheffczyk - 3242317
Sarah Khan - 3279206
Mahmoud Hashem - 3201329
Mohamed Saleh - 3201337

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1 Theoretical exercises

1.1 Define MLP

First we define the line equations such that points that are on the inside of the shape with respect to the line will receive a value less than 0.

$$\begin{aligned}\overline{KP} &= (x - x_1)(y_2 - y_1) - (y - y_1)(x_2 - x_1)\\ &= (x + 2)(-2 - 2) - (y - 2)(-4 + 2)\\ &= -4x - 8 + 2y - 4\\ &= -2x + y - 6\end{aligned}$$

$$\begin{aligned}\overline{KR} &= (x - x_1)(y_2 - y_1) - (y - y_1)(x_2 - x_1)\\ &= (x + 2)(4 - 2) - (y - 2)(2 + 2)\\ &= x - 2y + 6\\ &= -x + 2y - 6\end{aligned}$$

$$\begin{aligned}\overline{RQ} &= (x - x_1)(y_2 - y_1) - (y - y_1)(x_2 - x_1)\\ &= (x - 2)(4 - 2) - (y + 2)(8 - 2)\\ &= x - 3y - 14\end{aligned}$$

$$\begin{aligned}\overline{PQ} &= (x - x_1)(y_2 - y_1) - (y - y_1)(x_2 - x_1)\\ &= (x + 4)(2 + 2) - (y + 2)(8 + 4)\\ &= x - 3y - 2\end{aligned}$$

Then be can define a nonlinear activation function for the hidden neurons as:

$$\sigma(x) \begin{cases} 1 & , \forall x | x \geqq 0 \\ 0 & , \forall x | x < 0 \end{cases}$$

Thus we get 4 hidden neurons:

$$\begin{aligned} h_1 &= \sigma(-2x + y - 6) \\ h_2 &= \sigma(-x + 2y - 6) \\ h_3 &= \sigma(x - 3y - 14) \\ h_4 &= \sigma(x - 3y - 2) \end{aligned}$$

And a single output neuron:

$$o = \sigma(\sum_{i=1}^{4} h_i)$$

which will output 0 the input is within the shape.

1.2 Compute MLP

Compute the weigthed sum for the hidden neurons:

$$\begin{aligned} net_{h1} &= \sum_{n=1}^{N} w_{nh_1} \cdot x_n + w_{b_1} \cdot 1 \\ &= w_1 \cdot x_1 + w_2 \cdot x_2 + w_{b_1} \\ &= 0.1 \cdot 0.1 + 0.2 \cdot 0.4 + 0.3 \\ &= 0.39 \end{aligned}$$

$$\begin{aligned} net_{h2} &= \sum_{n=1}^{N} w_{nh_2} \cdot x_n + w_{b_1} \cdot 1 \\ &= w_3 \cdot x_1 + w_4 \cdot x_2 + w_{b_1} \\ &= 0.2 \cdot 0.1 + 0.3 \cdot 0.4 + 0.3 \\ &= 0.44 \end{aligned}$$

Apply the activation function to hidden neurons, in this case logisitc sigmoid:

$$out_{h1} = \sigma(net_{h1}) = \frac{1}{1 + \exp^{-0.39}} = 0.596$$

$$out_{h2} = \sigma(net_{h2}) = \frac{1}{1 + \exp^{-0.44}} = 0.608$$

Compute the weigthed sum for the output neurons:

$$\begin{aligned} net_{o1} &= \sum_{h=1}^{H} w_{ho_1} \cdot out_h + w_{b_2} \cdot 1 \\ &= w_7 \cdot out_{h_1} + w_8 \cdot out_{h_2} + w_{b_2} \\ &= 0.5 \cdot 0.596 + 0.6 \cdot 0.608 + 0.6 \\ &= 1.263 \end{aligned}$$

$$\begin{aligned} net_{o2} &= \sum_{h=1}^{H} w_{ho_1} \cdot out_h + w_{b_2} \cdot 1 \\ &= w_7 \cdot out_{h_1} + w_8 \cdot out_{h_2} + w_{b_2} \\ &= 0.5 \cdot 0.596 + 0.6 \cdot 0.608 + 0.6 \\ &= 1.263 \end{aligned}$$

Apply the activation function to output neurons:

$$out_{o1} = \sigma(net_{o1}) = \frac{1}{1 + \exp^{-1.142}} = 0.758$$

$$out_{o2} = \sigma(net_{o2}) = \frac{1}{1 + \exp^{-1.263}} = 0.780$$

Finally we can compare, using mse, the calculated output to the real output to compute the error:

$$
\begin{aligned}
E_{total} &= \frac{1}{n} \cdot \sum_{i=1}^{N} (\hat{Y_i} - Y_i)^2 \\
&= \frac{1}{2} \cdot \Big(E_{o_1} + E_{o_2} \Big) \\
&= \frac{1}{2} \cdot \Big((0.1 - 0.758)^2 + (0.9 - 0.780)^2 \Big) \\
&= \frac{1}{2} \cdot (0.433 + 0.014) \\
&= 0.224
\end{aligned}
$$