Expert Systems

Lecture 4:Uncertainty management in rule-based expert systems

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Outlines

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What is Uncertainty?

- Uncertainty refers to the lack of exact knowledge, preventing perfectly reliable conclusions.
- Expert systems must handle uncertainty to replicate human reasoning in decision-making.
- It assumes that perfect knowledge always exists and the law of the excluded middle can always be applied.

IF A is true **THEN** A is not false

and

IF B is false **THEN** B is not true

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Sources of Uncertainty

- **Weak Implications:** Rules often lack strong cause-effect relationships.
  Expert systems need to have the ability to handle vague associations, for example by accepting the degree of correlations as numerical certainty factors.
- **Imprecise Language:** Ambiguity arises from terms like "sometimes" or "rarely."
- **Unknown Data:** Missing or incomplete data can lead to errors in decision-making.
- **Conflicting Knowledge:** Experts may provide contradictory information.

Quantification of ambiguous

An expert system should be able to manage uncertainties because any real-world domain contains inexact knowledge and needs to cope with incomplete, inconsistent or even missing data.

Ray Simpson (1944)		Milton Hakel (1968)	
Term	**Mean value**	**Term**	**Mean value**
Always	99	Always	100
Very often	88	Very often	87
Usually	85	Usually	79
Often	78	Often	74
Generally	78	Rather often	74
Frequently	73	Frequently	72
Rather often	65	Generally	72
About as often as not	50	About as often as not	50
Now and then	20	Now and then	34
Sometimes	20	Sometimes	29
Occasionally	20	Occasionally	28
Once in a while	15	Once in a while	22
Not often	13	Not often	16
Usually not	10	Usually not	16
Seldom	10	Seldom	9
Hardly ever	7	Hardly ever	8
Very seldom	6	Very seldom	7
Rarely	5	Rarely	5
Almost never	3	Almost never	2
Never	0	Never	0

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Types of Uncertainty in Expert Systems

- **Aleatory Uncertainty**: Related to randomness and inherent variability.
- **Epistemic Uncertainty**: Stemming from incomplete or inaccurate knowledge.

Example

Aleatory: Coin toss outcomes.
Epistemic: Predicting disease progression without sufficient data.

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Challenges in Managing Uncertainty

- Complex systems often involve multiple, interacting uncertainties.
- Uncertainty increases with incomplete data and limited computational models.

Key Issue

Systems must balance computational efficiency and accuracy when dealing with uncertainty.

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Probability: A Refresher

- The probability of an event is the ratio of success outcomes to total possible outcomes.
- Events can be independent or dependent.

$$P(\text{success}) = \frac{\text{Number of successes}}{\text{Number of possible outcomes}}$$

$$P(\text{failure}) = \frac{\text{Number of failures}}{\text{Number of possible outcomes}}$$

Example

Rolling a dice: Probability of getting 6:

$$P(\text{Getting} = 6) = \frac{\text{success Outcomes (1)}}{\text{Total Outcomes (1+5)}} = \frac{1}{6} = 0.1666$$

$$P(\text{Not Getting} = 6) = \frac{\text{failure Outcomes (5)}}{\text{Total Outcomes (1+5)}} = \frac{5}{6} = 0.8333$$

Cup of Coffee

Example

Rolling two dice: Probability of getting a sum of 7:

$$P(\text{Sum} = 7) = \frac{\text{success Outcomes (6)}}{\text{Total Outcomes (36)}} = \frac{6}{36} = 0.167$$

Key Concepts in Probability

- **Sample Space**: The set of all possible outcomes.
- **Event**: A subset of the sample space.
- **Complementary Events**: $P(A') = 1 - P(A)$.

Example

For a die roll:

- Sample space: $\{1, 2, 3, 4, 5, 6\}$
- Event: Rolling a 3 ($P(3) = \frac{1}{6}$).

Conditional Probability: Expanded Example

- Scenario: You have 3 red balls and 2 blue balls in a bag.
- If one red ball is drawn, what is the probability the next ball is also red?

Solution

$$P(\text{Second Red—First Red}) = \frac{\text{Remaining Red Balls}}{\text{Remaining Total Balls}} = \frac{2}{4} = 0.5$$

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Conditional Probability

Let A and B be two events. The conditional probability of A occurring given B has occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Interpretation:** The likelihood of A occurring when B is known to have occurred.
- $P(A|B)$: Probability of A given B.
- $P(A \cap B)$: Joint probability of A and B.
- $P(B)$: Probability of B.

Joint Probability and Commutativity

- **Joint Probability:** Probability that both A and B occur.

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

- **Commutativity:** The joint probability is commutative:

$$P(A \cap B) = P(B \cap A)$$

- This relationship helps in understanding and calculating probabilities when events are dependent.

Law of Total Probability: Example

- Suppose there are two machines producing light bulbs:
  - Machine A produces 60% of bulbs; defect rate = 5%.
  - Machine B produces 40% of bulbs; defect rate = 2%.
- What is the probability that a randomly chosen bulb is defective?

Solution

$$P(\text{Defective}) = P(D|A)P(A) + P(D|B)P(B)$$

$$= (0.05)(0.6) + (0.02)(0.4) = 0.03\,(3\%)$$

Bayesian Rule

Conditional probability

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where:

- $P(A|B)$ is the conditional probability that event A occurs given that event B has occurred;
- $P(B|A)$ is the conditional probability of event B occurring given that event A has occurred;
- $P(A)$ is the probability of event A occurring;
- $P(B)$ is the probability of event B occurring.

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Multiple Events

Conditional Probability: The concept of conditional probability extends to events where A depends on multiple events $B_1, B_2, \ldots, B_n$:

$$p(A \cap B_1) = p(A|B_1) \cdot p(B_1)$$
$$p(A \cap B_2) = p(A|B_2) \cdot p(B_2)$$
$$\vdots$$
$$p(A \cap B_n) = p(A|B_n) \cdot p(B_n)$$

Combined Form:

$$\sum_{i=1}^{n} p(A \cap B_i) = \sum_{i=1}^{n} p(A|B_i) \cdot p(B_i)$$

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Total Probability and Bayesian Equation

If the events B_i form an exhaustive set:

$$\sum_{i=1}^{n} p(A \cap B_i) = p(A)$$

Bayesian Rule (Final Form):

$$p(A) = \sum_{i=1}^{n} p(A|B_i) \cdot p(B_i)$$

Interpretation: The probability of A is computed by considering all possible ways A can occur through the events $B_1, B_2, \ldots, B_n$, weighted by their respective probabilities.

Frame Title

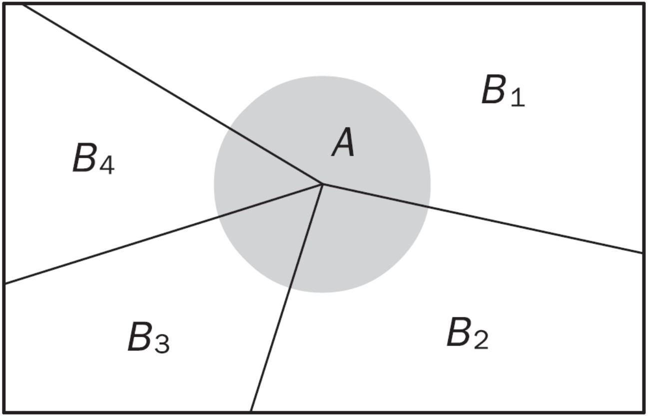

Figure: The joint probability

With NOT B

If the occurrence of event A depends on only two mutually exclusive events, that is B and NOT B, then Eq. (3.14) becomes

$$p(A) = p(A|B) \cdot p(B) + p(A|\neg B) \cdot p(\neg B) \tag{1}$$

where $\neg$ is the logical function NOT.
Similarly,

$$p(B) = p(B|A) \cdot p(A) + p(B|\neg A) \cdot p(\neg A) \tag{2}$$

Let us now substitute Eq. (3.16) into the Bayesian rule (3.11) to yield

$$p(A|B) = \frac{p(B|A) \cdot p(A)}{p(B|A) \cdot p(A) + p(B|\neg A) \cdot p(\neg A)} \tag{3}$$

Outlines

Bayesian Reasoning

- Bayesian reasoning uses prior probabilities and evidence to update beliefs.
- Key strength: Ability to incorporate new evidence incrementally.

IF E is true
THEN H is true **with probability p**

Cont.,

What if event E has occurred but we do not know whether event H has occurred? Can we compute the probability that event H has occurred as well?

$$p(H|E) = \frac{p(E|H) \cdot p(H)}{p(E|H) \cdot p(H) + p(E|\neg H) \cdot p(\neg H)} \quad (4)$$

where:

- $p(H)$ is the prior probability of hypothesis H being true;
- $p(E|H)$ is the probability that hypothesis H being true will result in evidence E;
- $p(\neg H)$ is the prior probability of hypothesis H being false;
- $p(E|\neg H)$ is the probability of finding evidence E even when hypothesis H is false.

Bayesian Reasoning Example

- Hypothesis: The patient has a disease.
- Evidence: Positive test result.
- Known Data:
  - Sensitivity: $P(\text{Positive—Disease}) = 0.9$
  - Prevalence: $P(\text{Disease}) = 0.01$
  - False positive rate: $P(\text{Positive—No Disease}) = 0.05$
- Posterior Probability:

$$P(\text{Disease—Positive}) = \frac{P(\text{Positive—Disease}) \cdot P(\text{Disease})}{P(\text{Positive})}$$

Posterior Probabilities in Expert Systems

How does an expert system compute all posterior probabilities and rank hypotheses?

Let us consider a simple example. An expert, given three conditionally independent evidences E_1, E_2, and E_3, creates three mutually exclusive and exhaustive hypotheses H_1, H_2, and H_3, and provides prior probabilities for these hypotheses $(p(H_1), p(H_2), p(H_3))$.

The expert also determines the conditional probabilities of observing each evidence for all possible hypotheses.

Posterior Probability Formula

The posterior probabilities for all hypotheses are computed using:

$$p(H_i|E_3) = \frac{p(E_3|H_i) \cdot p(H_i)}{\sum_{k=1}^{3} p(E_3|H_k) \cdot p(H_k)}, \quad i = 1, 2, 3 \tag{5}$$

Example Calculation:

$$p(H_1|E_3) = \frac{0.6 \cdot 0.40}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.34$$

$$p(H_2|E_3) = \frac{0.7 \cdot 0.35}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.34$$

$$p(H_3|E_3) = \frac{0.9 \cdot 0.25}{0.6 \cdot 0.40 + 0.7 \cdot 0.35 + 0.9 \cdot 0.25} = 0.32$$

Table 3.2: The Prior and Conditional Probabilities

Probability	$i = 1$	$i = 2$	$i = 3$
$p(H_i)$	0.40	0.35	0.25
$p(E_1\|H_i)$	0.3	0.8	0.5
$p(E_2\|H_i)$	0.9	0.0	0.7
$p(E_3\|H_i)$	0.6	0.7	0.9

The posterior probability for hypothesis H_i given evidence E_1 and E_3 is calculated as:

$$p(H_i|E_1E_3) = \frac{p(E_1|H_i) \cdot p(E_3|H_i) \cdot p(H_i)}{\sum_{k=1}^{3} p(E_1|H_k) \cdot p(E_3|H_k) \cdot p(H_k)}, \quad i = 1, 2, 3$$

Example Calculation:

$$p(H_1|E_1E_3) = \frac{0.3 \cdot 0.6 \cdot 0.40}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.19$$

$$p(H_2|E_1E_3) = \frac{0.8 \cdot 0.7 \cdot 0.35}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.52$$

$$p(H_3|E_1E_3) = \frac{0.5 \cdot 0.9 \cdot 0.25}{0.3 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.9 \cdot 0.25} = 0.29$$

Observations:

- H_2 (Tomorrow is dry) is now the most likely hypothesis with $p(H_2|E_1E_3) = 0.52$.
- Belief in H_1 (Tomorrow is rain) has decreased to $p(H_1|E_1E_3) = 0.19$.
- H_3 (Alternate hypothesis) has $p(H_3|E_1E_3) = 0.29$.

Posterior Probability with E_1, E_2, and E_3

Including additional evidence E_2, the posterior probability becomes:

$$p(H_i|E_1E_2E_3) = \frac{p(E_1|H_i) \cdot p(E_2|H_i) \cdot p(E_3|H_i) \cdot p(H_i)}{\sum_{k=1}^{3} p(E_1|H_k) \cdot p(E_2|H_k) \cdot p(E_3|H_k) \cdot p(H_k)}, \quad i = 1, 2, 3$$

Example Calculations:

- $p(H_1|E_1E_2E_3)$
- $p(H_2|E_1E_2E_3)$
- $p(H_3|E_1E_2E_3)$

The inclusion of E_2 allows for further refinement of the probabilities.

Final Posterior Probabilities for H_1, H_2, and H_3

The final posterior probabilities for hypotheses H_1, H_2, and H_3 are calculated as follows:

$$p(H_1|E_1E_2E_3) = \frac{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25}$$
$$= 0.45$$

$$p(H_2|E_1E_2E_3) = \frac{0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25}$$
$$= 0$$

$$p(H_3|E_1E_2E_3) = \frac{0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25}{0.3 \cdot 0.9 \cdot 0.6 \cdot 0.40 + 0.8 \cdot 0.0 \cdot 0.7 \cdot 0.35 + 0.5 \cdot 0.7 \cdot 0.9 \cdot 0.25}$$
$$= 0.55$$

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Final Posterior Probabilities: Conclusion

- $p(H_1|E_1E_2E_3) = 0.45$: Hypothesis H_1 retains significant belief.
- $p(H_2|E_1E_2E_3) = 0$: Hypothesis H_2 is no longer considered viable.
- $p(H_3|E_1E_2E_3) = 0.55$: Hypothesis H_3 becomes the most likely.

Outlines

Forecasting Using Bayesian Reasoning

- Bayesian reasoning allows for the integration of multiple pieces of evidence over time.
- Each new observation updates the posterior probability, which becomes the new prior.

Recursive Bayesian Update

$$P(H|E_1, E_2, ..., E_n) = \frac{P(E_n|H) \cdot P(H|E_1, E_2, ..., E_{n-1})}{P(E_n)}$$

Rain Prediction Example

Evidence:

- High humidity.
- Overcast skies.
- Low temperature.

Posterior probabilities refine with each observation.

Weather Prediction Using Expert Systems

Objective: Develop an expert system to predict if it will rain tomorrow.
System Requirements:

- Input: Real weather data (e.g., rainfall, temperature, sunshine).
- Output: Two possible outcomes:
  1. *Tomorrow is rain*.
  2. *Tomorrow is dry*.

Example: Table 3.3 summarizes London weather for March 1982. Rainfall is zero for dry days.

Frame Title

Table 3.3 London weather summary for March 1982

Day of month	Min. temp. °C	Max. temp. °C	Rainfall mm	Sunshine hours	Actual weather	Weather forecast
1	9.4	11.0	17.5	3.2	Rain	–
2	4.2	12.5	4.1	6.2	Rain	Rain
3	7.6	11.2	7.7	1.1	Rain	Rain
4	5.7	10.5	0.0	4.3	Dry	Rain*
5	3.0	12.0	0.0	9.5	Dry	Dry
6	4.4	9.6	0.0	3.5	Dry	Dry
7	4.8	9.4	4.6	10.1	Rain	Rain
8	1.8	9.2	5.5	7.8	Rain	Rain
9	2.4	10.2	4.8	4.1	Rain	Rain
10	5.5	12.7	4.2	3.8	Rain	Rain
11	3.7	10.9	4.4	9.2	Rain	Rain
12	5.9	10.0	4.8	7.1	Rain	Rain
13	3.0	11.9	0.0	8.3	Dry	Rain*
14	5.4	12.1	4.8	1.8	Rain	Dry*
15	8.8	9.1	8.8	0.0	Rain	Rain
16	2.4	8.4	3.0	3.1	Rain	Rain
17	4.3	10.8	0.0	4.3	Dry	Dry
18	3.4	11.1	4.2	6.6	Rain	Rain
19	4.4	8.4	5.4	0.7	Rain	Rain
20	5.1	7.9	3.0	0.1	Rain	Rain
21	4.4	7.3	0.0	0.0	Dry	Dry
22	5.6	14.0	0.0	6.8	Dry	Dry
23	5.7	14.0	0.0	8.8	Dry	Dry
24	2.9	13.9	0.0	9.5	Dry	Dry
25	5.8	16.4	0.0	10.3	Dry	Dry
26	3.9	17.0	0.0	9.9	Dry	Dry

Basic Rules for Prediction

Rules:

- **Rule 1:**

  IF *today is rain*, THEN *tomorrow is rain*.

- **Rule 2:**

  IF *today is dry*, THEN *tomorrow is dry*.

Outcome: Using these basic rules, the expert system would make 10 mistakes (e.g., when a wet day precedes a dry day or vice versa).

Improved Rules with Probabilities

To minimize mistakes, prior probabilities are assigned to the hypotheses:

- **Hypothesis 1:** Tomorrow is rain ($p = 0.5$).
- **Hypothesis 2:** Tomorrow is dry ($p = 0.5$).

Updated Rules:

- **Rule 1:**

  IF *today is rain* {LS = 2.5, LN = 0.6}, THEN *tomorrow is rain* {prior = 0.5}.

- **Rule 2:**

  IF *today is dry* {LS = 1.6, LN = 0.4}, THEN *tomorrow is dry* {prior = 0.5}.

Likelihood Ratios:Likelihood of Sufficiency

Likelihood of Sufficiency (LS):

$$LS = \frac{p(E|H)}{p(E|\neg H)}$$

Where:

- $p(E|H)$ is the probability of evidence E given hypothesis H.
- $p(E|\neg H)$ is the probability of evidence E given the negation of hypothesis H.

$$LS = \frac{p(\text{today is rain} \mid \text{tomorrow is rain})}{p(\text{today is rain} \mid \text{tomorrow is dry})}$$

Likelihood Ratios:Likelihood of Necessity

$$LN = \frac{p(\neg E|H)}{p(\neg E|\neg H)}$$

Where:

- $p(\neg E|H)$ is the probability of the absence of evidence $(\neg E)$ given hypothesis H.
- $p(\neg E|\neg H)$ is the probability of the absence of evidence $(\neg E)$ given the negation of hypothesis H.

$$LN = \frac{p(\text{today is dry} \mid \text{tomorrow is rain})}{p(\text{today is dry} \mid \text{tomorrow is dry})}$$

These values quantify the relationship between today's weather and tomorrow's prediction.

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Conclusion

Note that LN cannot be derived from LS. The domain expert must provide both values independently.

- Basic rules (IF-THEN) provide a foundation for prediction.
- Adding prior probabilities and likelihood ratios improves accuracy.
- Bayesian reasoning dynamically updates predictions based on new evidence.

Next Steps:

- Extend the rule set to include additional weather features (e.g., temperature, cloud cover).
- Test predictions with more comprehensive datasets.

Determining Likelihood Ratios: LS and LN

How does the domain expert determine values of LS and LN?
To provide values for LS and LN, an expert does not need to determine exact values of conditional probabilities. Instead, the expert directly decides likelihood ratios:

- High values of LS ($LS \gg 1$) indicate strong support for the hypothesis when evidence is observed.
- Low values of LN ($0 < LN < 1$) suggest strong opposition to the hypothesis when evidence is missing.

Since conditional probabilities can be computed from likelihood ratios, this approach allows the Bayesian rule to propagate evidence.

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Example: London Weather

Rule 1: If it is raining today, there is a high probability of rain tomorrow ($LS = 2.5$). If it is dry today, there is still a chance of rain tomorrow ($LN = 0.6$).
Rule 2: If it is dry today, the probability of a dry day tomorrow is also high ($LS = 1.6$). However, the probability of rain tomorrow (if it is raining today) is higher than the probability of a dry day tomorrow (if it is dry today). This is why LS and LN are typically determined by a domain expert or confirmed using statistical information (e.g., weather bureau data).

Converting $p(H)$ into Prior Odds

The rule-based expert system converts the prior probability $p(H)$ into prior odds:

$$O(H) = \frac{p(H)}{1 - p(H)} \tag{3.24}$$

Where:

- $p(H)$ is the prior probability of the hypothesis H.
- $O(H)$ is the odds of the hypothesis being true.

This representation helps simplify the combination of multiple pieces of evidence in Bayesian reasoning.

Prior Odds and Posterior Probabilities

The prior probability is used to adjust the uncertainty of the consequent for the first time. The prior odds are updated as follows:

$$O(H|E) = LS \times O(H)$$

$$O(H|\neg E) = LN \times O(H)$$

The posterior odds are then converted into posterior probabilities:

$$p(H|E) = \frac{O(H|E)}{1 + O(H|E)}$$

$$p(H|\neg E) = \frac{O(H|\neg E)}{1 + O(H|\neg E)}$$

London Weather Example

Suppose: *Today is rain.*

Step 1: Convert Prior Probability into Odds

$$O(\text{tomorrow is rain}) = \frac{0.5}{1 - 0.5} = 1.0$$

Step 2: Update Odds with Evidence

$$O(\text{tomorrow is rain — today is rain}) = 2.5 \times 1.0 = 2.5$$

Step 3: Convert Back to Probability

$$p(\text{tomorrow is rain — today is rain}) = \frac{2.5}{1 + 2.5} = 0.71$$

Applying Rule 2: Dry Day Probability

Suppose: *Today is rain.*

Step 1: Convert Prior Probability into Odds

$$O(\text{tomorrow is dry}) = \frac{0.5}{1 - 0.5} = 1.0$$

Step 2: Update Odds with Evidence

$$O(\text{tomorrow is dry — today is rain}) = 0.4 \times 1.0 = 0.4$$

Step 3: Convert Back to Probability

$$p(\text{tomorrow is dry — today is rain}) = \frac{0.4}{1 + 0.4} = 0.29$$

Control Rules:

- **Rule 1:**
  - If *today is rain* $\{LS = 2.5, LN = 0.6\}$,
  - Then *tomorrow is rain* $\{prior = 0.5\}$.
- **Rule 2:**
  - If *today is dry* $\{LS = 1.6, LN = 0.4\}$,
  - Then *tomorrow is dry* $\{prior = 0.5\}$.
- **Rule 3:**
  - If *today is rain* and *rainfall is low* $\{LS = 10, LN = 1\}$,
  - Then *tomorrow is dry* $\{prior = 0.5\}$.
- **Rule 4:**
  - If *today is rain*, *rainfall is low*, and *temperature is cold* $\{LS = 1.5, LN = 1\}$,
  - Then *tomorrow is dry* $\{prior = 0.5\}$.

Rules

- **Rule 5:**
  - If *today is dry* and *temperature is warm* $\{LS = 2.1, LN = 0.9\}$,
  - Then *tomorrow is rain* $\{\text{prior} = 0.5\}$.
- **Rule 6:**
  - If *today is dry*, *temperature is warm*, and *sky is overcast* $\{LS = 1.5, LN = 1\}$,
  - Then *tomorrow is rain* $\{\text{prior} = 0.5\}$.

Note: The `seek tomorrow` directive sets up the goal of the rule set.

Dialogue: Expert System for Weather Prediction

What is the weather today? $\Rightarrow$ *rain*

Rule 1: If *today is rain* $\{LS = 2.5,\ LN = 0.6\}$, then *tomorrow is rain* $\{prior = 0.5\}$.

Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is rain}) = \frac{0.5}{1 - 0.5} = 1.0$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is rain — today is rain}) = 2.5 \times 1.0 = 2.5$$

Step 3: Convert Odds to Probability:

$$p(\text{tomorrow is rain — today is rain}) = \frac{2.5}{1 + 2.5} = 0.71$$

Prediction:

Tomorrow is rain: [0.71]

Applying Rule 2: Dry Day Probability

Rule 2: If *today is dry* $\{LS = 1.6,\ LN = 0.4\}$, then *tomorrow is dry* $\{prior = 0.5\}$.

Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is dry}) = \frac{0.5}{1 - 0.5} = 1.0$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is dry — today is rain}) = 0.4 \times 1.0 = 0.4$$

Step 3: Convert Odds to Probability:

$$p(\text{tomorrow is dry — today is rain}) = \frac{0.4}{1 + 0.4} = 0.29$$

Prediction:

Tomorrow is dry: [0.29]

Adding Evidence: Low Rainfall

What is the rainfall today? $\Rightarrow$ *low*
Rule 3: If *today is rain and rainfall is low* $\{LS = 10,\ LN = 1\}$, then *tomorrow is dry* $\{prior = 0.5\}$.
Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is dry}) = \frac{0.29}{1 - 0.29} = 0.41$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is dry — today is rain and rainfall is low}) = 10 \times 0.41 = 4.1$$

Step 3: Convert Odds to Probability:

$$p(\text{tomorrow is dry — today is rain and rainfall is low}) = \frac{4.1}{1 + 4.1} = 0.80$$

Prediction:

Tomorrow is dry:[0.80]

Adding Evidence: Cold Temperature

What is the temperature today? $\Rightarrow$ *cold*

Rule 4: If *today is rain, rainfall is low, and temperature is cold* $\{LS = 1.5,\ LN = 1\}$, then *tomorrow is dry* $\{prior = 0.5\}$.

Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is dry}) = \frac{0.80}{1 - 0.80} = 4$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is dry — today is rain, rainfall is low, temperature is cold}) =$$

$$1.50 \times 4 = 6$$

Step 3: Convert Odds to Probability:

$$p(\text{tomorrow is dry — today is rain, rainfall is low, temperature is cold})$$

$$= \frac{6}{1 + 6} = 0.86$$

Prediction:

Tomorrow is dry: [0.86]

Rule 5: If *Today is dry, and Temperature is warm* {*LS 2 LN 0.9*}, Then *Tomorrow is rain* {*prior 0.5*}

Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is rain}) = \frac{0.71}{1 - 0.71} = 2.45$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is rain — today is not dry, temperature is not warm})$$

$$= 0.9 \times 2.45 = 2.21$$

Step 3: Convert Odds to Probability:

$$P(\text{tomorrow is rain — today is not dry, temperature is not warm})$$

$$= \frac{2.21}{1 + 2.21} = 0.69$$

Prediction:

$$\text{Tomorrow is dry: } [0.69]$$

Adding Evidence: Overcast Conditions

What is the cloud cover today? $\Rightarrow$ *overcast*
Rule 6: If *today is dry, temperature is warm, and sky is overcast* $\{LS = 5, LN = 1\}$, then *tomorrow is rain* $\{prior = 0.5\}$.
Step 1: Convert Prior Probability to Odds:

$$O(\text{tomorrow is rain}) = \frac{0.69}{1 - 0.69} = 2.23$$

Step 2: Update Odds with Evidence:

$$O(\text{tomorrow is rain — today is not dry, temperature is not warm}$$

$$\text{, sky is overcast}) = 1 \times 2.23 = 2.23$$

Step 3: Convert Odds to Probability:

$$p(\text{tomorrow is rain — today is not dry, temperature is not warm})$$

$$\text{, sky is overcast} = \frac{2.23}{1 + 2.23} = 0.69$$

Prediction:

$$\text{Tomorrow is rain: } [0.69]$$

Conclusion

Summary:

- After applying all rules and evidence, two hypotheses remain:
  - **Tomorrow is dry:** [0.86]
  - **Tomorrow is rain:** [0.69]
- The most likely hypothesis is that **tomorrow is dry**.

Success Rate: From Table 3.3, this expert system made only 4 mistakes, achieving an **86% success rate**.

Outlines

Bias of the Bayesian Method

Challenges in Bayesian Reasoning:

- Bayesian reasoning requires probability values as primary inputs.
- These values often involve human judgment, which may be inconsistent.
- Psychological research shows that humans struggle to elicit probability values in line with Bayesian rules.

Cont.,

Example: Diagnosing a Faulty Starter

- Rule 1:If the symptom is 'odd noises', THEN the starter is bad.

$$probability = 0.7$$

- Rule 2: IF the symptom is 'odd noises', THEN the starter is good

$$probability = 0.3.$$

Observation: Experts often deny the existence of implicit probabilities like 0.3.

Inconsistency in Probabilities

Empirical Rules:

- Rule 1: IF the starter is bad, THEN the symptom is 'odd noises'

$$\textit{probability} = 0.85.$$

- Rule 2: IF the starter is bad, THEN the symptom is not 'odd noises'

$$\textit{probability} = 0.15.$$

Calculating Posterior Probability:

$$p(\text{starter is bad — odd noises}) = \frac{0.85 \cdot 0.05}{0.85 \cdot 0.05 + 0.15 \cdot 0.95} = 0.23$$

Result: The posterior probability is significantly lower than the expert's estimate of 0.7.

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Backward Calculation:

$$p(H) = \frac{p(H|E) \cdot p(E|\neg H)}{p(H|E) \cdot p(E|\neg H) + p(E|H) \cdot [1 - p(H|E)]}$$

Given Values:

- $p(H|E) = 0.7$ (starter is bad — odd noises).
- $p(E|H) = 0.85$ (odd noises — starter is bad).
- $p(E|\neg H) = 0.15$ (odd noises — starter is good).

Result:

$$p(H) = \frac{0.7 \cdot 0.15}{0.7 \cdot 0.15 + 0.85 \cdot (1 - 0.7)} = 0.29$$

This value is inconsistent with the expert's prior probability of 5%.

Conclusion on Bayesian Bias

Key Issues:

- Experts often make inconsistent assumptions when estimating prior and conditional probabilities.
- Inconsistencies can lead to significant deviations in posterior probabilities.

Alternative Methods:

- Use statistical information and empirical studies to improve consistency.
- Apply piecewise linear interpolation models, as used in PROSPECTOR.

Limitation:

- Assumes conditional independence of evidence, which is rarely satisfied in real-world problems.

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THANKS