Expert Systems

Lecture 5: Uncertainty management in rule-based expert systems

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Outlines

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Introduction to Certainty factors

- Expert systems require handling uncertain or incomplete data.
- Two key approaches:
  1. Certainty Factors (CF)
  2. Bayesian Reasoning
- These methods enable systems to make probabilistic or heuristic decisions.

Syntax for Rules with Certainty Factors:

- In expert systems using certainty factors, rules are represented as:

$$\text{IF } \langle\text{evidence}\rangle \text{ THEN } \langle\text{hypothesis}\rangle\{\text{CF}\}$$

- Where:
  - $\langle$evidence$\rangle$: Observed data or facts.
  - $\langle$hypothesis$\rangle$: The conclusion or belief.
  - CF: Certainty factor indicating belief strength in the hypothesis.

Key Components:

- *Measure of Belief (MB)*: Degree to which evidence supports the hypothesis.
- *Measure of Disbelief (MD)*: Degree to which evidence contradicts the hypothesis.

Frame Title

Term	Certainty factor
Definitely not	−1.0
Almost certainly not	−0.8
Probably not	−0.6
Maybe not	−0.4
Unknown	−0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

Figure: Uncertain terms and their interpretation

Measure of Belief and Disbelief in Certainty Factors

Definition: The measure of belief (MB) and disbelief (MD) represents the degree to which evidence E supports or contradicts a hypothesis H.

Formulas:

$$MB(H,E) = \begin{cases} 1, & \text{if } p(H) = 1, \\ \frac{\max\left(p(H|E), p(H)\right) - p(H)}{\max(1,0) - p(H)}, & \text{otherwise.} \end{cases}$$

$$MD(H,E) = \begin{cases} 1, & \text{if } p(H) = 0, \\ \frac{\min\left(p(H|E), p(H)\right) - p(H)}{\min(1,0) - p(H)}, & \text{otherwise.} \end{cases}$$

Where:

- $p(H)$: Prior probability of hypothesis H being true.
- $p(H|E)$: Conditional probability of H given evidence E.

Range:

- $MB(H,E)$ and $MD(H,E)$ range from 0 to 1.
- These values reflect the strength of evidence in supporting or contradicting the hypothesis.

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Total strength of belief or disbelief Determination

- CF measures belief (MB) and disbelief (MD) about a hypothesis H.
- Range: -1.0 (definitely false) to $+1.0$ (definitely true).
- Formula:

$$CF(H, E) = \frac{MB(H, E) - MD(H, E)}{1 - \min(MB(H, E), MD(H, E))}$$

Associating Certainty Factors with Possible Values

Certainty Factors for Multiple Outcomes:

- Experts can associate different certainty factors (CF) for each possible value of a variable.
- Rules take the form:

$$\text{IF } A \text{ is } X \begin{cases} \text{THEN } B \text{ is } Y \, \{CF 0.7\}, \\ \text{THEN } B \text{ is } Z \, \{CF 0.2\}. \end{cases}$$

Example

Rule:

IF Patient has a fever
THEN Diagnosis is Flu {CF 0.8},
THEN Diagnosis is Common Cold {CF 0.3}.

What Does it Mean?

- When A receives the value X, B can take multiple possible outcomes:
  - 70% chance of $B = Y$
  - 20% chance of $B = Z$
  - Remaining 10% may represent unknown outcomes.

Key Concept: Propagation of Certainty

The net certainty of a rule's conclusion is calculated as:

$$cf(H, E) = cf(E) \times cf(\text{rule})$$

This propagates the certainty from evidence to conclusion through the rule.

Example

Example: Weather Forecasting

Rule:

IF the sky is clear THEN the forecast is sunny {cf 0.8}.

Evidence:

$$cf(\text{sky is clear}) = 0.5$$

Calculation:

$$cf(H, E) = 0.5 \times 0.8 = 0.4$$

Result: This means the certainty of the forecast being sunny is 0.4, interpreted as "It may be sunny."

Conjunctive Rule Syntax

Structure of Conjunctive Rules in Certainty Factors:

- Conjunctive rules are used when multiple pieces of evidence must all be true for a hypothesis to be supported.
- The general syntax is as follows:

$$\text{IF } \langle \text{evidence } E_1 \rangle \text{ AND } \langle \text{evidence } E_2 \rangle \ \ldots \text{ AND } \langle \text{evidence } E_n \rangle$$

$$\text{THEN } \langle \text{hypothesis } H \rangle \ \{cf\}.$$

Key Concept

In conjunctive rules, all conditions (evidence) must be satisfied simultaneously, and the certainty factor (cf) reflects the combined confidence in the hypothesis.

Frame Title

Example

Rule:

IF temperature is cold AND it is raining THEN tomorrow is wet {cf 0.85}.

Certainty Factors for Conjunctive Rules

Definition:

- For rules with multiple conditions joined by **AND**, the certainty factor is calculated as:

$$cf(H, E_1 \cap E_2 \cap \cdots \cap E_n) = \min\big(cf(E_1), cf(E_2), \ldots\big) \times cf(\text{rule})$$

Cont.,

Example: Conjunctive Rule

Rule: IF sky is clear AND the forecast is sunny THEN the action is "wear sunglasses" cf 0.8.

Certainty Values:

$$cf(\text{sky is clear}) = 0.9, \quad cf(\text{forecast is sunny}) = 0.7$$

Calculation:

$$cf(H, E_1 \cap E_2) = \min(0.9, 0.7) \times 0.8 = 0.7 \times 0.8 = 0.56$$

Result: The action "wear sunglasses" has a certainty factor of 0.56, meaning it is "probably a good idea."

Disjunctive Rule Syntax

Structure of Disjunctive Rules in Certainty Factors:

- Disjunctive rules are used when any one of multiple pieces of evidence can support a hypothesis.
- The general syntax is as follows:

$$\text{IF } \langle \text{evidence } E_1 \rangle \text{ OR } \langle \text{evidence } E_2 \rangle \ldots \text{ OR } \langle \text{evidence } E_n \rangle$$
$$\text{THEN } \langle \text{hypothesis } H \rangle \, \{cf\}.$$

Formula for Certainty:

$$cf(H, E_1 \cup E_2 \cup \cdots \cup E_n) = \max\left(cf(E_1), cf(E_2), \ldots\right) \times cf(\text{rule})$$

Key Concept

In disjunctive rules, the certainty factor (cf) is based on the strongest piece of evidence supporting the hypothesis.

Example: Disjunctive Rule

Rule:

Example: Disjunctive Rule Calculation

Rule:

IF sky is overcast OR the forecast is rain THEN the action is "take an umbre

Certainty Values:

- $cf(\text{sky is overcast}) = 0.6$
- $cf(\text{forecast is rain}) = 0.8$

Calculation:

$$cf(H, E_1 \cup E_2) = \max(0.6, 0.8) \times 0.9$$

$$cf(H, E_1 \cup E_2) = 0.8 \times 0.9 = 0.72$$

Result

The certainty factor for the action "take an umbrella" is 0.72. This can be interpreted as:

"Almost certainly, an umbrella should be taken today."

Key Insight

For disjunctive rules, the strongest evidence ($\max(cf)$) determines the overall certainty, multiplied by the rule's cf.

Handling Multiple Rules in Expert Systems

Problem:
Sometimes two or more rules affect the same hypothesis. How does an expert system handle such situations?

When the same conclusion is obtained as a result of the execution of two or more rules, the certainty factors (CFs) of these rules must be combined to calculate a single certainty factor for the hypothesis.

Rules in the Knowledge Base:

- **Rule 1:** IF A is X, THEN C is Z with $cf = 0.8$.
- **Rule 2:** IF B is Y, THEN C is Z with $cf = 0.6$.

Challenge: What certainty factor should be assigned to C having value Z if both Rule 1 and Rule 2 are fired?

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Formula for Combining Certainty Factors

Combining Certainty Factors:
To calculate a combined certainty factor, we use the following formula (Durkin, 1994):

$$cf(cf_1, cf_2) = \begin{cases} cf_1 + cf_2 \times (1 - cf_1), & \text{if } cf_1 > 0 \text{ and } cf_2 > 0 \\ \frac{cf_1 + cf_2}{1 - \min(|cf_1|, |cf_2|)}, & \text{if } cf_1 < 0 \text{ or } cf_2 < 0 \\ cf_1 + cf_2 \times (1 + cf_1), & \text{if } cf_1 < 0 \text{ and } cf_2 < 0 \end{cases}$$

Where:

- cf_1: Certainty factor for hypothesis H established by Rule 1.
- cf_2: Certainty factor for hypothesis H established by Rule 2.
- $|cf_1|$ and $|cf_2|$: Absolute values of the certainty factors.

Explanation of the Formula

Case 1: Both CFs are positive ($cf_1 > 0,\ cf_2 > 0$)

$$cf = cf_1 + cf_2 \times (1 - cf_1)$$

Case 2: One CF is negative ($cf_1 < 0$ or $cf_2 < 0$)

$$cf = \frac{cf_1 + cf_2}{1 - \min(|cf_1|, |cf_2|)}$$

Case 3: Both CFs are negative ($cf_1 < 0,\ cf_2 < 0$)

$$cf = cf_1 + cf_2 \times (1 + cf_1)$$

Intuition: The formula ensures that evidence supporting the hypothesis strengthens the certainty, while conflicting evidence adjusts it accordingly.

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Example: Both Certainty Factors are Positive

Given:

$$cf(E_1) = cf(E_2) = 1.0$$

Step 1: Calculate Individual Certainty Factors

$$cf_1(H, E_1) = cf(E_1) \times cf_1 = 1.0 \times 0.8 = 0.8$$

$$cf_2(H, E_2) = cf(E_2) \times cf_2 = 1.0 \times 0.6 = 0.6$$

Step 2: Combine Certainty Factors (Using Equation 3.35):

$$cf(cf_1, cf_2) = cf_1(H, E_1) + cf_2(H, E_2) \times [1 - cf_1(H, E_1)]$$

$$cf(cf_1, cf_2) = 0.8 + 0.6 \times (1 - 0.8) = 0.8 + 0.12 = 0.92$$

Result: Combined certainty factor is $cf = 0.92$.

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Example: Opposite Signs of Certainty Factors

Given:

$$cf(E_1) = 1.0, \quad cf(E_2) = -1.0$$

Step 1: Calculate Individual Certainty Factors

$$cf_1(H, E_1) = cf(E_1) \times cf_1 = 1.0 \times 0.8 = 0.8$$

$$cf_2(H, E_2) = cf(E_2) \times cf_2 = -1.0 \times 0.6 = -0.6$$

Step 2: Combine Certainty Factors (Using Equation 3.35):

$$cf(cf_1, cf_2) = \frac{cf_1(H, E_1) + cf_2(H, E_2)}{1 - \min(|cf_1(H, E_1)|, |cf_2(H, E_2)|)}$$

$$cf(cf_1, cf_2) = \frac{0.8 - 0.6}{1 - \min(0.8, 0.6)} = \frac{0.2}{1 - 0.6} = \frac{0.2}{0.4} = 0.5$$

Result: Combined certainty factor is $cf = 0.5$.

Example: Both Certainty Factors are Negative

Given:

$$cf(E_1) = cf(E_2) = -1.0$$

Step 1: Calculate Individual Certainty Factors

$$cf_1(H, E_1) = cf(E_1) \times cf_1 = -1.0 \times 0.8 = -0.8$$

$$cf_2(H, E_2) = cf(E_2) \times cf_2 = -1.0 \times 0.6 = -0.6$$

Step 2: Combine Certainty Factors (Using Equation 3.35):

$$cf(cf_1, cf_2) = cf_1(H, E_1) + cf_2(H, E_2) \times [1 + cf_1(H, E_1)]$$

$$cf(cf_1, cf_2) = -0.8 + (-0.6) \times (1 + (-0.8)) = -0.8 - 0.6 \times 0.2 = -0.8 - 0.12 = -$$

Result: Combined certainty factor is $cf = -0.92$.

Certainty Factors Theory vs Bayesian Reasoning

Certainty Factors Theory

The certainty factors theory provides a **practical alternative** to Bayesian reasoning. It offers a heuristic approach to combine certainty factors, mimicking the thinking process of a human expert.

Key Difference

Unlike Bayesian reasoning, the method for combining certainty factors is not mathematically rigorous. Certainty factors are not treated as probabilities, but as measures of confidence.

Illustration

To demonstrate evidential reasoning and the propagation of certainty factors through rules, consider the example of the **FORECAST expert system** discussed in section 3.4.

Takeaway: Certainty factors are a practical and intuitive tool, designed to work closely with the way experts reason in real-world scenarios.

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FORECAST: An Application of Certainty Factors

Objective:

Purpose

The expert system predicts whether it will rain tomorrow by establishing certainty factors for the multivalued object **tomorrow**.

Approach:

- Rules are used to predict tomorrow's weather based on today's conditions.
- Certainty factors (CFs) indicate the confidence level of each rule.

Simplification

For this task, the same set of rules as in Section 3.4 is used to simplify calculations and analysis.

Knowledge Base - Rules 1 and 2

Rule 1:

Condition and Outcome

If today is rain, then tomorrow is rain {cf 0.5}.

Rule 2:

Condition and Outcome

If today is dry, then tomorrow is dry {cf 0.5}.

Key Points:

- These rules establish the baseline conditions for predicting tomorrow's weather.
- Certainty factors of 0.5 indicate moderate confidence.

Knowledge Base - Rules 3 and 4

Rule 3:

Condition and Outcome

If today is rain **and** rainfall is low, then tomorrow is dry {cf 0.6}.

Rule 4:

Condition and Outcome

If today is rain **and** rainfall is low **and** temperature is cold, then tomorrow is dry {cf 0.7}.

Increased Confidence

Rules 3 and 4 refine predictions by adding more conditions, increasing the certainty factor (0.6 and 0.7, respectively).

Knowledge Base - Rules 5 and 6

Rule 5:

Condition and Outcome

If today is dry **and** temperature is warm, then tomorrow is rain {cf 0.65}.

Rule 6:

Condition and Outcome

If today is dry **and** temperature is warm **and** sky is overcast, then tomorrow is rain {cf 0.55}.

Dynamic Conditions

Rules 5 and 6 introduce warm temperatures and overcast skies as factors for predicting rain, with varying certainty factors.

Dialogue Example - Step 1: What is the Weather Today?

Question: What is the weather today?
Answer: rain

Rule 1

If today is rain, then tomorrow is rain {cf 0.5}.

Calculation:

$$cf(\text{tomorrow is rain, today is rain}) = cf(\text{today is rain}) \times cf = 1.0 \times 0.5 = 0.5$$

Intermediate Conclusion:

- Tomorrow is rain {0.50}.

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Dialogue Example - Step 2: What is the Rainfall Today?

Question: What is the rainfall today?
Answer: low
Degree of Certainty: 0.8

Rule 3

If today is rain **and** rainfall is low, then tomorrow is dry {cf 0.6}.

Calculation:

$$cf(\text{tomorrow is dry, today is rain and rainfall is low}) = \min(cf(\text{today is rain}),$$

$$= \min(1, 0.8) \times 0.6 = 0.48$$

Intermediate Conclusion:

- Tomorrow is rain {0.50}.
- Tomorrow is dry {0.48}.

Dialogue Example - Step 3: What is the Temperature Today?

Question: What is the temperature today?
Answer: cold
Degree of Certainty: 0.9

Rule 4

If today is rain **and** rainfall is low **and** temperature is cold, then tomorrow is dry {cf 0.7}.

Calculation:

$$cf(\text{tomorrow is dry, today is rain and rainfall is low and temperature is cold})$$
$$= \min(cf(\text{today is rain}), cf(\text{rainfall is low}), cf(\text{temperature is cold})) \times cf$$
$$= \min(1, 0.8, 0.9) \times 0.7 = 0.56$$

Intermediate Conclusion:

- Tomorrow is dry {0.56}.
- Tomorrow is rain {0.50}.

Final Combination of Certainty Factors

Combine Results from Rule 3 and Rule 4:

$$cf(cf_{\text{Rule 3}}, cf_{\text{Rule 4}}) = cf_{\text{Rule 3}} + cf_{\text{Rule 4}} \times (1 - cf_{\text{Rule 3}})$$

$$= 0.48 + 0.56 \times (1 - 0.48) = 0.48 + 0.56 \times 0.52 = 0.77$$

Final Conclusion:

- Tomorrow is dry $\{0.77\}$.
- Tomorrow is rain $\{0.50\}$.

Key Insight

The probability of having a dry day tomorrow is almost certain ($\{0.77\}$); however, there is still a moderate chance of rain ($\{0.50\}$).

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Comparison of Bayesian Reasoning and Certainty Factors

Overview

Two popular techniques for uncertainty management in expert systems:

- **Bayesian Reasoning:** Probability-based approach.
- **Certainty Factors:** Heuristic and intuitive approach.

Objective

Determine which approach is better suited for specific types of problems in expert systems.

Bayesian Reasoning: Strengths and Applications

Strengths

- Oldest and best-established technique for handling inexact knowledge and random data.
- Effective in forecasting and planning with reliable statistical data.

Application: PROSPECTOR System

Used for aiding geologists in ore exploration:

- Predicted the existence of molybdenum near Mount Tolman, Washington.
- Relied on statistical independence and valid data for probabilities.

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Limitations of Bayesian Reasoning

Key Issues

- Assumes conditional independence of evidence, which is not always valid.
- Requires reliable statistical information, often unavailable in certain domains.

Example: MYCIN System

- Bayesian methods were unsuitable for medical diagnostics due to the lack of required statistical data.
- This limitation motivated the development of certainty factors.

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Certainty Factors: Strengths

Advantages Over Bayesian Reasoning

- Designed for scenarios where probabilities are unknown, difficult, or expensive to obtain.
- Supports intuitive judgments and incremental evidence acquisition.

Diagnostic Systems: MYCIN

- Uses expert knowledge and certainty factors to manage hypotheses.
- Provides better explanations of reasoning in rule-based systems.

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Challenges and Comparisons

Common Challenges

- Both techniques require an expert for personal, subjective, and qualitative information.
- Human bias influences the choice of approach.

Bayesian Reasoning

Most suitable when:

- Reliable statistical data exists.
- The knowledge engineer can lead serious analytical conversations.

Certainty Factors

- Practical and intuitive for large systems lacking statistical data.
- Delivers results acceptable in many applications despite its heuristic nature.

Comparison: Bayesian Reasoning vs Certainty Factors (Part 1)

Aspect	Bayesian Reasoning	Certainty Factors
Approach	Probability-based; mathematically rigorous.	Heuristic and intuitive; rule-based.
Data Requirements	Requires reliable statistical data and assumes conditional independence of evidence.	Works with partial, subjective, or unavailable probability data.

Comparison of Bayesian Reasoning and Certainty Factors (Part 1)

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Comparison: Bayesian Reasoning vs Certainty Factors (Part 2)

Aspect	Bayesian Reasoning	Certainty Factors
Strengths	Works well in forecasting and planning where reliable statistical data is available.	Effective in diagnostics (e.g., MYCIN), handling incremental evidence, and providing better explanations.
Limitations	• Requires statistical independence of evidence. • Computationally expensive for large knowledge bases.	• Lacks mathematical rigor. • Relies heavily on expert intuition and qualitative judgments.

Comparison of Bayesian Reasoning and Certainty Factors (Part 2)

Comparison: Bayesian Reasoning vs Certainty Factors (Part 3)

Aspect	Bayesian Reasoning	Certainty Factors
Applications	Used in fields like geology and exploration (e.g., PROSPECTOR).	Common in medical diagnostics and systems with uncertain or unavailable data (e.g., MYCIN).
Best Use Cases	When statistical data is reliable and conditional independence can be assumed.	When probabilities are unknown, too complex, or expensive to determine.

Comparison of Bayesian Reasoning and Certainty Factors (Part 3)

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Questions and Discussion

THANKS