Digital Image Fundamentals

A SIMPLE IMAGE FORMATION MODEL

Images are two-dimensional functions of the form f (x ,y). The value of f at spatial coordinates (x, y) is a scalar quantity whose physical meaning is determined by the source of the image, and whose values are proportional to energy radiated by a physical source (e.g., electromagnetic waves).

As a consequence, f(x,y) must be nonnegative, and finite.

$f(x,y) = i(x,y)r(x,y)$

$i(x,y)$ is the illumination

$r(x,y)$ is the reflected illumination

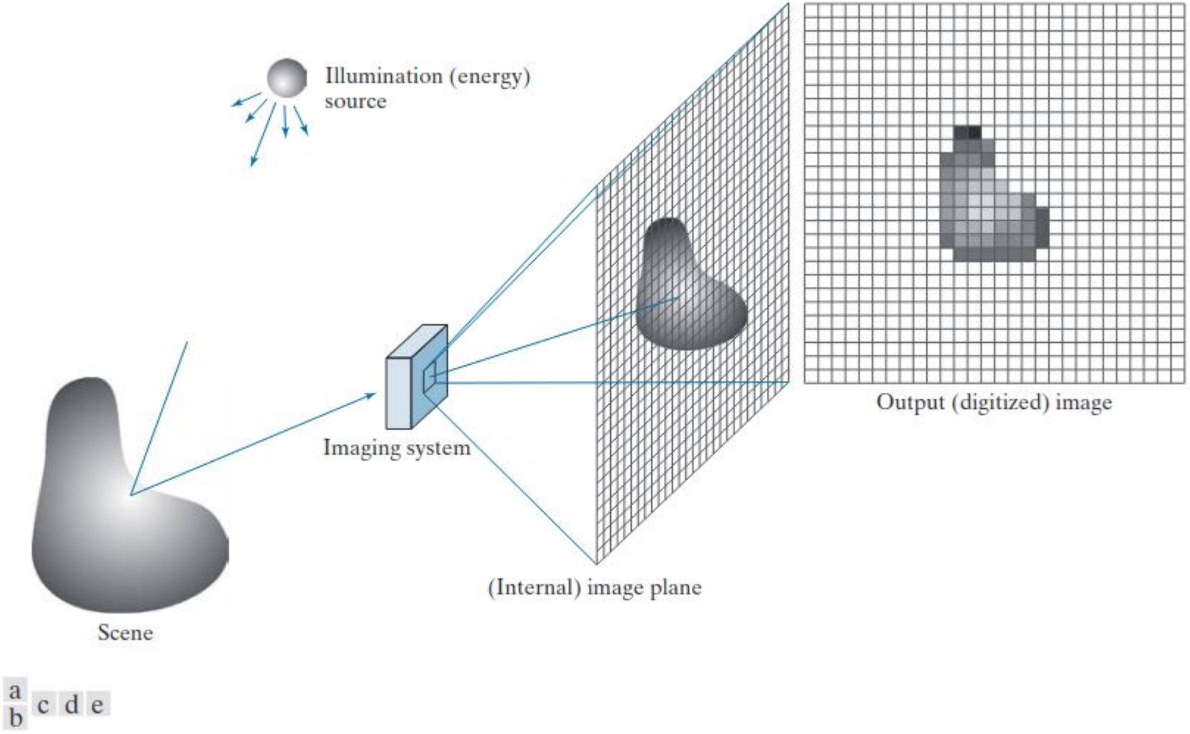


FIGURE 2.15 An example of digital image acquisition. (a) Illumination (energy) source. (b) A scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image sampling and quantization

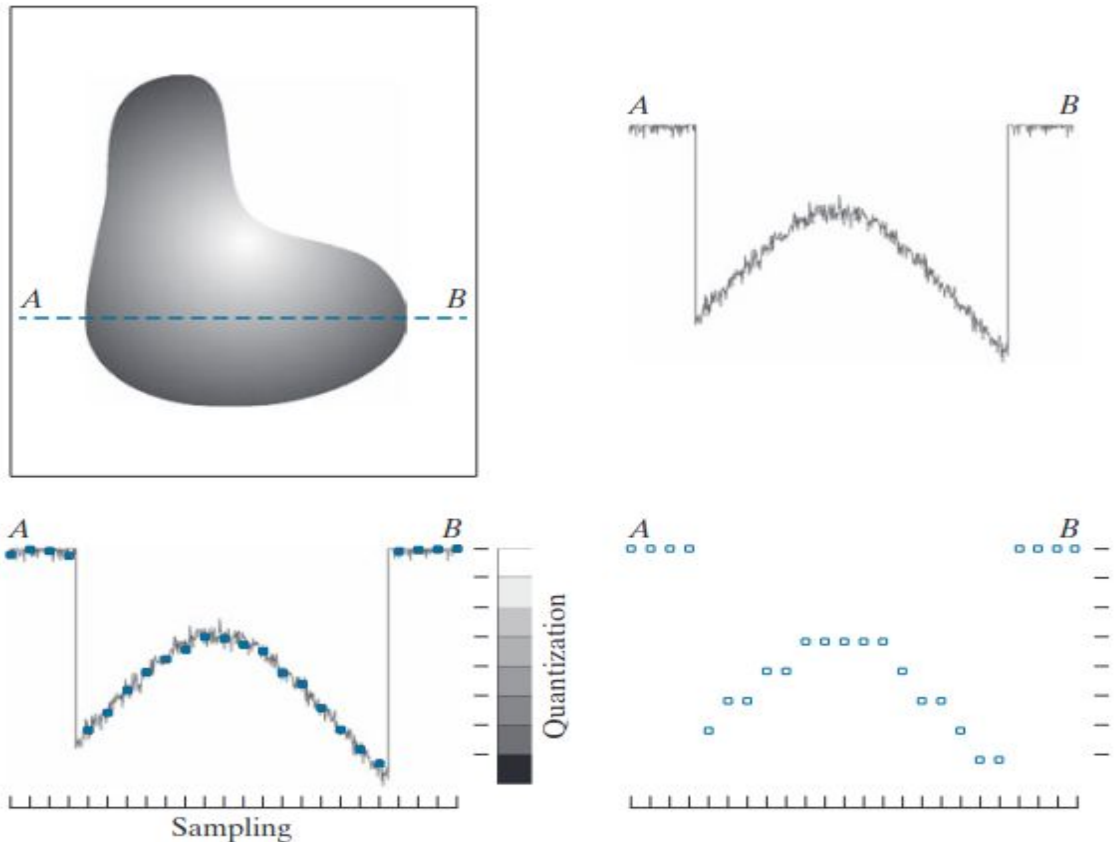


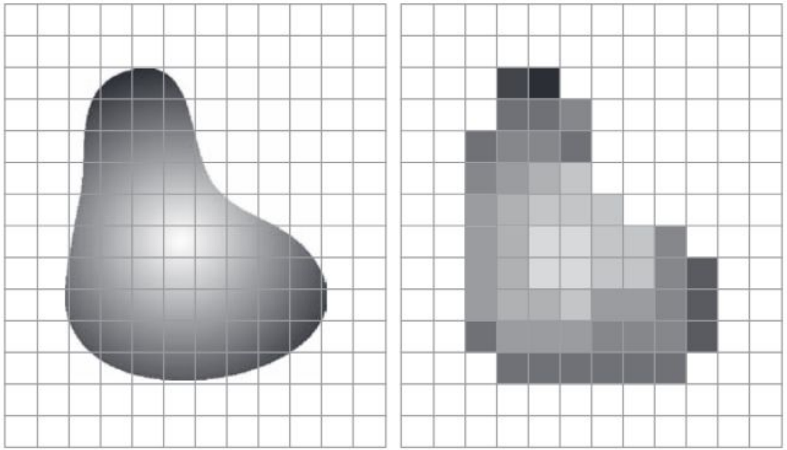
FIGURE 2.16 (a) Continuous image. (b) A scan line showing intensity variations along line AB in the continuous image. (c) Sampling and quantization. (d) Digital scan line. (The black border in (a) is included for clarity. It is not part of the image).

Image sampling and Quantization

To digitize an image, we have to sample the function in both coordinates and also in amplitude. Digitizing the coordinate values is called sampling. Digitizing the amplitude values is called quantization.

a b

FIGURE 2.17
(a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



Digital Image representation

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

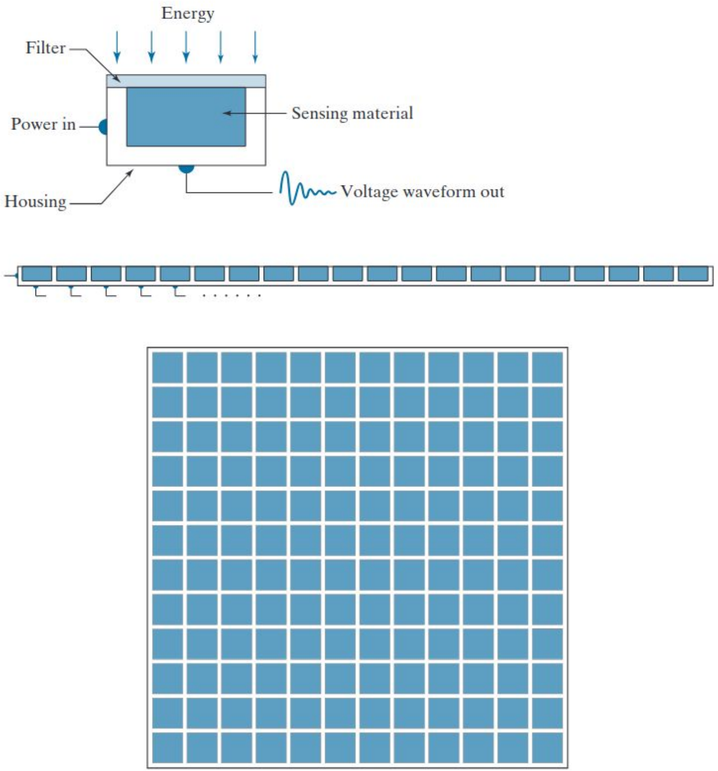
$$\mathbf{A}=\begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

We assume that the discrete levels are equally spaced and that they are integers in the range [0, L-1]

K is the number of bits assigned to each pixel $L = 2^k$

b is the total bit needed to store the image $b = M \times N \times k$

Sensing elements



a
b
c

FIGURE 2.12
(a) Single sensing element.
(b) Line sensor.
(c) Array sensor.

Spatial resolution

spatial resolution can be stated in several ways, with line pairs per unit distance, and dots (pixels) per unit distance.

Dots per unit distance is a measure of image resolution used in the printing and publishing industry.

Example: dots per inch (dpi).

In computer vision we use image resolution i.e. 5Mpixels

a b
c d

FIGURE 2.23 Effects of reducing spatial resolution. The images shown are at: (a) 930 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi.

Intensity resolution

Intensity resolution is number of intensity levels used to represent the image.

The more intensity levels used, the finer the level of detail discernable in an image.

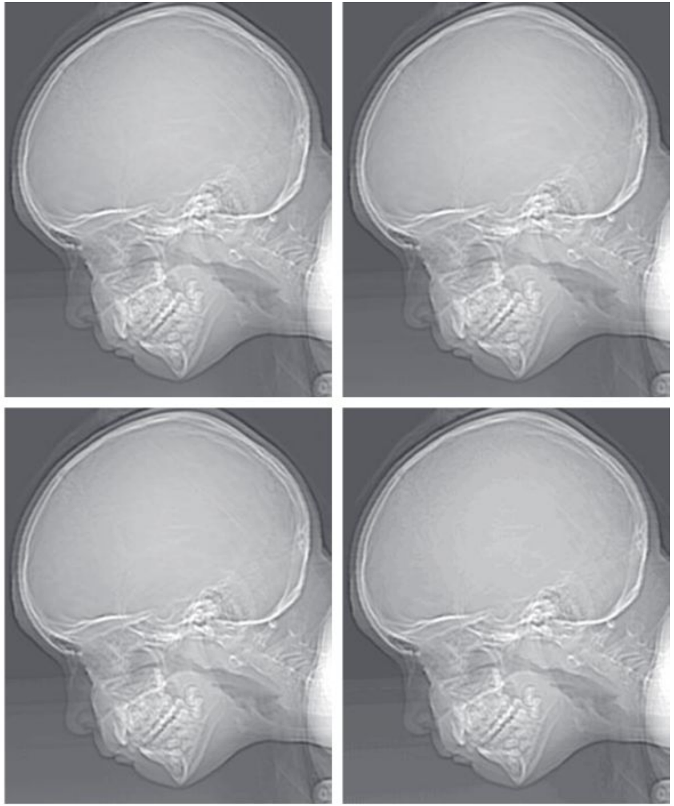
Intensity level resolution usually given in terms of number of bits used to store each intensity level.

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

Intensity resolution

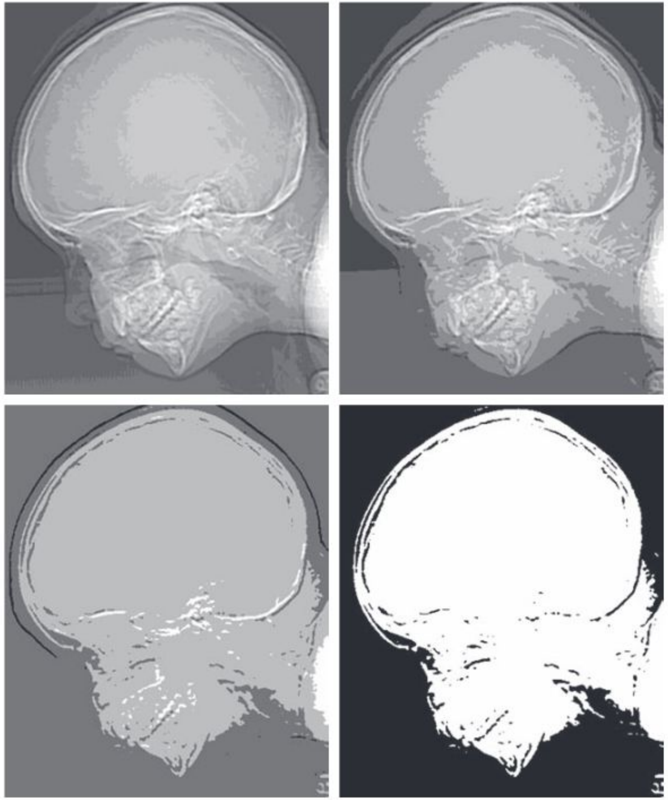
a b
c d

FIGURE 2.24
(a) 774 × 640, 256-level image. (b)-(d) Image displayed in 128, 64, and 32 intensity levels, while keeping the spatial resolution constant. (Original image courtesy of the Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



e f
g h

FIGURE 2.24
(*Continued*)
(e)-(h) Image displayed in 16, 8, 4, and 2 intensity levels.



Using an insufficient number of intensity levels(less than 16 intensity levels) in smooth areas of a digital image causes false contouring

Relationships between pixels

- a pixel p at coordinate (x,y) has
  - $N_4(p)$: 4-neighbors of p
    (x+1, y), (x-1,y),(x,y+1), (x,y-1)

$$\begin{array}{ccc} & x & \\ x & p & x \\ & x & \end{array}$$

  - $N_D(p)$: 4-diagonal neighbors of p
    (x+1, y+1), (x+1,y-1),(x-1,y+1), (x-1,y-1)

$$\begin{array}{ccc} x & & x \\ & p & \\ x & & x \end{array}$$

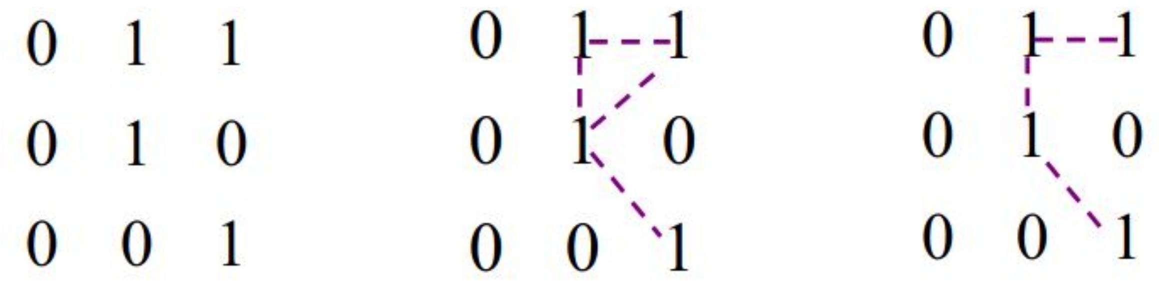
  - $N_8(p)$: 8-neighbors of p :
    a combination of $N_4(p)$ and $N_D(p)$

$$\begin{array}{ccc} x & x & x \\ x & p & x \\ x & x & x \end{array}$$

Relationships between pixels

- Let V be the set of gray-level values used to defined connectivity
  - 4-connectivity :
    - 2 pixels p and q with values from V are 4-connected if q is in the set $N_4(p)$
  - 8-connectivity :
    - 2 pixels p and q with values from V are 8-connected if q is in the set $N_8(p)$
  - m-connectivity (mixed connectivity):
    - 2 pixels p and q with values from V are m-connected if
      - q is in the set $N_4(p)$ or
      - q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty.

Relationships between pixels



(a) An arrangement of pixels. (b) Pixels that are 8-adjacent (c) m-adjacency.

m-connectivity eliminates the multiple path connections that arise in 8-connectivity

Distance measure

The Euclidean distance between p = (x,y) and q = (u,v) is defined as:

$$D_e(p,q) = \left[(x-u)^2 + (y-v)^2\right]^{\frac{1}{2}}$$

The D4 distance, (called the city-block distance) between p and q is defined as

$$D_4(p,q) = |x-u| + |y-v|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

The D8 distance (called the chessboard distance) between p and q is defined as

$$D_8(p,q) = \max(|x-u|, |y-v|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance measure

For pixels p, q, and s, with coordinates $(x, y), (u, v)$, and (w, z), respectively, D is a *distance function* or *metric* if

(a) $D(p,q) \geq 0 \quad (D(p,q) = 0 \text{ iff } p = q)$,

(b) $D(p,q) = D(q,p)$, and

(c) $D(p,s) \leq D(p,q) + D(q,s)$.

Arithmetic operations

used extensively in most branches of image processing.

 Arithmetic operations for 2 pixels p and q :

- Addition : p+q used in image average to reduce noise.
- Subtraction : p-q basic tool in medical imaging.
- Multiplication : pxq
- Division : p/q

 Arithmetic Operation entire images are carried out pixel by pixel.

2	3	4	0	0
0	3	5	1	0
0	2	6	1	7
0	0	0	1	0
0	0	1	0	0

1	0	0	0	0
0	4	7	0	0
0	0	3	6	0
0	0	2	4	5
0	0	0	1	3

Logic operations

- AND : p AND q (p•q)
- □ OR : p OR q (p+q)
- □ COMPLEMENT : NOT q , q’
- □ logic operations apply only to binary images.
- □ arithmetic operations apply to multivalued pixels.
- □ logic operations used for tasks such as masking, feature detection, and shape analysis.
- □ logic operations perform pixel by pixel.

Mask operations

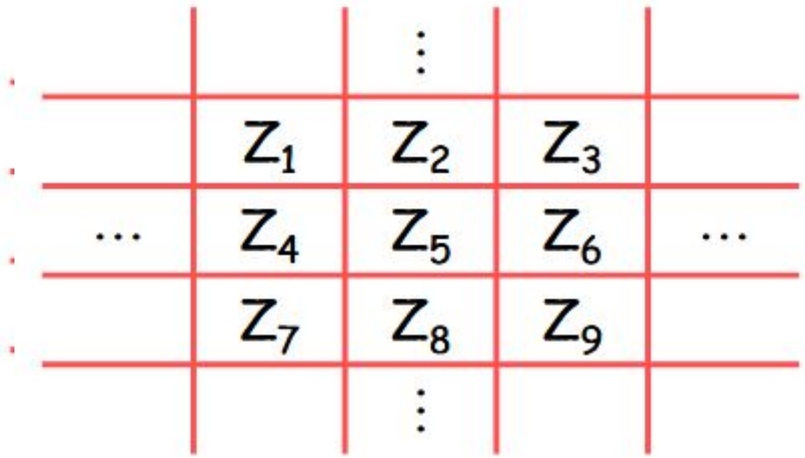
Besides pixel-by-pixel processing on entire images, arithmetic and Logical operations are used in neighborhood oriented operations.

Let the value assigned to a pixel be a function of its gray level and the gray level of its neighbors.

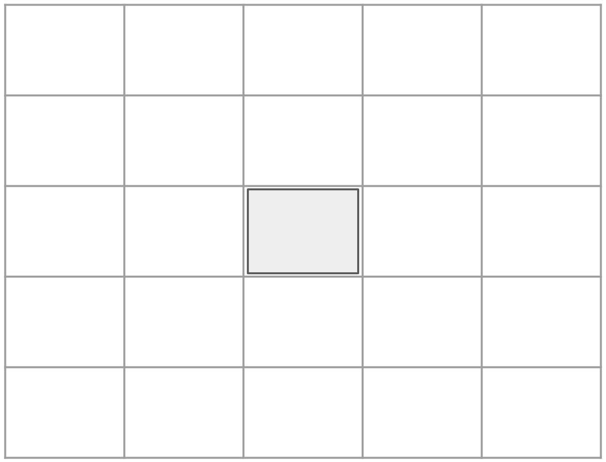
		⋮		
	Z_1	Z_2	Z_3	
...	Z_4	Z_5	Z_6	...
	Z_7	Z_8	Z_9	
		⋮		

Mask operations

replace the gray value of pixel Z5 with the average gray values of it’s neighborhood within a 3x3 mask



Input image



Output image

Mask operations

$$Z = \frac{1}{9}(Z_1 + Z_2 + Z_3 + \ldots + Z_9)$$

$$Z = \frac{1}{9}Z_1 + \frac{1}{9}Z_2 + \frac{1}{9}Z_3 + \ldots + \frac{1}{9}Z_9$$
$$= w_1Z_1 + w_2Z_2 + w_3Z_3 + \ldots w_9Z_9$$
$$= \sum_{i=1}^{9} w_i Z_i$$