COMSCI/ECON 206 — Problem Set 1

Chenlei Tao

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Github link

https://github.com/Sarah8214/An-Interdisciplinary-Study.git

Acknowledgments

I would like to express my sincere gratitude to Professor Luyao Zhang for her valuable feedback and guidance, which greatly helped me refine both the theoretical framing and methodological rigor of this work. I am also thankful to Zijun for the thoughtful comments and suggestions, which inspired me to clarify my arguments and improve the overall clarity of the analysis. Their insights have been instrumental in shaping the revised version of this paper.

Part 1 — Economist

1. Game and Equilibrium Concept

We study a two-player bargaining game adapted from the standard oTree bargaining demo (Chen, Schonger & Wickens, 2016). In the original version, two players simultaneously demand an integer share of 100 tokens. If the sum of demands does not exceed 100, each receives their demand; otherwise, both receive zero. For our project, we modified the app by adjusting Constants.num_rounds to create 1-, 2-, and 5-round versions of the game. We also revised the payoff logic in models.py to track cumulative earnings across rounds and updated the HTML templates so that the user interface displays both the cumulative score after each round and a final payoff summary. These revisions allow us to investigate how players' strategies evolve over repeated interactions. Repetition enables players to adjust their choices in response to prior outcomes, capturing potential learning effects. Moreover, by presenting cumulative earnings, the design emphasizes long-term outcomes, thereby testing how fairness perceptions and conditional cooperation shape behavior while keeping the underlying payoff structure fixed.

Formally, let the set of players be $I = 1,2$. Each player i chooses a demand

$s_i \in S_i = \{0,1,2,\ldots,100\}$.

The payoff function is

$$u_i(s_1, s_2) = s_i, if s_1 + s_2 \leq 100; 0, if s_1 + s_2 > 100.$$

We adopt the concept of Nash equilibrium (Nash, 1950). A strategy profile $s^* = (s_1^*, s_2^*)$ is a Nash equilibrium if, for each player i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad \forall s_i \in S_i$$

Existence theorem. For any finite normal-form game, a mixed-strategy Nash equilibrium exists (Nash, 1950). The proof embeds the mixed strategy set into a compact convex simplex and applies Kakutani's fixed point theorem to the best-response correspondence. For continuous strategy spaces with continuous payoffs, existence extends by Glicksberg's theorem (1952).

2. Analytical Solution

Pure-strategy equilibria.
All pairs (s_1^*, s_2^*), such that $s_1^* + s_2^* = 100$ are pure-strategy Nash equilibria.

- If one player increases her demand, the total exceeds 100 and her payoff drops to 0.
- If she decreases her demand, her payoff falls directly.
  Thus no unilateral deviation is profitable.

Symmetry and focal point.
Although many equilibria exist, (50,50) is a natural symmetric focal equilibrium because it is both equal and easily recognized by players.

Efficiency.
Every equilibrium with $s_1 + s_2 = 100$ is Pareto efficient, since the full resource is allocated and the total surplus equals 100. From a utilitarian perspective, all equilibria are equivalent ($\sum_i u_i = 100$).

Fairness.

- Extreme equilibria such as (100,0) are highly unequal.
- The symmetric split (50,50) is perfectly equal and uniquely satisfies envy-freeness and proportionality (each receives at least half the pie).
  Thus efficiency is universal, but fairness varies greatly.

3. Interpretation

- **Realism and coordination**
  In the one-shot version (R=1), equilibrium multiplicity creates a coordination problem. While theory predicts many equilibria, experiments often show clustering around equal splits, reflecting fairness norms and focal-point effects (Roth, 1995).
- **Multiplicity and refinements**
  The large set of equilibria is not equally compelling. Standard refinements (e.g., trembling-hand perfection) suggest that extreme allocations are less stable, while the equal split is more robust and commonly observed.
- **Repeated rounds (R=2,5)**
  In finitely repeated games, any stage-game Nash equilibrium repeated each round forms a subgame perfect equilibrium by backward induction. Yet in practice, repeated interaction enables learning and history dependence: experimental studies find that more rounds encourage convergence toward fairer and more stable splits. We therefore predict outcomes closer to (50,50) in the 2- and 5-round treatments.
- **Bounded rationality**
  Because players cannot enumerate all equilibria in such a large action space, they use heuristics such as "demand half" or "repeat last round." This explains the empirical prevalence of equal splits despite the theoretical multiplicity.
- **Computational tractability**
  Our game is analytically simple ($s_1 + s_2 = 100$), but in general, computing Nash equilibria is PPAD-complete (Daskalakis, Goldberg & Papadimitriou, 2009). This example thus serves as a tractable case to illustrate the gap between theory and computation.

Part 2 — Computational Scientist

2a — Normal Form & Computation (Google Colab)

In this experiment, two players simultaneously choose a demand $xi \in \{1,2,\dots,100\}$ If the sum of the demands $\mathrm{x1} + \mathrm{x2} \leq 100$, each player receives their demanded amount; otherwise, both receive zero.

Discretization note:

In our experiment, players can demand any integer between 1 and 100. For computational feasibility, I discretized the strategy space to 5 representative actions $\{0,25,50,75,100\}$. This simplification preserves the structure of the game but makes the matrix tractable for visualization and solver computation.

```
Payoff matrix for Player 1:
 [[  0.    0.    0.    0.    0.]
 [ 25.   25.   25.   25.    0.]
 [ 50.   50.   50.    0.    0.]
 [ 75.   75.    0.    0.    0.]
 [100.    0.    0.    0.    0.]]

Payoff matrix for Player 2:
 [[  0.   25.   50.   75.  100.]
 [  0.   25.   50.   75.    0.]
 [  0.   25.   50.    0.    0.]
 [  0.   25.    0.    0.    0.]
 [  0.    0.    0.    0.    0.]]
```

Figure 1. The payoff matrices for Player 1 and Player 2 in the reduced action grid.

```
Nash equilibria (strategy profiles):
(array([1., 0., 0., 0., 0.]), array([0., 0., 0., 0., 1.]))
(array([0., 1., 0., 0., 0.]), array([0., 0., 0., 1., 0.]))
(array([0., 0., 1., 0., 0.]), array([0., 0., 1., 0., 0.]))
(array([0., 0., 0., 1., 0.]), array([0., 1., 0., 0., 0.]))
(array([0., 0., 0., 0., 1.]), array([1., 0., 0., 0., 0.]))
(array([0., 0., 0., 0., 1.]), array([0., 0., 0., 0., 1.]))
(array([-0.        , 0.66666667, 0.33333333, 0.        , 0.        ]), array([-0. , -0. , 0.5, 0.5, 0. ]))
(array([-0.        , 0.33333333, 0.        , 0.66666667, 0.        ]), array([-0.        , 0.33333333, 0.        , 0.66666667, 0.        ]))
(array([-0. , -0. , 0.5, 0.5, 0. ]), array([-0.        , 0.66666667, 0.33333333, 0.        , 0.        ]))
(array([-0.        , 0.33333333, 0.16666667, 0.5       , 0.        ]), array([-0.        , 0.33333333, 0.16666667, 0.5       , 0.        ]))
```

Figure 2. Nash equilibria of the reduced grid computed using Nashpy's support enumeration method (Garrison, 2018).

Interpretation:

- The first few equilibria correspond to pure-strategy profiles where each player's demand sums to 100 (e.g., 50+50, 75+25, etc.).
- Later equilibria show mixed strategies where players randomize over multiple actions to satisfy the Nash equilibrium conditions.
- The runtime warning indicating degeneracy reflects the game's multiple symmetric equilibria due to the structure of payoffs.

2b — Extensive Form & SPNE (Game Theory Explorer)

An extensive-form version of the bargaining game was also constructed using Game Theory Explorer (GTE) (Savani & von Stengel, 2015). In the original setup, both players simultaneously choose an integer demand $xi \in \{1,2,\dots,100\}$. For computational tractability and readability of the tree, we discretized the action space to three representative choices $\{1,50,100\}$. To model simultaneous moves in GTE, the game tree is drawn in a sequential form, but Player 2's information set ensures that they do not observe Player 1's choice before acting. Payoffs at each terminal node follow the same rule as in the normal form: if $x1 + x2 \leq 100$, then payoffs = (x1,x2); otherwise, both receive (0,0).

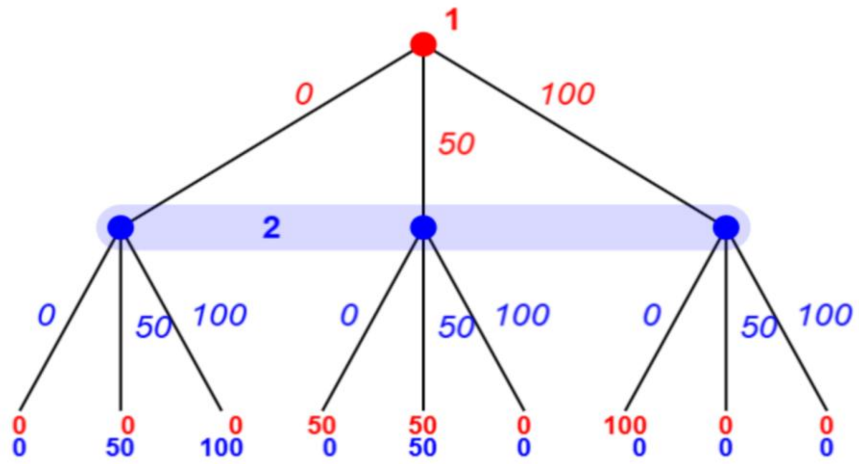

Figure 3: Extensive-form of the 2-player bargaining game in GTE. Player 1 moves first, choosing from $\{0,50,100\}$, and Player 2 observes before responding. Payoffs at terminal nodes follow: if total demand ≤100, payoffs = (x1, x2); otherwise, both get 0.

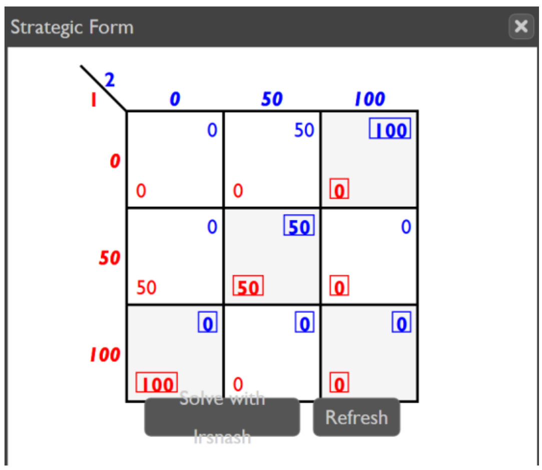

Figure 4. Strategic-form matrix of the 2-player bargaining game, showing payoffs for cross-check with the extensive form.

Interpretation:

- SPNE refines the multiplicity observed in the simultaneous-move normal form by selecting strategy profiles consistent with sequential rationality.

- Player 2's best response maximizes their payoff given Player 1's observed demand, and Player 1 anticipates this response.
- While the normal form admits many equilibria with total demand = 100, the sequential SPNE identifies specific profiles consistent with credible best responses, demonstrating first-mover advantage and backward induction reasoning.

Part 3 — Behavioral Scientist (experiment & AI comparison; 30 points)

3a — oTree Deployment & Human Experiment

Screenshots placeholders:

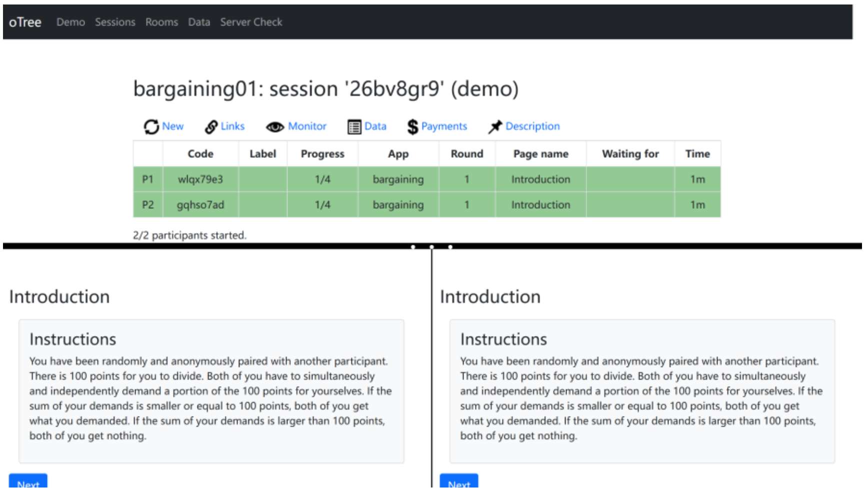

Figure 4. Instructions page, explains game rules and payoffs.

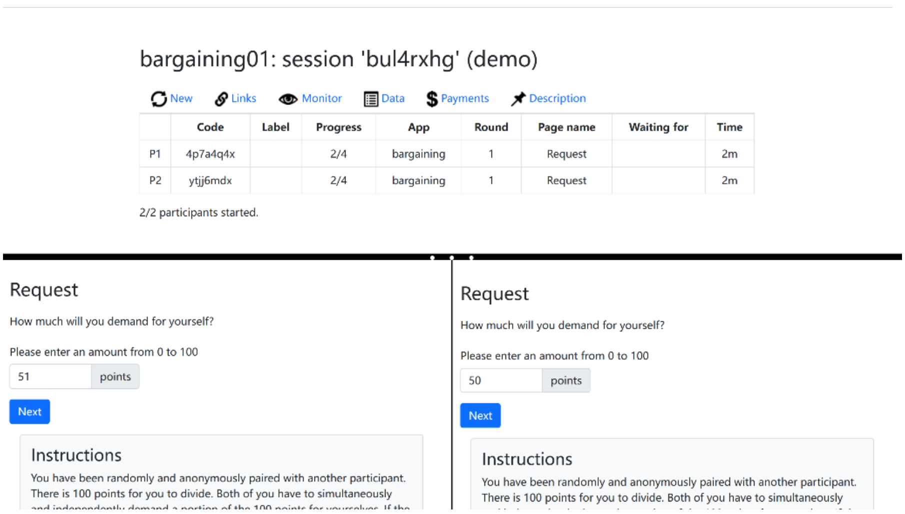

Figure 5. Decision page, each participant selects an integer demand from 1–100.

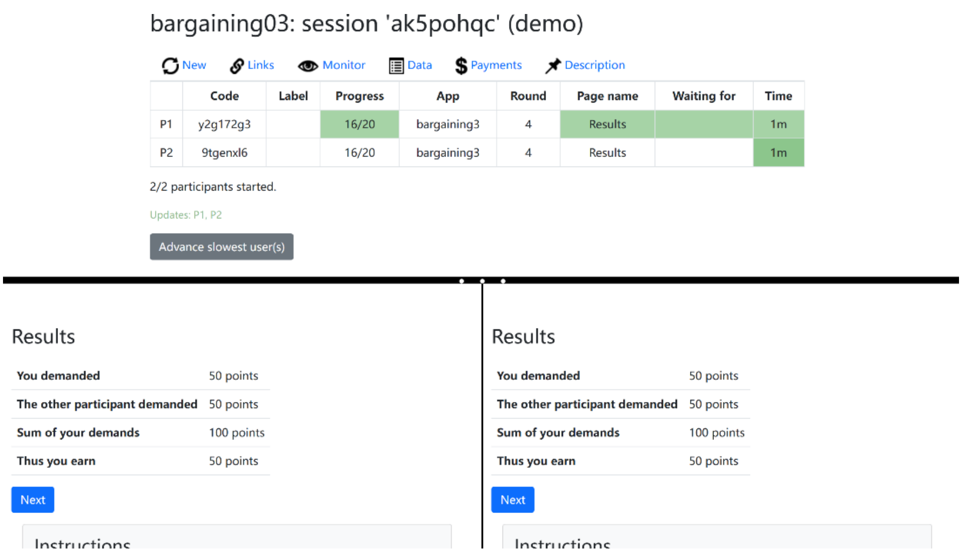

Figure 6. Results page, displays chosen demands, total payoff, and whether each player received their demand.

Human session (two classmates, pre-assigned group):

Round	Player 1 Demand	Player 2 Demand	Player 1 Payoff	Player 2 Payoff
Experiment 1 – Round 1	51	50	0	0
Experiment 2 – Round 1	51	50	0	0

Experiment 2 – Round 1	51	25	51	25
Experiment 3 – Round 1	51	25	51	25
Experiment 3 – Round 2	49	51	49	51
Experiment 3 – Round 3	50	50	50	50
Experiment 3 – Round 4	50	50	50	50
Experiment 3 – Round 5	50	50	50	50

Table 1. Player demands and payoffs across rounds of the bargaining game.

Post-play interviews:

Participants reported that after some initial trial-and-error, it became natural to settle at 50–50 in later rounds, which was perceived as a stable and fair solution.

Interpretation:

The results show that while early rounds involved coordination failures or unequal splits, repeated interaction gradually led players toward the fair 50–50 outcome. This pattern suggests learning and adaptation: initial experimentation gave way to convergence on an equitable and stable solution, consistent with participants' post-play reports. Although the sequential SPNE predicts a potential first-mover advantage, the human data indicate that any such advantage was not clearly observed in practice, as repeated play and fairness considerations quickly guided both players to equal splits.

3b — LLM "ChatBot" Session

Model & settings: [Model name = ChatGPT-5].

The LLM sessions were conducted with ChatGPT (OpenAI, 2025), using the system prompt and interaction protocol shown in Figure 7.

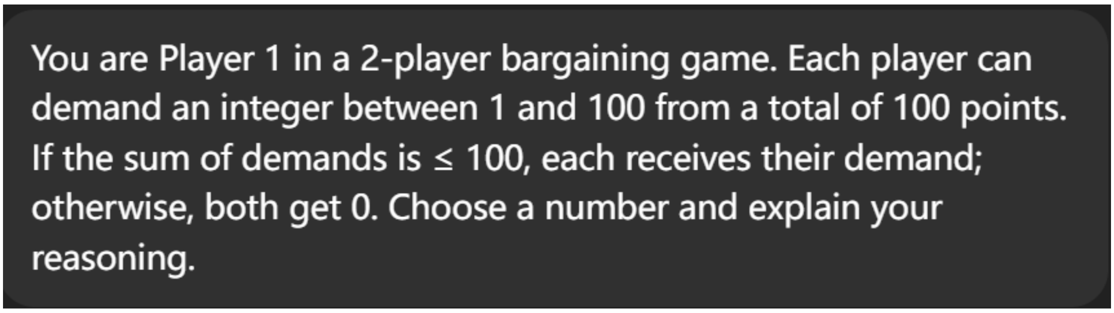

Figure 7. Screenshot of the LLM interaction for the 2-player bargaining game. The prompt asks the LLM to act as Player 1, choose a demand, and explain the reasoning behind the choice.

AI behavior:

Round	LLM Demand	Reasoning Summary
1	50	Chooses fair split to maximize guaranteed payoff
2	50	Maintains fairness and stability across repeated rounds
3	50	Consistently selects NE-aligned choice
4	50	Consistency due to payoff visibility and rational reasoning
5	50	Same as above, converging immediately to fair equilibrium

Table 2. LLM (ChatGPT) demands and reasoning across repeated rounds

Interpretation:

- LLM chooses the NE-aligned fair split (50–50) from the first round and maintains it across repeated rounds.
- Contrasts with human behavior, where trial-and-error and bounded rationality delayed convergence to the stable 50–50 allocation.
- Although the sequential SPNE allows for first-mover advantage in theory, the LLM's immediate adherence to the fair split shows that no such advantage was exploited, as the model consistently selects the equilibrium regardless of move order.

3c — Comparative Analysis & Theory Building

Comparison Table:

Source	Observed Behavior	Alignment with NE
Theoretical NE (Part 1/2)	Any demand pair summing to 100	Matches pure-strategy NE
Human session	Initial over- or under-demand (51+50, 51+25), eventually stabilized to 50–50	Partial: trial-and-error before reaching stable NE
LLM session	Immediate fair split (50–50) across all rounds	Fully aligned with NE

Table 3. Comparative behavior of theoretical, human, and LLM play

Observations: Humans vs. NE
Humans often began by over- or under-demanding (e.g., 51+50, 51+25). This reflects bounded rationality and uncertainty about the opponent's strategy.

After receiving zero payoffs in early rounds, participants adapted. They converged to the stable and fair 50–50 split, which is consistent with a Nash equilibrium.

Observations: LLM (ChatGPT-5) vs. NE
In contrast, the LLM immediately selected the fair split (50–50). It repeated this choice in every round.

This behavior shows full alignment with the theoretical equilibrium, without the trial-and-error phase observed in human play.

Behavioral Explanation
Human decision-making is shaped by fairness preferences, risk aversion, and limited strategic foresight (Camerer et al., 2004; Fehr & Schmidt, 1999). These factors explain the initial deviations from NE.

The LLM, however, reasons through payoff calculations consistently. Its behavior matches equilibrium predictions from the start, demonstrating rational and systematic evaluation.

Potential Refinement: Bounded-Rationality Equilibrium
To capture human behavior, we propose a "bounded-rationality equilibrium." This framework incorporates risk aversion and adaptive learning.

It predicts that humans may deviate from NE in early rounds to avoid the risk of zero payoff. Over time, feedback and repeated play guide them toward the equilibrium outcome.

Formally:

$$x_i^{BR} = \arg\max_{x_i \le \theta_i} E\left[\, u_i\left(x_i, x_j\right)\right], \quad \theta_i < 100 - \min\left(x_j\right)$$

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Appendix A: Point-by-Point Response to Feedback

1. Original Comment (Professor Luyao Zhang):
"Missing citation and detailed description of oTree adaptation for multi-round bargaining."

Response:
I fully agree with this feedback. In the original submission, I neglected to include a proper citation for the oTree platform and did not clearly describe the modifications made to the app. I appreciate Professor Zhang for pointing out the correct citation format, which I have now incorporated as Chen, D. L., Schonger, M., & Wickens, C. (2016). Additionally, while I briefly mentioned the multi-round adaptation, I did not explain in detail what specific code or UI changes were implemented, nor did I justify why these changes enhance the experiment's ability to test learning and fairness over time.

Revision Made:

- Added Chen et al. (2016) to the references.
- Expanded Section X with detailed description of code/UI changes.
- Highlighted the rationale for testing learning and fairness over multiple rounds.

2. Original Comment (Professor Luyao Zhang):
"Missing citations for open-source software used, such as Nashpy, Game Theory Explorer, and LLM interfaces."

Response:
I agree with this feedback. In the original submission, I overlooked citing the essential software tools used in the analysis, which is important for reproducibility and academic transparency. Properly citing these tools not only acknowledges the work of the developers but also allows readers to reference the software if they wish to replicate the experiments.

Revision Made:

- Updated the references section with citations for Nashpy and Game Theory Explorer.
- Explicitly mentioned the use of ChatGPT in the methodology section to

improve transparency and reproducibility.

3. Original Comment (Professor Luyao Zhang):
"GitHub repository lacks the three clearly organized folders (/economist, /computational_scientist, /behavioral_scientist) expected for modular readability."

Response:
I partially agree with this feedback. In my original submission, I organized the repository using three separate branches instead of folders, with each branch containing its own README and corresponding materials. This structure did allow for modular access to the different parts, but I understand that using top-level folders improves immediate readability and navigation for reviewers or collaborators.

To address this, I have now modified the main branch of the repository to include the three recommended folders—/economist, /computational_scientist, /behavioral_scientist—each containing the relevant README and materials. I also added a top-level README.md summarizing the three parts and providing links to each folder, along with setup instructions (e.g., requirements.txt) to ensure code reproducibility.

Revision Made:

- Added three top-level folders in the main branch: /economist, /computational_scientist, /behavioral_scientist.
- Added a top-level README.md summarizing each folder and linking to them.
- Included setup instructions for reproducibility.

4. Original Comment (Professor Luyao Zhang):
"Writing and presentation could be improved: some figures are not labeled or referenced, captions should explain relevance, and shorter paragraphs are recommended for readability."

Response:
I acknowledge this feedback. I have now added labels and references for all figures, ensuring each figure is clearly mentioned in the main text. Additionally, I revised figure captions to explain what each figure shows and why it is relevant to the analysis. For readability, I have also shortened paragraphs in Part 3, especially in the behavioral comparison section, to make the discussion clearer and easier to follow.

Revision Made:

- Added labels and references for all figures in the main text.
- Updated figure captions to clarify content and relevance.
- Shortened paragraphs in Part 3's behavioral comparisons for improved readability.

5. Original Comment (Zijun):
"Round labels: The human-results panel mixes '2 rounds' and '2nd round' (p.7). It was a bit confusing. Standardize naming and ensure the repository contains raw logs matching the table exactly."

Response:
I agree with this feedback. In the original submission, the inconsistent round labels could cause confusion. I have now standardized all round labels throughout the tables and text for clarity. Additionally, I have revised the table to make it more concise and easier to understand. The repository has been updated to ensure that the raw logs exactly match the revised table, improving transparency and reproducibility.

Revision Made:

- Standardized round labels in all tables and text.
- Simplified and clarified the human-results table.
- Ensured raw logs in the repository correspond exactly to the updated table.

6. Original Comment (Zijun):
"Bridge SPNE to behavior. The SPNE discussion (p.5) mentions first-mover advantage, but the human/LLM sections (p.7–9) don't revisit it. Add 1–2 sentences on whether a first-mover advantage actually appeared and why/why not in your data."

Response:
I appreciate this careful feedback. I have now added 1–2 sentences in both the human and LLM interpretation sections, explicitly discussing whether a first-mover advantage appeared. In the human sessions, any potential advantage was not clearly observed due to early trial-and-error and fairness considerations, which led participants to converge on the stable 50–50 split. In the LLM sessions, the model immediately selected the NE-aligned fair split, showing that no first-mover advantage was exploited.

Revision Made:

- Added discussion of first-mover advantage in the human session interpretation.
- Added discussion of first-mover advantage in the LLM session interpretation.
- Ensured these sentences clearly bridge the SPNE theory with observed behavior.