Part 1 — Economist

1. Game and Equilibrium Concept

We study a two-player bargaining game adapted from the standard *oTree bargaining demo*. In the original version, two players simultaneously demand an integer share of 100 tokens. If the sum of demands does not exceed 100, each receives their demand; otherwise, both receive zero.
For our project, we modified the demo by changing the number of rounds: instead of a one-shot interaction, players now play in 2-round and 5-round versions. This allows us to compare outcomes across different horizons while keeping the payoff structure fixed.

Formally, let the set of players be $I = 1,2$. Each player i chooses a demand

$s_i \in S_i = \{0,1,2,\dots,100\}$.

The payoff function is

$$u_i(s_1, s_2) = s_i, if s_1 + s_2 \leq 100; 0, if s_1 + s_2 > 100.$$

We adopt the concept of Nash equilibrium (Nash, 1950). A strategy profile $s^* = (s_1^*, s_2^*)$ is a Nash equilibrium if, for each player i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad \forall s_i \in S_i$$

Existence theorem. For any finite normal-form game, a mixed-strategy Nash equilibrium exists (Nash, 1950). The proof embeds the mixed strategy set into a compact convex simplex and applies Kakutani's fixed point theorem to the best-response correspondence. For continuous strategy spaces with continuous payoffs, existence extends by Glicksberg's theorem (1952).

2. Analytical Solution

Pure-strategy equilibria.
All pairs (s_1^*, s_2^*), such that $s_1^* + s_2^* = 100$ are pure-strategy Nash equilibria.

- If one player increases her demand, the total exceeds 100 and her payoff drops to 0.
- If she decreases her demand, her payoff falls directly.
  Thus no unilateral deviation is profitable.

Symmetry and focal point.
Although many equilibria exist, (50,50) is a natural symmetric focal equilibrium because it is both equal and easily recognized by players.

Efficiency.
Every equilibrium with $s_1 + s_2 = 100$ is Pareto efficient, since the full resource is allocated and the total surplus equals 100. From a utilitarian perspective, all equilibria are equivalent ($\sum_i u_i = 100$).

Fairness.

- Extreme equilibria such as (100,0) are highly unequal.
- The symmetric split (50,50) is perfectly equal and uniquely satisfies envy-freeness and proportionality (each receives at least half the pie).
  Thus efficiency is universal, but fairness varies greatly.

3. Interpretation

Realism and coordination.
In the one-shot version (R=1), equilibrium multiplicity creates a coordination problem. While theory predicts many equilibria, experiments often show clustering around equal splits, reflecting fairness norms and focal-point effects (Roth, 1995).

Multiplicity and refinements.
The large set of equilibria is not equally compelling. Standard refinements (e.g., trembling-hand perfection) suggest that extreme allocations are less stable, while the equal split is more robust and commonly observed.

Repeated rounds (R=2,5).
In finitely repeated games, any stage-game Nash equilibrium repeated each round forms a subgame perfect equilibrium by backward induction. Yet in practice, repeated interaction enables learning and history dependence: experimental studies find that more rounds encourage convergence toward fairer and more stable splits. We therefore predict outcomes closer to (50,50) in the 2- and 5-round treatments.

Bounded rationality.
Because players cannot enumerate all equilibria in such a large action space, they use heuristics such as “demand half” or “repeat last round.” This explains the empirical prevalence of equal splits despite the theoretical multiplicity.

Computational tractability.
Our game is analytically simple ($s_1 + s_2 = 100$), but in general, computing Nash equilibria is PPAD-complete (Daskalakis, Goldberg & Papadimitriou, 2009). This example thus serves as a tractable case to illustrate the gap between theory and computation.

Part 2 — Computational Scientist (coding & tools; 40 points)

2a — Normal Form & Computation (Google Colab)

In this experiment, two players simultaneously choose a demand $xi \in \{1,2,\ldots,100\}$ If the sum of the demands x1+x2≤100, each player receives their demanded amount; otherwise, both receive zero.

Discretization note:

In our experiment, players can demand any integer between 1 and 100. For computational feasibility, I discretized the strategy space to 5 representative actions $\{0,25,50,75,100\}$. This simplification preserves the structure of the game but makes the matrix tractable for visualization and solver computation.

Payoff Matrices

The payoff matrices for Player 1 and Player 2 in the reduced action grid are shown below.

```
Payoff matrix for Player 1:
 [[  0.    0.    0.    0.    0.]
 [ 25.   25.   25.   25.    0.]
 [ 50.   50.   50.    0.    0.]
 [ 75.   75.    0.    0.    0.]
 [100.    0.    0.    0.    0.]]

Payoff matrix for Player 2:
 [[  0.   25.   50.   75.  100.]
 [   0.   25.   50.   75.    0.]
 [   0.   25.   50.    0.    0.]
 [   0.   25.    0.    0.    0.]
 [   0.    0.    0.    0.    0.]]
```

Nash Equilibria

Using NashPy's support enumeration, the solver computed the following equilibria for the reduced grid:

```
Nash equilibria (strategy profiles):
(array([1., 0., 0., 0., 0.]), array([0., 0., 0., 0., 1.]))
(array([0., 1., 0., 0., 0.]), array([0., 0., 0., 1., 0.]))
(array([0., 0., 1., 0., 0.]), array([0., 0., 1., 0., 0.]))
(array([0., 0., 0., 1., 0.]), array([0., 1., 0., 0., 0.]))
(array([0., 0., 0., 0., 1.]), array([1., 0., 0., 0., 0.]))
(array([0., 0., 0., 0., 1.]), array([0., 0., 0., 0., 1.]))
(array([-0.        ,  0.66666667,  0.33333333,  0.        ,  0.        ]), array([-0. , -0. ,  0.5,  0.5,  0. ]))
(array([-0.        ,  0.33333333,  0.        ,  0.66666667,  0.        ]), array([-0.        ,  0.33333333,  0.        ,  0.66666667,  0.        ]))
(array([-0. , -0. ,  0.5,  0.5,  0. ]), array([-0.        ,  0.66666667,  0.33333333,  0.        ,  0.        ]))
(array([-0.        ,  0.33333333,  0.16666667,  0.5       ,  0.        ]), array([-0.        ,  0.33333333,  0.16666667,  0.5       ,  0.        ]))
```

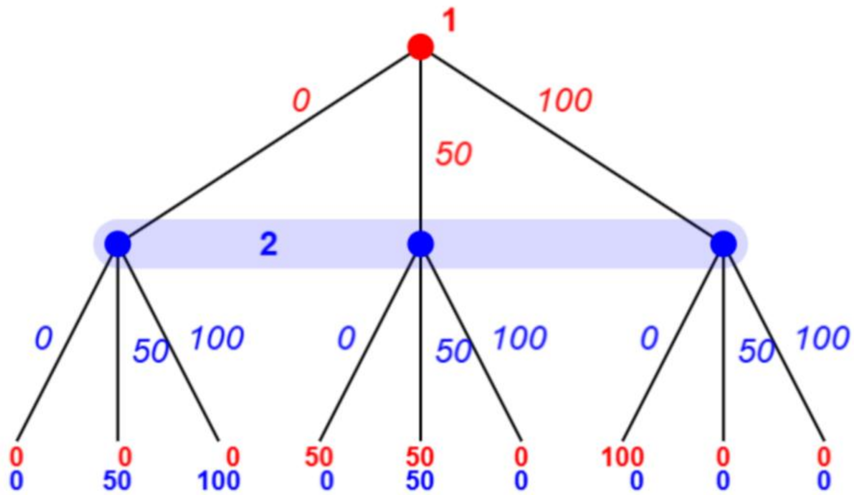
Interpretation:

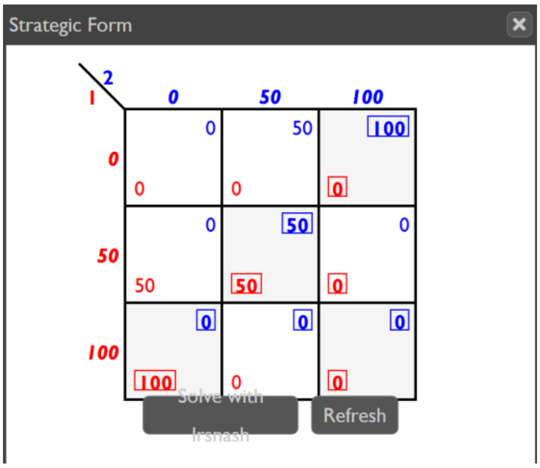
- The first few equilibria correspond to pure-strategy profiles where each player's demand sums to 100 (e.g., 50+50, 75+25, etc.).
- Later equilibria show mixed strategies where players randomize over multiple actions to satisfy the Nash equilibrium conditions.
- The runtime warning indicating degeneracy reflects the game's multiple symmetric equilibria due to the structure of payoffs.

2b — Extensive Form & SPNE (Game Theory Explorer)

An extensive-form version of the game was built in Game Theory Explorer (GTE), with Player 1 moving first and Player 2 observing Player 1's demand before choosing their own. A reduced action set {0,25,50,75,100} was used to keep the tree readable. Payoffs at each terminal node follow the same rule as the normal form: if $x1+x2\leq100$, payoffs = (x1,x2); otherwise, (0,0).

Game Tree





1 \ 2	0	50	100
0	0, 0	0, 50	0, 100
50	50, 0	50, 50	0, 0
100	100, 0	0, 0	0, 0

Interpretation:

- SPNE refines the multiplicity observed in the simultaneous-move normal form by selecting strategy profiles consistent with sequential rationality.
- Player 2's best response maximizes their payoff given Player 1's observed demand, and Player 1 anticipates this response.
- While the normal form admits many equilibria with total demand = 100, the sequential SPNE identifies specific profiles consistent with credible best responses, demonstrating first-mover advantage and backward induction reasoning.

Part 3 — Behavioral Scientist (experiment & AI comparison; 30 points)

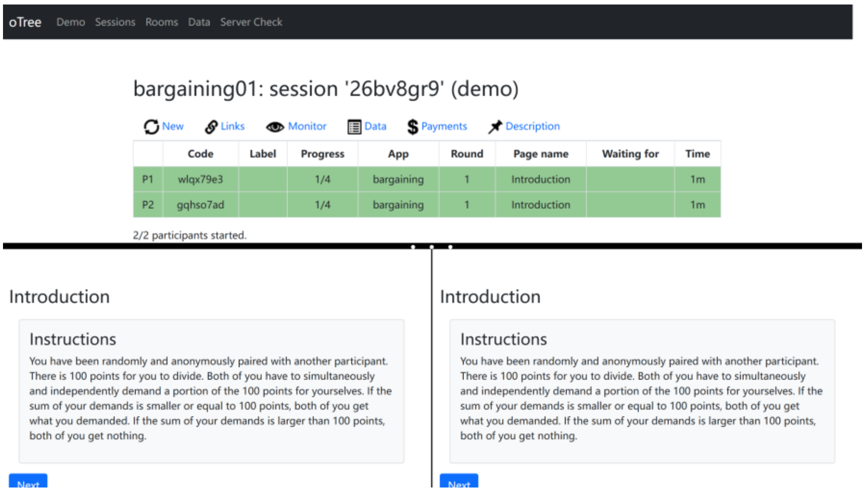
3a — oTree Deployment & Human Experiment

The bargaining game was implemented as an oTree app with:

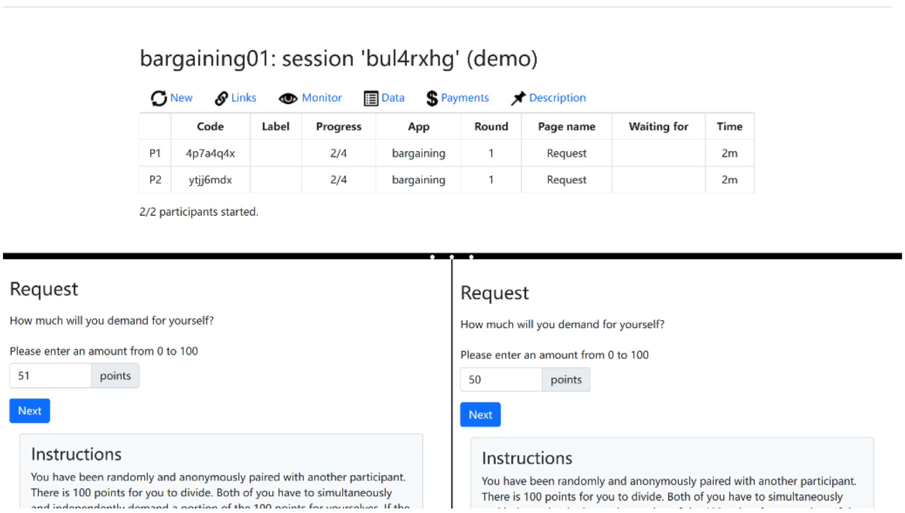
- **Instructions page:** Explains game rules and payoffs.
- **Decision page:** Each participant selects an integer demand from 1–100.
- **Results page:** Displays chosen demands, total payoff, and whether each player received their demand.

Screenshots placeholders:

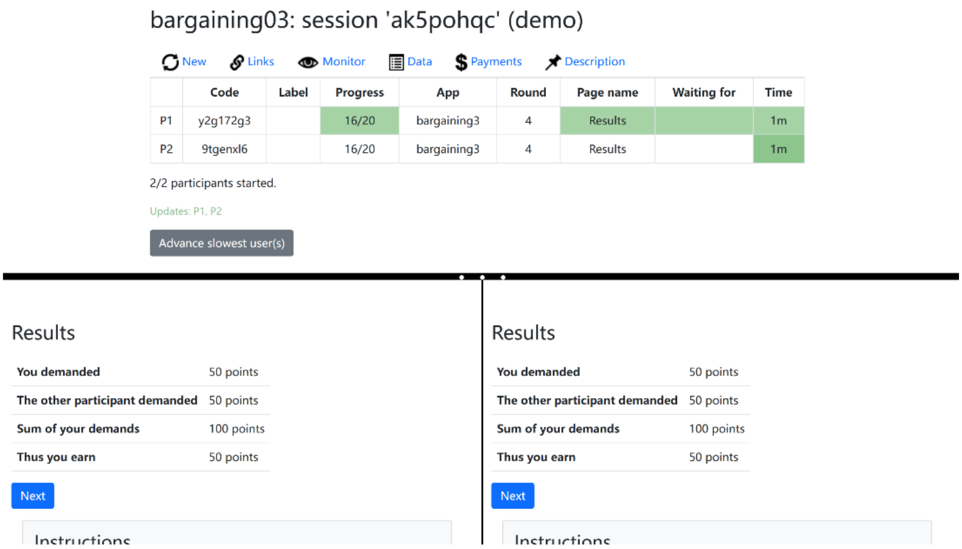
- **Instructions page**



- **Decision page**



- **Results page**



Human session (two classmates, pre-assigned group):

Round	Player 1 Demand	Player 2 Demand	Player 1 Payoff	Player 2 Payoff
1 round	51	50	0	0
2 rounds	51	50	0	0
2nd round	51	25	0	0
5 rounds	1st round: 51	25	0	0
	2nd round: 49	51	0	0
	3rd round: 50	50	50	50
	4th round: 50	50	50	50
	5th round: 50	50	50	50

Post-play interviews:

- Participants reported that after some initial trial-and-error, it became natural to settle at 50–50 in later rounds, which was perceived as a stable and fair solution.

3b — LLM "ChatBot" Session

The same game was played with an LLM using the following prompt:

You are Player 1 in a 2-player bargaining game. Each player can demand an integer between 1 and 100 from a total of 100 points. If the sum of demands is $\leqslant$ 100, each receives their demand; otherwise, both get 0. Choose a number and explain your reasoning.

AI behavior:

Round	LLM Demand	Reasoning Summary
1	50	Chooses fair split to maximize guaranteed payoff
2	50	Maintains fairness and stability across repeated rounds
3	50	Consistently selects NE-aligned choice
4	50	Consistency due to payoff visibility and rational reasoning
5	50	Same as above, converging immediately to fair equilibrium

Interpretation:

- LLM chooses the NE-aligned fair split (50–50) from the first round and maintains it across repeated rounds.
- Contrasts with human behavior, where trial-and-error and bounded rationality delayed convergence to the stable 50–50 allocation.

3c — Comparative Analysis & Theory Building

Comparison Table:

Source	Observed Behavior	Alignment with NE
Theoretical NE (Part 1/2)	Any demand pair summing to 100	Matches pure-strategy NE
Human session	Initial over- or under-demand (51+50, 51+25), eventually stabilized to 50–50	Partial: trial-and-error before reaching stable NE

LLM session	Immediate fair split (50–50) across all rounds	Fully aligned with NE

Observations:

1. **Humans vs NE:**

- Humans initially over- or under-demand due to bounded rationality and risk considerations.
- After experiencing zero payoff, they converge to a stable fair split (50–50).

2. **LLM vs NE:**

- LLM selects the equilibrium immediately, reflecting rational reasoning without trial-and-error.
- Human convergence lag highlights the effect of bounded rationality, learning, and fairness considerations.

Behavioral explanation:

- Humans are influenced by fairness, risk aversion, and limited strategic foresight (Camerer et al., 2004; Fehr & Schmidt, 1999).
- LLM reasoning aligns closely with equilibrium predictions due to rational and consistent calculation of payoffs.

Potential refinement/new solution concept:

- Introduce a **"bounded-rationality equilibrium"** to account for human risk aversion and learning across rounds.
- Predicts that humans may initially deviate from NE to avoid zero payoff but converge to NE after feedback.
- Can be formalized as:

$$x_i^{BR} = \arg\max_{x_i \leq \theta_i} E\left[u_i\left(x_i, x_j\right)\right], \quad \theta_i < 100 - \min\left(x_j\right)$$

References

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