

# SarahBrown - CV - HW4

April 30, 2021

0.1 In this assignment, we will get acquainted with essential matrix and fundamental matrix. And recover the essential matrix from matched points of two scene with OpenCV. Furthermore, we will try to localize these matched points in the 3D space

```
[106]: # Read matched points from two scenes
```

```
import pickle
with open('data_points.pickle','rb') as handle:
    data=pickle.load(handle)
```

```
[107]: # Find fundamental matrix from the match points using OpenCV
```

```
import cv2

pts1 = data['x1']
pts2 = data['x2']

F, mask = cv2.findFundamentalMat(pts1,pts2,cv2.FM_8POINT)
pts1 = pts1[mask.ravel()==1] # remove outliers by only keeping inliers
pts2 = pts2[mask.ravel()==1] #
```

```
[108]: # "Find" essential matrix
```

```
import numpy as np

K = np.eye(3) # Take the camera intrinsic matrix matrix K to be identity.
E = K.T @ F @ K # The essential matrix is just the same as the fundamental
↳matrix when K = I

#E, mask = cv2.findEssentialMat(pts1,pts2,K,cv2.RANSAC) # alternatively, we may
↳try to find essential matrix directly
```

```
[109]: # Sanity check of the essential matrix
```

```
import numpy as np
hx1=np.hstack((pts1,np.ones((2000,1))))
hx2=np.hstack((pts2,np.ones((2000,1))))

rint=np.random.randint(hx1.shape[0])
hx2[rint].T @ E @hx1[rint] # this should be almost zero
```

[109]: 2.909400498296577e-11

- 1 Q1 (5 points) Compute a potential solution of  $R$  and  $t$  from  $E$ . Note that you will only get half of the points if you use `cv2.decomposeEssentialMat`

```
[110]: import numpy as np

# this function should be helpful. You probably want to call the function below
# instead of np.linalg.svd
def mySVD(E): # compute SVD  $E = U S V$  and enforcing  $\det(U)=\det(V)=1$ 
    U,S,V = np.linalg.svd(E)
    detU=np.linalg.det(U)
    detV=np.linalg.det(V)
    U=U/detU
    V=V/detV
    S=S*detU*detV
    return U,S,V

def compute_one_R_and_t_from_E(E):
    # Input:
    # E: essential matrix
    # Output:
    # R: rotation matrix (3x3)
    # t: translation (3x1)
    W = np.array([[0,-1,0],
                  [1,0,0],
                  [0,0,1]])
    #from Simon J. D Prince CV Text book equations 16.18 and 16.19
    #U, L, VT = np.linalg.svd(E)
    #tx = U @ np.diag(L) @ W @ np.transpose(U)
    #t = np.transpose(np.array([tx[2,1],tx[0,2],tx[1,0]]))
    #R = U @ np.linalg.inv(W) @ (VT)

    #from slides
    U, S, VT = mySVD(E)
    tx = np.transpose(VT) @ (W) @ np.diag(S) @ VT
    t = np.transpose(np.array([(tx[1,2]-tx[2,1])/2],((tx[2,0]-tx[0,2])/
    →2),((tx[0,1]-tx[1,0])/2)]))
    R = U @ np.transpose(W) @ VT

    return R,t
```

## 1.1 Testing solution of Q.1

```
[111]: R,t = compute_one_R_and_t_from_E(E)
```

```
tx= np.array([[0,t[2],-t[1]],  
              [-t[2],0,t[0]],  
              [t[1],-t[0],0]])
```

```
R@tx-E
```

```
[111]: array([[ -1.03683028e-06,  1.28662268e-05,  1.28662304e-05],  
              [-1.28662298e-05, -5.18415111e-07, -5.18415249e-07],  
              [ 1.28662273e-05,  5.18415023e-07,  5.18415160e-07]])
```

2 Q2.a (5 points) For two lines  $a_1 + \lambda_1 b_1$  and  $a_2 + \lambda_2 b_2$  in the 3-D space parametrized by  $\lambda_1$  and  $\lambda_2$  ( $a_1, a_2, b_1, b_2$  are length-3 vectors). Find the intersecting point between the two lines (the mid point between the closest points of the two lines) by derivating the expressions of the optimum  $\lambda_1$  and  $\lambda_2$ .

```
[112]: from matplotlib import pyplot as plt
```

```
import cvui
```

```
%matplotlib inline
```

```
math1 = cv2.imread('2a-1.jpg')
```

```
math2 = cv2.imread('2a-2.jpg')
```

```
math1 = cv2.cvtColor(math1, cv2.COLOR_BGR2RGB)
```

```
math2 = cv2.cvtColor(math2, cv2.COLOR_BGR2RGB)
```

```
plt.figure(figsize=(40,40))
```

```
plt.subplot(121), plt.imshow(math1), plt.title("Minimization"), plt.xticks([],  
↪plt.yticks([])
```

```
plt.subplot(122), plt.imshow(math2), plt.title("Solving Linear Equation"), plt.  
↪xticks([], plt.yticks([])
```

```
[112]: (<AxesSubplot:title={'center':'Solving Linear Equation'}>,  
       <matplotlib.image.AxesImage at 0x7fd777063460>,  
       Text(0.5, 1.0, 'Solving Linear Equation'),  
       ([], []),  
       ([], []))
```

Minimization

$$\min_{\lambda_1, \lambda_2} \|a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2\|^2$$

$\lambda_1, \lambda_2$  - scalar  
 $a_1, a_2, b_1, b_2$  - 1x3

$$= \min_{\lambda_1, \lambda_2} \langle a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2, a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2 \rangle$$

$$= \min_{\lambda_1, \lambda_2} \langle a_1, a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2 \rangle + \langle \lambda_1 b_1, a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2 \rangle + \langle -a_2, a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2 \rangle + \langle \lambda_2 b_2, a_1 + \lambda_1 b_1 - a_2 - \lambda_2 b_2 \rangle$$

$$= \min_{\lambda_1, \lambda_2} \langle a_1, a_1 \rangle + \lambda_1 \langle a_1, b_1 \rangle - a_2 \langle a_1, a_1 \rangle - \lambda_2 \langle a_1, b_2 \rangle + \lambda_1 \langle b_1, a_1 \rangle + \lambda_1^2 \langle b_1, b_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle - \lambda_1 \lambda_2 \langle b_1, b_2 \rangle + a_2 \langle a_1, b_2 \rangle + \lambda_2 \langle a_2, b_1 \rangle - \lambda_2^2 \langle b_2, b_2 \rangle$$

$$= \min_{\lambda_1, \lambda_2} A \quad \text{where } A \text{ equals all of these}$$

$$\frac{\partial A}{\partial \lambda_1} = \frac{\partial}{\partial \lambda_1} \left[ \langle a_1, a_1 \rangle + \lambda_1 \langle a_1, b_1 \rangle - a_2 \langle a_1, a_1 \rangle - \lambda_2 \langle a_1, b_2 \rangle + \lambda_1 \langle b_1, a_1 \rangle + \lambda_1^2 \langle b_1, b_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle - \lambda_1 \lambda_2 \langle b_1, b_2 \rangle + a_2 \langle a_1, b_2 \rangle + \lambda_2 \langle a_2, b_1 \rangle - \lambda_2^2 \langle b_2, b_2 \rangle \right]$$

$$= \langle a_1, b_1 \rangle + 2\lambda_1 \langle b_1, b_1 \rangle - \lambda_2 \langle b_1, b_2 \rangle - a_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle$$

$$= \langle a_1 - a_2, b_1 \rangle + 2\lambda_1 \langle b_1, b_1 \rangle - \lambda_2 \langle b_1, b_2 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle$$

$$= \langle a_1 - a_2, b_1 \rangle + 2\lambda_1 \langle b_1, b_1 \rangle + a_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, b_2 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle$$

$$= 2\langle a_1 - a_2, b_1 \rangle + 2\lambda_1 \langle b_1, b_1 \rangle - 2\lambda_2 \langle b_1, a_1 \rangle - 2\lambda_2 \langle b_1, a_2 \rangle$$

To find min set equal to 0.  $0 = 2\langle a_1 - a_2, b_1 \rangle + 2\lambda_1 \langle b_1, b_1 \rangle - 2\lambda_2 \langle b_1, a_1 \rangle - 2\lambda_2 \langle b_1, a_2 \rangle$

$$\rightarrow \lambda_1 \langle b_1, b_1 \rangle + \langle a_1 - a_2, b_1 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle = 0$$

$$\frac{\partial A}{\partial \lambda_2} = \frac{\partial}{\partial \lambda_2} \left[ \langle a_1, a_1 \rangle + \lambda_1 \langle a_1, b_1 \rangle - a_2 \langle a_1, a_1 \rangle - \lambda_2 \langle a_1, b_2 \rangle + \lambda_1 \langle b_1, a_1 \rangle + \lambda_1^2 \langle b_1, b_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle - \lambda_1 \lambda_2 \langle b_1, b_2 \rangle + a_2 \langle a_1, b_2 \rangle + \lambda_2 \langle a_2, b_1 \rangle - \lambda_2^2 \langle b_2, b_2 \rangle \right]$$

$$= -\langle a_1, b_2 \rangle - \lambda_1 \langle b_1, b_2 \rangle - a_2 \langle b_2, a_1 \rangle - \lambda_2 \langle b_2, a_2 \rangle + \langle a_2, b_1 \rangle$$

$$= -\langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, b_2 \rangle - \lambda_2 \langle b_2, a_1 \rangle - \lambda_2 \langle b_2, a_2 \rangle + \langle a_2, b_1 \rangle$$

$$= -\langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, b_2 \rangle - 2\lambda_2 \langle b_2, a_1 \rangle - 2\lambda_2 \langle b_2, a_2 \rangle + \langle a_2, b_1 \rangle$$

set equal to 0

$$0 = -\langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, b_2 \rangle - 2\lambda_2 \langle b_2, a_1 \rangle - 2\lambda_2 \langle b_2, a_2 \rangle + \langle a_2, b_1 \rangle$$

$$\lambda_1 \langle b_1, b_2 \rangle + \langle a_1 - a_2, b_2 \rangle - \lambda_2 \langle b_2, a_1 \rangle - \lambda_2 \langle b_2, a_2 \rangle = 0$$

so we have:

$$\lambda_1 \langle b_1, b_1 \rangle + \langle a_1 - a_2, b_1 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle = 0$$

$$\lambda_2 \langle b_2, b_2 \rangle + \langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle = 0$$

Solving Linear Equation

$$\lambda_1 \|b_1\|^2 + \langle a_1 - a_2, b_1 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle = 0$$

$$\lambda_1 \|b_1\|^2 = \lambda_2 \langle b_1, a_1 \rangle + \lambda_2 \langle b_1, a_2 \rangle - \langle a_1 - a_2, b_1 \rangle$$

$$\lambda_1 (\|b_1\|^2) (\|b_1\|^2)^{-1} = (\lambda_2 \langle b_1, a_1 \rangle + \lambda_2 \langle b_1, a_2 \rangle - \langle a_1 - a_2, b_1 \rangle) (\|b_1\|^2)^{-1}$$

$$\lambda_1 = (\lambda_2 \langle b_1, a_1 \rangle + \lambda_2 \langle b_1, a_2 \rangle - \langle a_1 - a_2, b_1 \rangle) (\|b_1\|^2)^{-1}$$

$$\lambda_2 \|b_2\|^2 - \langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle = 0$$

$$\lambda_2 \|b_2\|^2 - \langle a_1 - a_2, b_2 \rangle - \lambda_2 \langle b_1, a_1 \rangle - \lambda_2 \langle b_1, a_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle = 0$$

$$\lambda_2 (\|b_2\|^2 - \langle b_1, a_1 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_1 \rangle - \langle b_1, a_2 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_2 \rangle) - \langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle = 0$$

$$\lambda_2 (\|b_2\|^2 - \langle b_1, a_1 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_1 \rangle - \langle b_1, a_2 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_2 \rangle) = \langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle$$

$$\lambda_2 = \left( \langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle \right) (\|b_2\|^2 - \langle b_1, a_1 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_1 \rangle - \langle b_1, a_2 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_2 \rangle)^{-1}$$

$\lambda_2$  can be found with this equation which can then be used to calculate  $\lambda_1$  with its equation

$$\lambda_2 = \frac{\langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle}{\|b_2\|^2 - \langle b_1, a_1 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_1 \rangle - \langle b_1, a_2 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_2 \rangle}$$

$$\lambda_2 = \frac{\langle a_1 - a_2, b_2 \rangle - \lambda_1 \langle b_1, a_1 \rangle - \lambda_1 \langle b_1, a_2 \rangle}{\|b_2\|^2 - \langle b_1, a_1 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_1 \rangle - \langle b_1, a_2 \rangle \langle b_1, a_1 \rangle^{-1} \langle b_1, a_2 \rangle}$$

$$\lambda_1 = (\lambda_2 \langle b_1, a_1 \rangle + \lambda_2 \langle b_1, a_2 \rangle - \langle a_1 - a_2, b_1 \rangle) (\|b_1\|^2)^{-1}$$

$$\lambda_1 = \frac{\lambda_2 \langle b_1, a_1 \rangle + \lambda_2 \langle b_1, a_2 \rangle - \langle a_1 - a_2, b_1 \rangle}{\|b_1\|^2}$$

### 3 Q2.b (5 points) Implement the solution of Q2.a by completing the function below

```
[113]: def computeIntersection(a1,b1,a2,b2):
    # Input:
    # a1: Nx3 matrix (a1[i] = ith a1)
    # b1: Nx3 matrix (b1[i] = ith b1)
    # a2: Nx3 matrix (a2[i] = ith a2)
    # b2: Nx3 matrix (b2[i] = ith b2)
    # N.B. for the ith pair of lines, line 1: a1[i]+lambda1[i] b1[i] and
    # line 2: a2[i]+lambda2[i] b2[i]
    # Output:
    # points: Nx3 matrix (points[i] = the intersecting point for ith pair)
    points = np.empty(shape=np.shape(a1))

    n = len(a1)
    for i in range(n):
        a1i = a1[i]; a2i = a2[i]; b1i = b1[i]; b2i = b2[i]
        b1normsq = np.dot(b1i,b1i); b2normsq = np.dot(b2i, b2i)
        a1a2 = (a1i-a2i)

        lambda2 = (np.dot(a1a2,b2i)*b1normsq - np.dot(a1a2,b1i)*np.
        dot(b1i,b2i))/(b1normsq*b2normsq - np.dot(b1i,b2i)**2)
        lambda1 = (lambda2*(np.dot(b1i,b2i))-np.dot(a1a2,b1i)) / ((b1normsq))
```

```

    point = (a1i + (lambda1 * b1i) + a2i + (lambda2 * b2i))/2.0
    points[i] = point
return points

```

### 3.1 Testing solution of Q2.b

We will take camera center of view 1 as origin, so  $\mathbf{a}_1 = \mathbf{0}$ ,  $\mathbf{b}_1 = [x_1[0], x_1[1], 1]^\top$

And  $\mathbf{a}_2 = \mathbf{t}$ ,  $\mathbf{b}_2 = R[x_2[0], x_2[1], 0]^\top$

```

[114]: a1 = np.zeros((pts1.shape[0],3))
      b1 = np.hstack((pts1,np.ones((pts1.shape[0],1))))
      a2 = np.tile(t,(pts1.shape[0],1))
      b2 = np.hstack((pts2,np.ones((pts2.shape[0],1))))
      b2 = (R.T @ b2.T).T

      Xs=(computeIntersection(a1,b1,a2,b2)) # 3D coordinates of points in the first
      ↪ camera view
      Xps=(R@(Xs-t).T).T # 3D coordinates of points in the second camera view
      Xs,Xps

```

```

[114]: (array([[ -7.43604951,   48.66761496, -2829.18397952],
               [ -8.15478157,   13.76484284, -3733.82471541],
               [-12.49381847,   15.95569632, -3688.7495735 ]],
        ...,
        [ -40.50867914,   58.77384587, -7080.43115241],
        [ -52.5995618 ,   60.82071569, -6708.83273944],
        [ -36.81195278,   69.25851921, -5232.83036157]]),
      array([[ 7.43603602, -58.23522507, 2722.28181318],
               [ 8.15476354, -93.13821581, 3626.92254063],
               [12.49380066, -90.94735145, 3581.84739928],
               ...,
               [40.50864514, -48.13002166, 6973.52898867],
               [52.5995296 , -46.0830621 , 6601.93057625],
               [36.81192776, -37.64490182, 5125.92820034]]))

```

## 4 Chirality

4.0.1 Note that  $X_s$  and  $X_p$ s are the points in the 3D space with camera centers as origins and with z-axis pointing from the camera centers to the objects. So the z-component (third column of  $X_s$  and  $X_p$ s) should be both non-negative because the object points suppose to be in front of the cameras. But with only 1/4 of the chance you would be lucky. Because there are four possible combinations of  $R$  and  $t$  and only one is correct (satisfies chirality).

5 Q2.c (4 points) Find the correct  $R$  and  $t$  by adjusting your solution in Q1. It is okay to provide a “buggy” solution that only works for the current dataset. Please redefine `compute_one_R_and_t_from_E(E)` below

```
[115]: import numpy as np

# this function should be helpful. You probably want to function below instead
# of np.linalg.svd
def mySVD(E): # compute SVD  $E = U S V$  and enforcing  $\det(U)=\det(V)=1$ 
    U,S,V = np.linalg.svd(E)
    detU=np.linalg.det(U)
    detV=np.linalg.det(V)
    U=U/detU
    V=V/detV
    S=S*detU*detV
    return U,S,V

def compute_one_R_and_t_from_E(E):
    # Input:
    # E: essential matrix
    # Output:
    # R: rotation matrix (3x3)
    # t: translation (3x1)
    W = np.array([[0,-1,0],
                  [1,0,0],
                  [0,0,1]])

    #from slides
    U, S, VT = mySVD(E)
    tx = np.transpose(VT) @ np.transpose(W) @ np.diag(S) @ VT
    t = np.transpose(np.array([((tx[1,2]-tx[2,1])/2),((tx[2,0]-tx[0,2])/
    ↪2),((tx[0,1]-tx[1,0])/2)]))
    R = U @ (W) @ VT

    return R,t
```

[116]: *# rerun everything below*

```
R,t = compute_one_R_and_t_from_E(E)

tx= np.array([[0,t[2],-t[1]],[-t[2],0,t[0]],[t[1],-t[0],0]])
rint=np.random.randint(hx1.shape[0])
hx2[rint].T @ R@tx @hx1[rint] # this should be almost zero
```

[116]: 4.630454680421177e-07

```
[117]: a1 = np.zeros((pts1.shape[0],3))
b1 = np.hstack((pts1,np.ones((pts1.shape[0],1))))
a2 = np.tile(t,(pts2.shape[0],1))
b2 = np.hstack((pts2,np.ones((pts2.shape[0],1))))
b2 = (R.T @ b2.T).T

Xs=computeIntersection(a1,b1,a2,b2)
Xps=(R@(Xs-t).T).T
Xs,Xps # note that third third columns should be positive
```

[117]: (array([[ 0.28687998, -1.8775782, 109.14884922],  
[ 0.23945037, -0.40417965, 109.63699108],  
[ 0.37123112, -0.47409453, 109.60449159],  
...,  
[ 0.61579632, -0.89345589, 107.63380911],  
[ 0.84394685, -0.97585321, 107.6415475 ],  
[ 0.7574854 , -1.42514355, 107.67678137]]),  
array([[ 0.28687946, -2.24669299, 105.02460402],  
[ 0.23944984, -2.73483479, 106.49800259],  
[ 0.37123059, -2.70233531, 106.42808771],  
...,  
[ 0.6157958 , -0.73165284, 106.00872628],  
[ 0.84394633, -0.73939123, 105.92632896],  
[ 0.75748488, -0.77462512, 105.47703862]]))

[118]: *# Sanity check: the projection to cameras should get back the original*

```
print(pts1[0],Xs[0]/Xs[0,2]) # the numbers should match if you did correctly
print(pts2[0],Xps[0]/Xps[0,2])
```

```
[ 0.00262833 -0.017202 ] [ 0.00262834 -0.017202  1.          ]
[ 0.00273155 -0.02139206] [ 0.00273155 -0.02139206  1.          ]
```

[119]: *#!pip install open3d # install open3d if you don't have it installed before*  
*# visualize your point cloud*

```
import open3d as o3d
```

```
cloud=o3d.geometry.PointCloud(o3d.utility.Vector3dVector(Xps))
o3d.visualization.draw_geometries([cloud])
```

**6 Q2.d (1 point): What is the object that you are seeing? Click the screen and move the mouse around and you can rotate it**

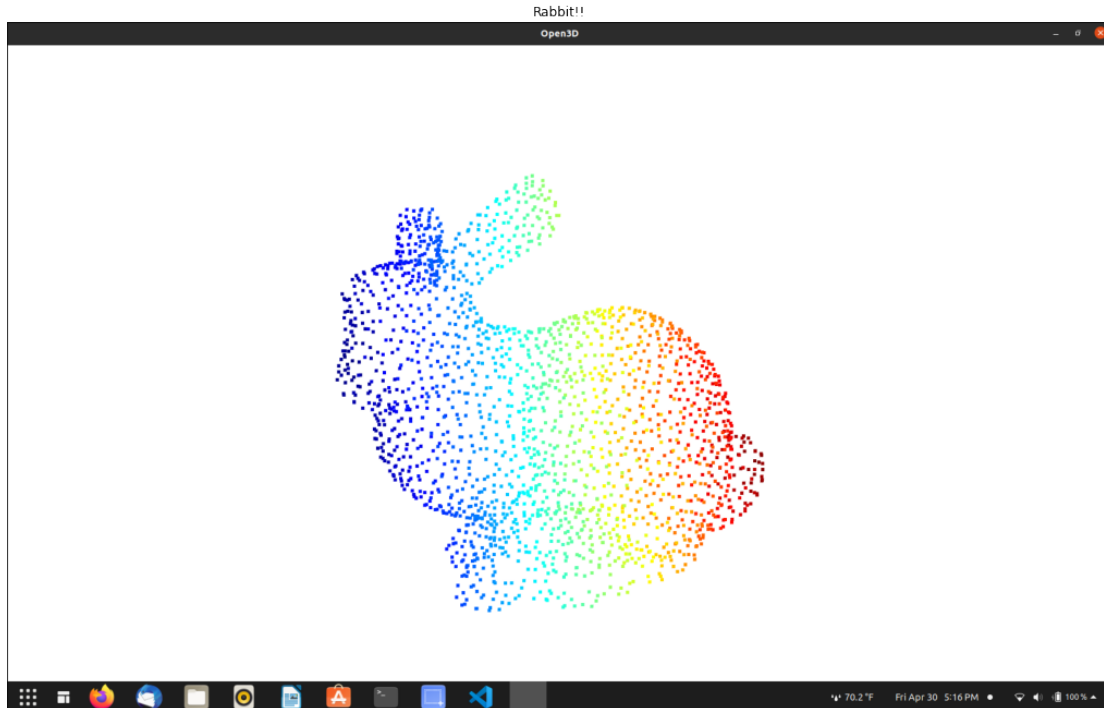
It's a rabbit! The blocks are rainbow colored depending on location.

```
[120]: rabbit = cv2.imread('rabbit.png')

rabbit = cv2.cvtColor(rabbit, cv2.COLOR_BGR2RGB)

plt.figure(figsize=(40,40))
plt.subplot(121), plt.imshow(rabbit), plt.title("Rabbit!!"), plt.xticks([], ),
plt.yticks([])
```

```
[120]: (<AxesSubplot:title={'center':'Rabbit!!'}>,
<matplotlib.image.AxesImage at 0x7fd776f23700>,
Text(0.5, 1.0, 'Rabbit!!'),
([], []),
([], []))
```





## 7 Q3 (10 points, extra credit): Capture two images and try to reconstruct 3D shape with the tools you developed.

It does something, I couldn't get good matches on my pictures of things so I reused the pics I took for homework 3 and got some matches at least.

```
[121]: import cv2
import numpy as np
from matplotlib import pyplot as plt
from scipy.stats import norm
import scipy.io as sio
%matplotlib inline

img1 = cv2.imread('File_000.jpg')
img2 = cv2.imread('File_001.jpg')

img1c = np.array(img1)
img2c = np.array(img2)

img1c=cv2.resize(img1c,(0,0),fx=0.5,fy=0.5)
img2c=cv2.resize(img2c,(0,0),fx=0.5,fy=0.5)

img1 = cv2.cvtColor(img1c, cv2.COLOR_RGB2GRAY)
img2 = cv2.cvtColor(img2c, cv2.COLOR_RGB2GRAY)

[122]: sift = cv2.xfeatures2d.SIFT_create() #SIFT()

# find the keypoints and descriptors with SIFT
kp1, des1 = sift.detectAndCompute(img1, None)
kp2, des2 = sift.detectAndCompute(img2, None)

# FLANN parameters
FLANN_INDEX_KDTREE = 0
index_params = dict(algorithm = FLANN_INDEX_KDTREE, trees = 5)
search_params = dict(checks=50)

flann = cv2.FlannBasedMatcher(index_params, search_params)
matches = flann.knnMatch(des1, des2, k=2)

good = []
points1 = []
points2 = []

# ratio test as per Lowe's paper
for i, (m, n) in enumerate(matches):
    if m.distance < 0.3*n.distance:
```

```

        good.append(m)
        points2.append(kp2[m.trainIdx].pt)
        points1.append(kp1[m.queryIdx].pt)
    print(len(good))

```

75

```

[123]: points1 = np.int32(points1)
        points2 = np.int32(points2)
        F, mask = cv2.findFundamentalMat(points1, points2, cv2.FM_LMEDS)
        # We select only inlier points
        points1 = points1[mask.ravel()==1]
        points2 = points2[mask.ravel()==1]

        print(len(points1))

        K = np.eye(3) # Take the camera intrinsic matrix K to be identity.
        E = K.T @ F @ K

```

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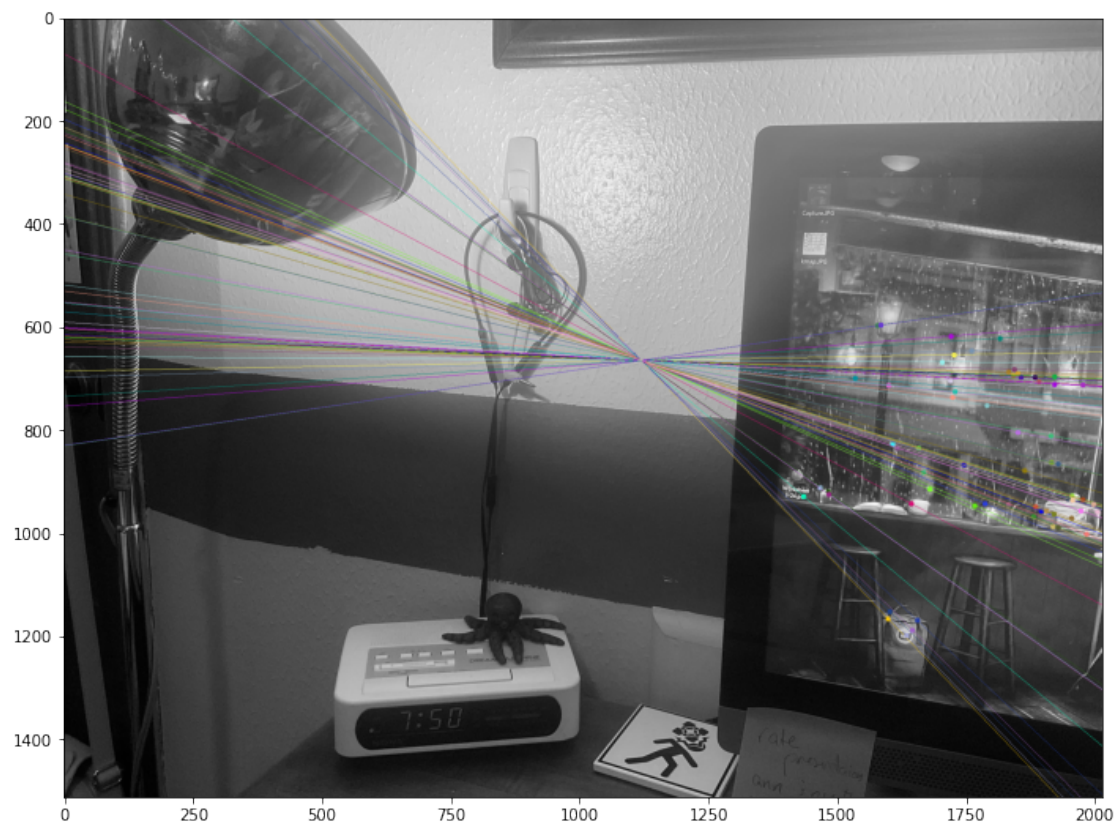
```

[124]: def drawlines(img1, img2, lines, pts1, pts2):
        ''' img1 - image on which we draw the epilines for the points in img2
            lines - corresponding epilines '''
        r, c = img1.shape
        img1 = cv2.cvtColor(img1, cv2.COLOR_GRAY2BGR)
        img2 = cv2.cvtColor(img2, cv2.COLOR_GRAY2BGR)
        for r, pt1, pt2 in zip(lines, pts1, pts2):
            color = tuple(np.random.randint(0, 255, 3).tolist())
            x0, y0 = map(int, [0, -r[2]/r[1] ])
            x1, y1 = map(int, [c, -(r[2]+r[0]*c)/r[1] ])
            img1 = cv2.line(img1, (x0, y0), (x1, y1), color, 1)
            img1 = cv2.circle(img1, tuple(pt1), 5, color, -1)
            img2 = cv2.circle(img2, tuple(pt2), 5, color, -1)
        return img1, img2

        # Find epilines corresponding to points in right image (second image) and
        # drawing its lines on left image
        lines1 = cv2.computeCorrespondEpilines(points2.reshape(-1, 1, 2), 2, F)
        lines1 = lines1.reshape(-1, 3)
        img5, img6 = drawlines(img1, img2, lines1, points1, points2)
        # Find epilines corresponding to points in left image (first image) and
        # drawing its lines on right image
        lines2 = cv2.computeCorrespondEpilines(pts1.reshape(-1, 1, 2), 1, F)
        lines2 = lines2.reshape(-1, 3)
        img3, img4 = drawlines(img2, img1, lines2, points2, points1)
        plt.figure(figsize=(40, 20))

```

```
plt.subplot(211),plt.imshow(img5)
plt.subplot(212),plt.imshow(img3)
plt.show()
```



```
[125]: import numpy as np

# this function should be helpful. You probably want to function below instead
→ of np.linalg.svd
def mySVD(E): # compute SVD  $E = U S V$  and enforcing  $\det(U)=\det(V)=1$ 
    U,S,V = np.linalg.svd(E)
    detU=np.linalg.det(U)
    detV=np.linalg.det(V)
    U=U/detU
    V=V/detV
    S=S*detU*detV
    return U,S,V

def compute_one_R_and_t_from_E(E):
    # Input:
    # E: essential matrix
    # Output:
    # R: rotation matrix (3x3)
    # t: translation (3x1)
    W = np.array([[0,-1,0],
                  [1,0,0],
                  [0,0,1]])

    #from slides
    U, S, VT = mySVD(E)
    tx = np.transpose(VT) @ np.transpose(W) @ np.diag(S) @ VT
    t = np.transpose(np.array([((tx[1,2]-tx[2,1])/2),((tx[2,0]-tx[0,2])/
→ 2),((tx[0,1]-tx[1,0])/2)]))
    R = U @ np.transpose(W) @ VT

    return R,t
```

```
[126]: R,t = compute_one_R_and_t_from_E(E)

tx= np.array([[0,t[2],-t[1]],
              [-t[2],0,t[0]],
              [t[1],-t[0],0]])
R@tx-E
```

```
[126]: array([[ -2.52325830e-01,  4.25325717e-01, -1.07580466e-03],
               [-3.76059159e-02,  6.33858196e-02,  2.17051297e-03],
               [ 1.26915358e-03,  1.14345838e-04, -1.50000051e+00]])
```

```
[127]: R,t = compute_one_R_and_t_from_E(E)
hx1=np.hstack((points1,np.ones((len(points1),1))))
hx2=np.hstack((points2,np.ones((len(points2),1))))

rint=np.random.randint(hx1.shape[0])
hx2[rint].T @ E @hx1[rint] # this should be almost zero
tx= np.array([[0,t[2],-t[1]],[-t[2],0,t[0]],[t[1],-t[0],0]])
rint=np.random.randint(hx1.shape[0])

hx2[rint].T @ R@tx @hx1[rint] # this should be almost zero
```

```
[127]: -83264.21678039154
```

```
[128]: a1 = np.zeros((points1.shape[0],3))
b1 = np.hstack((points1,np.ones((points1.shape[0],1))))
a2 = np.tile(t,(points2.shape[0],1))
b2 = np.hstack((points2,np.ones((points2.shape[0],1))))
b2 = (R.T @ b2.T).T

Xs=computeIntersection(a1,b1,a2,b2)
Xps=(R@(Xs-t).T).T
Xs,Xps # note that third third columns should be positive
```

```
[128]: (array([[ -4.22696758e-01,  -2.61971844e-01,   3.60352655e-04],
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```

```

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```

```
[129]: cloud=o3d.geometry.PointCloud(o3d.utility.Vector3dVector(Xps))
o3d.visualization.draw_geometries([cloud])
```

```
[130]: thing = cv2.imread('something.png')

thing = cv2.cvtColor(thing, cv2.COLOR_BGR2RGB)

plt.figure(figsize=(40,40))
plt.subplot(121), plt.imshow(thing), plt.title("Something?"), plt.xticks([],
↪plt.yticks([]))
```

```
[130]: (<AxesSubplot:title={'center':'Something?'}>,
<matplotlib.image.AxesImage at 0x7fd7a0a3d7f0>,
Text(0.5, 1.0, 'Something?'),
([], []),
([], []))
```

