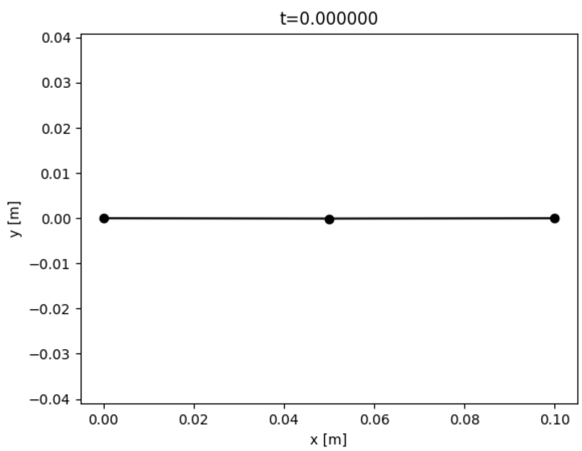
MAE 263F: Homework 1

I. Assignment 1

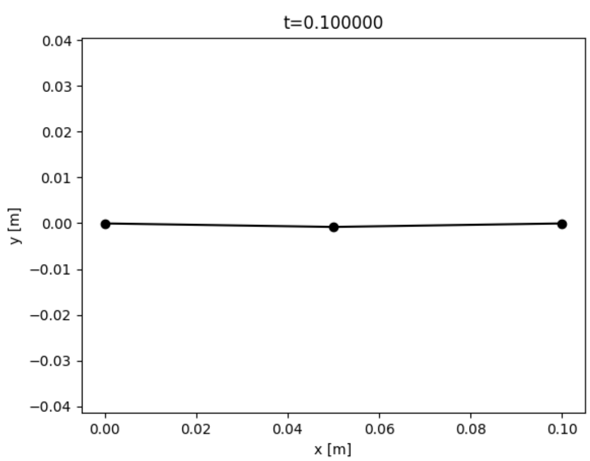
In assignment 1, we are asked to develop a solver that simulates the position and velocity of a sphere as a function of time (1) implicitly and (2) explicitly.

A. Show the shape of the structure at t = {0, 0.01, 0.05, 0.10, 1.0, 10} seconds. Then plot position and velocity of the second ball (R2) as a function of time.

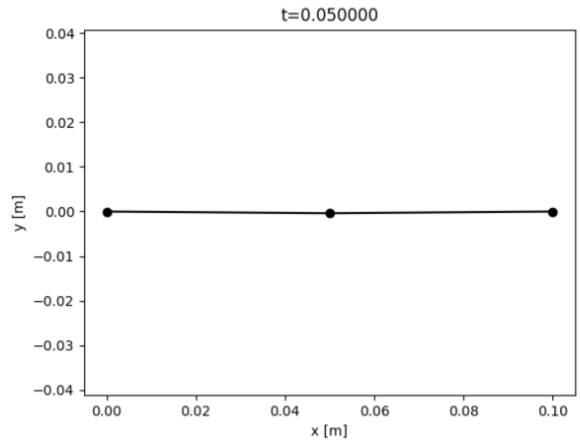
The following plots of the shape of the structure were generated using the implicit solver. We note that the three balls are initially arranged in a straight line until sphere 2 begins to drop. Sphere 2 initially drops slowly and then the velocity increases.



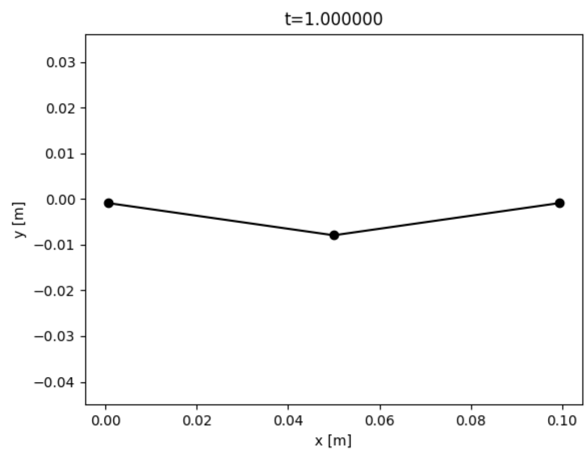
(a)



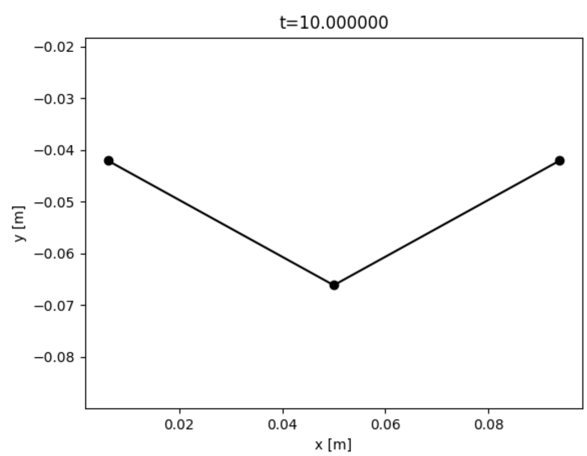
(b)



(c)



(d)



(d)

Figure 1: Plots (a)-(d) represent the shape of three rigid spheres are on an elastic beam that is falling under gravity in a viscous fluid.at t = {0, 0.01, 0.05, 0.10, 1.0, 10} seconds, respectively.

We now plot the position and the velocity of the second sphere as a function of time. We note that the velocity of the second sphere is initialized to 0, which explains the sharp "decrease" in the velocity observed in the following plot.

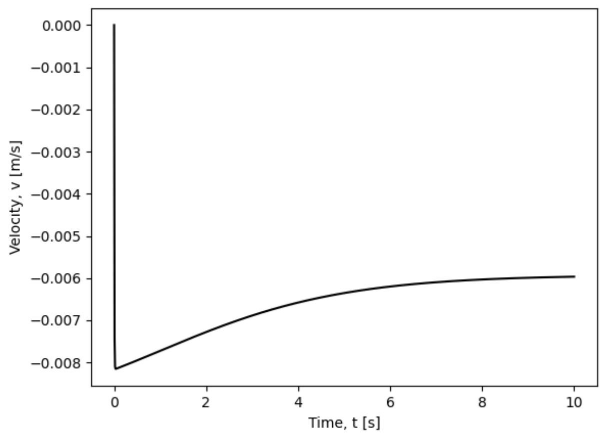


Figure 2: Velocity as a function of time for the second sphere when three rigid spheres are on an elastic beam that is falling under gravity in a viscous fluid.

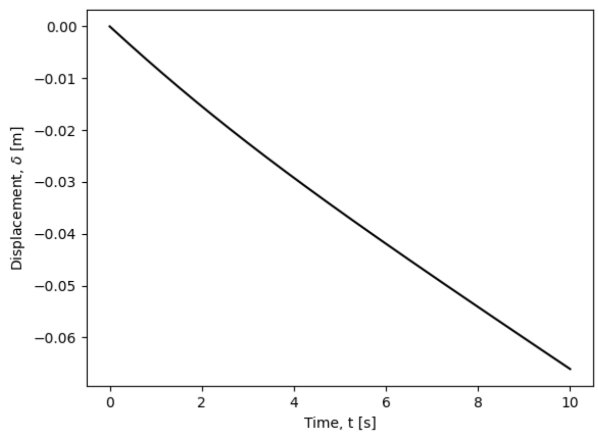


Figure 3: Displacement as a function of time for the second sphere when three rigid spheres are on an elastic beam that is falling under gravity in a viscous fluid.

B. What is the terminal velocity of the system?

We observe the terminal velocity of the system to be $v = -0.007\ m/s$.

C. What happens to the turning angle if all the radii (R1, R2, R3) are the same and does the simulation agree with your intuition?

Let's begin by assuming the radii for the three spheres are 0.005 m. Since all three spheres are the same size/density/material/weight, we expect them to sink at the same rate, and therefore for the turning angle to be equal to 0. We now plot the turning angle; we can observe that the turning angle begins at a very small value (which we initialized it to), and immediately goes to zero. These observations match our intuition.

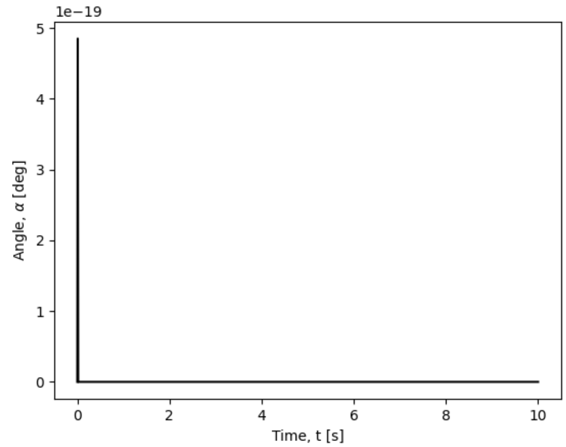


Figure 4: Turning angle as a function of time when three rigid spheres of equal radii are on an elastic beam that is falling under gravity in a viscous fluid.

D. Try changing the time step for your explicit simulation and use the observation to elaborate the benefits and drawbacks of the explicit and implicit approach.

As we decrease the timestep of our explicit solver, we note that it takes significantly longer to find a solution, which is a major drawback of the explicit approach. It will converge to the correct solution; however, to do so, it requires a timestep that is so computationally intensive that it is impractical. On the other hand, the implicit solver finds an approach almost simultaneously. Thus, the implicit method is more practical.

II. ASSIGNMENT 2

In assignment 2, we are tasked with writing a solver that simulates the system of 21 spheres as a function of time, implicitly.

A. Include two plots showing the vertical position and velocity of the middle node with time. What is the terminal velocity?

We plot the position and velocity of the middle node as a function of time; we observe similar behavior to the three spheres case presented in Assignment 1.

The velocity is initialized to $v = 0\ m/s$, and then jumps to roughly $-0.007\ m/s$ until it approaches terminal velocity. The terminal velocity of the middle node is around $v = -0.0022$.

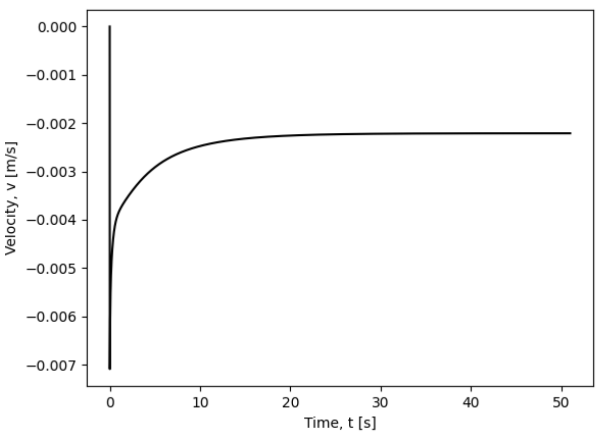


Figure 5: Velocity as a function of time for N = 21 rigid spheres attached to an elastic beam falling in a viscous fluid.

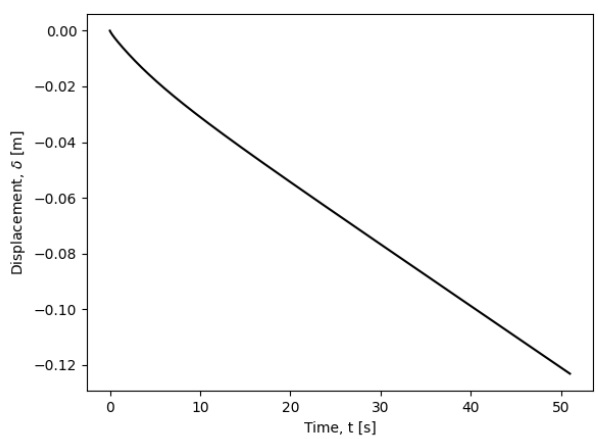


Figure 6: Displacement as a function of time for N = 21 rigid spheres attached to an elastic beam falling in a viscous fluid.

In terms of displacement, we observe a gradually increasingly rapid and negative displacement for the middle node.

B. Include the final deformed shape.

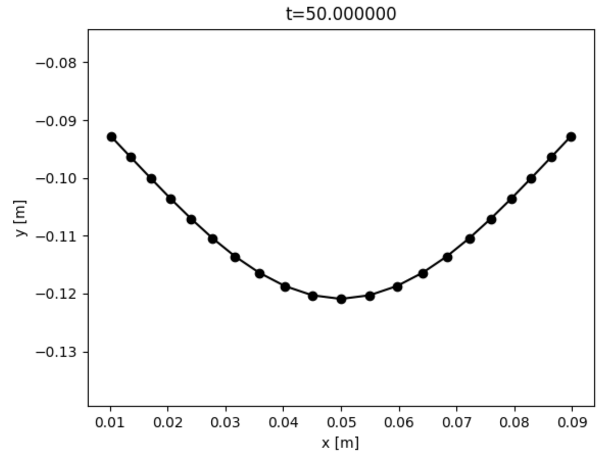


Figure 7: Final deformed structure of N = 21 rigid spheres attached to an elastic beam falling in a viscous fluid.

We observe a uniformly deformed shape in figure 7 for the N=21 spheres case.

C. Discuss the significance of spatial discretization and temporal discretization.

As we increase the number of nodes, we represent our system better and so our accuracy increases; unfortunately, so does our computational effort. The same occurs when we increase our time step: the accuracy and the computational effort both increase.

We plot the terminal velocity versus number of nodes to show how accuracy increases. Note, the change in terminal velocity is minimal, as expected, but the scale is adjusted so we can observe the change. We plot this for N = {20, 25, 30, 35, 40, 45} nodes. We keep the timestep constant at dt = 10e-2.

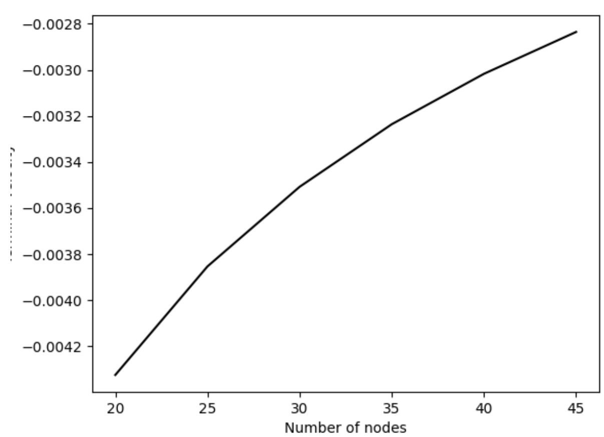


Figure 8: Terminal velocity as a function of the number of nodes in the system for N = {20, 25, 30, 35, 40, 45} rigid spheres attached to an elastic beam falling in a viscous fluid.

We repeat this for the time step, to observe the affect that decreasing the time step has on the accuracy. We keep the number of nodes constant at N = 21. For simplicity, we choose the time steps t = {0.1, 0.01, 0.005, 0.001}. We can clearly see the accuracy of the terminal velocity increasing as the time step decreases.

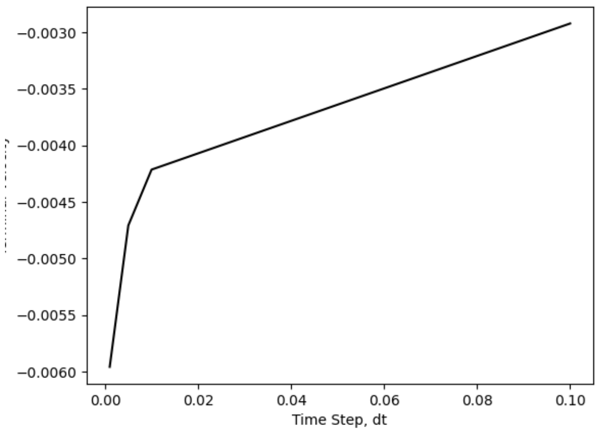


Figure 9: Terminal velocity as a function of the time step in the system for N = 21 rigid spheres attached to an elastic beam falling in a viscous fluid.

III. ASSIGNMENT 3

In Assignment 3, we are asked to write a solver that simulates elastic beam bending as a function of time implicitly.

A. Plot the maximum vertical displacement of the beam as a function of time.

In figure 10, we plot the maximum vertical displacement of an elastic beam bending as a function of time. We note that the maximum vertical displacement, y_{max}, reaches steady state at approximately t = 0.4 seconds and $y_{max} = -0.035$.

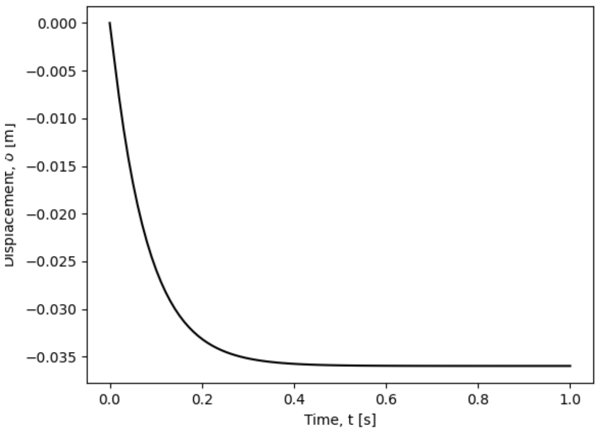


Figure 10: Displacement as a function of time for N= 50 spheres for an elastic beam bending with a point load of P=2000 applied at d = 0.075 m.

We wish to examine the accuracy of this simulation compared to Euler beam theory. In Euler beam theory, we utilize the equation:

$$y_{max} = \frac{Pc(L^2-c^2)^{1.5}}{9\sqrt{3}EIl} \quad \text{where } c = \min\,(d, l-d) \qquad (1)$$

Plugging in the provided values for this situation, we find $y_{max} = \frac{Pc(L^2-c^2)^{1.5}}{9\sqrt{3}EIl} = -0.038$, or an error of 7.8%.

B. Benefits of simulation over the predictions from beam theory.

Euler beam theory is valid only for small deformations. We consider a higher load $P = 20000$ such that the beam undergoes a large deformation. We simulate the result and note that the maximum vertical displacement at steady state is $y_{max} = -0.023$ according to our simulation (see figure 11).

However, according to Euler beam theory and equation (1), if we plug in the requested values we obtain $y_{max} = -0.38$, or a 1552% error (quite large).

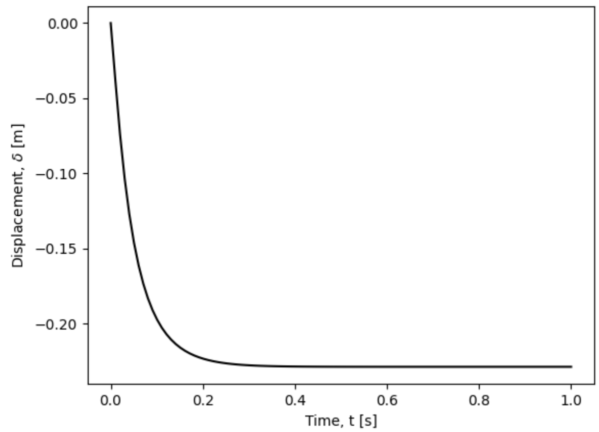


Figure 11: Displacement as a function of time for N= 50 spheres for an elastic beam bending with a point load of P=20,000 applied at d = 0.075 m.