Midterm Presentation:
Snake Project

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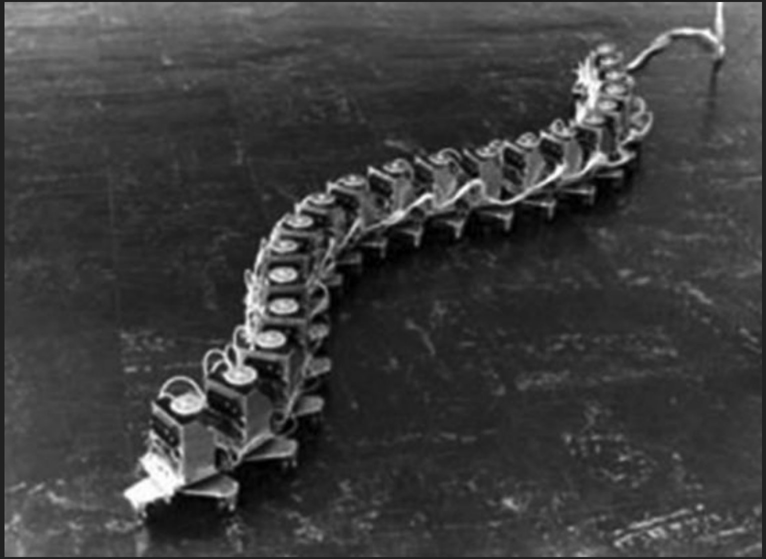
Introduction

Project Goal: develop an analytical approach to simulating snake-like motion in viscous fluid



ACM III - world's first snake robot developed by Prof. Shigeo Hirose in 1972[1].

Mamba - Amphibious snake robot [1]

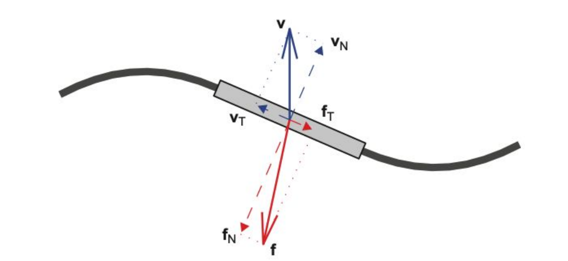
Two Mathematical Models Considered for Simulating Snake Motion in Fluid

Resistive Force Theory - Anisotropic Drag

- Hancock [2] - two forms of singular solutions to flow around a slender body including the (1) “Stokeslet” and (2) a potential dipole. By placing both solutions along centerline of a slender body the approximate solution for flow near this body is given by:

$$C_t = \frac{2\pi\mu}{\ln\left(\frac{2\lambda}{R}\right) - 0.5}$$, where Cn / Ct ~ 2

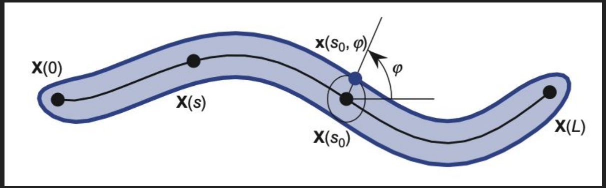
- Takes advantage of the linearity of Stokes flow. Many variations of this theory by Gray and Hancock, Lighthill, and Cortez et al.



RFT uses local tangential and normal components of velocity to compute the anisotropic drag which enables forward motion [3].

Slender Body Theory - Long Range Hydrodynamics

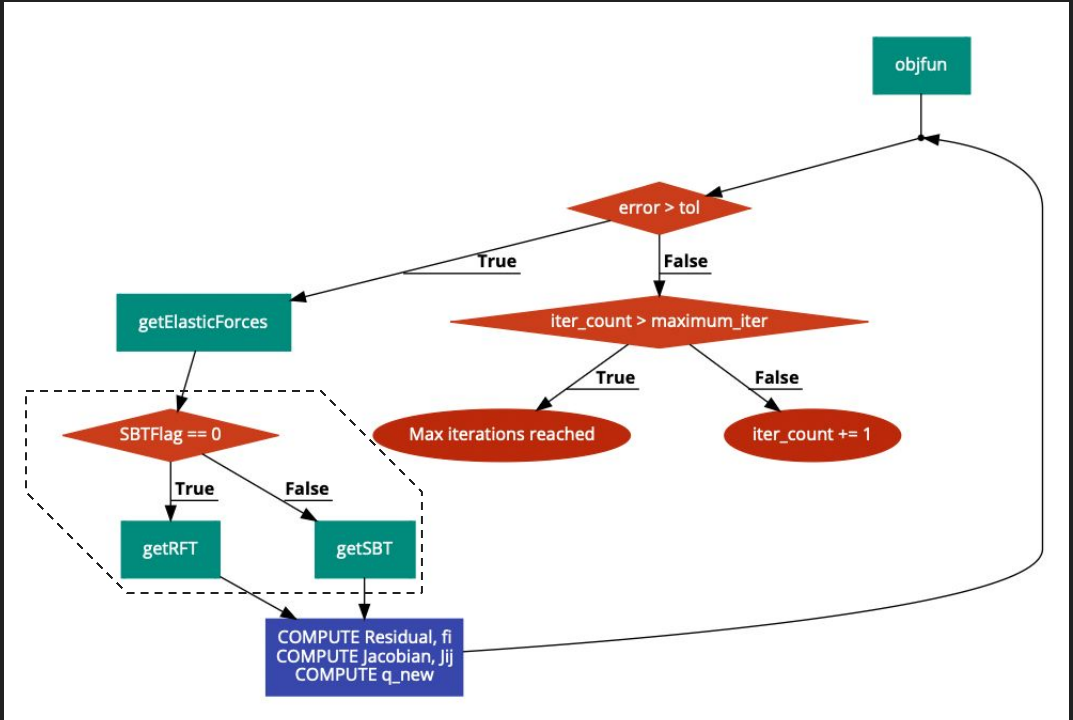
- More general theory, that involve summing the singular forces along a discrete body based on proximity to nonlocal flow-fields [3].
- More computationally complex.



SBT sums a set of singularities along a centerline to capture local and nonlocal flow [3].

DER Objective Function Updated to Include Extraction of Hydrodynamic Forces

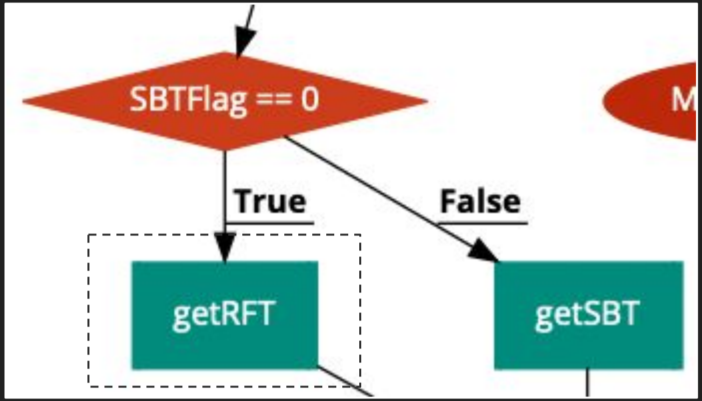
1. User-defined *SBTFlag* governs choice between *RFT* and *SBT*
2. Hydrodynamic viscous forces derived using chosen theory
3. Forces added to the residual calculation
4. Jacobian updated
5. Error and *q_new* computed



Hydrodynamics Extraction Added to Objective Function

getRFT() Functionality

- Takes inputs for μ, R, and L in addition to DOF values
- Computes local tangent and normal vectors between edges and accounts for end effects
    - If Re <= 1:
        - $F\square = -C\square u\square \Delta L$
        - $F\square = -C\square u\square \Delta L$
    - Elif Re > 1:
        - $F\square = -C\square \rho_x A|u\square|u\square$
        - $F\square = -C\square \rho_x A|u\square|u\square$
    - Fhydro += $F\square + F\square$
- Returns Fhydro



getRFT is called when SBTFlag == 0

getSBT() Functionality For 2D Motion

- Takes inputs for $\boldsymbol{\mu}$, R, and L in addition to DOF values
- Computes the normal drag for each node
  - $f_{\square,i} = -4\pi\mu u_{\square}$
- Computes force per unit length due to stokeslet interaction
  - Regularized Green's Function [4]:

    $S_{i\square}(r) = \delta_{i\square}\,(r^2 + 2\varepsilon^2 + r_i r_{\square}) / (r^2 + \varepsilon^2)^{3/2}$

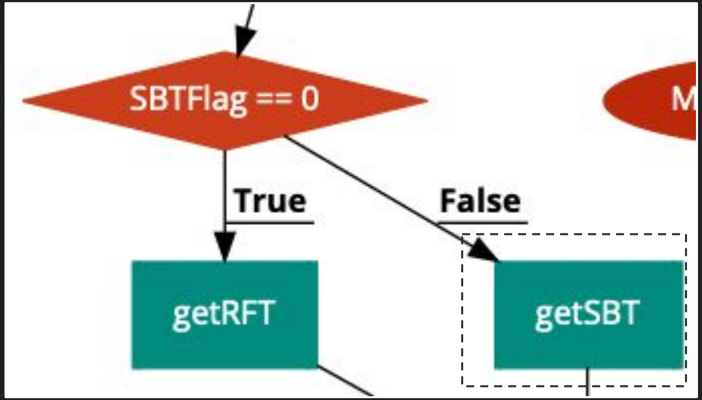
  - Stokeslet Fluid Flow:

    $u_{\square}(r) = 1 / (8\pi\mu)\ \Sigma_{\square}\Sigma_i\ S_{i\square}(r - r_{\square})f_{\square,i} \rightarrow 2k \times 1$

    Let $A_{\square\ i\square} = \Sigma_{\square}\Sigma_i\ S_{i\square}(r - r_{\square}) \rightarrow 2k \times 2k$

    $f_{\square,i} = (8\pi\mu) \cdot A_{\square\ i\square} \backslash\ u_{\square}(r) \rightarrow 2k \times 1$

- Fhydro = $(f_{\square,i} + f_{\square,i}) \cdot \Delta L$
- Returns Fhydro



getSBT is called when SBTFlag == 1

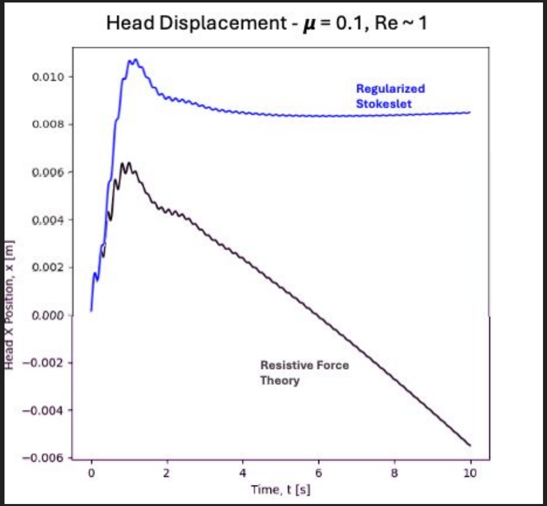
Parameters for Verification

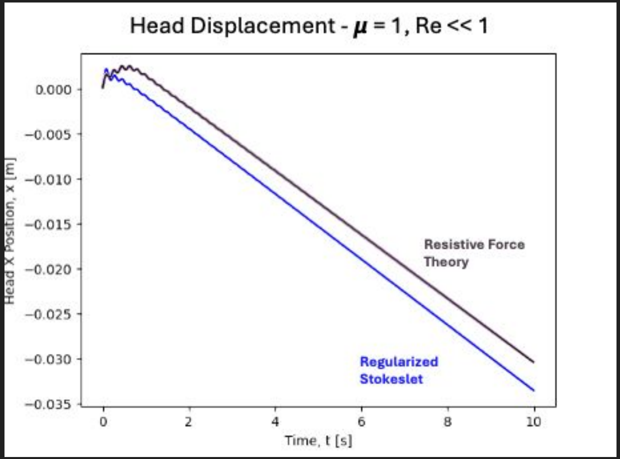
- Studies:
  - Do viscous force dissipate with lower viscosity?
  - Does SBT approach RFT at low Re?
  - How does body motion symmetry affect net thrust?
  - Do the net viscous forces achieve propulsion?

- $Y = 7e10$ Pa
- $\rho = 2700$ kg / m^3
- $R = 0.01$ m
- $L = 4$ m
- $Nv = 15$
- $Dt = 5$ ms
- Head Amplitude = 0.1 m
- Head Frequency = 2.83 Hz

Verification #1 (1 / 2): RFT and Regularized Stokeslet (RS) Convergence at Low Re (< 1)

- RFT and RS results should converge if Re << 1
- For slender bodies (L/R >> 1) RFT and RS methods generally match closely at low Re
- Divergence expected as Re approaches and exceeds 1 as long range hydrodynamics further dampen motion in RS (RFT ignores these effects)

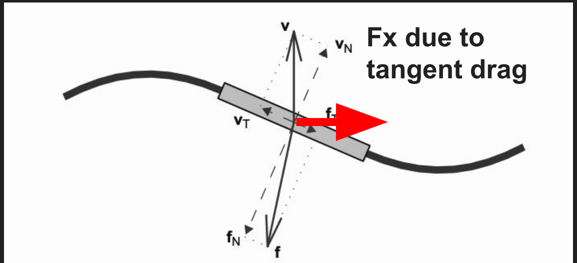


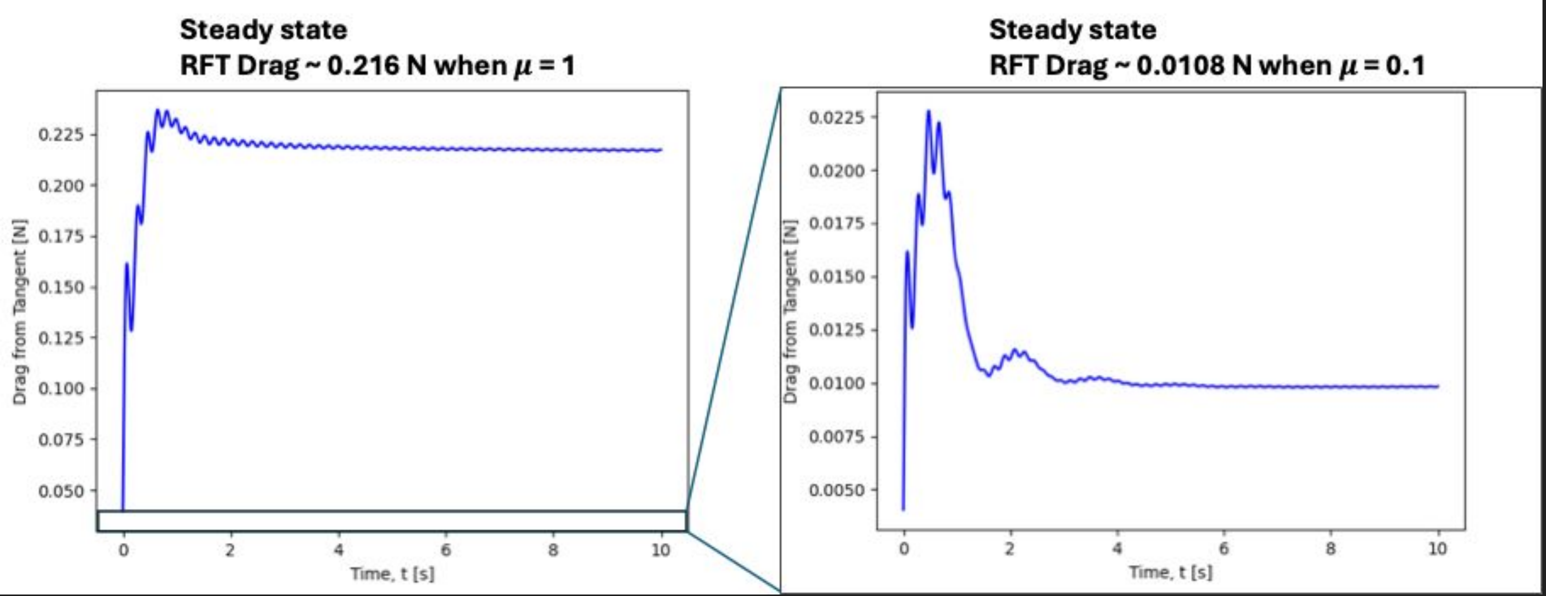


But why is total displacement increasing for RFT?

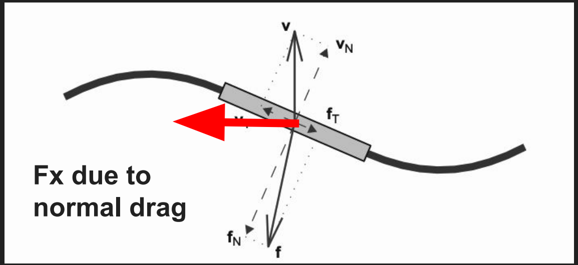
RFT and SBT Converge at Higher Viscosities and Lower Re

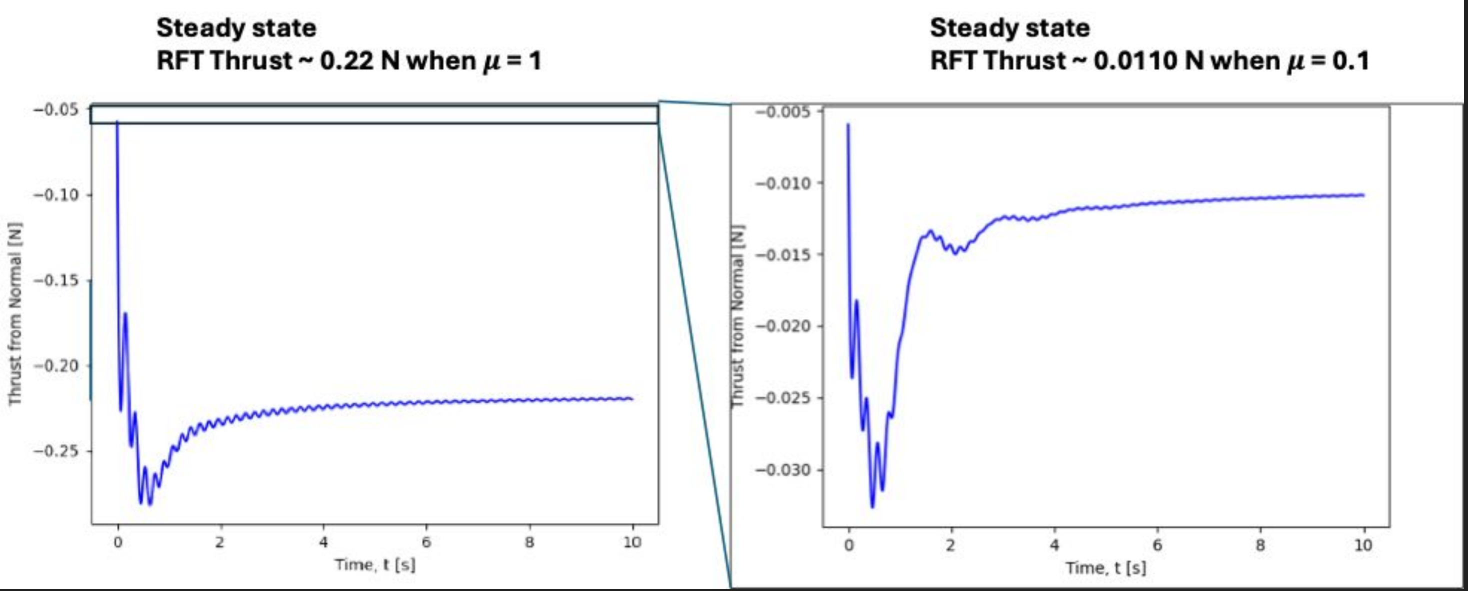
RFT Thrust Due to tangent Increases with Mu



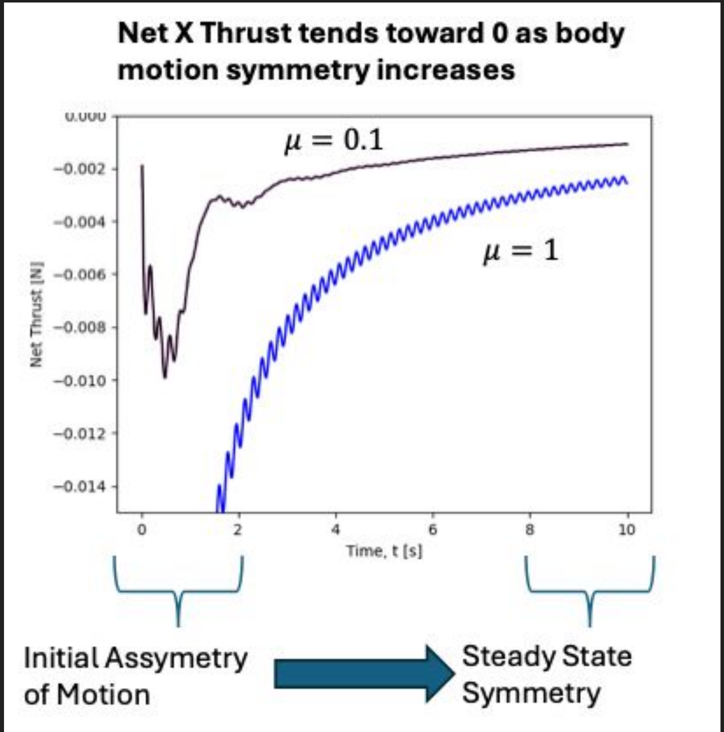


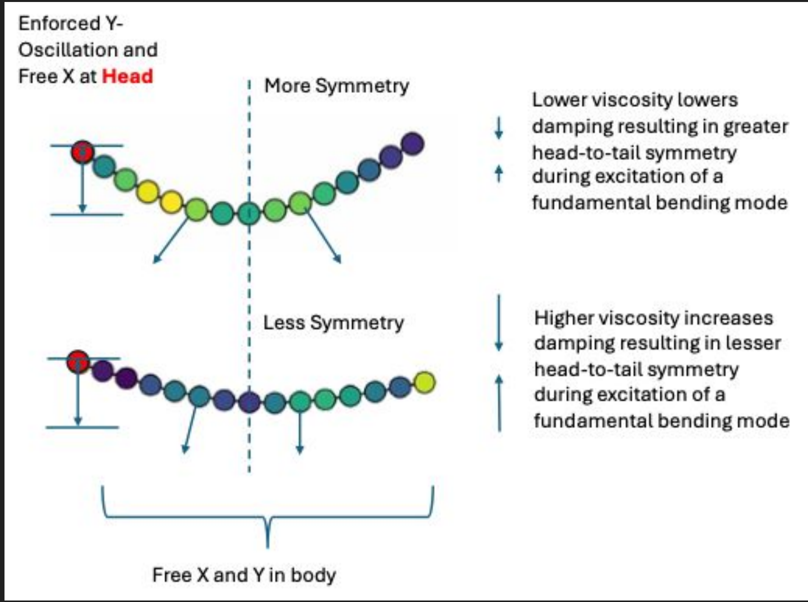
RFT Thrust Due to Normal Increases with Mu





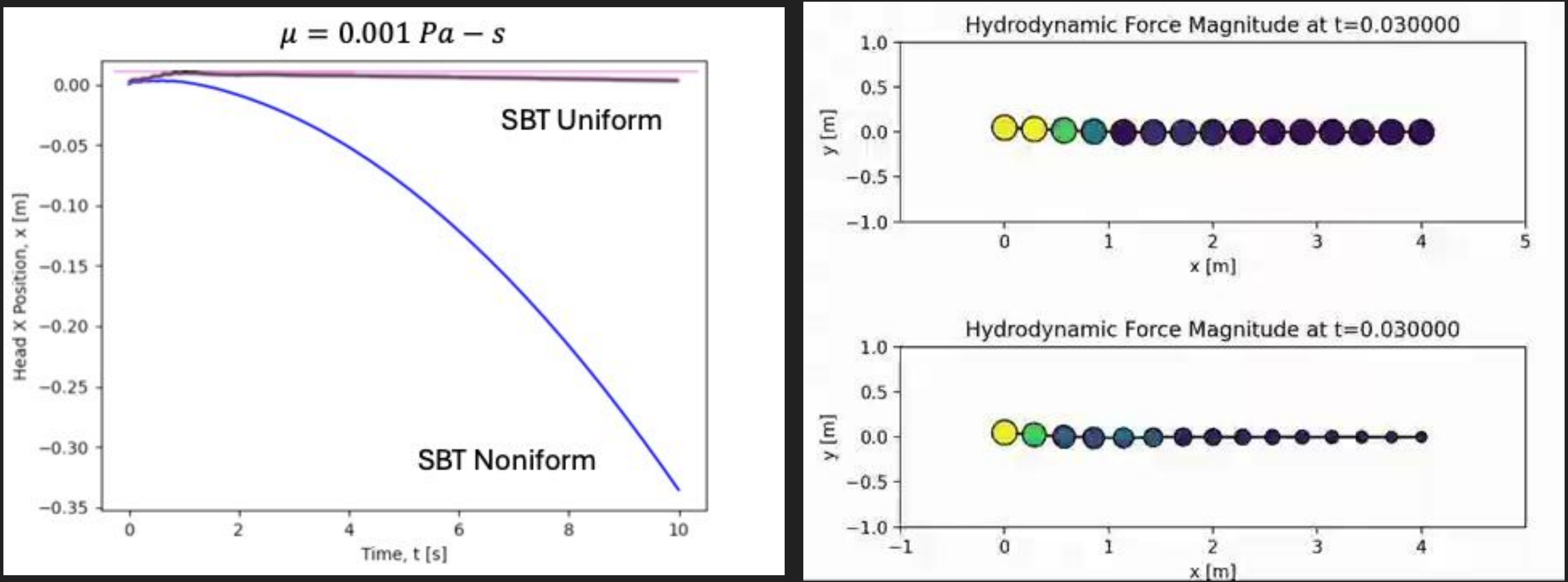
Body Motion Symmetry Dampens Net Thrust for a Flexible Body





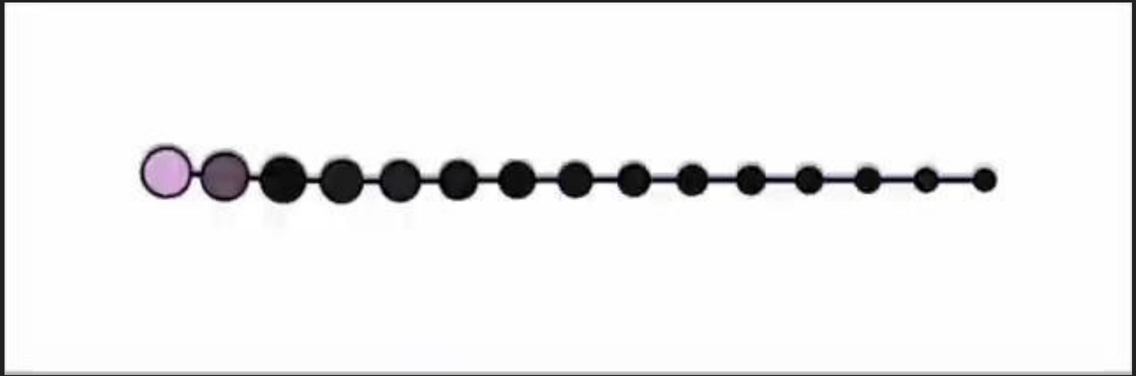
Takeaway: As viscosity increases so does the tangential and normal components of drag. The net drag however is dependent on the symmetry of the body motion in RFT which tends to increase with lower viscosity for a flexible body

When Redistributing the Same Total Mass to Slim Down Towards the Tail the Body Exhibits Greater Motion Assymmetry



Adding Assymetry Produces Greater Sensitivity to Long Range Hydrodynamics

- Narrowing the snake body as you approach the tail makes for differential mass and stiffness across body
- Therefore modal behavior is asymmetric and results in greater net thrust for low viscosities

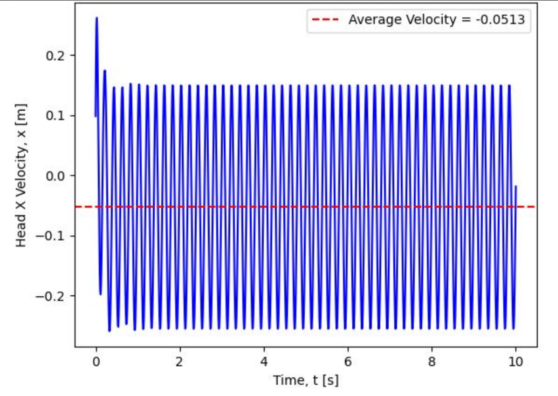


Verification #2 (RFT): Viscosity Effects

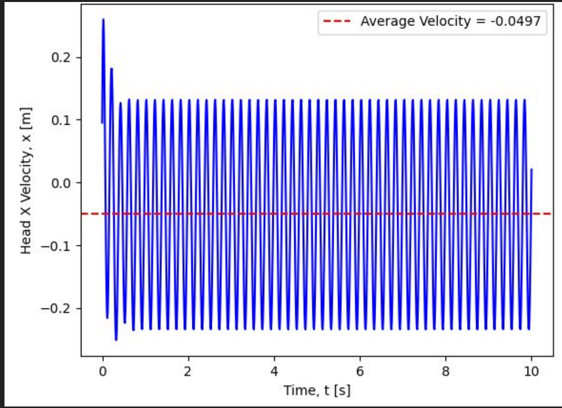
$$F_{drag} \propto \mu * v$$

- As viscosity increases, the drag force should also increase → decrease in velocity

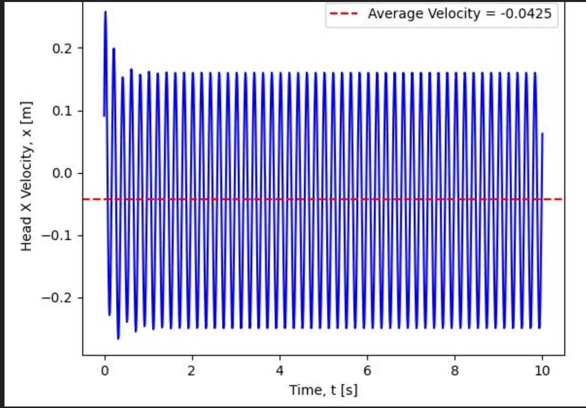
Case 1: mu = 1.1 Pa-s



Case 2: mu = 1.9 Pa-s

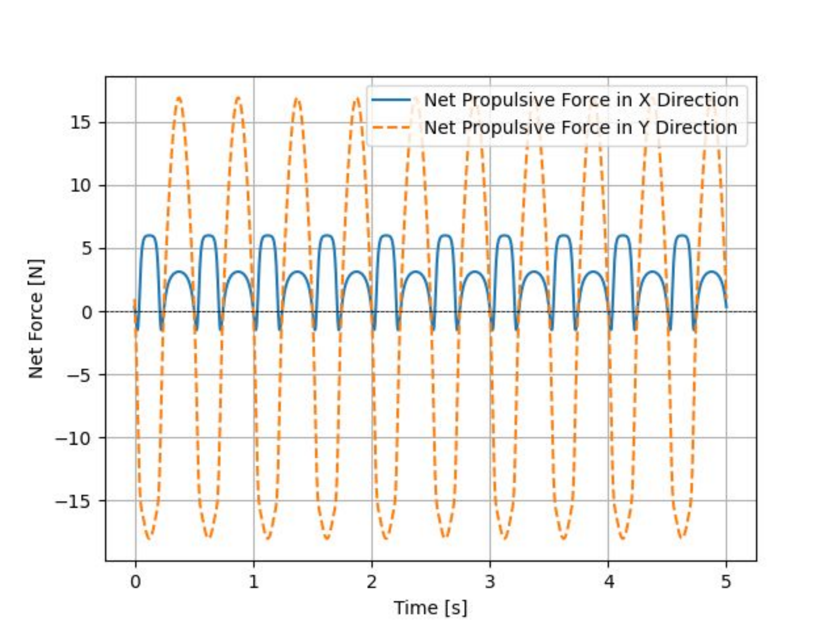


Case 3: mu = 3 Pa-s



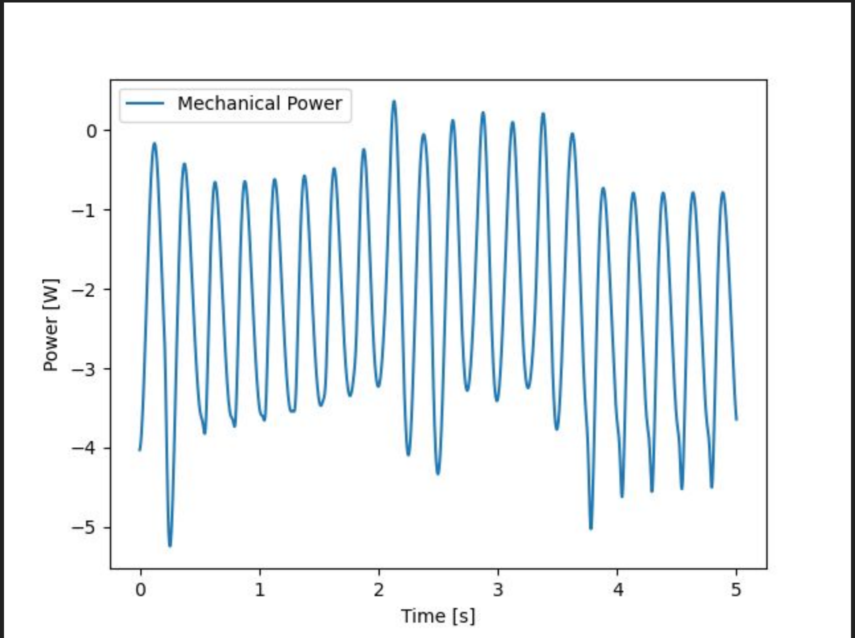
Validation #3 (RFT): Sum of Net Forces

- Oscillatory but has net positive force in x-direction (indicates forward propulsion)
- Net zero force in y-direction, as expected
- Strong oscillations in y-direction expected for lateral undulary motion
  - Indicates snake is pushing laterally against surrounding fluid to generate thrust



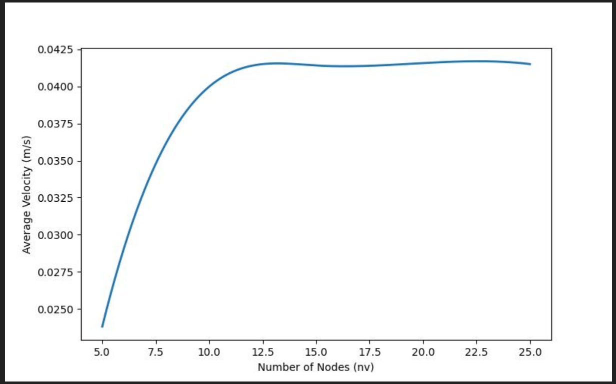
Verification #4 (RFT): Power over Time

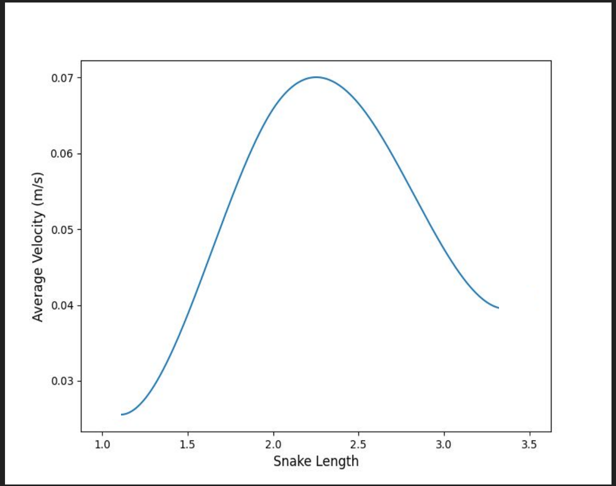
- Power consumption is at first **transient** before reaching **steady state oscillations** (initial increase in power to power the snake)
    - Expected since its starting from rest
- Likely much power being lost resisting drag forces (low Reynolds number means drag is significant compared to inertia)



Sensitivity Analysis: Parameter Determination (Using RFT)

- Sensitivity analysis using average velocity of head
  - Number of nodes: 12
    - Higher number → smoother motion → better thrust
    - But also more computational efficiency
  - Length of snake:
    - Longer → more undulation cycles → more propulsion
    - But also increase in drag (more surface area)





References

[1] Liljebäck, Pål, et al. *Snake robots: modelling, mechatronics, and control*. London: Springer, 2013.

[2] Gray, J., Hancock. *The Propulsion of Sea Urchin Spermatozoa.* 1955.

[3] Shum, H., and E. A. Gaffney. "Mathematical Models for Individual Swimming Bacteria." *Microbiorobotics: Biologically Inspired Microscale Robotic Systems* (2012): 29.

[4] Cortez, Ricardo. "Regularized stokeslet segments." *Journal of Computational Physics* 375 (2018): 783-796.

[5] G. Taylor, "Analysis of the Swimming of Long and Narrow Animals," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 214, no. 1117, pp. 158-183, Feb. 1952.

[6] Rodenborn, Bruce, et al. "Propulsion of microorganisms by a helical flagellum." *Proceedings of the National Academy of Sciences* 110.5 (2013): E338-E347.

[7] Gray, James. "The mechanism of locomotion in snakes." Journal of experimental biology 23.2 (1946): 101-120.

Backup: Biomechanics Model of Snake Undulation [3]