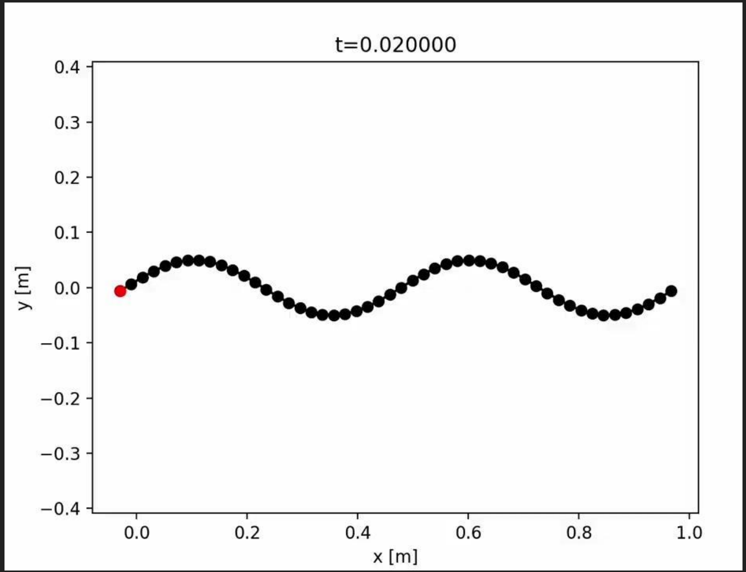
Midterm Presentation: *Snake Project*

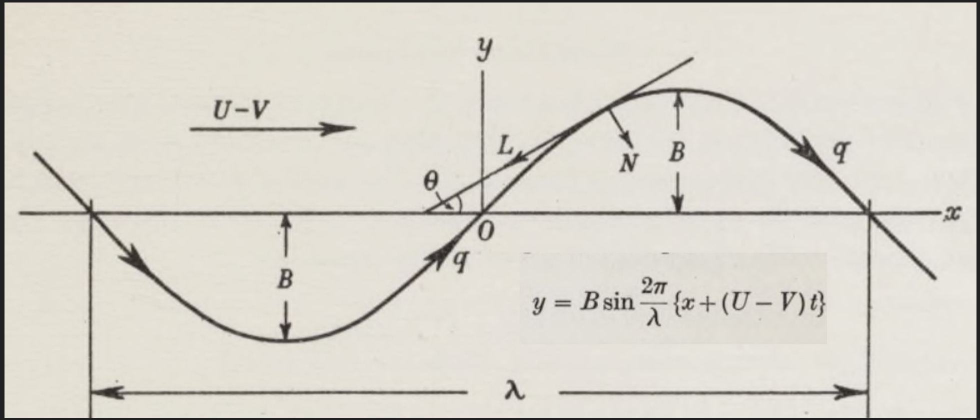
Sarah Enayati and Jonathan Gray

Introduction

Project Goal: develop an analytical approach to optimizing snake-like motion in viscous fluid



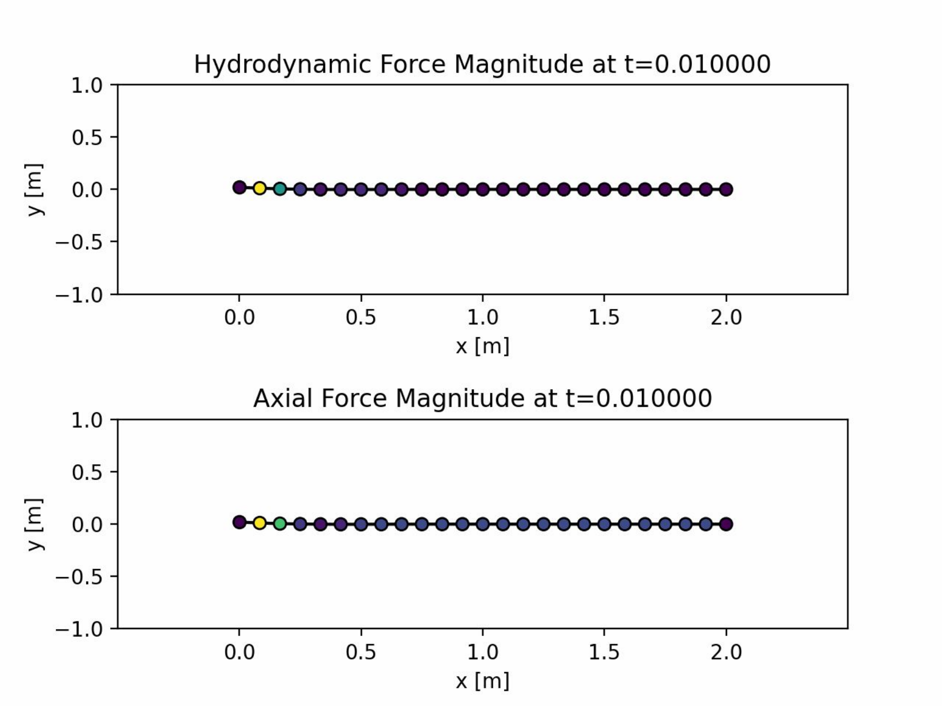
Project Proposal Feasibility Study [1]



Dominant Forces in RFT [2]

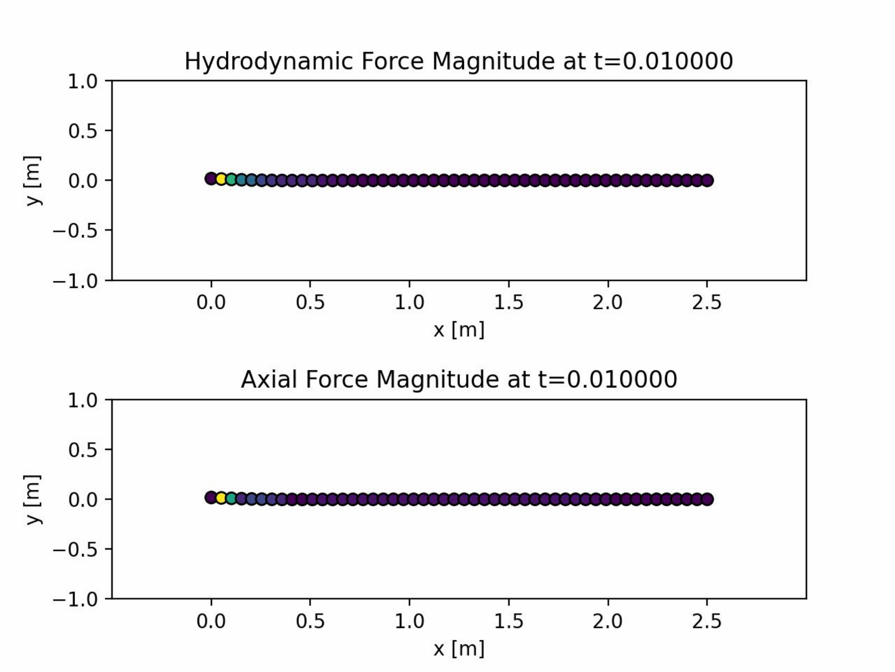
Code Enhancements

- Head node enforced Y-Oscillation, Body nodes free
- Introducing long range hydrodynamics using Stokeslet Method
- Tracking Hydrodynamic and axial force contours
- Empirically based correlation method for long range hydrodynamics.



Code Enhancements

- Head node enforced Y-Oscillation, Body nodes free
- Introducing long range hydrodynamics using Stokeslet Method
- Tracking Hydrodynamic and axial force contours
- Empirically based correlation method for long range hydrodynamics.



Comparing Effects of Short and Long Range Hydrodynamics

Resistive Force Theory:

for each node, k, in snake body: do

// tangents and normals are calculated

$\hat{t} = \frac{q_{k+1}-q_k}{|q_{k+1}-q_k|}$

$\hat{n} = [-\hat{t}[1], \hat{t}[0]]$

// tangential and normal velocity and forces are calculated

$v_t = u_k \cdot \hat{t}$

$v_n = u_k \cdot \hat{n}$

$F_t = -C_t * v_t$

$F_n = -C_n * v_n$

// Hydrodynamic force is updated for all DOF

$F_{hydro} = F_{hydro} + F_t + F_n$

// Hydrodynamic Jacobian is updated for all DOF

$J_t = \frac{-C_t}{dt} * \hat{t} \otimes \hat{t}$

$J_n = \frac{-C_n}{dt} * \hat{n} \otimes \hat{n}$

$J_{hydro} = J_{hydro} + J_t + J_n$

Long-Range Hydrodynamics (Empirical Approach):

for each node, j, in range: do

$F_{interaction} = \frac{(C_n(u_j-u_k)\cdot\hat{r})*\hat{r}+C_t((u_j-u_k)-(u_j-u_k)\cdot\hat{r}*\hat{r})}{|r|}$

$F_{hydro}[k] = F_{hydro}[k] + F_{interaction}$

$F_{hydro}[j] = F_{hydro}[j] - F_{interaction}$

$J_{interaction} = \frac{C_n*\hat{r}\otimes\hat{r}+C_t*(I-\hat{r}\otimes\hat{r})}{|r|}$

// Update Jacobian

$J_{hydro}[k,k] = J_{hydro}[k,k] + J_{interaction}$

$J_{hydro}[k,j] = J_{hydro}[k,j] - J_{interaction}$

$J_{hydro}[j,j] = J_{hydro}[k,k] - J_{interaction}$

$J_{hydro}[j,j] = J_{hydro}[j,j] + J_{interaction}$

end for

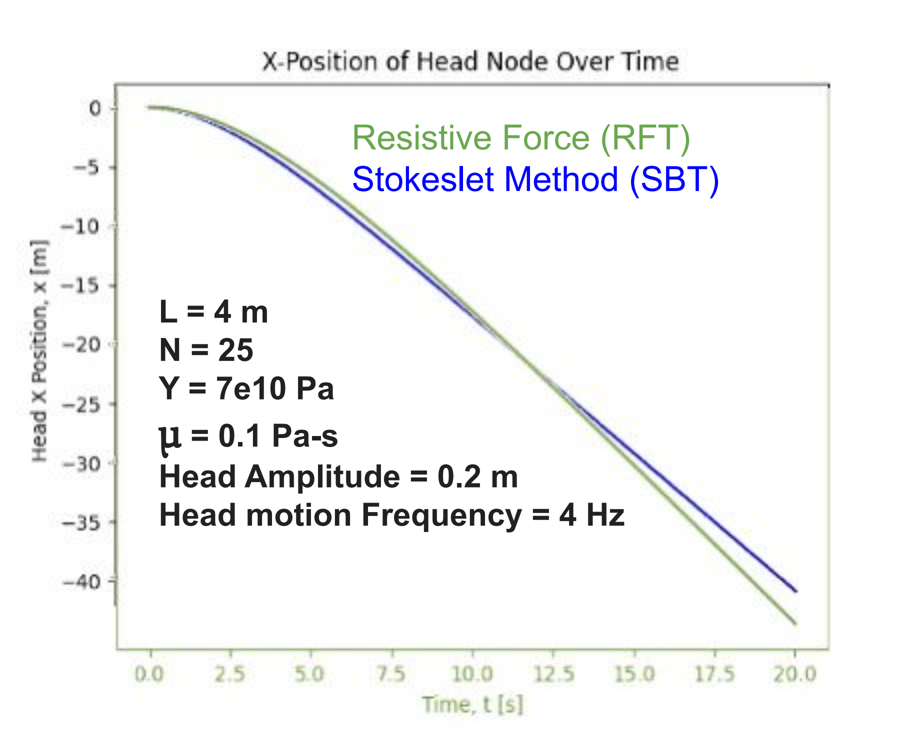
Stokeslet Method [2] (SBT):

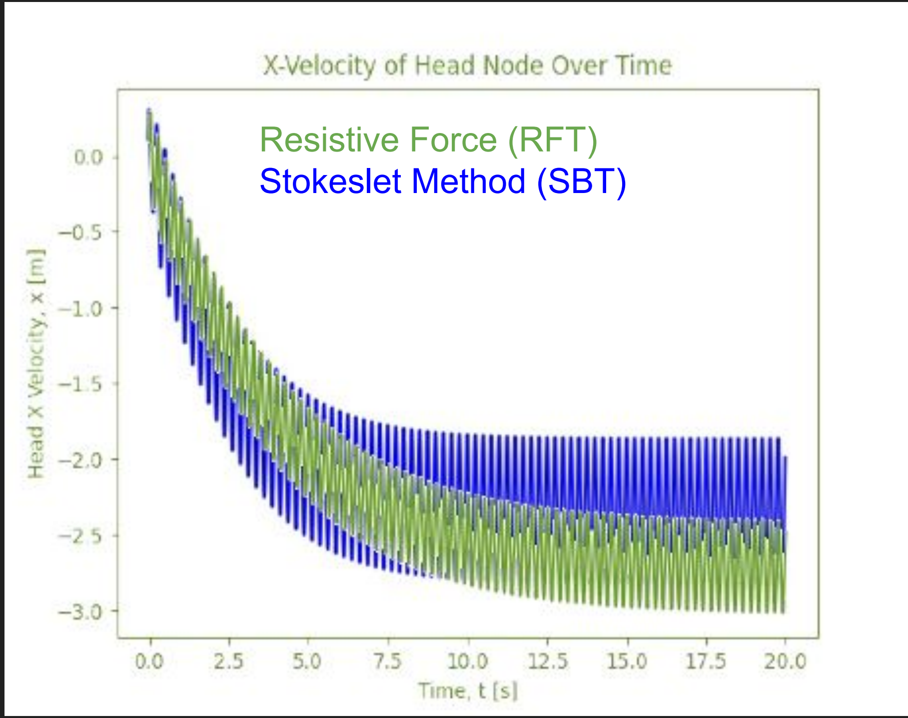
$$\mathbf{u}(\mathbf{r}) = \mathbf{f} \cdot \mathbb{J}(\mathbf{r}) \quad \text{and} \quad p(\mathbf{r}) = \frac{\mathbf{f} \cdot \mathbf{r}}{4\pi|\mathbf{r}|^3},$$

where $\mathbb{J}(\mathbf{r})$ is the Oseen tensor,

$$\mathbb{J}(\mathbf{r}) \equiv \frac{1}{8\pi\mu}\left(\frac{\mathbb{I}}{|\mathbf{r}|} + \frac{\mathbf{r}\mathbf{r}^{\mathrm{T}}}{|\mathbf{r}|^3}\right).$$

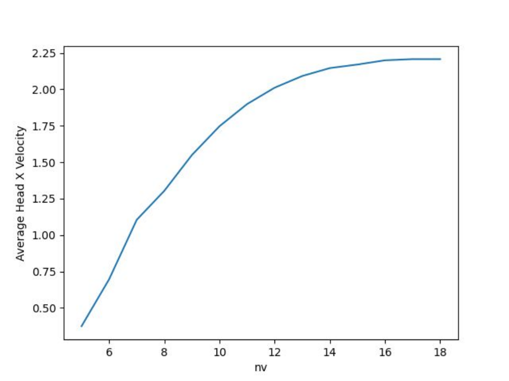
RFT and SBT Give Approximately the Same Solution for Re ~ 2

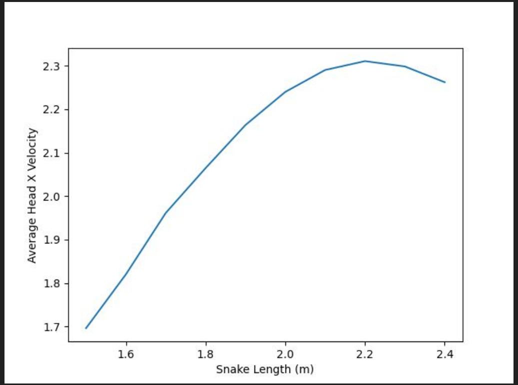


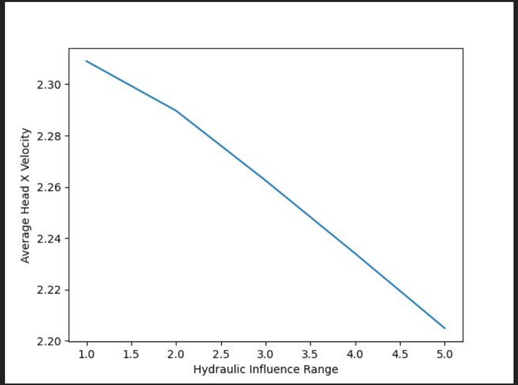


Preliminary Results: Parameter Determination

- Sensitivity analysis using average velocity of head
  - Number of nodes: 16
    - Higher number → smoother motion → better thrust
    - But also more computational efficiency
  - Length of snake: 2.2 m
    - Longer → more undulation cycles → more propulsion
    - But also increase in drag (more surface area)
  - Hydrodynamic Influence Range: 3
    - Higher number → more accurate
    - But also more interactions with environment (slower velocity)







References

[1] S. Enayati, J. Gray , "Project Proposal: Locomotion of Snake in Water,” MAE 263F, Oct. 2024..

[2] G. Taylor, "Analysis of the Swimming of Long and Narrow Animals,” *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 214, no. 1117, pp. 158-183, Feb. 1952.

[3] Rodenborn, Bruce, et al. "Propulsion of microorganisms by a helical flagellum." *Proceedings of the National Academy of Sciences* 110.5 (2013): E338-E347.

[4] Gray, James. "The mechanism of locomotion in snakes." Journal of experimental biology 23.2 (1946): 101-120.

Backup: Biomechanics Model of Snake Undulation [3]