Experiment 5: Beam Vibrations

City College of New York

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ME 31100

Fundamentals of Mechatronics

Group: 4

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Abstract

This study aimed to gain a comprehensive understanding of the principles and methodologies governing the vibrational behavior of Cantilever beams. We investigated two scenarios: one involving a magnet (mass) not affixed to the free end of the beam, and another with the magnet (mass) attached to the free end. Employing Analytical, LabView, Experimental, and MATLAB techniques, we analyzed these situations. Through these methodologies, we derived natural frequencies, which facilitated an exploration of mode shapes and damped natural frequencies, thereby enhancing our understanding of the system.

Objective

Using an impulse force, the damping coefficient and the natural frequency of the system will be found for Cantilever beams of different materials. The goal is to understand how the cut and material of the beam affects its frequency response characteristics and the effect the speed of a wave traveling through the material has.

Introduction

When considering a project, one of the factors examined is the stability of the material chosen. To put it into perspective; all solid objects experience impact differently and depending on the frequencies of the vibration the impact generates, the integrity of the material might not hold up. This is the reason behind why choosing the most suitable material for a project is so important. The experiment being done for this lab is for students to gain some understanding on the difference in behavior different materials express under a similar force of impact.

Theory

Prior to calculating the important factors that characterize the beam vibrations, it's necessary to convert the data collected to its proper physical units. Such values can be obtained with the following equation:

$$P_{trans} = sens * (V_{trans} - V_{offset}) \quad (1)$$

To obtain the damping ratio of the beam vibrations, the use of the logarithmic decrement can be used. Such a value is dependent on the amplitude of the peaks of the signal (after conversions to the proper physical units) denoted by A_1 and A_n. For the purpose of this lab, only the first two peak (n = 2)

$$\zeta = \frac{ln(A_1/A_2)}{\sqrt{4\pi^2 + [ln(A_1/A_2)]^2}} \quad (2)$$

The lab looks to find the acceleration of the beams through the accelerometer, where the acceleration can directly be found after converting the data into its physical units as well as through the strain measurements. The latter method can be found using the equation below:

$$a = \frac{E * I}{S * M_{eq} * c} * \varepsilon \quad (3)$$

The total mass of the of the contraption is denoted by M_{eq} and is found from the equation below:

$$M_{eq} = M + 0.24m \quad (4)$$

The natural frequency is characterized as the non-disturbed oscillations of the beam without external influences. Such value for the case of the beams can be found with the equation below.

$$w_n = \sqrt{k/M_{eq}} \quad (5)$$

The stiffness of the beam is denoted by k. Such value is a function of the modulus of elasticity, moment of inertia and length of the beam. The value can be calculated with the following formula:

$$k = \frac{3E*I}{L^2} \quad (6)$$

The damping frequency is the more realistic case where the dissipated energy of the oscillations is taken into account. Such value is a function of both the natural frequency and the damping ratio and can be calculated with the formula below.

$$w_d = w_n\sqrt{1-\zeta^2} \quad (7)$$

Experimental Apparatus and procedure

This lab includes the use of two Cantilever beams; one is made of aluminum and another with a combination of several metals. Both beams are equipped with a 350 Ohm, CEA-06-062UW-350 Measurements Group strain gauge with a gauge factor of 2.135. Additionally, a PCB piezoelectric accelerometer is provided for each beam along with a PCB impact hammer that will be used to generate the impact force. A Model 482A10 ICP AMP/SUPPLY signal conditioner, a Vishay Model P3 strain indicator and recorder and a Fluke 179 TRUE RMS Digital Multimeter (DMM) will also be used.

<u>*Procedure:*</u>

1) The measurements for the beams are recorded according to Image 1.

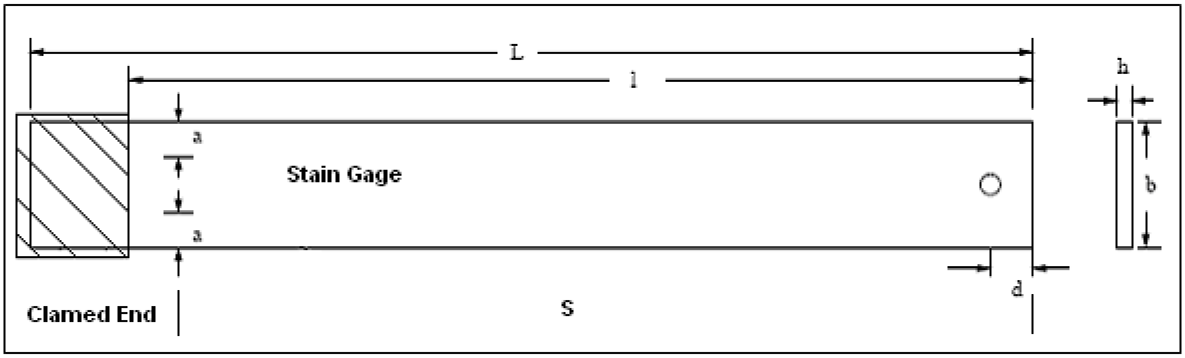


Image 1: This image shows all the measurements to be taken for each Cantilever beam.

2) Next, the beams are mounted onto the bending fixture and secured with the Micrometer. Students are to take note that the micrometer head is centered over the beam but not touching it.
3) Connect the Vishay Model P3 strain indicator to the system (there are wires attached to the strain gauge) and set up the indicator in preparation to record data.
4) The DMM is used to measure and record the output values with each output value corresponding to a microstrain value. This is done in increments of $50\mu\varepsilon$.
5) In the next part of the lab, the micrometer is removed and MATLAB is used to record impact data. The first set of data will be without anything being done to the beams.
6) In the second set of data, the impact hammer is used to strike the beams on their free ends.
7) All the data from MATLAB is recorded and saved for later analysis.

Results

The sensitivity of the impact and accelerometer is known prior to the experiment. The sensitivity of the strain gauge can be calculated by calibrations to transform the units to microstrain. Such value is obtained through linear regression done in (Figure A3).

Strain gauge: 2446 $\mu\varepsilon/V$
Accelerometer: 89.9999 $m/s^2/V$
Impact hammer: 432.9004 N/V

To convert each graph of the data collected from voltage to the physical units, MATLAB was used to apply equation (1) onto each data point. The voltage is also offsetted from the non-impact measurements taken.

Composite beam data

parameter	Dimension (in)
h	1/8
b	1
d	1 6/16
a	5/16
L	18

l	17 2/32

(Refer to image 1 for corresponding parameter measurement on the beam)

Total Mass:

$M_{eq} \;\; = = 0.01 + (0.24 * .9)$ (4)

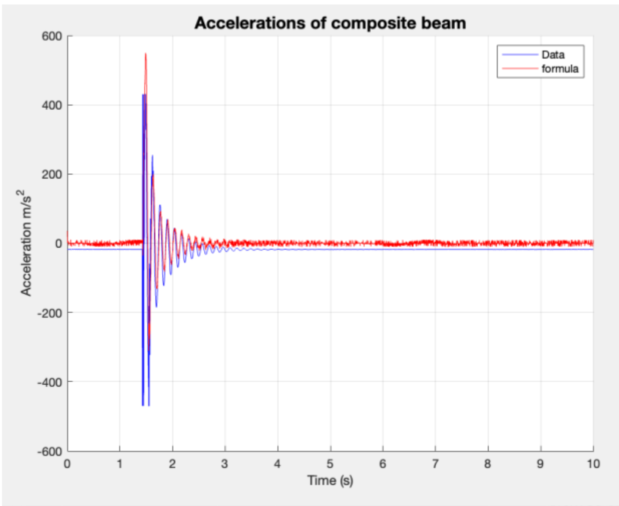
$M_{eq} \;\; = 0.226$ kg

Acceleration through strain gauge:

$a = (20000000000 * 6.77460003E - 11)/(2.5 * 0.0015875 * 0.226) * \varepsilon$ (3)

$$a \;=\; 1510.606933 * \varepsilon \; (m/s^2)$$

The graph is then plotted against acceleration obtained through the accelerometer in (Figure C4)



C4: comparison of accelerations of the composite beam obtained from accelerometer vs from conversion of the strain measurements

Damping ratio:

$$\zeta = \frac{ln(-503.078/-210.088)}{\sqrt{4\pi^2 + [ln(-503.078/-210.088)]^2}} \quad (2)$$

ζ =0.13765

Stiffness:

$k \;= (3 * 20000000000 * 6.77460003E - 11)/0.4572^3$ (6)

$k = 42.53204306$

Natural frequency:

$$w_n = \sqrt{(42.53204306/0.226)} \quad (5)$$

$$w_n = 31.27858605\ hz$$

Damping frequency:

$$w_{d} = 31.27858605\sqrt{(1 - 0.13765406^2)} \quad (7)$$

$$w_d = 30.98082539\ hz$$

Aluminum beam data

Parameter	Dimension (in)
h	3/16
b	1
d	1 6/8
a	5/16
L	17 24/32
l	16 24/32

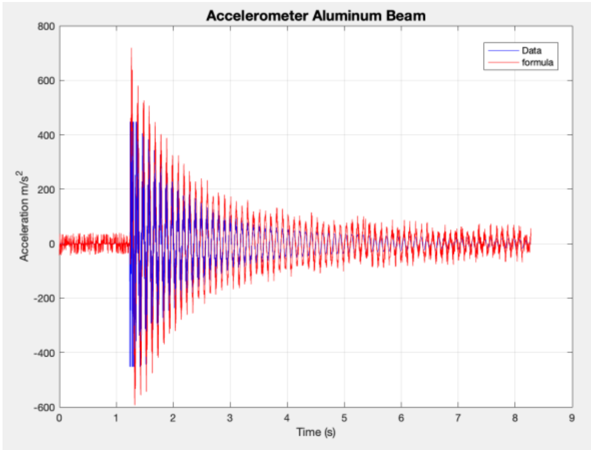
(Refer to image 1 for corresponding parameter measurement on the beam)

Acceleration through strain gauge:

$$a = (69000000000 * 0.0000000022864275509)/(2.5 * 0.0434732411(0.0238125) * \varepsilon \quad (3)$$

$$a = 6095.923464 * \varepsilon$$

The graph is then plotted against acceleration obtained through the accelerometer in (Figure S4)



S4: comparison of accelerations of the Aluminum beam obtained from accelerometer vs from conversion of the strain measurements

Total mass:

$M_{eq} = 0.01 + (0.24 * 0.139471838)$ (4)

$M_{eq} = 0.04347324112$ kg

Damping ratio:

$$\zeta = \frac{ln(288.649/233.037)}{\sqrt{4\pi^2 + [ln(288.649/233.037)]^2}} \quad (2)$$

$\zeta = 0.03404167$

Stiffness:

$k = (3 * 69000000000 * 0.0000000002286427509)/0.45085^3$ (6)

$k = 516.4538985$

Natural frequency:

$w_n = \sqrt{516.4538985/0.04347324112}$ (5)

$w_n = 108.9945468\ hz$

Damping frequency:

$w_d = 108.9945468 * \sqrt{(1 - 0.03404167\text{^}2)}$ (7)

$w_d = 108.9313751\ hz$

Discussion of Results

After converting to physical units, the corresponding sensors are plotted against time for each beam as seen in (Figure C1-3) for the composite beams and (Figure S1-3) for the aluminum beam.

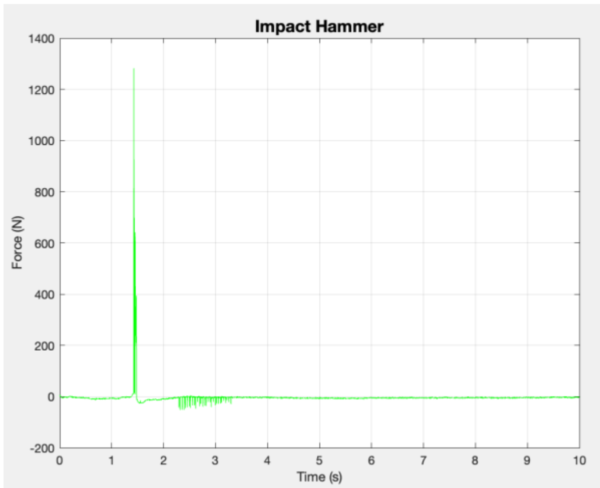


Figure C1: Impact hammer Force vs time graph of the composite beam

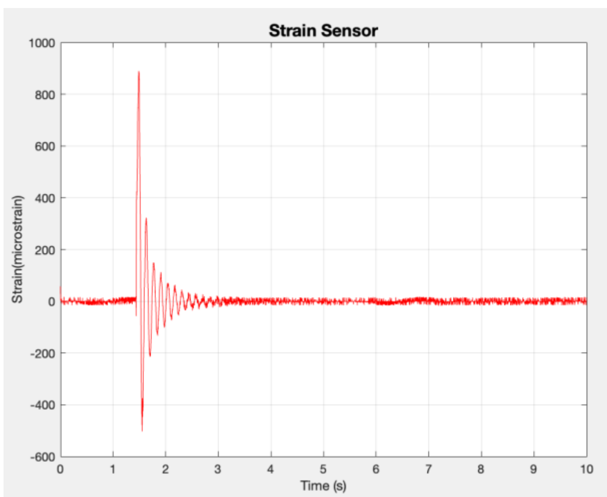


Figure C2: Strain gauge sensor strain vs time graph of the composite beam

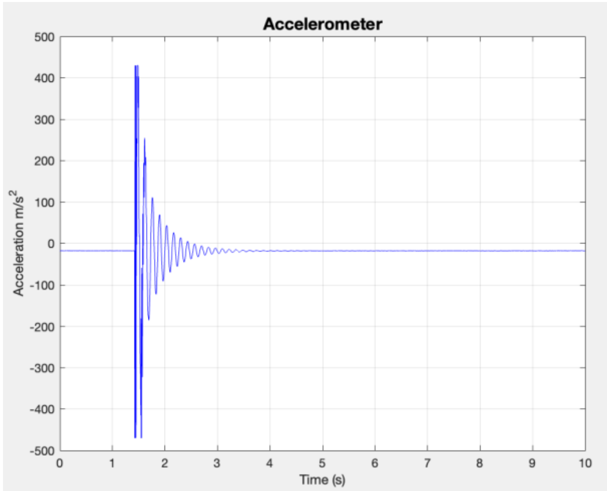


Figure C3: Accelerometer Acceleration vs time graph of the composite beam

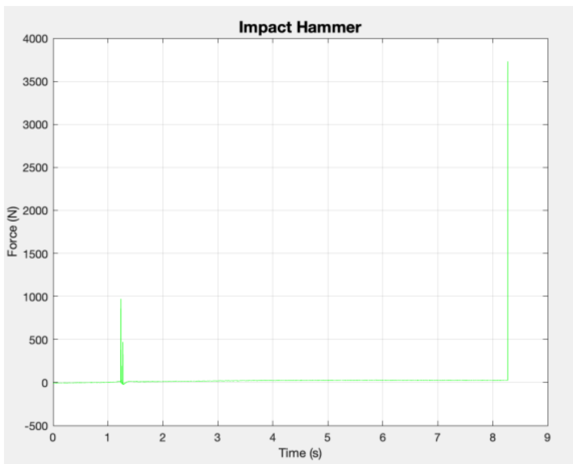


Figure S1: Impact hammer Force vs time graph of the Aluminum beam

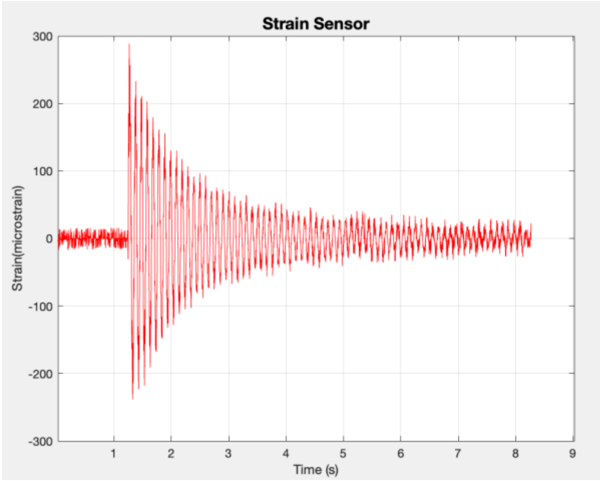


Figure S2: Strain gauge sensor strain vs time graph of the Aluminum beam

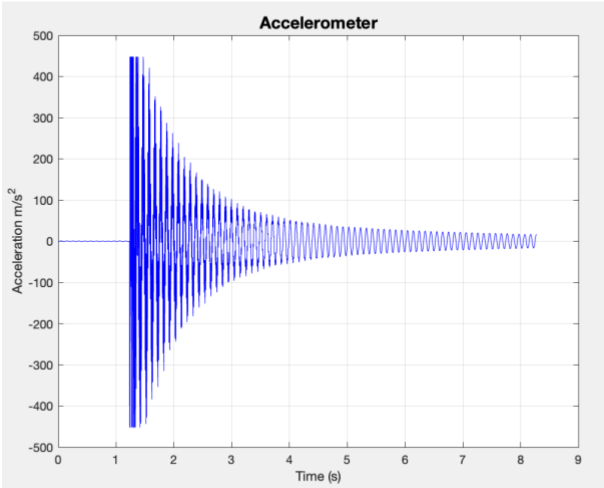


Figure S3: Accelerometer Acceleration vs time graph of the Aluminum beam

The stress curves are plotted in (Figure C5&S5). For the aluminum beam, the maximum stress is observed to be at 19.91 MPa and for the composite bar, the value is to be observed around 60.497 MPa. Maximum yield strength of an Aluminum alloy is seen to be a value between 40 Mpa to 700 MPa [1], depending on the alloy which is way above the limits of the Aluminum beam upon impact of the hammer in this experiment.

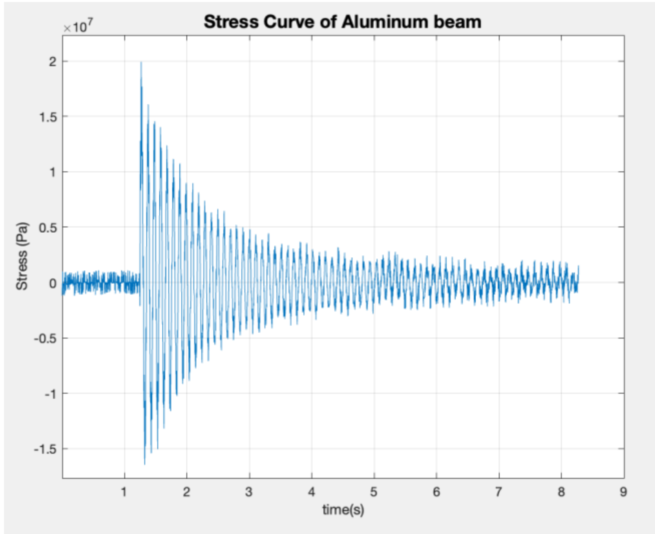


Figure S5: Stress plotted vs time of aluminum beam

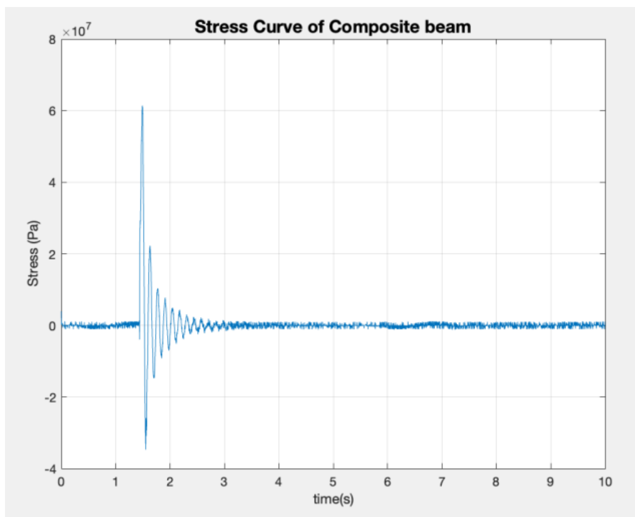


Figure S5: Stress plotted vs time of composite beam

There is a definite lag between the times when the student first hit the beam versus when the strain gage began to record motion because it takes time for the impact to travel through one end of the beam to another. This is called the elastic wave speed. The calculated elastic wave speed is $v = \frac{0.4572m}{7s} = 0.0653\frac{m}{s}$. The theoretical velocity is $c = \sqrt{\frac{70\times10^9}{2710}} = 5082.34\frac{m}{s}$. The error percentage is 99.9987%.

The damped natural frequency and damping coefficients of the composite beam are: $w_d = 30.98082539\ hz$ and $f_d = 4.93075hz$.

The damped natural frequency and damping coefficients of the aluminum beam are: $w_d = 108.9313751\ hz$ and and $f_d = 17.33696hz$.

The natural frequency of the composite beam is: $w_n = 31.27858605\ hz$ and the theoretical value of the composite beam is: $\frac{1}{12.0625^2} \times (\frac{220\times10^9\times0.010417}{1500\times0.125})^{\frac{1}{2}} = 12.0087hz$. The percentage error is $\frac{31.278-24.0274}{24.0274} \times 100\% = 30.176\%$

The natural frequency of the aluminum beam is: $w_n = 108.9945468\ hz$ and the theoretical value of the aluminum beam is $\frac{1}{12.75^2} \times (\frac{70\times10^9\times0.1865}{2710\times0.1875})^{\frac{1}{2}}$= 34.836hz. The percentage error is

$$\frac{108.9945468 - 34.836}{34.836} \times 100\% = 212.879\%$$

Conclusion

Material selection plays a big role in any/all engineering projects. One of the components that are taken into consideration during material selection is the impact resistance. This lab plays a role in teaching students about the effect an impact may have on different materials. For the purpose of this lab, Aluminum and an alloy made of a composition of metals were tested and analyzed. By doing this, the difference in how each Cantilever beam reacted to the frequency of vibration applied (the impact exerted was of a similar magnitude for both beams). All this testing is to understand how the choice of material may suit the needs of a project depending on the results wanted.

References

[1] "Aluminum Alloys - Yield Strength and Tensile Strength". Amesweb https://amesweb.info/Materials/Aluminum-Yield-Tensile-Strength.aspx

Appendix

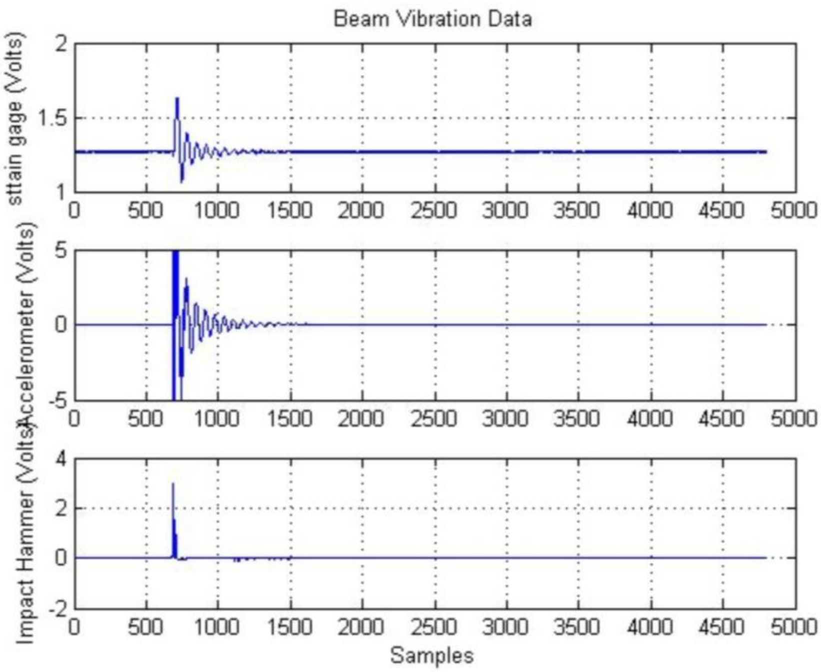


Figure A1: raw data of voltage vs samples of the strain gauge, accelerometer and impact hammer taken for the composite beam

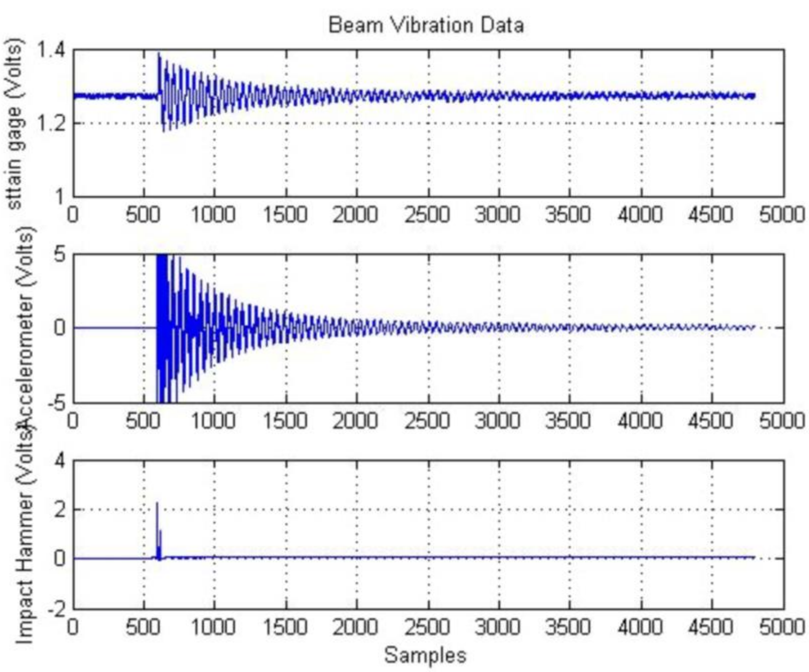


Figure A2: raw data of voltage vs samples of the strain gauge, accelerometer and impact hammer taken for the Aluminum beam

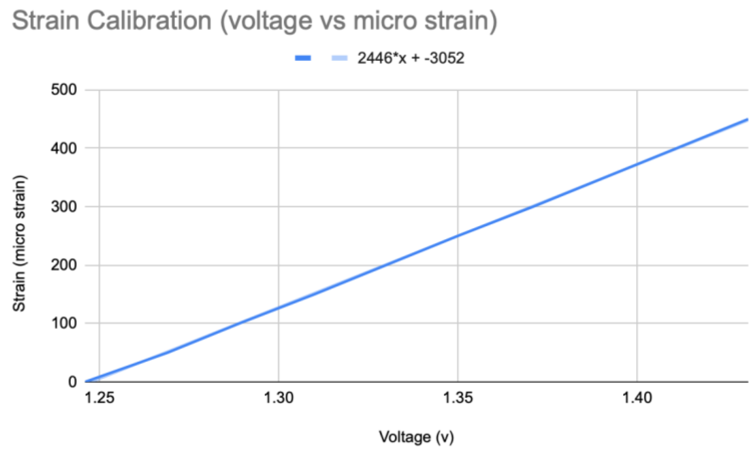


Figure A3: Strain calibrations to obtain slope for sensitivity for strain gauge. Conversions to microstrain from voltage.