# Experiment 2: Optoelectronic Sensors use in Analyzing Free-Falling Motion

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**ME 31100**
Fundamentals of Mechatronics

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## Abstract

This study investigates the application of optoelectronic sensors in analyzing the motion of free-falling balls, specifically focusing on tennis, racquet, and ping pong balls. The experiment aims to determine the position, velocity, damping coefficient, and coefficient of restitution of each ball during free fall. Data collected from the experiment is processed using MATLAB to generate time vs position and time vs velocity graphs, enabling the calculation of damping coefficients. The results are compared with theoretical predictions based on particle dynamics. The findings provide insights into the behavior of different balls during free fall and demonstrate the efficacy of optoelectronic sensors in motion analysis.

## Objectives

This lab looks to explore the use of optoelectronics-based emitter-detector sensors to help observe the motion of various free-falling balls, more specifically that of a ping pong, racquet and tennis ball. Such data is processed via MATLAB in order to obtain data for time vs position and time vs velocity to obtain values of our damping coefficients. Such experimental results are compared with values obtained through the theory of particle dynamic

## Introduction

For this lab, data will be recorded using a provided MATLAB script on three different balls: a tennis ball, a racquetball, and a ping pong ball. In order for this to happen, the program is connected to a contraption with eight different optoelectronic sensors providing statistics on the three samples in free fall.

Data will be processed through MATLAB to get information about the positions of the balls over time by converting the data of the samples of voltages to time. Such data collected from the experiment which is then processed would help to determine the variables of damping coefficients and coefficient of restitution.

## Theory

In order to measure the position of the different balls as they fall down, an optoelectronic sensor is utilized to measure voltage samples. Directly across the sensor is an LED. The interference of the ball as it blocks the path of the light from the optoelectronic sensor will create a disturbance from the sample collected. Such a graph will show a rectangular hump in the graph for each sensor (Figure 1-3). This disturbance can then be plotted and such data can be analyzed to determine where a ball is at a given time in its free fall.

To analyze the position as a function of time of the experimental data collected, polynomial fitting is used. Such data is matched up to the experimental collected data. The equation below will also be used in calculating the first approaching position (ya) and rebounding positions (yr) to calculate the coefficient of restitution.

$$y(t)_{polynomial\ fit} = a_0 + a_1 t + a_2 t^2 \quad (1)$$

The damping coefficients of the balls can also be determined by fitting the position equation below to match the experimental position vs time graph. Note that this equation is to be applied only for the approaching case. The inviscid case of finding position vs time (Figure A4, B4, C4) is used to show necessity of a damping coefficient when calculating positions vs time without an experimental trial.

$$y(t)=\begin{cases}-\frac{1}{2}gt^2+y_0 & \text{for inviscid case}\\ -\left(\frac{m}{b}\right)gt+\left(\frac{m}{b}\right)^2 g\left(1-e^{-bt/m}\right)+y_0 & \text{for viscous case}\end{cases} \quad (2)$$

The plot of the velocity of each ball as a function of time can be seen in Figure A1, B1, C1. To derive the function,take the first derivative of the position equation (1):

$$v(t) = a_1 + 2a_2t \quad (3)$$

Alternatively, one can find the velocity by dividing the Diameter of the ball by the change in time which is found by the amount of time from the start of the disturbance to the end of one approaching/rebounding rectangular hump (Fig 1,2,3).

$$v(t) = \frac{-D}{\Delta t} \quad (4)$$

The coefficient of restitution (e) is a value that tells us the plasticity or elasticity of the object upon collision. A value of 0 would be a perfectly plastic collision whereas an value of one would be a perfectly elastic collision.

To find the e, the approaching and rebounding velocity must be calculated. Such values make use of the first approaching and rebounding position (1). Note the time that will be used in the calculation is denoted as the time the disturbance is first tracked. As for the first approaching case, that would be in the first sensor whereas the rebound would take the time from the last sensor.

Approaching velocity:

$$v_a = -\sqrt{2gy_a} \quad (5)$$

Rebounding velocity :

$$v_r = \sqrt{2gy_r} \quad (6)$$

The coefficient of restitution is a ratio of the absolute value of the rebounding velocity to the approaching velocity. Take note that the rebounding velocity should be less than the approaching velocity (except in the case of perfectly elastic collision in which the two values would be equal to each other).

$$e = \frac{(first)\ rebounding\ velocity}{(first)\ approaching\ velocity} = \left|\frac{v_r}{v_a}\right| \quad (7)$$

## Experimental Apparatus and procedure

Experimental Setup:

1) Three balls with distinct characteristics:
    a) A tennis ball, vibrantly green-yellow with a diameter of 64.8 mm (2.55 inches), weighing 54.8 grams, and featuring a rigid surface with a distinctive felt-like texture.
    b) A ping-pong ball, pure white, measuring 38.1 mm (1.5 inches) in diameter, weighing a mere 2.8 grams, and boasting an exceptionally smooth surface that renders it buoyant due to its lightness.
    c) A racquet ball, blue with a diameter of around 55.9 mm (2.2 inches), weighing 40.2 g, and has a somewhat smooth surface that deforms easily.
2) Sensors:

    Eight sensors placed at different heights to measure the voltage as a function of time during the motion of the ball(s).

3) LED Light and Phototransistor Setup (Figure 8)
    a) LED lights placed above the sensors to illuminate the path of the falling ball.
    b) Phototransistors positioned below the sensors to detect changes in light intensity as the ball passes through.

Experimental Procedure:

1. Drop the ball from a known initial height.
2. Record the voltage readings from each sensor as the ball descends.
3. Convert the voltage readings to time values using appropriate calibration data.
4. Measure the position (height) of each sensor from the initial drop point.

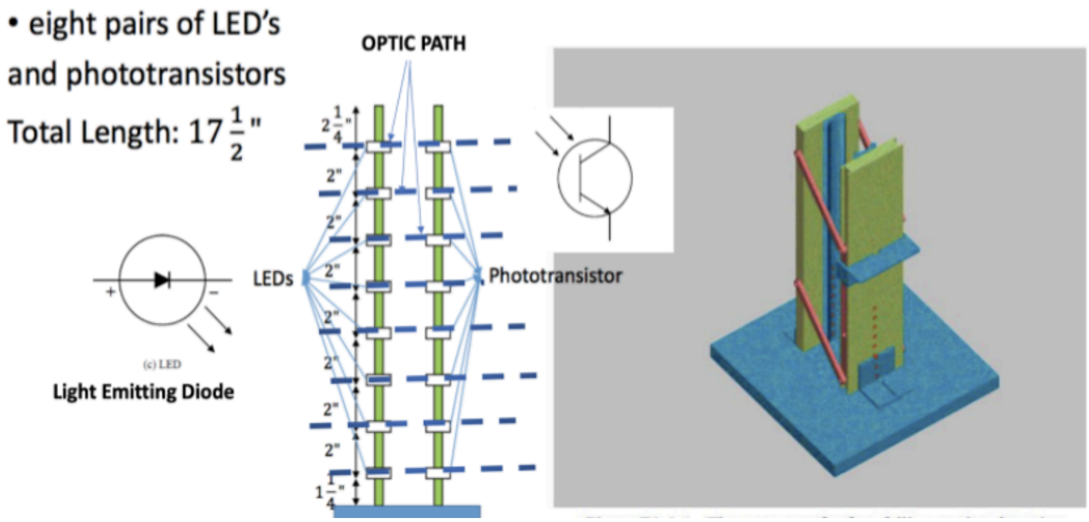


Figure 8: optoelectronic sensor apparatus consisting of 8 pairs of photo transistor and LED.

MATLAB Data Processing Procedures:

- Import the data collected from the experiment.
- The data collected measures voltage as a function of the sample. The samples must be converted to time and the optic path (Fig 1-3) of each sensor should be plotted in the same window for ease of data interpretation.
- Take the value of time at the leftmost corner of the first approaching case (as the balls descend). The first approaching case should have 8 time values for the 8 sensors.
- The first rebounding case would measure the same corners but on the 2nd rectangles going up. Data values may vary based on the ball.
- Take the values of the height at which the ball was first dropped to each descending sensor height as the positions and the collected time from steps 3-4 as the time value in your plot.
- Using equation (1), calculate the coefficients $a_0$, $a_1$, $a_2$ from the 'polyfit' function of MATLAB. Plug the coefficients back into the equation to get your polynomial fitting. Make sure to use the 'hold' function to have the plots on the same graphs.
- To calculate the damping coefficient (approaching case), using equation (3), vary the value of your b coefficient until the plot overlaps with the experimental plot. To calculate the velocity vs time plot, using equation (4), plot the function using the same time as with the positions graphs.
- To plot the experimental data to the inviscid model (approaching case), plot the equation (2) on the same graph as the experimental data.

**Results**

*Racquetball*

Damping coefficient b = 0.00000006

$$y_a(0.202) = 0.4443 + 1.0657(0.202) + (-5.2616)(0.202)^2 \text{ (1)}$$

$$y_a(0.202) = 0.4449\, m$$

$$v_a = -\sqrt{2(9.81)(0.4444)} \text{ (5)}$$

$$v_a = -2.953\, \frac{m}{s}$$

$$y_r(0.395) = -1.8211 + 6.7257(0.395) + (-5.1081)(0.395)^2 \text{ (1)}$$

$$y_r = 0.0386$$

$$v_r = \sqrt{2(9.81)(0.0386)} \text{ (6)}$$

$$v_r = 0.869\, \frac{m}{s}$$

$$e = \left|\frac{0.869}{-2.953}\right| \text{ (7)}$$

$$e = 0.294$$

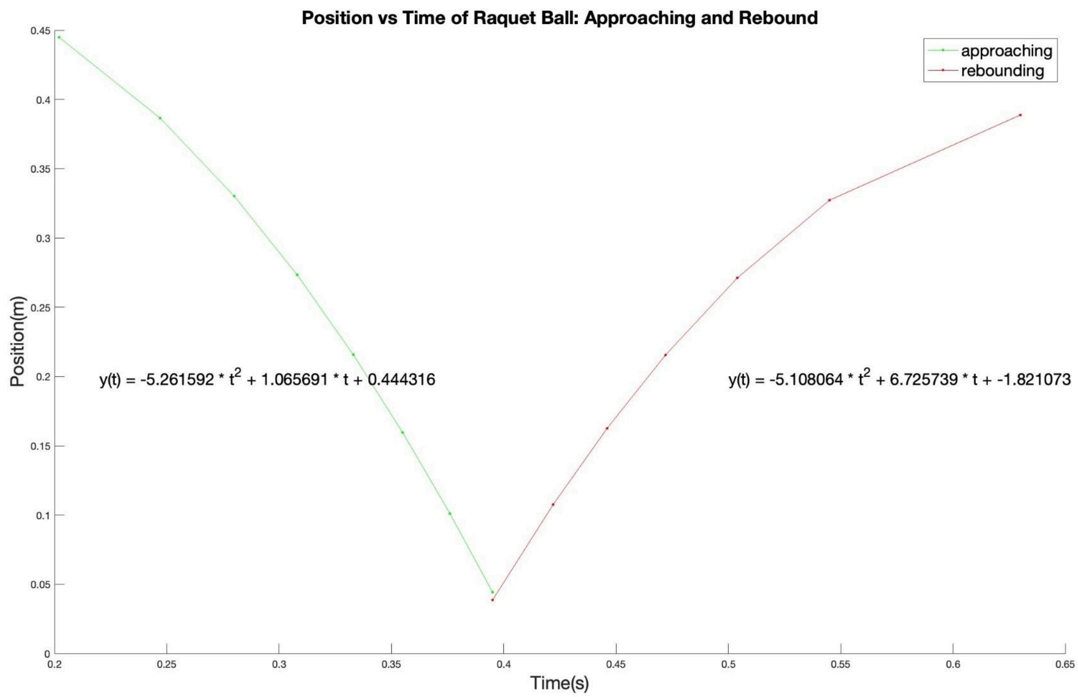


Figure A2: The Position vs Time graph of the Racquet ball upon its first approaching and rebound case (after curve fitting)

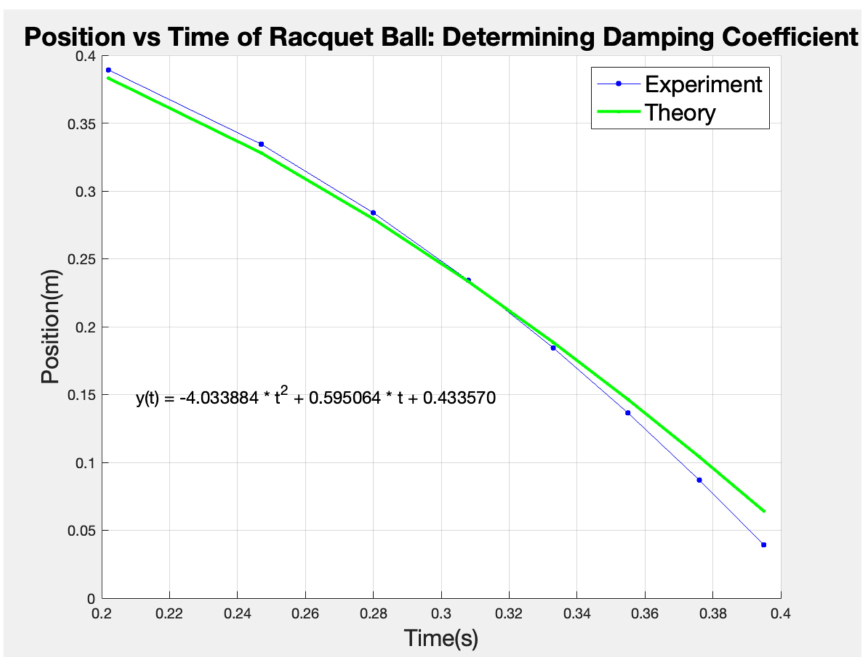


Figure A3: Graph comparing the experimental and theoretical Position vs Time of the Racquet Ball in the first approaching case. Theoretical case is fitted for the inviscid case, fitted for the damping coefficient.

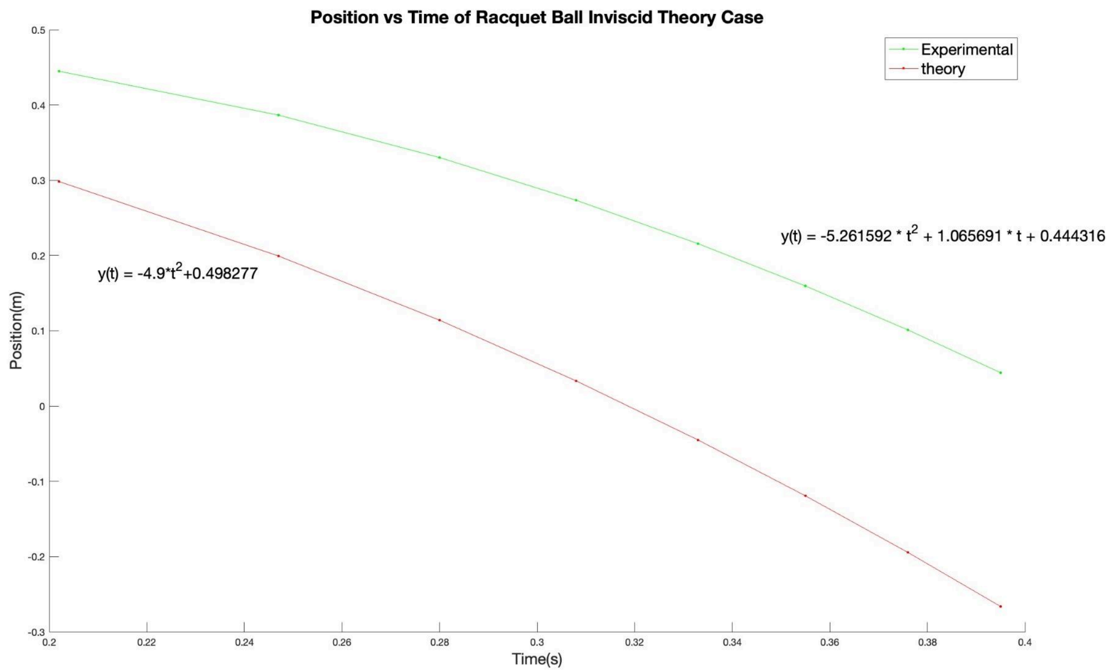


Figure A4: Graph indicates the time vs position of the Racquet ball upon its first rebound. This graph compares the experimental result and the inviscid theory case.

*Tennis Ball*

Damping coefficient b = 0.193

$$y_a(0.202) = 0.3944 + 0.7482(0.202) + 3.8528(0.202)^2 \quad (1)$$

$$y_a(0.202) = 0.3888\, m$$

$$v_a = -\sqrt{2(9.81)(0.3888)} \quad (5)$$

$$v_a = -2.7606\ \frac{m}{s}$$

$$y_r(0.416) = -2.1244 + 8.0591(0.416) + (-6.8399)(0.416)^2$$

$$y_r = 0.0445$$

$$v_r = \sqrt{2(9.81)(0.0445)} \quad (6)$$

$$v_r = 0.9339\ \frac{m}{s}$$

$$e = \left|\frac{0.9339}{-2.7606}\right| \quad (7)$$

$$e = 0.338$$

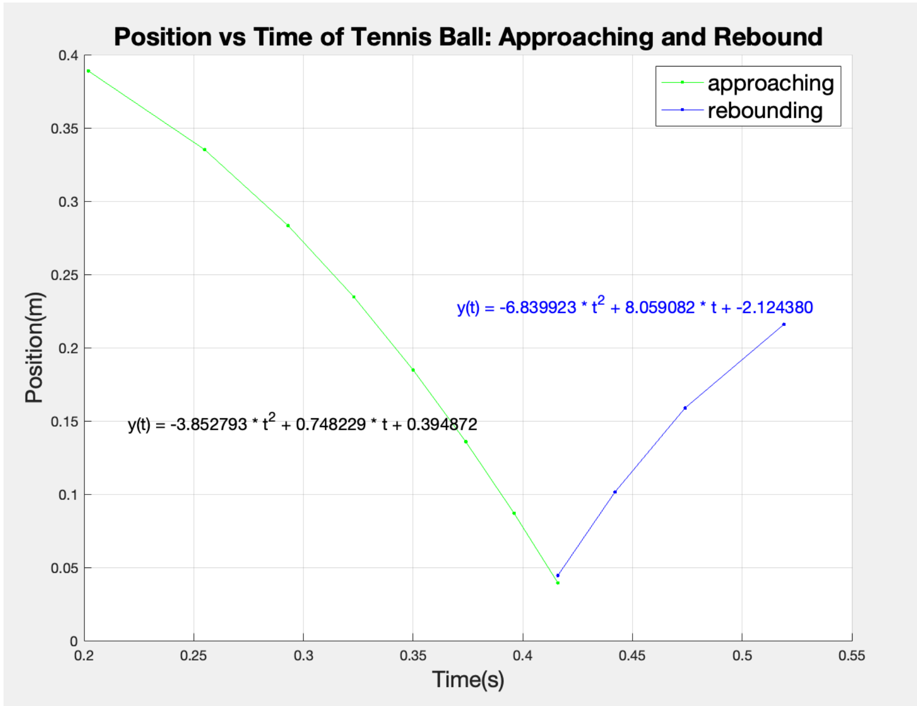


Figure B2: The Position vs Time graph of the Tennis ball upon its first approaching and rebound case (after curve fitting)

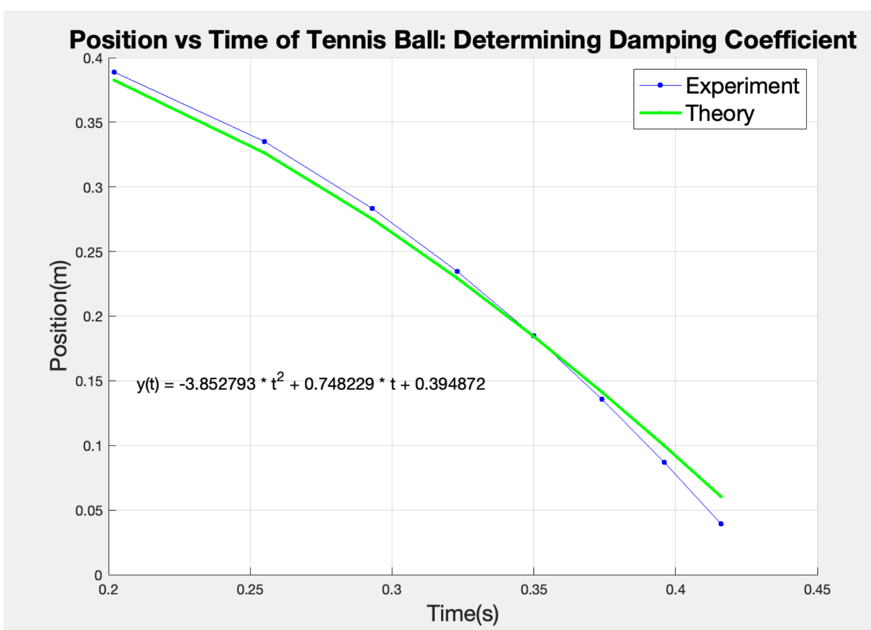


Figure B3: Graph comparing the experimental and theoretical Position vs Time of the Tennis Ball in the first approaching case. Theoretical case is fitted for the inviscid case, fitted for the damping coefficient.

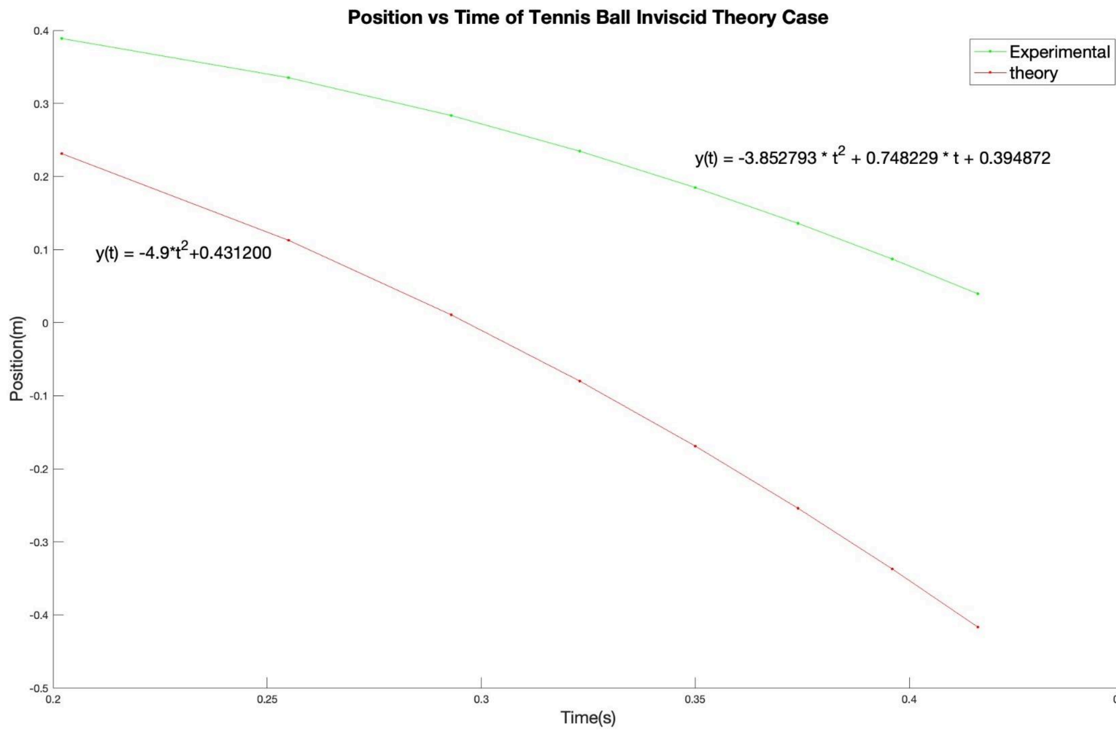


Figure B4: Graph indicates the time vs position of the Tennis ball upon its first rebound. This graph compares the experimental result and the inviscid theory case.

*Ping Pong*

Damping coefficient b = 0.0021

$$y_a(0.202) = 0.4654 + 0.8326(0.202) + 4.5933(0.202)^2 \text{ (1)}$$

$$y_a(0.202) = 0.4461\, m$$

$$v_a = -\sqrt{2(9.81)(0.4461)} \text{ (5)}$$

$$v_a = -2.957\, \frac{m}{s}$$

$$y_r(0.407) = -1.6207 + 5.7103(0.407) + (-4.0122)(0.407)^2$$

$$y_r = 0.434$$

$$v_r = \sqrt{2(9.81)(0.434)} \text{ (6)}$$

$$v_r = 2.9197\, \frac{m}{s}$$

$$e = \left|\frac{2.9197}{-2.7606}\right| \text{ (7)}$$

$$e = 0.987$$

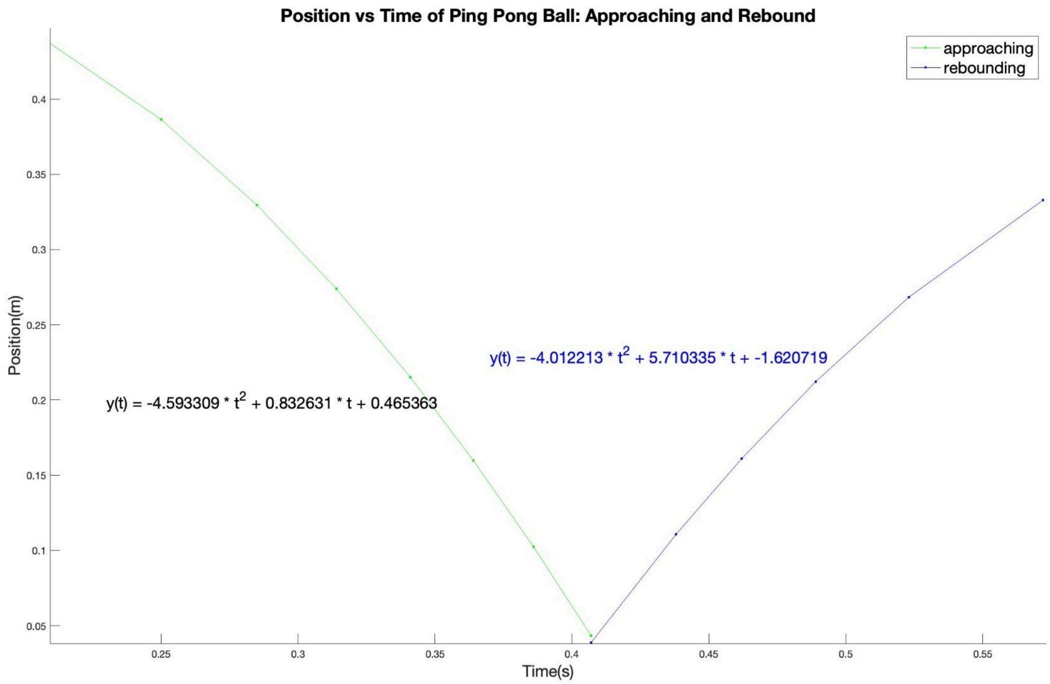


Figure C2: The Position vs Time graph of the Ping Pong ball upon its first approaching and rebound case (after curve fitting)

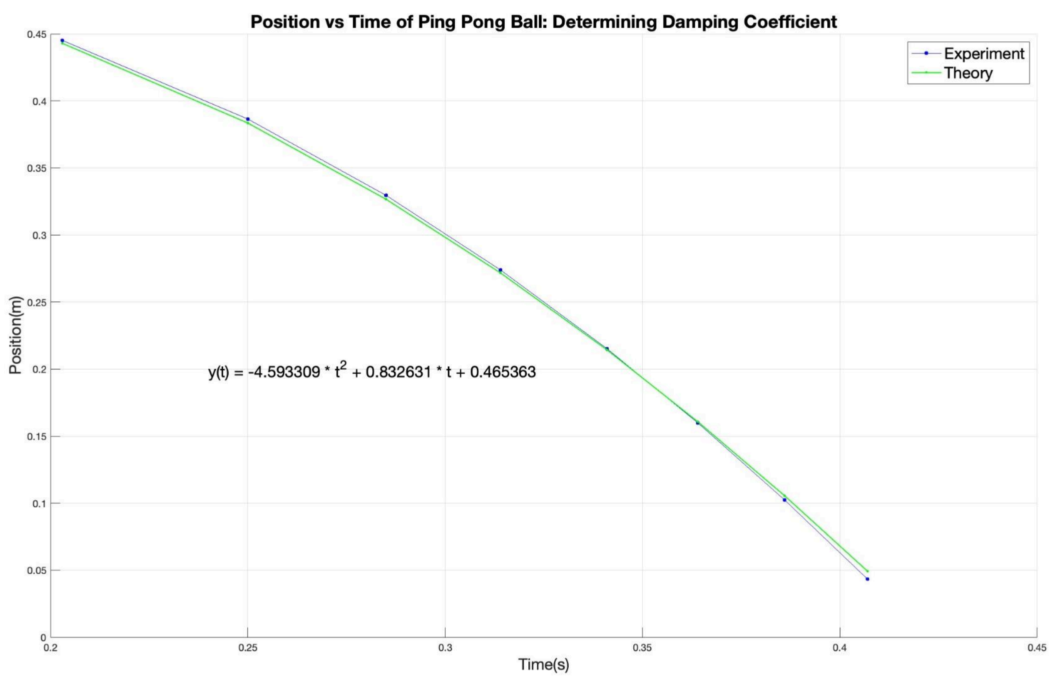


Figure C3: Graph comparing the experimental and theoretical Position vs Time of the Ping Pong Ball in the first approaching case. Theoretical case is fitted for the inviscid case, fitted for the damping coefficient.

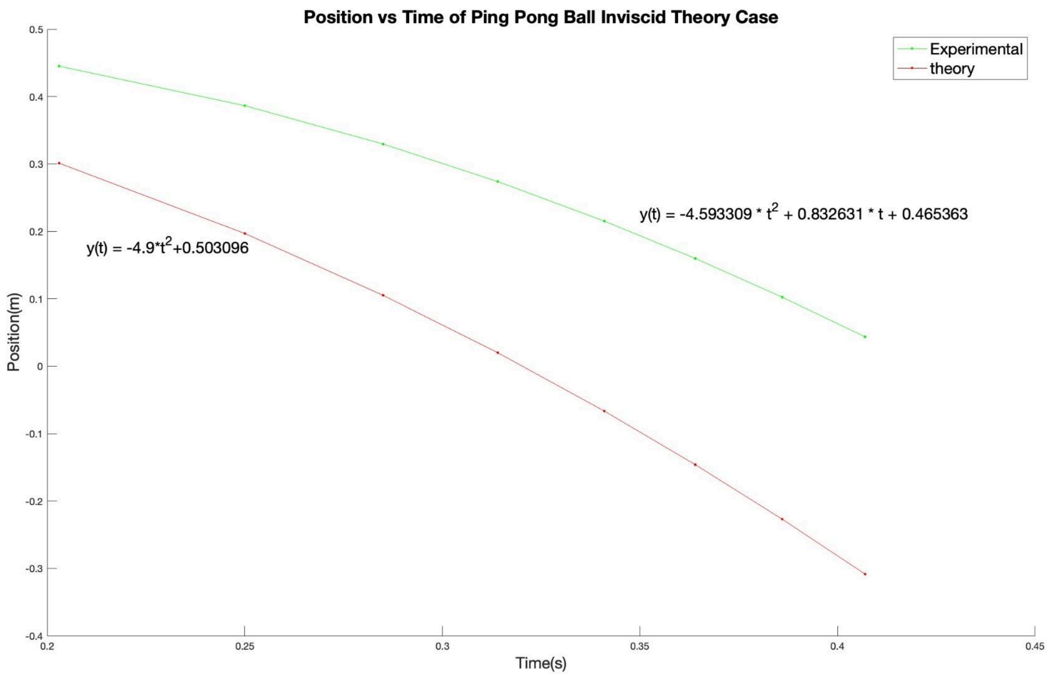


Figure C4: Graph comparing the experimental and theoretical Position vs Time of the Ping Pong Ball in the first approaching case. Theoretical case is fitted for the inviscid case, fitted for the damping coefficient.

## Discussions of Results

For the racquetball, tennis ball, and ping-pong ball, the times the measurement was first tracked by the sensors are 0.202 s, 0.202 s, and 0.203 s, respectively (Figures 6A, 8A, and 7A).

For the racquetball, the initial height is $y_0$ = 0.4983 m, and the first rebound height is $y_1$ = 0.044 m. For the tennis ball and ping-pong ball, the initial height is $y_0$ = 0.4312 m, and the first rebound height is $y_1$ = 0.044 m.

The first approaching velocities ($v_a$) and rebounding velocities ($v_r$) provide insight into the kinetic behavior of the balls during collision. For the racquetball, $v_a$ = - 2.953 m/s and $v_r$ = 0.869 m/s. For the tennis ball, $v_a$ = -2.7606 m/s and vr = 0.9339 m/s. For the ping-pong ball, $v_a$ = -2.957 m/s and $v_r$ = 2.919m/s.

The coefficients of restitution (e) indicate whether collisions are elastic or plastic. For the racquetball, e = 0.294, suggesting a more plastic collision. For the tennis ball, e = 0.338, also indicating a more plastic collision. However, for the ping-pong ball, e = 0.987, suggesting a collision closer to elastic.

The velocity of the different balls can be calculated directly from the polynomial fitted position equation by taking the first derivative. Results of the plotting is shown below as it is plotted against the theory velocity case.

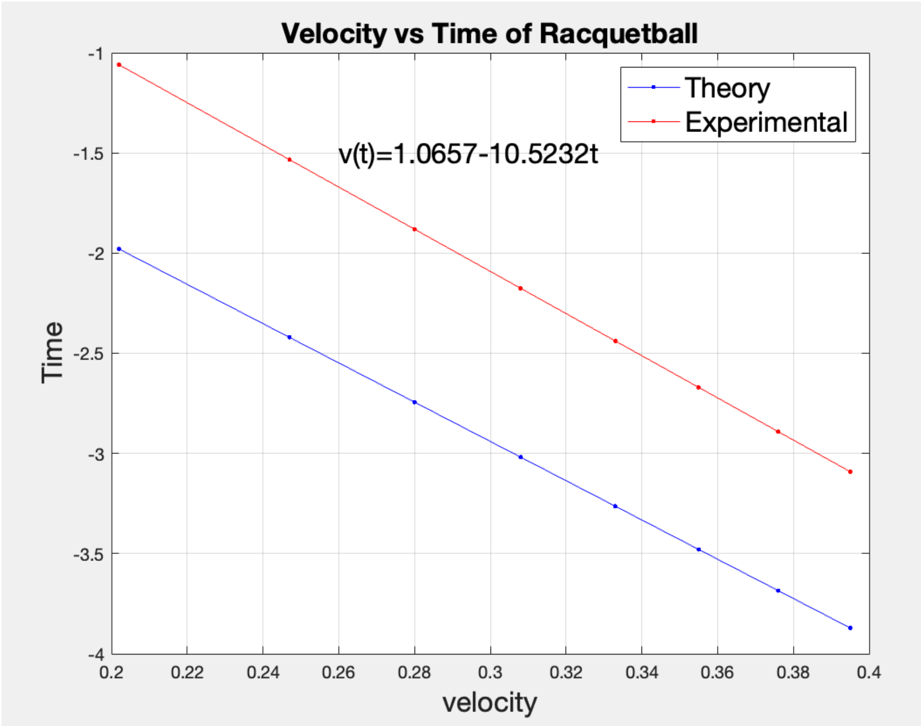


Figure A1: Graph of the velocity vs time of the Racquetball on its first approaching case. Graph shows both experimental and theory case (v(t)=-gt)

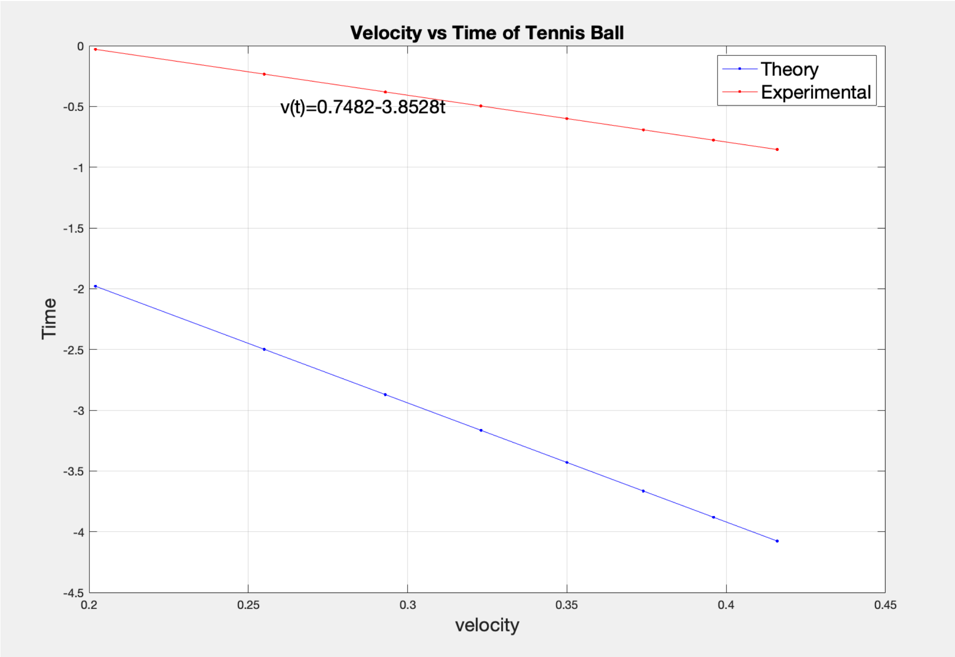


Figure B1: Graph of the velocity vs time of the Tennis ball on its first approaching case. Graph shows both experimental and theory case (v(t)=-gt)

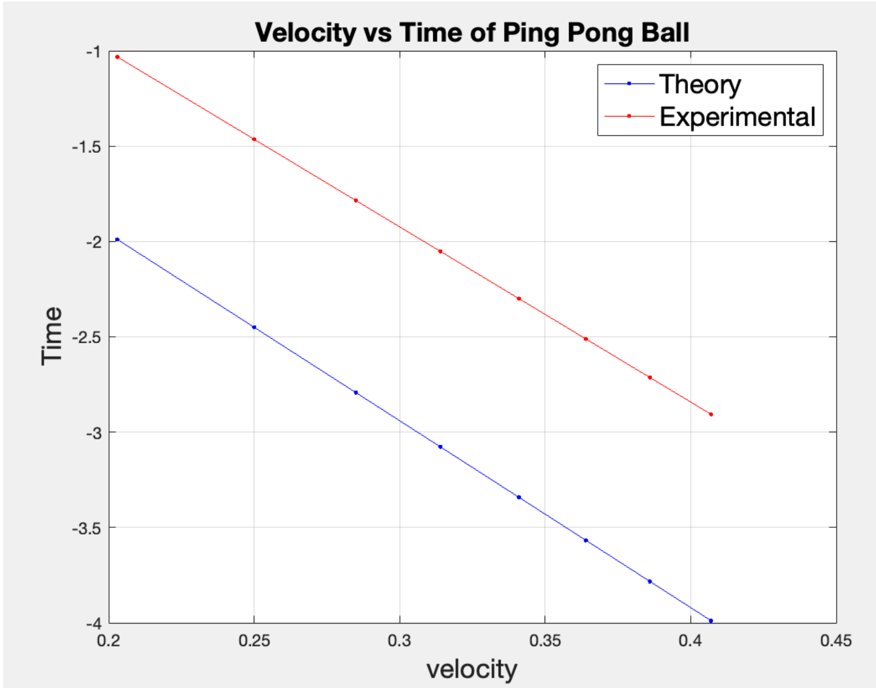


Figure C1: Graph of the velocity vs time of the Ping Pong ball on its first approaching case. Graph shows both experimental and theory case (v(t)=-gt)

Other methods of calculating the velocity are using the experimental data and plugging into equation(4). In the equation, the D is the diameter of the specific ball being used and change of time is the amount of time the ball was blocking the LED emitted light from the phototransistors. It can be seen as a 'rectangle' from Figures 1-3. Such value may be more off due to inaccuracy of the experiment. For this lab specifically, each ball is based through the optical path once which is not enough data to create an accurate testing.

Velocity can be determined by differentiating the position vs time graph (1), resulting in equation (3). Another method is to calculate using experimental data. This would utilize equation (4) where change in time is the change of time as the ball interferes with the sensor's optic path. This can then be plotted in the same time values of position vs time graphs. Figure A1, B1, C1 represents the plotted  velocity found with the differential method compared with the theoretical velocity vs time equation.

Some attributes of the tennis, racquet, and ping pong ball that may have caused differences in the time histories of position and velocity are different surface materials, different size, different weight, and slight difference in initial position dropped. The surface of a tennis ball consists of uniform felt-covered rubber compound with wool, nylon, and cotton which helps reduce drag when thrown. The racquetball is covered in pure rubber which causes it to bounce nearly to its initial position. Ping pong ball is covered with a thin layer of ABS plastic which means the ping pong ball mainly consists of air. This helps the ball bounce multiple times before stopping. The different size and weight will also affect the position and velocity because it directly correlates with how buoyant the balls will be.

To find the viscous damping constant, $b$, for all three balls in the approaching case, the equation (2) was utilized. The damping coefficient for the ping pong ball was found to be 0.0021, the tennis ball at 0.193, and the racquetball at 0.00000006. The value calculated using the inviscid equation does not fit with the model of experimental data collected as illustrated in Figures A4, B4, C4. This proves the need to account for the frictional forces within the atmosphere as the ball falls. The small error percentage may be caused by the data's significant figures. The terminal velocity is calculated using $v_t =- \frac{m}{b} g$, plug in the values for tennis ball $v_t =- \frac{53.8}{0.193} g$, $v_t =- 2785.43$. The ball will approach 99% of its terminal velocity using $t_t = 5\tau = \frac{5m}{b}$, and the value is calculated to be 1419.69 m/s. The sources of friction and viscosity are based on the balls' materials. This can be reduced by using material with lower coefficient of friction.

## Conclusion

This study successfully implemented an optoelectronic sensor system to measure the position of a free-falling ball, enabling the calculation of damping coefficients and coefficient of restitution. Through MATLAB processing, polynomial fitting, and velocity calculations, the experimental results were analyzed, and the inviscid model was compared. The findings showed that the racquetball had the lowest damping coefficient (b = 0.00000006) and the tennis ball had the lowest (b = 0.193). Additionally, the coefficient of restitution revealed that the racquetball had the most plastic collision (e = 0.294) and the ping pong ball approached an elastic collision (e = 0.987). These results demonstrate the effectiveness of optoelectronic sensors in measuring free-falling balls and provide valuable insights into the physics of ball motion. Future studies can explore improvements to the sensor system and experimental design to enhance accuracy and expand the scope of the investigation.

# Appendix

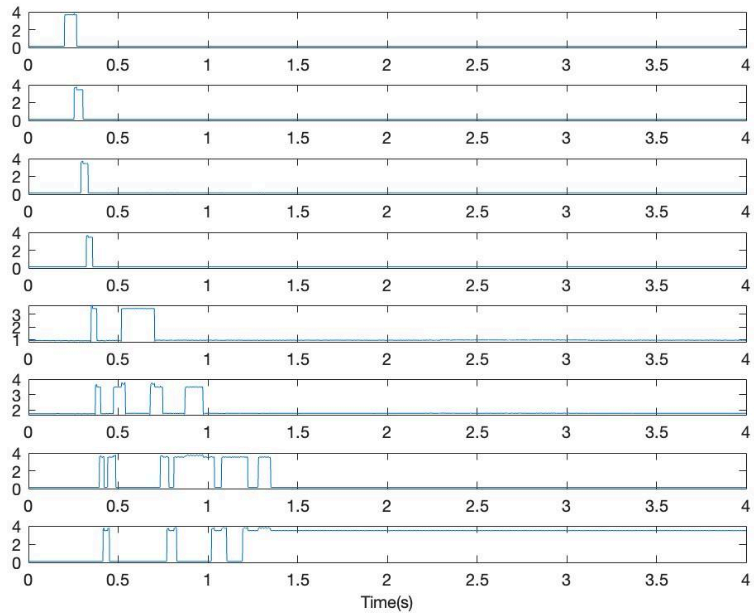


Figure 1: Tennis Ball Optic Path.

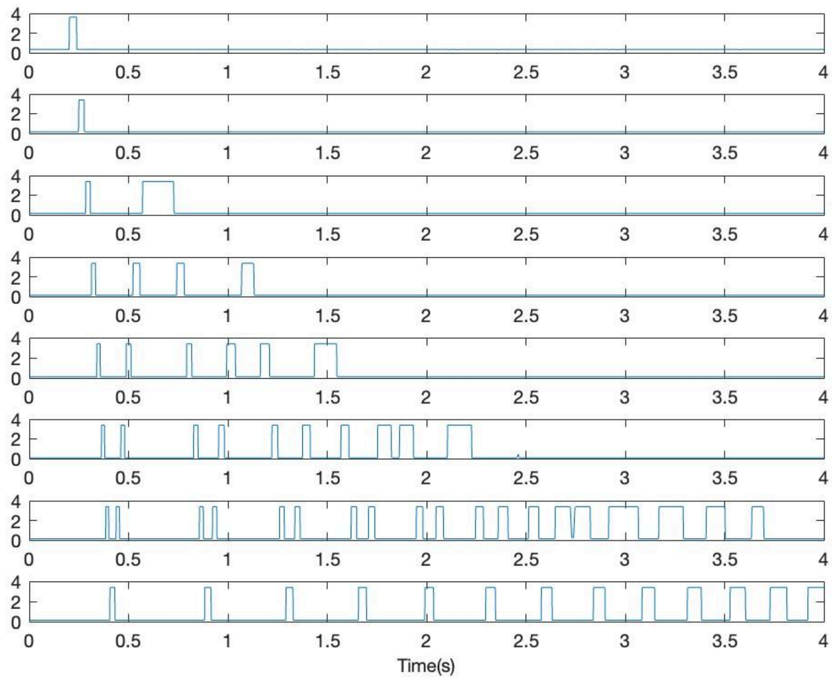


Figure 2: Ping Pong Ball Optic Path

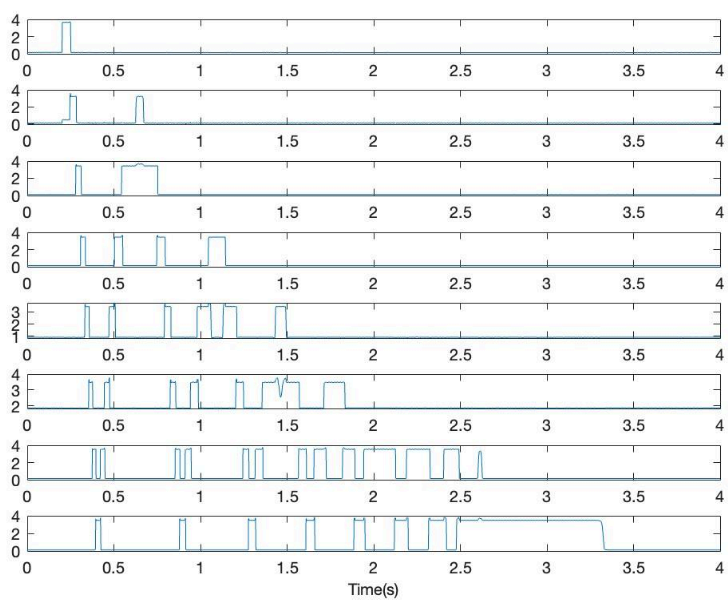


Figure 3: RacquetBall Optic Path

```
% for plotting raw data of 8 sensors
clear;      % Clear variables and functions from memory
close all;  % Closes all the open figure windows
% Read data from an excel file
filename = '';
A = xlsread(filename);
% Create time array
t=;
% Complete for statement block
for i=1:8

end
xlabel('Time(s)', 'FontSize', 10);
% Now read 8 time data for the first approach
%%
% for plotting position and finding b
clear;      % Clear variables and functions from memory
close all;  % Closes all the open figure windows
% Create time array and position array
% Convert inch to international unit
time = [];
position = 0.001*25.4*[];
% Plot experimental position
plot (time, position,'b.-')
xlabel('Time(s)', 'FontSize', 10);
ylabel('Position(m)', 'FontSize', 10);
hold on
% Plot fitted porition
p = polyfit(time,position,2); % Fit data to polynomial
position_fit = ;
plot (, ,'g.-')
hold on
% Calculate tv=0 and y0
velocity = 2*p(1).*time + p(2);
tv_0 = ;
y0 = ;
% Define parameters
m = ;
b = ; % Verify this value to find the best b
g = 9.8;
% Plot position considering b
position_b = ;
plot (, ,'r.-')
legend ('experiment','polyfit','theory');
```

Figure 5: Blank MATLAB code used to create the polynomial fitting of data as well as the damping coefficients of the three balls.

| Time (s) | 0.202 | 0.247 | 0.28 | 0.308 | 0.333 | 0.355 | 0.376 | 0.395 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Position (in) | 17.5 | 15.25 | 13 | 10.75 | 8.5 | 6.25 | 4 | 1.75 |

Figure 6A: Position vs time data of the Racquetball in the first approaching case.

| Time (s) | 0.395 | 0.422 | 0.446 | 0.472 | 0.504 | 0.545 | 0.63 |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Position (in) | 1.75 | 4 | 6.25 | 8.5 | 10.75 | 13 | 15.25 |

Figure 6B: Position vs time data of the Racquetball in the first rebound case.

| Time (s) | 0.203 | 0.25 | 0.285 | 0.314 | 0.341 | 0.364 | 0.386 | 0.407 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Position (in) | 17.5 | 15.25 | 13 | 10.75 | 8.5 | 6.25 | 4 | 1.75 |

Figure 7A: Position vs time data of the Ping Pong Ball in the first approaching case.

| Time (s) | 0.407 | 0.438 | 0.462 | 0.489 | 0.523 | 0.572 |
| --- | --- | --- | --- | --- | --- | --- |
| Position (in) | 1.75 | 4 | 6.25 | 8.5 | 10.75 | 13 |

Figure 7B: Position vs time data of the Ping Pong Ball in first rebound case.

| Time (s) | 0.202 | 0.255 | 0.293 | 0.323 | 0.35 | 0.374 | 0.396 | 0.416 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Position (in) | 17.5 | 15.25 | 13 | 10.75 | 8.5 | 6.25 | 4 | 1.75 |

Figure 8A: Position vs time data of the Tennis Ball in the first approaching case.

| Time (s) | 0.416 | 0.442 | 0.474 | 0.519 |
| --- | --- | --- | --- | --- |
| Position (in) | 1.75 | 4 | 6.25 | 8.5 |

Figure 8B: Position vs time data of the Tennis Ball in the first rebound case.