Experiment 1: Strain Measurements and the Wheatstone Bridge Circuit

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Fundamentals of Mechatronics

Group: 4

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Abstract:

This experiment aims to understand how objects respond to various forces by measuring surface deformation and quantifying mechanical stress using strain gauges. Strain gauges are resistive sensors attached to structural components that measure changes in length or deformation. We will use a quarter-bridge Wheatstone circuit to record longitudinal and transverse strains, which will allow us to calculate the mechanical stress experienced by the component. By analyzing the data using statistical methods and linear regression, we can determine the Young's modulus (E) and Poisson's ratio (v) of the material. Young's modulus represents the material's stiffness, while Poisson's ratio describes its lateral strain response to axial tensile loading. Understanding these mechanical properties is crucial in designing and optimizing structures, materials, and mechanical systems. By performing this experiment, we will gain hands-on experience with strain measurement techniques and data analysis, enhancing our understanding of mechanical testing and measurement principles.

Introduction:

Strain, a fundamental concept in mechanics, refers to the ratio of a material's change in length to its initial length. It can be either tensile (positive) or compressive (negative), resulting from elongation or contraction. When a material is subjected to an external force, the Poisson effect occurs, causing lateral expansion or contraction in the transverse directions. Poisson's ratio (v) quantifies this effect, representing the negative ratio of transverse strain to axial strain. Although dimensionless, strain is often expressed in units like in/in or mm/mm, but due to its small magnitude, micro-strain (με) is used, equivalent to ε x 10^{-6}. Strain can arise from external forces (mechanical strain), thermal effects (thermal strain), or internal forces from non-uniform cooling (residual strain). This experiment focuses on mechanical strain, measuring the deformation caused by an applied external force. The outcome will be determined through both analytical and experimental calculations, allowing for a comparison of accuracy.

Objectives:

- Employing a quarter-bridge Wheatstone circuit with a strain gauge as the active arm to accurately measure both longitudinal and transverse strains in a 7.5-inch long bar subjected to uniaxial tensile force.
- Utilize engineering statistics and regression analysis techniques to calculate the fundamental material properties of the specimen, including Young's Modulus (*E*) and Poisson's Ratio (ν). This analysis aims to provide a comprehensive understanding of the mechanical behavior of the tested material under the applied forces, contributing valuable insights to the material's performance in various conditions.

Theory:

Calculating the Longitudinal stress is a direct relationship between the force and the area. The equation $A = b \cdot h$ represents the area (A) of the material, where both base (b) and height (h) were measured using vernier calipers before the experiment. The area of the bar is measured to have an area of 0.1719 in^2. The forces are added at a 4 lb increment. The relationship can be described below:

$$\sigma_{LK} = \frac{F_k}{A}$$

where k was the k^{th} data point collected during experimentation, and varies: $k \varepsilon \{1, 25\}$.

The Modulus of Elasticity (Young's Modulus), as per Britannica, is a measure of a material's ability to withstand changes in length under tension or compression. While such value can be given by the vendor, such value can also be verified experimentally. This modulus can be derived from another expression related to longitudinal stress.

To derive the Young's Modulus (E) through statistical means, we must first calculate the E for each force interval as a function of longitudinal stress and strain we get: $E = \frac{\sigma_L}{\varepsilon_l}$. Each E value can be seen with its respective force in Figure 3.

In order to find the mean E value, each calculated Young's modulus can be summed up and divided by the number of trials which is expressed in the equation:

$$\overline{E} = \frac{1}{N} \sum E_k \quad \text{eq. (1)}$$

where N is the number of data points.

Another method of calculating the experimental value of Young's Modulus (E_{LR}) is by using linear regression. The slope or the linear Strain can be defined with the equation:

$$E_{LR} = \frac{N \sum_{k=1}^{N} \sigma_{L_k} \varepsilon_{L_k} - \sum_{k=1}^{N} \sigma_{L_k} \sum_{k=1}^{N} \varepsilon_{L_k}}{N \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \left[\sum_{k=1}^{N} \varepsilon_{L_k} \right]^2} \quad (2)$$

To determine the 'zero-offset' stress (σ_{L0}) of the equation plotted in Figure 1, such values can be found with the equation:

$$\sigma_{L0} = \frac{\sum_{k=1}^{N} \sigma_{L_k} \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \sum_{k=1}^{N} \varepsilon_{L_k} \sum_{k=1}^{N} \varepsilon_{L_k} \sigma_{L_k}}{N \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \left[\sum_{k=1}^{N} \varepsilon_{L_k}\right]^2} \quad (3)$$

The same methods applied to find Young's modulus can be done to calculate Poisson's Ratio. The first method described below utilizes statistical means and the equation can be found below:

$$\bar{v} = \frac{1}{N} \sum v_k \quad (4)$$

Linear regression can also be used to calculate another experimental value of Poisson's Ratio. The value can be determined by the equation:

$$v_{LR} = \frac{\sum_{k=1}^{N} \varepsilon_{T_k} \sum_{k=1}^{N} \varepsilon_{L_k} - N \sum_{k=1}^{N} \varepsilon_{L_k} \varepsilon_{T_k}}{N \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \left[\sum_{k=1}^{N} \varepsilon_{L_k}\right]^2} \quad (5)$$

To determine the 'zero-offset' stress (ε_{T0}) of the equation plotted in Figure 2, such value can be found with the equation:

$$\varepsilon_{T0} = \frac{\sum_{k=1}^{N} \varepsilon_{T_k} \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \sum_{k=1}^{N} \varepsilon_{L_k} \sum_{k=1}^{N} \varepsilon_{L_k} \varepsilon_{T_k}}{N \sum_{k=1}^{N} \varepsilon_{L_k}^2 - \left[\sum_{k=1}^{N} \varepsilon_{L_k}\right]^2} \quad (6)$$

The equations used to calculate shear strain utilizes the strain rosette setup. Such a setup is modeled in Figure B2. The subsequent equations is the generalized formula for the strain rosette:

$$\varepsilon_A = \varepsilon_x cos^2\theta_A + \varepsilon_y sin^2\theta_A + \gamma_{xy} cos\theta_A sin\theta_A \quad (7)$$

$$\varepsilon_B = \varepsilon_x cos^2\theta_B + \varepsilon_y sin^2\theta_B + \gamma_{xy} cos\theta_B sin\theta_B \quad (8)$$

$$\varepsilon_c = \varepsilon_x cos^2\theta_c + \varepsilon_y sin^2\theta_c + \gamma_{xy} cos\theta_c sin\theta_c \quad (9)$$

Due to limitations regarding placements of strain gauges on our apparatus the longitudinal strain must be used in place of the third strain gauge of the rosette. The equation (7) could be simplified by relating $\varepsilon_A = \varepsilon_y$, as we utilized the longitudinal strain gauge as part of our strain rosette:

$$\varepsilon_y = \varepsilon_x cos^2\theta_A + \varepsilon_y sin^2\theta_A + \gamma_{xy} cos\theta_A sin\theta_A \quad (10)$$

Also note that by using the longitudinal strain gauge, the corresponding angle would be 90 degrees, rendering the equation useless.

Equipment:

1. Four strain gauges with resistance values of 350 ohms mounted on a acrylic bar (1 longitudinal, 1 transverse, 2 at angles of $\theta_B = 51^{\circ}$ and $\theta_C = 132^{\circ}$)
2. Measurements Group P-3 Wheatstone-bridge strain indicator
3. Measurements Group loading stage (4 lbs/division; total capacity: 200 lbs)
4. Ruler
5. Protractor

Experimental Apparatus and Procedure:

The provided acrylic bar is attached to the loading stage which is then calibrated to a 0 force output using the dial attached to the stage. Once it is confirmed that there is a zero load being applied to the bar, the four different strain gauges are connected to the P-3 strain indicator. This connection is done by following the quarter-bridge wiring diagram seen on the indicator. The wires connected to the gauges are split into red, white and yellow wires that are then affixed to the P+ terminal, the S-terminal, and the D120 terminal respectively. The first and second channels correspond to the longitudinal and transverse strain gauges, while the third and fourth channels are connected to the angled gauges.

Once all the gauges are connected, the menu is used to set the units to microstrain (με) and enter a gauge factor of 2.135. Finally, the measurements for both channels are balanced using the balance button available on the indicator.

Now, loads are being applied! The load is increased by increments of 4 until 100 pounds while the data for the readings are recorded (this can be seen in Figure 3). These figures are then used to calculate Young's modulus, Poisson's ratio, and longitudinal stress through the use of multiple different calculation methods

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The experimental apparatuses listed in the Equipment section above are used to collect all the data needed for calculations in this lab. The two element rosette strain gauge sheen in Figure A below is used to measure values of longitudinal and transverse strain and are one parallel with the x and y axes respectively. The channels on the Measurements Group P-3 Wheatstone-bridge strain indicator providing the corresponding data are channels 1 and 2.

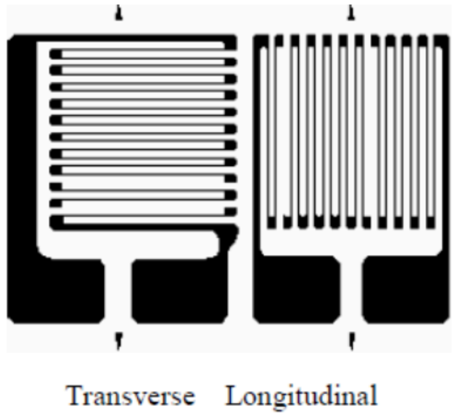

Figure A: Two element strain rosette

Additionally, there are have two pairs of strain gauge on the acrylic bar at the angles $\theta_1 = 51^{\circ}$ and $\theta_2 = 132^{\circ}$ whose placement can be seen in Figure B1. This pair of gauges provide the data to calculate the values of shear strain and are connected to channels 3 and 4 on the Wheatstone-bridge strain indicator. Figure B2 shows a close up photo of the gauges used for this lab.

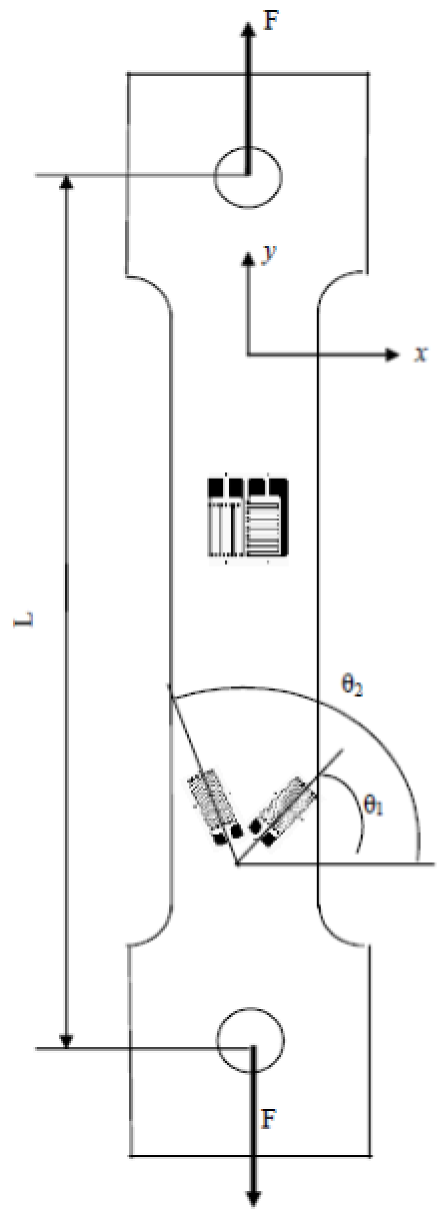

Figure B1: Bar with two pair of strain gauge mounted (lower part of bar) as well as the longitudinal and transverse strain gauge (upper part of bar)

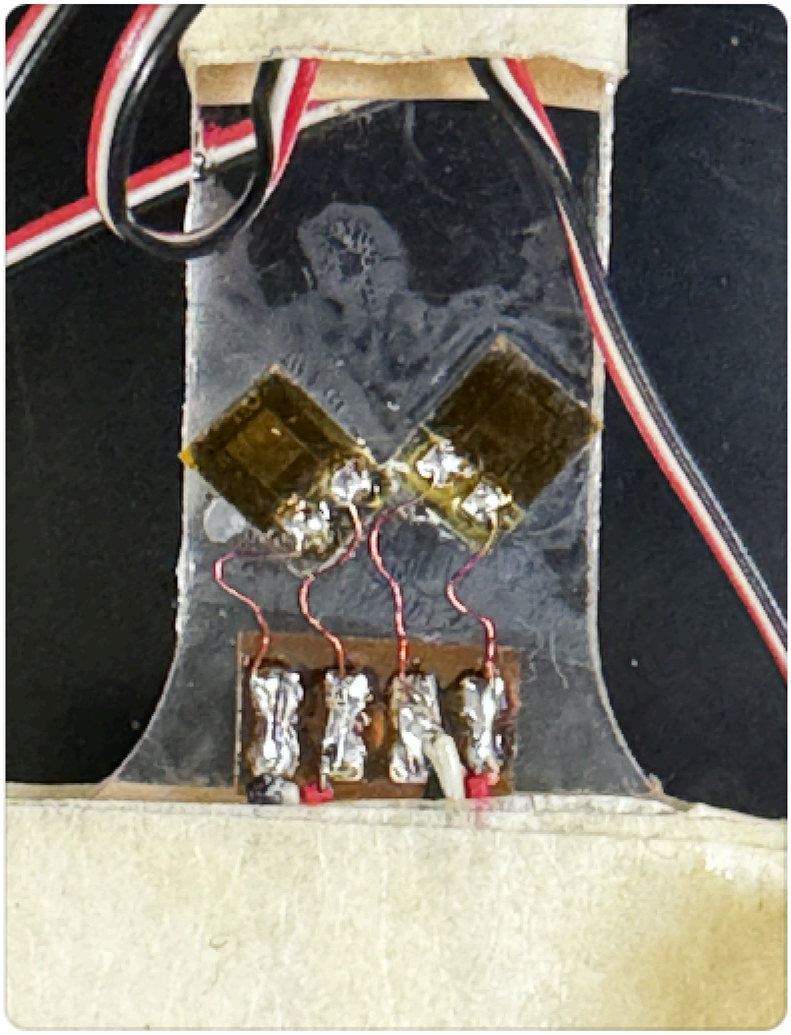

Figure B2: Photo of experimental strain gauges corresponding (left strain gauge corresponds with channel 3, right strain gauge corresponds with channel 4)

Experimental Results:

In calculating the Young's Modulus, the calculation in obtaining the value through statistical means is found below:

$$\overline{E} = \frac{1}{24} * 13.9512 \text{ eq. (1)}$$

$$\overline{E} = 0.5813\, Msi$$

$$\overline{E} = 581.3\, ksi$$

Young's Modulus through Linear regression:

$$E_{LR} = \frac{24(5.8017E6)-(7576.63)(14431)}{24(1.1266E7)-(14431)^2} \text{ (2)}$$

$$E_{LR} = 0.4844\, Msi$$

$$E_{LR} = 484.4\, ksi$$

Longitudinal stress (zero-offset) through linear regression:

$$\sigma_{L0} = \frac{(7576.63)(1.1266E7)-(14431)(5.8017E6)}{24(1.1266E7)-(14431)^2} \text{ (3)}$$

$$\sigma_{L0} = 23.8934\, psi$$

Poisson's Ratio through statistical means:

$$\overline{v} = \frac{1}{24} * 8.5308 \text{ (4)}$$

$$\overline{v} = 0.3556$$

Poisson's Ratio through linear regression:

$$v_{LR} = \frac{(-4846)(14431)-24(-3.7617E6)}{24(1.1266E7)-(14431)^2} \text{ (5)}$$

$$v_{LR} = 0.3276$$

Transverse strain (zero-offset) through linear regression:

$$\varepsilon_{T0} = \frac{(-4846)(1.1266E7)-(14431)(-3.7617E6)}{24(1.1266E7)-(14431)^2} \quad (6)$$

$$\varepsilon_{T0} = -4.9633\ \mu\varepsilon$$

When $F = 60lb$, $\varepsilon_B = 177\ \mu\varepsilon$, $\varepsilon_C = 288\ \mu\varepsilon$, $\theta_C = 132^\circ$, $\theta_B = 51^\circ$ $\varepsilon_y = 651\ \mu\varepsilon$. Note due to constraints, value for $\varepsilon_A = \varepsilon_y$ which is a measured value. Calculations for the shear strain and transverse strain is then found in the following calculations:

$$651 = \varepsilon_x cos^2 90^\circ + \varepsilon_y sin^2 90^\circ + \gamma_{xy} cos 90^\circ sin 90^\circ \quad (10)$$

$$177 = \varepsilon_x cos^2 51^\circ + \varepsilon_y sin^2 51^\circ + \gamma_{xy} cos 51^\circ sin 51^\circ \quad (8)$$

$$288 = \varepsilon_x cos^2 132^\circ + \varepsilon_y sin^2 132^\circ + \gamma_{xy} cos 132^\circ sin 132^\circ \quad (9)$$

The solution of these equations are shown below:

$$\varepsilon_y = 651\ \mu\varepsilon$$

$$\varepsilon_x = -348.25\mu\varepsilon$$

$$\gamma_{xy} = -169.49\mu\varepsilon$$

Error calculations

The error calculation is done in comparison with the values found experimentally and through the handbook. Each value obtained experimentally through linear regression and statistical means is calculated below:

Linear regression (v_{LR})

$$100 - (\frac{0.3276}{0.4} \times 100)$$
$$= 18.1\%$$

Statistical means $(\overline{v})$

$$100 - (\frac{0.3556}{0.4} \times 100)$$
$$= 11.1\%$$

Linear regression (E_{LR})

$$100 - (\frac{484.4}{470}) \times 100)$$
$$= 3.06\%$$

Statistical means $(\overline{E})$

$$100 - (\frac{581.3}{470}) \times 100)$$
$$= 23.6\%$$

Experimental Data

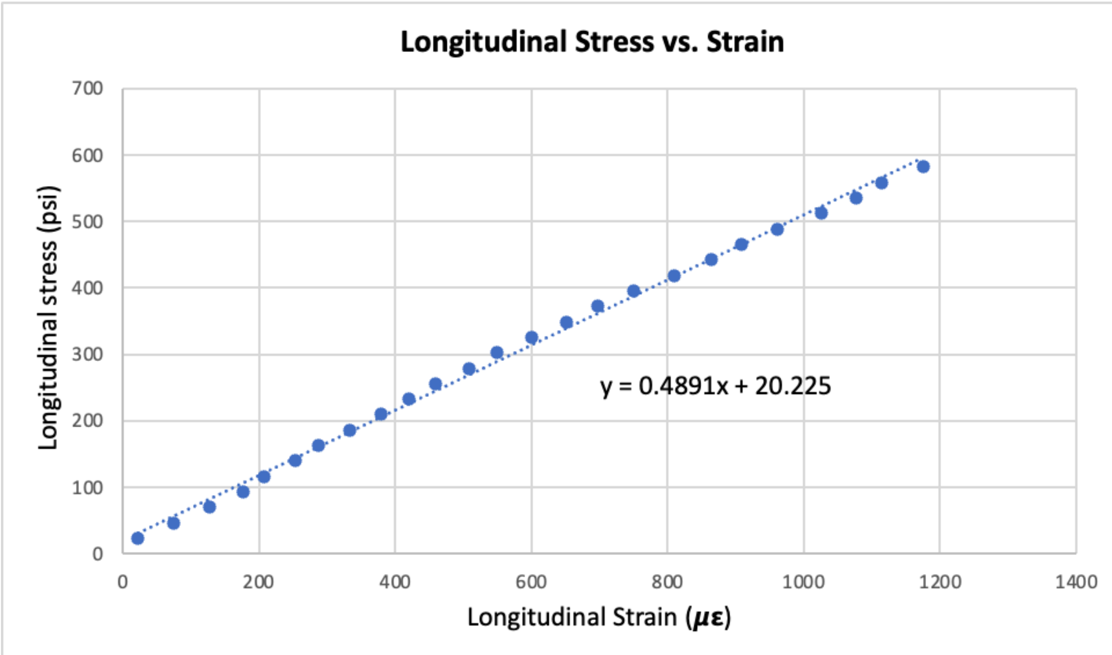

Figure 1: Graph plotting the Longitudinal Strain vs Stress

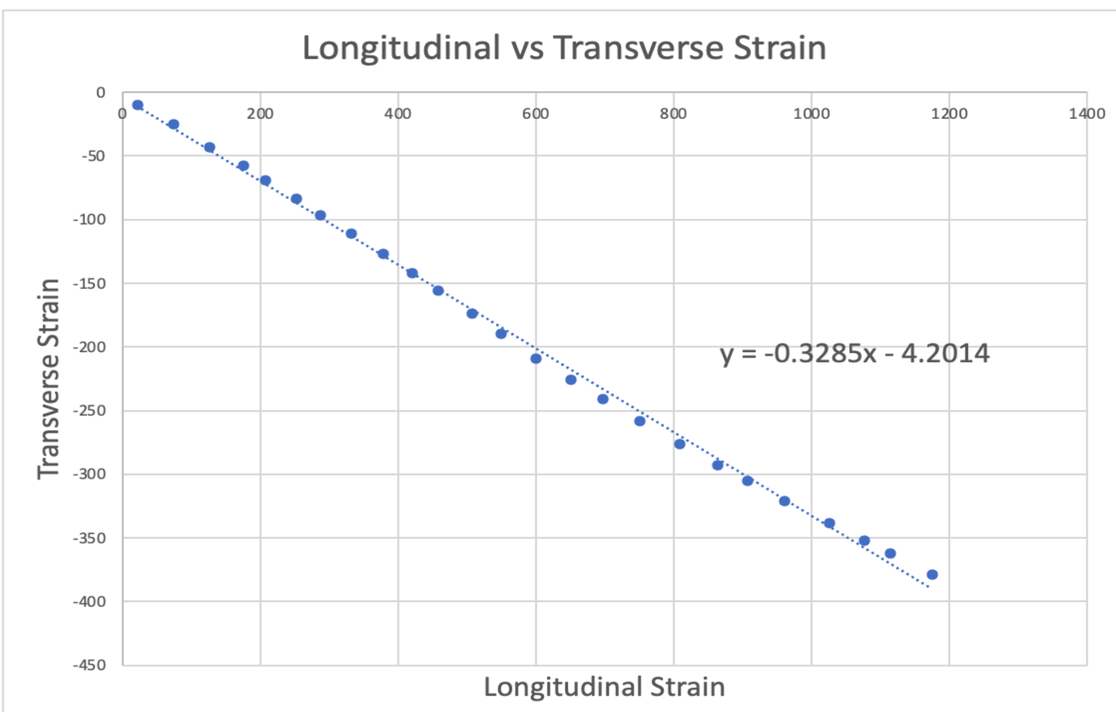

Figure 2: Graph plotting the Transverse vs Longitudinal Strain

Force (lb)	Longitudinal Strain (με)	Transverse Strain (με)	Channel 3 (με)	Channel 4 (με)	Longitudinal stress (psi)	Young's Modulus (psi)	Poisson's Ratio
0	0	0	0	0	0	-	-
4	22	-10	13	14	23.27272727	1.05785124	0.454545455
8	74	-25	27	39	46.54545455	0.628992629	0.337837838
12	127	-43	43	52	69.81818182	0.549749463	0.338582677
16	176	-58	47	80	93.09090909	0.52892562	0.329545455
20	207	-69	57	105	116.3636364	0.562143171	0.333333333
24	253	-84	52	138	139.6363636	0.551922386	0.33201581
28	288	-97	59	163	162.9090909	0.565656566	0.336805556
32	333	-111	63	188	186.1818182	0.559104559	0.333333333
36	379	-127	72	208	209.4545455	0.552650516	0.335092348
40	420	-142	87	224	232.7272727	0.554112554	0.338095238
44	459	-156	103	238	256	0.557734205	0.339869281
48	508	-174	124	247	279.2727273	0.549749463	0.342519685
52	549	-190	152	251	302.5454545	0.551084617	0.346083789
56	601	-209	164	270	325.8181818	0.542126758	0.347753744
60	651	-226	177	288	349.0909091	0.536237956	0.347158218

64	698	-241	186	314	372.3636364	0.533472258	0.345272206
68	751	-258	192	336	395.6363636	0.526812735	0.343541944
72	809	-276	190	373	418.9090909	0.51781099	0.341161928
76	864	-293	188	405	442.1818182	0.511784512	0.33912037
80	908	-305	203	418	465.4545455	0.512615138	0.335903084
84	961	-321	221	435	488.7272727	0.508561158	0.334027055
88	1026	-338	236	453	512	0.499025341	0.329434698
92	1077	-352	257	469	535.2727273	0.497003461	0.326833798
96	1115	-362	276	481	558.5454545	0.500937627	0.324663677
100	1175	-379	292	497	581.8181818	0.49516441	0.322553191

Figure 3: Table of experimental data collected with 25 samples collected. (Note in calculations, the N value disregards 0 lb force)

Discussion of Results:

In the calculations for the Poisson's Ratio, the method of statistical means yield a $\bar{v}$ $of\ 0.3556$ and through the method of linear regression the obtained v_{LR} of 0.3276. The expected value would be ~0.4, giving us an error percentage to be 18.1% for the linear regression method and a 11.1% error for the statistical means method.

The method employed in the calculations of Young's Modulus. The expected value of E would be in the range 325-470 ksi and calculated E = 484.4 ksi giving us an error of 3.06%. The value is slightly out of bound from the range which could be due to variation in the strain measurements. Through statistical means, the value of our E is about 581.4 ksi with an error of 23.6%. It is apparent in this case that the linear regression method is more accurate.

In addition our zero-offset values can be found through linear regression. The offset for the transverse strain (ε_{T0}) corresponding to Figure 2 has a value of $-\ 4.9633$ με. The zero-offset corresponding to Figure 1 (σ_{L0}) can be seen to be around $23.8934\ psi$. This value is expected to be the same in comparison to our Poisson's Ratio and Young's Modulus but due to calibration errors, it does not start at 0.

In examining the relationship between the longitudinal strain and stress, it is observed that there is a linear relationship between the two (Figure 1) in which the slope defines our Young's Modulus. Similarly, such an observation is made of the graph of our Transverse Stress vs. Strain (Figure 2). The graph can also be observed to be a negative linear relationship between the two, related by its Poisson's Ratio as the slope.

In the calculations for the shear strain, one force value is used (Figure 3) to define our inputs for (7-9), which is chosen to be 60 lb. The experiment is limited in the execution of a strain rosette, therefore $\varepsilon_A = \varepsilon_y$ which is measured to be 651 με**.** ε_B can be observed from channel 3 with a value of 177 με and ε_c can be observed from channel 4 measured at 288 με. The angles of the strain gauge placed are measured as $\theta_B = 51^{\circ}$ and $\theta_C = 132^{\circ}$. The calculated transverse strain (ε_x) and shear strain (γ_{xy}) (7-9) are as following:

$$\varepsilon_y = 651\ \mu\varepsilon$$

$$\varepsilon_x = -\ 348.25\mu\varepsilon$$

$$\gamma_{xy} = -\ 169.49\mu\varepsilon$$

If the experiment were to be performed under harsh conditions, after installation, the strain gauges should be protected by a coating to prevent any external factors from affecting the measurements.

Conclusion:

In this experiment, students gain a more in-depth understanding on the mechanical behavior of the provided specimen under the condition of applied forces. Using the techniques of Young's modulus (E) and Poisson's ratio (v) of the material by analyzing the data using statistical methods and linear regression. The values found throughout the experience were slightly different or out of the range when compared to the actual values due to measurement error or material defection. After this experiment, it is obvious that the mechanical behavior of materials under the condition of applied forces is well understood and being more cautious in measurements and calculations are crucial for this type of experiment.

References:

[1] “Young's Modulus.” Encyclopædia Britannica, Encyclopædia Britannica, Inc., https:
Appendix