Lasso estimation in multiple linear regression

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1. Lasso for the Boston Housing data

The Boston House-price dataset concerns housing values in 506 suburbs of Boston corresponding to year 1978. They are available here: https://archive.ics.uci.edu/ml/datasets/Housing

The Boston House-price corrected dataset (available in boston.Rdata) con- tains the same data (with some corrections) and it also includes the UTM coor- dinates of the geographical centers of each neighborhood.

1.1 Lasso estimation using the package 'glmnet'

After loading the right package, the response and explanatory variables from the Boston Housing data are set.

```
#install.packages("glmnet")
library(glmnet)
```

```
## Loading required package: Matrix
## Loaded glmnet 3.0-2
```

```
library(Matrix)

boston <- load("boston.Rdata")

response <- "CMEDV"

explanatory <- c("CRIM", "ZN", "INDUS", "CHAS", "NOX", "RM", "AGE", "DIS", "RAD", "TAX", "PTRATIO", "B"

# glmnet cannot handle factors -> "CHAS" is a factor
boston.c$CHAS <- as.numeric(boston.c$CHAS)
```

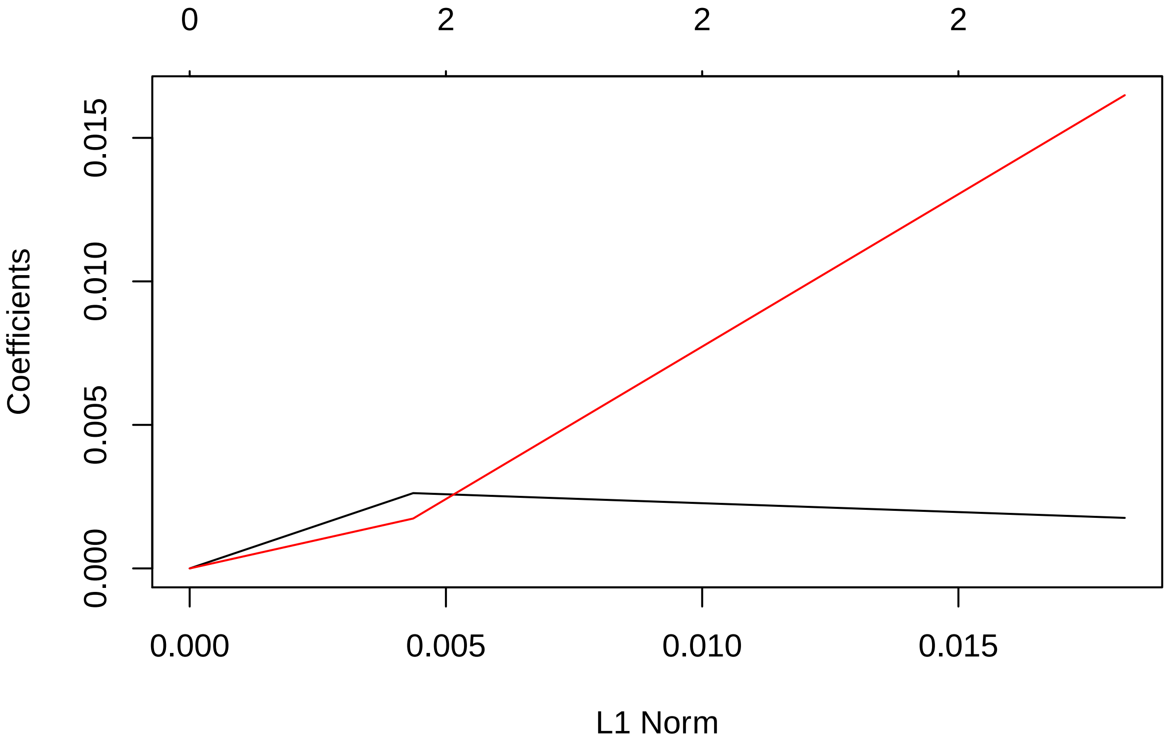
Fitting the Lasso Regression model.

```
lasso_boston <- glmnet(x = as.matrix(boston.c[, explanatory]),
                y = boston.c$CMEDV,
                alpha = 1,     # specifying alpha = 1: Lasso Regression
                standardize = FALSE,
                intercept = FALSE)
lasso_boston$beta
```

```
## 13 x 5 sparse Matrix of class "dgCMatrix"
##         s0          s1          s2          s3          s4
## CRIM     . .           .           .           .
## ZN       . .           .           .           .
## INDUS    . .           .           .           .
## CHAS     . .           .           .           .
```

```
## NOX      . .           .           .           .
## RM       . .           .           .           .
## AGE      . .           .           .           .
## DIS      . .           .           .           .
## RAD      . .           .           .           .
## TAX      . 0.002623633 0.002307334 0.002022783 0.001760304
## PTRATIO  . .           .           .           .
## B        . 0.001737741 0.007119139 0.012018732 0.016486342
## LSTAT    . .           .           .           .
```

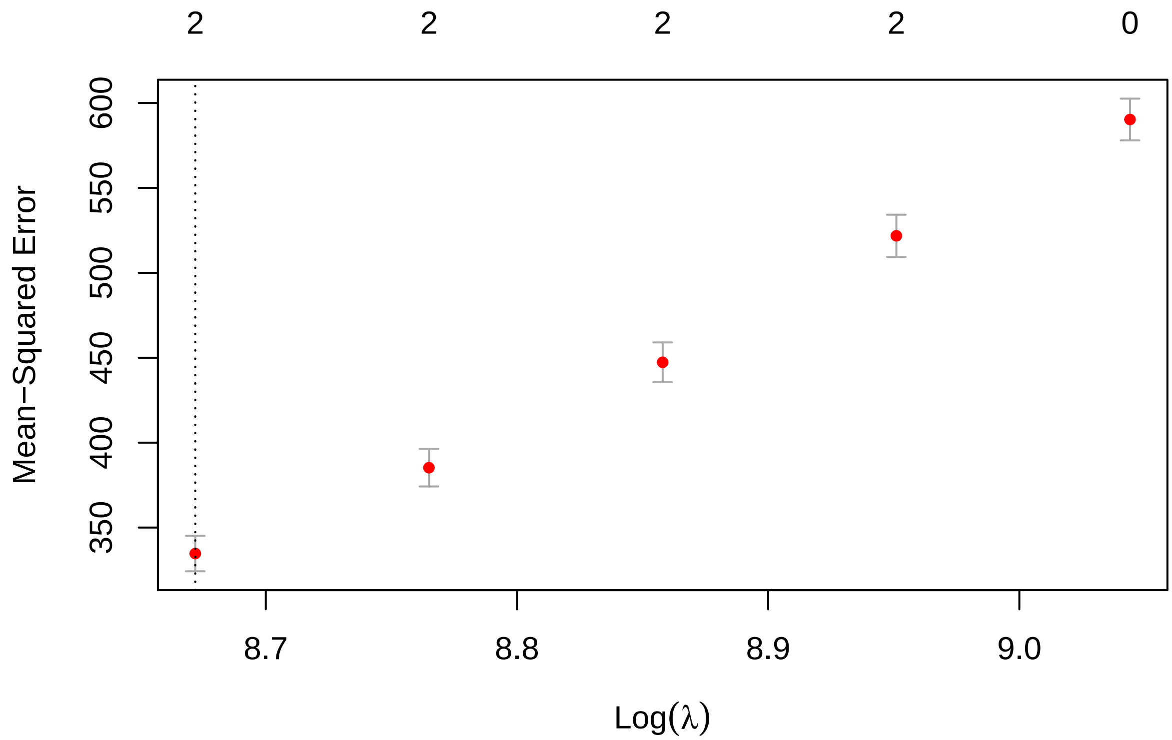
```r
plot(lasso_boston)
```




```r
#text(locator(), labels = c("TAX", "B"))
# names of the betas missing in the plot

cv_lasso_boston <- cv.glmnet(x = as.matrix(boston.c[, explanatory]),
                y = boston.c$CMEDV,
                alpha = 1,      # specifying alpha = 1: Lasso Regression
                standardize = FALSE,
                intercept = FALSE)
cv_lasso_boston
```

```
##
## Call:  cv.glmnet(x = as.matrix(boston.c[, explanatory]), y = boston.c$CMEDV,      alpha = 1, standard
##
## Measure: Mean-Squared Error
##
##     Lambda Measure    SE Nonzero
## min   5837   334.7 10.41       2
## 1se   5837   334.7 10.41       2
```

```r
plot(cv_lasso_boston)
```



The Lasso Estimation does variable selection automatically. This results in a model with two non-zero explanatory variables. The variables affecting the model are the full-value property tax rates ('TAX') and the proportion of blacks by town ('B').

1.2 Ridge Regression model using glmnet

Task description:

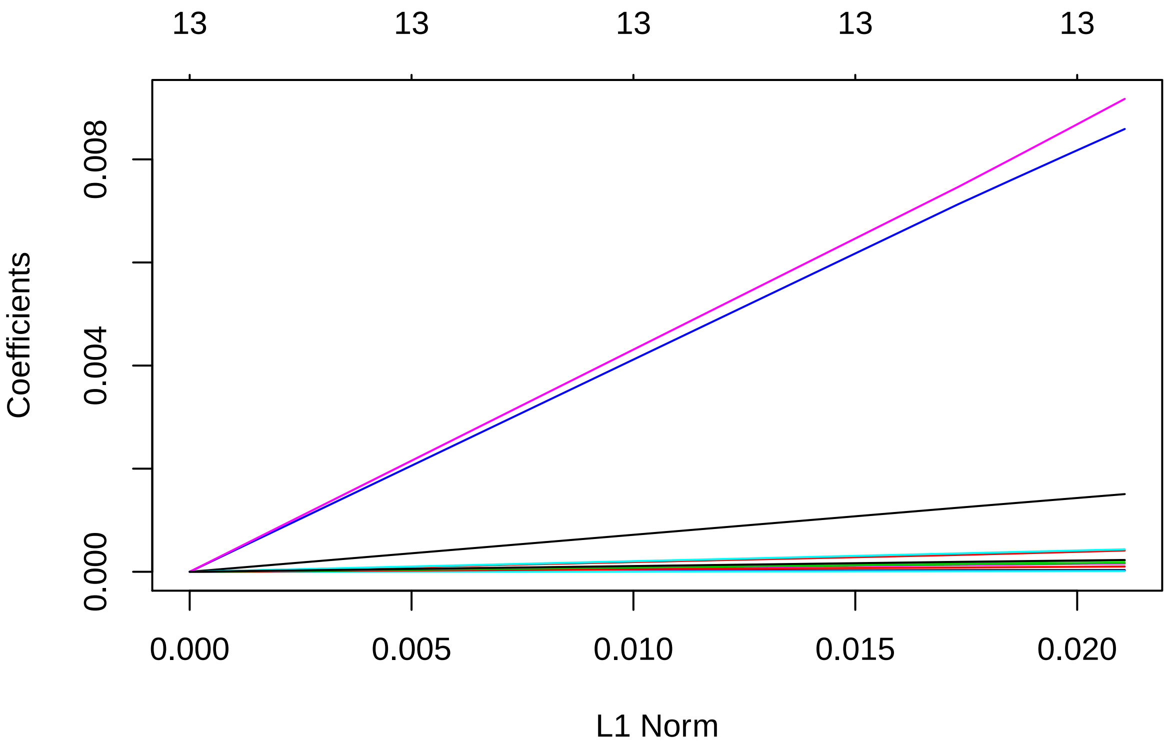
Use glmnet to fit the previous model using ridge regression. Compare the 10-fold cross validation results from function cv.glmnet with those you obtained in the previous practice with your own functions.

```r
ridge_boston <- glmnet(x = as.matrix(boston.c[, explanatory]),
                y = boston.c$CMEDV,
                alpha = 0,     # specifying alpha = 0: Ridge Regression
                standardize = FALSE,
                intercept = FALSE)
ridge_boston$beta
```

```
## 13 x 5 sparse Matrix of class "dgCMatrix"
##                    s0           s1           s2           s3           s4
## CRIM    5.121190e-35 3.129122e-05 3.217069e-05 3.283579e-05 3.323520e-05
## ZN      3.363982e-34 3.267988e-04 3.533479e-04 3.817224e-04 4.120141e-04
## INDUS   2.226496e-34 1.816651e-04 1.932905e-04 2.051945e-04 2.173131e-04
## CHAS    2.474348e-35 2.191053e-05 2.349778e-05 2.516203e-05 2.690234e-05
## NOX     1.216239e-35 1.049404e-05 1.122691e-05 1.199023e-05 1.278256e-05
## RM      1.475444e-34 1.314780e-04 1.410854e-04 1.511744e-04 1.617426e-04
## AGE     1.462027e-33 1.242872e-03 1.327772e-03 1.415833e-03 1.506813e-03
## DIS     9.122115e-35 8.292515e-05 8.914248e-05 9.569996e-05 1.026014e-04
## RAD     1.862984e-34 1.464932e-04 1.552873e-04 1.641726e-04 1.730775e-04
```

```
## TAX     8.553691e-33 7.129210e-03 7.601712e-03 8.088988e-03 8.589147e-03
## PTRATIO 4.098485e-34 3.567226e-04 3.819446e-04 4.082736e-04 4.356713e-04
## B       8.399607e-33 7.464300e-03 8.007256e-03 8.576883e-03 9.172900e-03
## LSTAT   2.389677e-34 1.922832e-04 2.042857e-04 2.165093e-04 2.288740e-04
```

```
plot(ridge_boston)
```



The ridge regression of boston data using cv.glmnet

```
cv_ridge_boston <- cv.glmnet(x = as.matrix(boston.c[, explanatory]),
                y = boston.c$CMEDV,
                alpha = 0,     # specifying alpha = 0: Ridge Regression
                standardize = FALSE,
                intercept = FALSE)
cv_ridge_boston$glmnet.fit$beta
```

```
## 13 x 5 sparse Matrix of class "dgCMatrix"
##                   s0           s1           s2           s3           s4
## CRIM    5.121190e-35 3.129122e-05 3.217069e-05 3.283579e-05 3.323520e-05
## ZN      3.363982e-34 3.267988e-04 3.533479e-04 3.817224e-04 4.120141e-04
## INDUS   2.226496e-34 1.816651e-04 1.932905e-04 2.051945e-04 2.173131e-04
## CHAS    2.474348e-35 2.191053e-05 2.349778e-05 2.516203e-05 2.690234e-05
## NOX     1.216239e-35 1.049404e-05 1.122691e-05 1.199023e-05 1.278256e-05
## RM      1.475444e-34 1.314780e-04 1.410854e-04 1.511744e-04 1.617426e-04
## AGE     1.462027e-33 1.242872e-03 1.327772e-03 1.415833e-03 1.506813e-03
## DIS     9.122115e-35 8.292515e-05 8.914248e-05 9.569996e-05 1.026014e-04
## RAD     1.862984e-34 1.464932e-04 1.552873e-04 1.641726e-04 1.730775e-04
## TAX     8.553691e-33 7.129210e-03 7.601712e-03 8.088988e-03 8.589147e-03
## PTRATIO 4.098485e-34 3.567226e-04 3.819446e-04 4.082736e-04 4.356713e-04
## B       8.399607e-33 7.464300e-03 8.007256e-03 8.576883e-03 9.172900e-03
## LSTAT   2.389677e-34 1.922832e-04 2.042857e-04 2.165093e-04 2.288740e-04
```

The function that we have used in the other practice

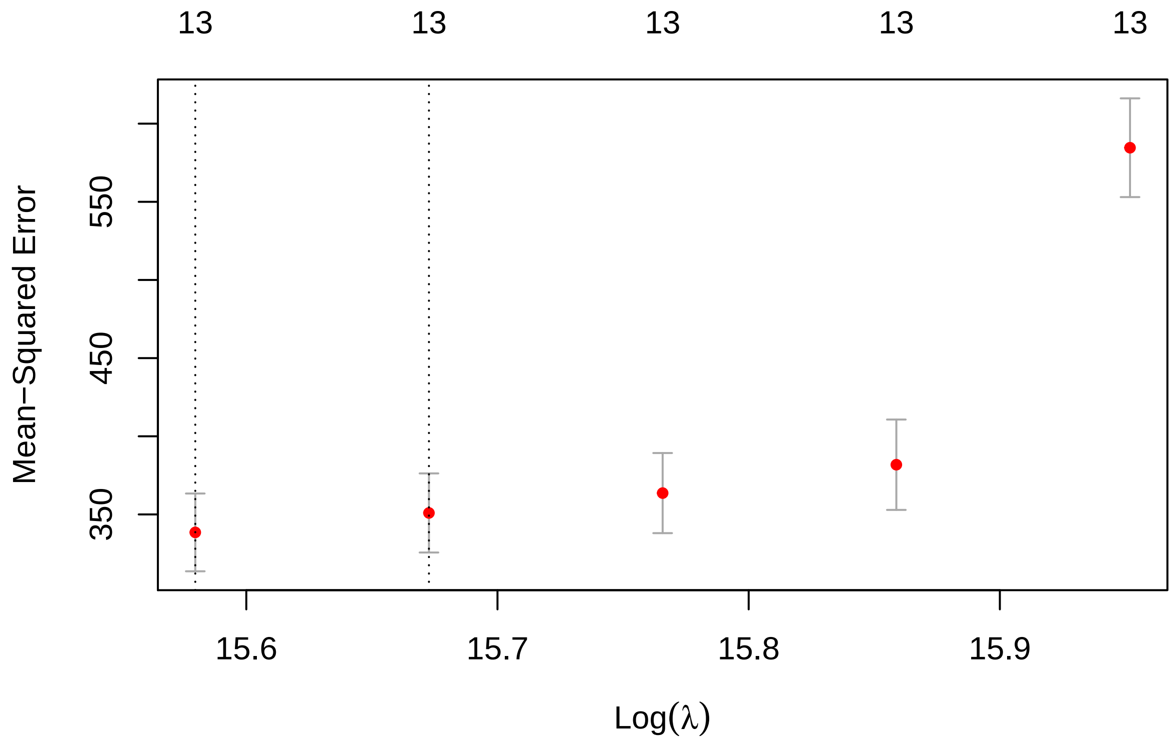
```
PMSE_k_fold <- function(X_t, Y_t, lambda.v, k=10){

  n_X_t <- dim(X_t)[1]
  p_X_t <- dim(X_t)[2] #length of columns
  n_subset <- as.integer((n_X_t)/k)+ 1
  group <- rep(seq(1,k), times = n_subset)
  group <- group[1:n_X_t]
group_random <- sample(group)
X_t_group <- cbind(X_t, group_random)
Y_t_group <- cbind(Y_t, group_random)

#now we can start:

PMSE <- list()

for (la in 1:n_lambdas){

  group_random <- sample(group)

  X_t_group <- cbind(X_t, group_random)

  Y_t_group <- cbind(Y_t, group_random)

  #now we can start:

  PMSE <- list()

  for (la in 1:n_lambdas){

    lambda <- lambda.v[la]

    y_hat <- list()
    beta <- list()
    h <- list()
    y <- list()

    for (l in 1:k){

      new_X_t_val <- subset(X_t_group, group_random==l)[ ,1:p_X_t]
      new_X_t_test <- subset(X_t_group, group_random!=l)[ ,1:p_X_t]

      new_Y_t_val <- subset.matrix(Y_t_group, group_random==l)[,1]
      new_Y_t_test <- subset.matrix(Y_t_group, group_random!=l)[,1]

      p_new <- dim(new_X_t_test)[2]
      p_new_v <- dim(new_X_t_val)[2]

      beta[[l]] <- solve(t(new_X_t_test)%*%new_X_t_test + lambda*diag(1,p_new))%*% t(new_X_t_test)%*%(n

      H_val <- new_X_t_val%*%solve(t(new_X_t_val)%*%new_X_t_val + (lambda+1e-13)*diag(1,p_new_v))%*% t(n

      # singular matrix for lambda = 0 -> trick: add a very small number
```

```
      y_hat[[l]] <-  (new_X_t_val)%*%beta[[l]]
      h[[l]] <- diag(H_val)
      y[[l]] <- new_Y_t_val


  }

  y_hat <- c(do.call(rbind, y_hat))
  beta <- c(do.call(cbind, beta))
  h <- c(do.call(rbind, h))
  y <- c(do.call(rbind, y))

  PMSE[[la]] <- 1/n_X_t * sum(((y-y_hat)/(1-h))^2)

  }

}

return(PMSE)

}
lambda.max = 2e4
n_lambdas <- 25
lambda.v <- exp(seq(0,log(lambda.max+1),length=n_lambdas))-1
```

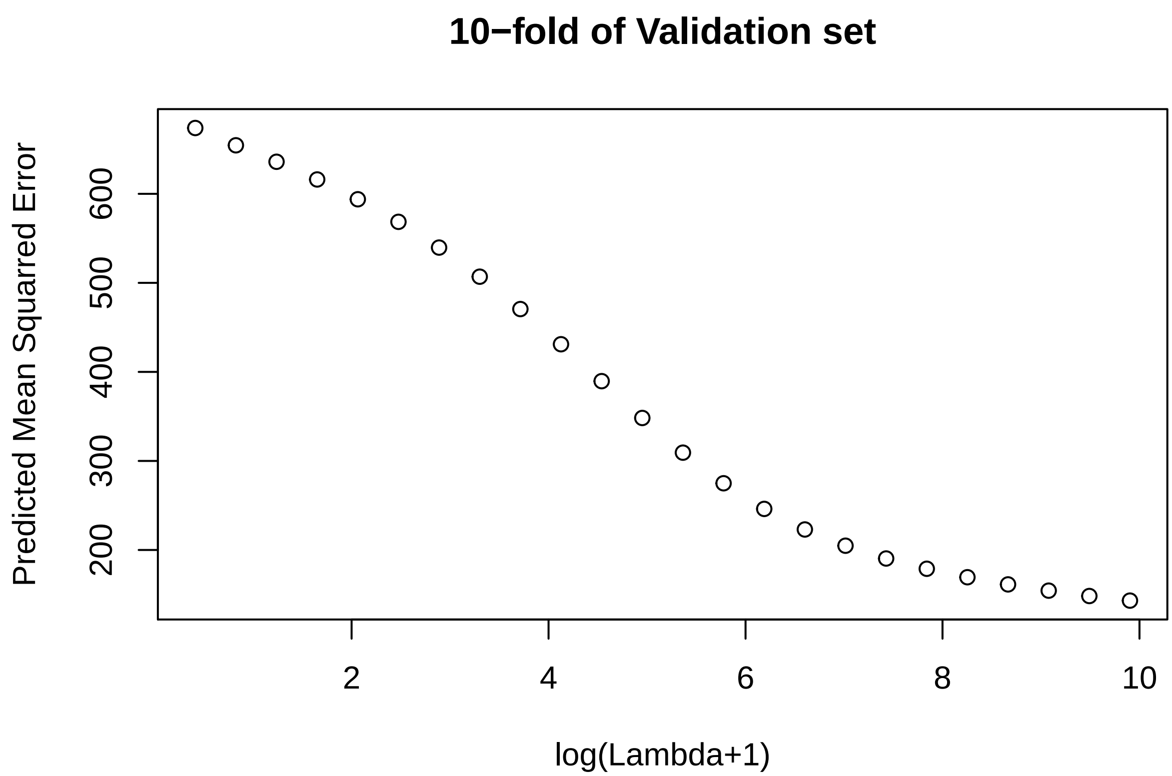
```
cv_ridge_boston
```

```
##
## Call:  cv.glmnet(x = as.matrix(boston.c[, explanatory]), y = boston.c$CMEDV,      alpha = 0, standard
##
## Measure: Mean-Squared Error
##
##      Lambda Measure    SE Nonzero
## min 5836770   338.5 24.87      13
## 1se 6405848   351.0 25.30      13
```

```
plot(cv_ridge_boston)
```




```r
PMSE_val_10 <- PMSE_k_fold(X_t = as.matrix(boston.c[, explanatory]), Y_t = boston.c$CMEDV, lambda.v = la
plot(log(lambda.v[-1]+1), PMSE_val_10[-1], ylab = "Predicted Mean Squarred Error", xlab = "log(Lambda+1)
```




```r
#I am not sure if we have to keep or skip the +1 of this plot in the log(lambda)
```

2. A regression model with $p >> n$

Reading in the data.

```
express <- read.csv("journal.pbio.0020108.sd012.CSV",header=FALSE)
surv <- read.csv("journal.pbio.0020108.sd013.CSV",header=FALSE)
death <- (surv[,2]==1)
log.surv <- log(surv[death,1]+.05)
expr <- as.matrix(t(express[,death]))
colnames(expr) <- paste("V", 1:nrow(express), sep = "")
```

2.1 Lasso estimation using glmnet for regressing 'log.surv' against 'expr'

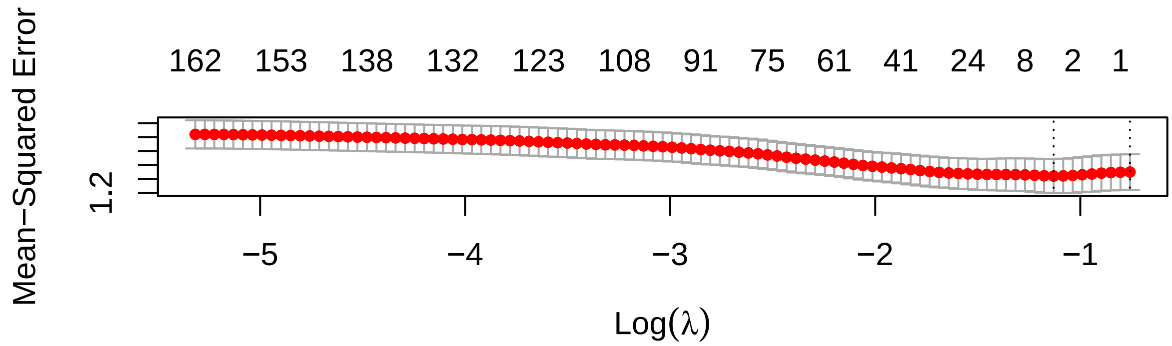
Task decsription:

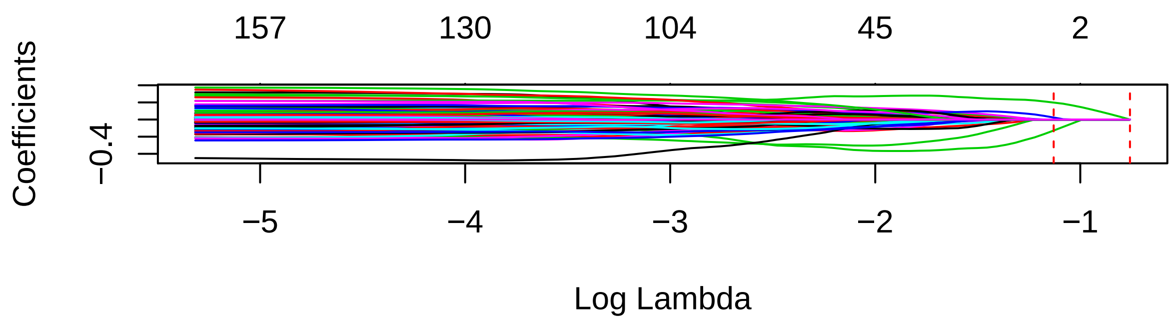
Use glmnet and glmnet to obtain the Lasso estimation for regressing log.surv against expr. How many coefficient different from zero are in the Lasso estimator? Illustrate the result with two graphics.

```
lasso_surv <- glmnet(x = expr, y = log.surv,
                     alpha = 1)

cv_lasso_surv <- cv.glmnet(x = expr, y = log.surv,
                     alpha = 1)

# Number of non-zero coefficients
length(rownames(coef(cv_lasso_surv, s = "lambda.min"))[coef(cv_lasso_surv, s = "lambda.min")[,1] != 0])
```

```
## [1] 4
```

```
# Plot two graphics
par(mfrow=c(2,1))
plot(cv_lasso_surv)
plot(lasso_surv,xvar="lambda")
abline(v=log(cv_lasso_surv$lambda.min),col=2,lty=2)
abline(v=log(cv_lasso_surv$lambda.1se),col=2,lty=2)
```





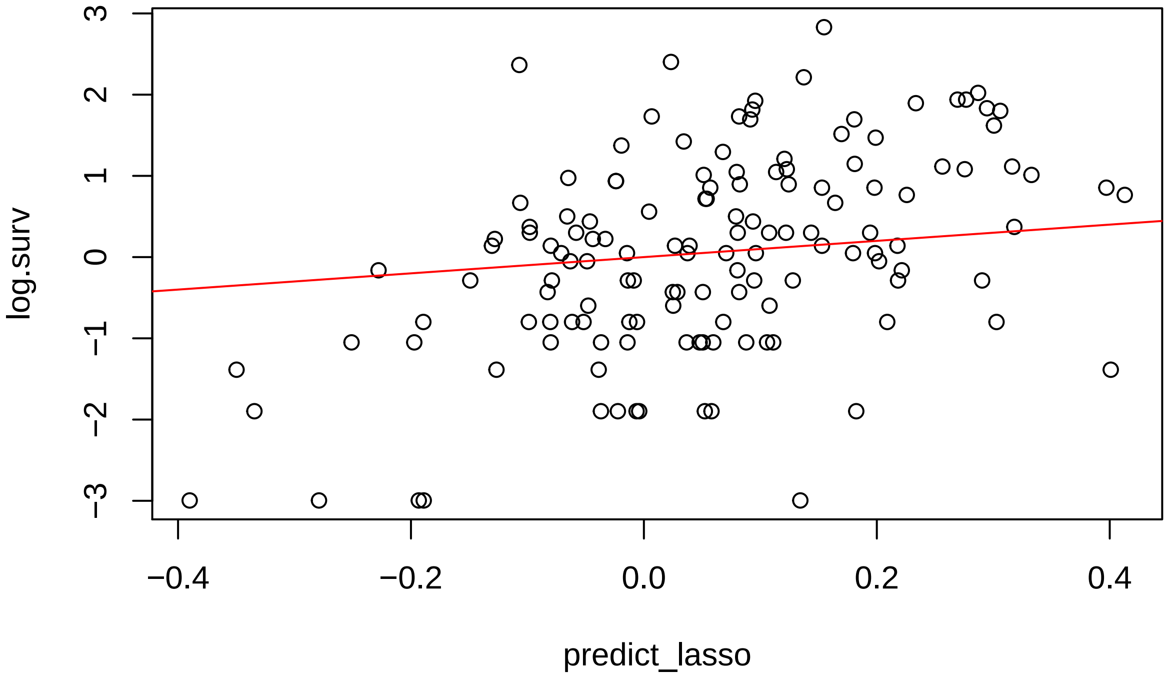

```
par(mfrow=c(1,1))
```

2.2 Computation of the responding fitted values

Task description:

Compute the fitted values with the Lasso estimated model (you can use predict). Plot the observed values for the response variable against the Lasso fitted values.

```
predict_lasso <- predict(lasso_surv,
                  newx = expr,
                  s = cv_lasso_surv$lambda.min)

plot(predict_lasso, log.surv)
abline(a=0, b=1, col = 2)
```



Scale is not good.

2.3 OLS regression model for 'log.surv' against 'expr'

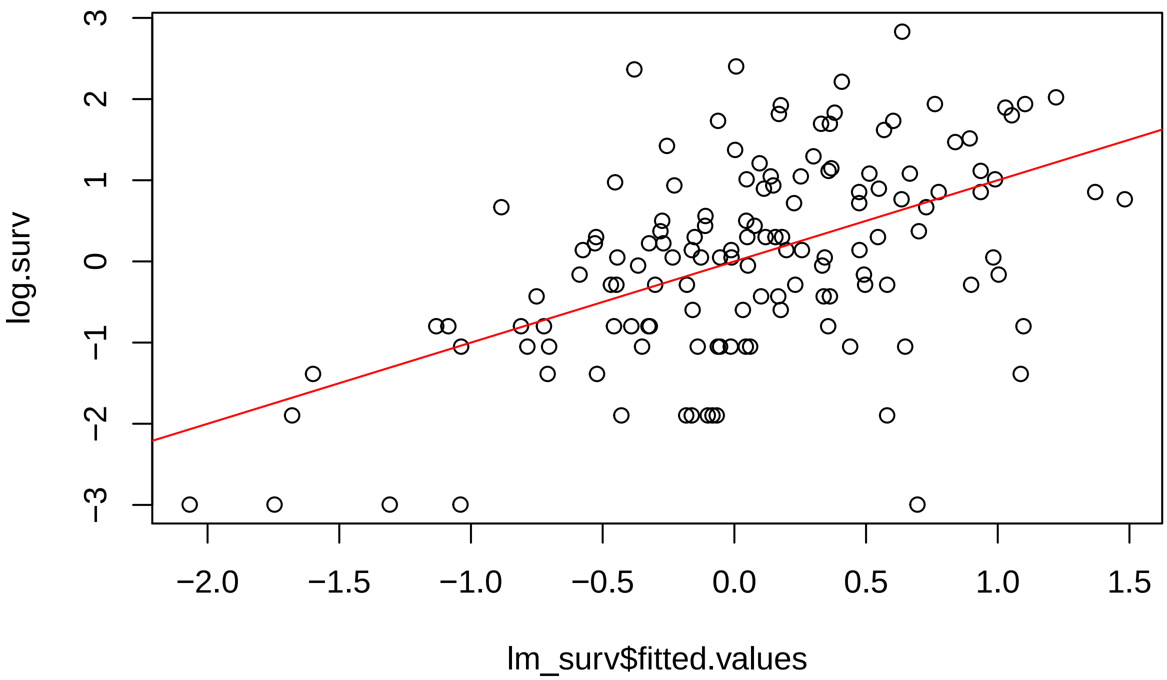
Task description:

Consider the set S0 of non-zero estimated Lasso coefficients. Use OLS to fit a regression model with response log.surv and explanatory variables the columns of expr with indexes in S0. Plot the observed values for the response variable against the OLS fitted values.

```
coeff_lasso <- rownames(coef(cv_lasso_surv, s = "lambda.min"))[coef(cv_lasso_surv, s = "lambda.min")[,1

lm_surv <- lm(log.surv ~ expr[, coeff_lasso[-1]])
lm_surv
```

```
##
## Call:
## lm(formula = log.surv ~ expr[, coeff_lasso[-1]])
##
## Coefficients:
##                    (Intercept)  expr[, coeff_lasso[-1]]V2252
##                         0.0662                       -1.0309
## expr[, coeff_lasso[-1]]V3787  expr[, coeff_lasso[-1]]V5352
##                         0.4836                        0.4467
```

```
plot(lm_surv$fitted.values, log.surv)
abline(a = 0, b = 1, col = 2)
```



2.4 Comparison of Lasso and OLS Regression

Task description:

Compare the OLS and Lasso fitted values. Do a plot for that.

```
plot(lm_surv$fitted.values, predict_lasso)
abline(a = 0, b = 1, col = 2)
```