

$$D_{0b} = \text{X} \text{ CZ}^{2b}$$

$b \in \mathbb{Z}_3$

Def 3:  $\text{X} :=$

$R_B: (1)$

$(2)$

$R_{21}: (1)$

$(2)$

Lem 13 By Def 3,  $R_B$  &  $R_{21}$ , 20. (1)

$(1)$

$(2)$

$(3)$

Proof: 20. (1)/(2)/(3). LHS :=

$\stackrel{R_{21}}{=}$

$\stackrel{R_{13}}{=}$

$\stackrel{\text{def}}{=} D_{0b} \text{ with Z on bottom} = 20. (1)/(2)/(3). \text{ RHS.}$



$$\boxed{D_{ab}}_{a \neq 0} = \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array}$$

Def 3:  $\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} := \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \end{array} \begin{array}{c} \text{H} \text{---} \bullet \text{---} \text{H} \text{---} \bullet \text{---} \text{H} \text{---} \bullet \text{---} \\ \text{H} \text{---} \bullet \text{---} \text{H} \text{---} \bullet \text{---} \text{H} \text{---} \bullet \text{---} \end{array}$

$R_{13}$ : (1)  $\begin{array}{c} \bullet \text{---} \boxed{Z} \text{---} \\ \bullet \text{---} \end{array} = \begin{array}{c} \text{---} \boxed{Z} \text{---} \bullet \\ \text{---} \bullet \end{array}$  (2)  $\begin{array}{c} \bullet \text{---} \\ \bullet \text{---} \boxed{Z} \end{array} = \begin{array}{c} \text{---} \bullet \\ \text{---} \boxed{Z} \bullet \end{array}$

$R_{21}$ : (1)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \text{---} \text{---} \\ \text{---} \boxed{Z} \end{array}$  (2)  $\begin{array}{c} \text{---} \text{---} \\ \text{---} \boxed{Z} \end{array} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \text{---} \text{---} \boxed{Z} \\ \text{---} \text{---} \end{array}$

Lem 14 By Def 3,  $R_{13}$  &  $R_{21}$ ,

20.(4)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{10}} = \boxed{D_{10}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

(7)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{20}} = \boxed{D_{20}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

(5)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{11}} = \boxed{D_{11}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

(8)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{21}} = \boxed{D_{21}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

(6)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{12}} = \boxed{D_{12}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

(9)  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{22}} = \boxed{D_{22}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array}$

Proof: 20.(4)-(9). LHS :=  $\begin{array}{c} \boxed{Z} \text{---} \\ \text{---} \end{array} \boxed{D_{ab}} = \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array} \boxed{Z}$

$\underline{\underline{R_{21}}}$   $\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array} \boxed{Z}$

$\underline{\underline{R_{13}}}$   $\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array} \boxed{Z}$

$\underline{\underline{\text{def}}}$   $\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \boxed{D_{ab}} \begin{array}{c} \text{---} \\ \text{---} \boxed{Z} \end{array} = 20.(4)-(9). \text{ RHS.}$

