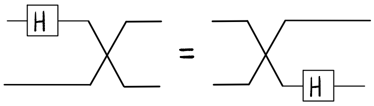
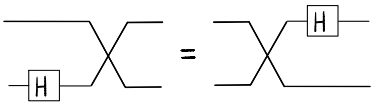
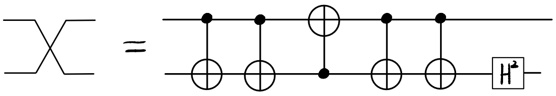
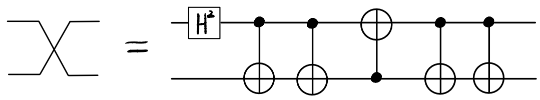
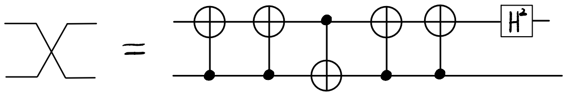
R_{19}: (1)  (2) 

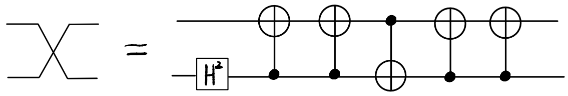
C2: $H^4 = I$

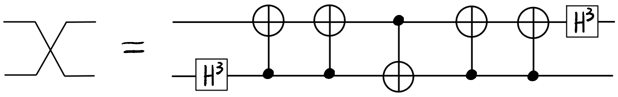
Lem P Def 2, Def 3, C_2, C_8, R_{19} & R_{32} imply

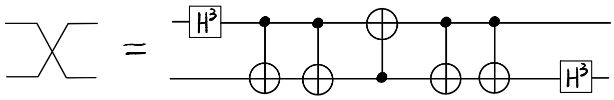
Def 4:  ✓

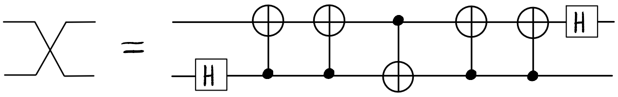
Def 6:  ✓

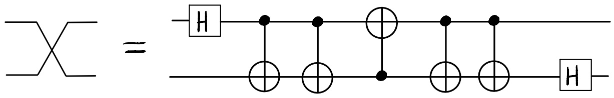
Def 5:  ✓

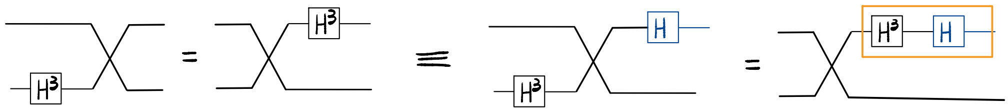
Def 7:  ✓

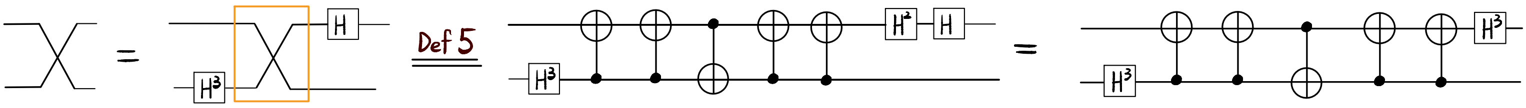
Def 8:  ✓

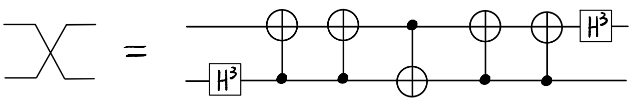
Def 9:  ✓

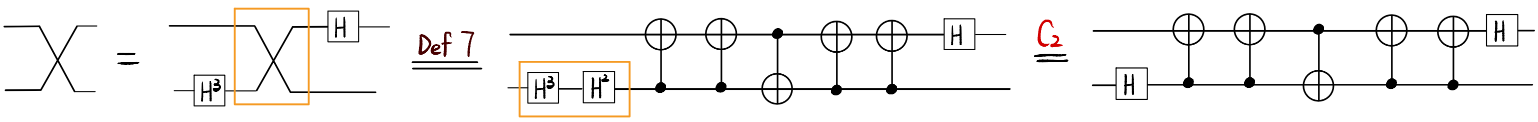
Def 10:  ✓

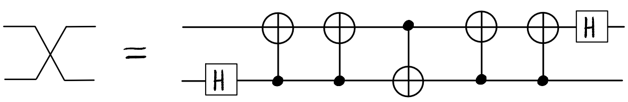
Def 11:  ✓

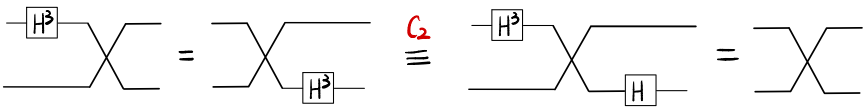
Proof cont. R_{19}: (2)  . By C_2,

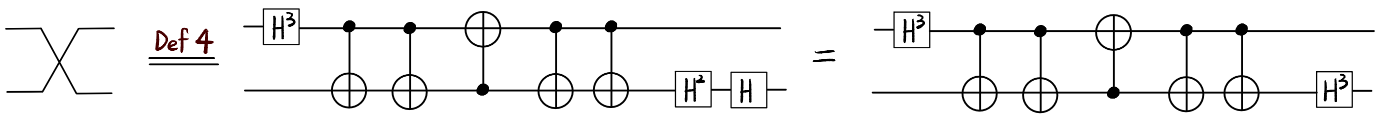


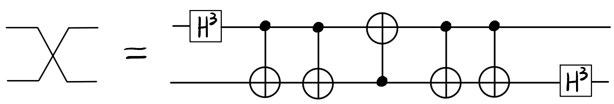
Hence Def 8: 

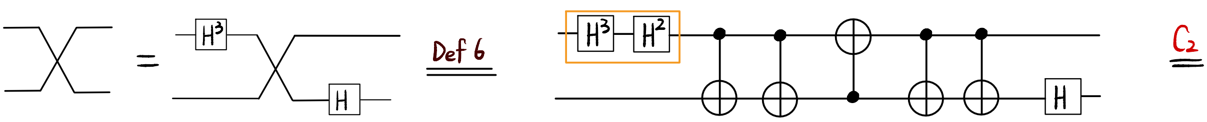


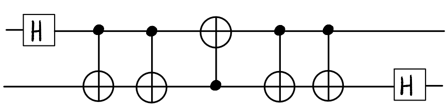
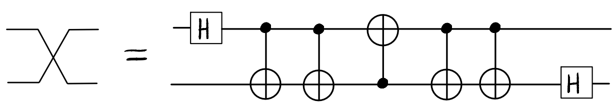
Hence Def 10: 

R_{19}: (1) 

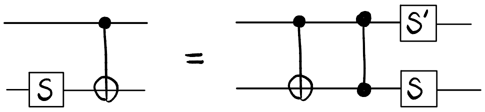
Then 

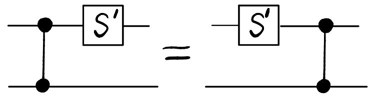
Hence Def 9: 

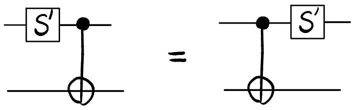
Alternatively, 

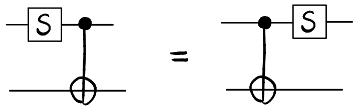
 . Hence Def 11: 

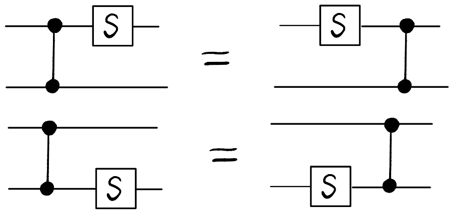
∎

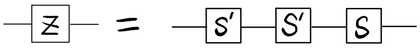
R_{23}^4:  $\cdot\, w^2$

R_{15}: 

R_{24}^3: 

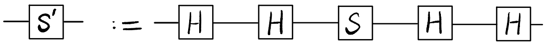
R_{24}^2: 

C_7: 

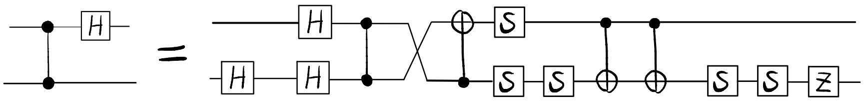
R_{10}: 

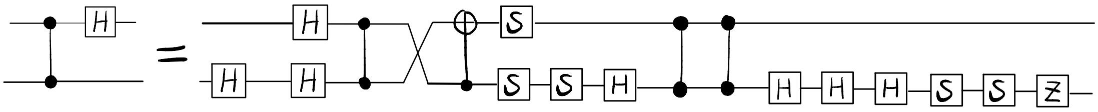
$C1: w^3 = 1$ $\quad$ $C2: H^4 = I$ $\quad$ $C3: S^3 = I$

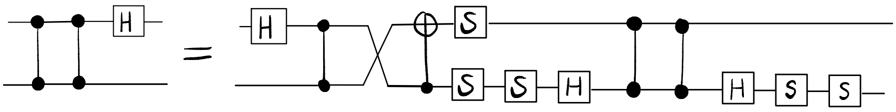
$C5: SH^2SH^2 = H^2SH^2S$ $\quad$ $SS' = S'S$

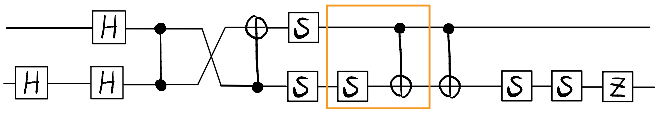
Def 1: 

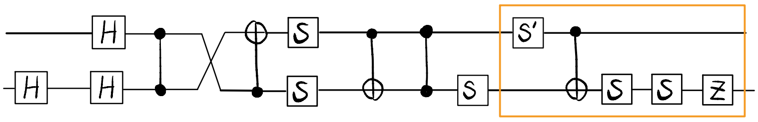
Lem Q Def 1–3, $C_1 - C_8$, R_{10}, R_{15}, R_{23}, R_{24} & R_{25} imply

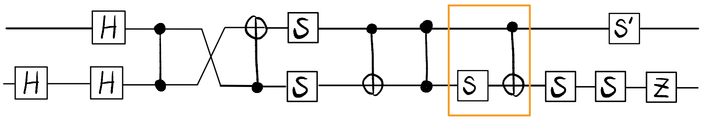
R_{27}:  $\cdot\, w$

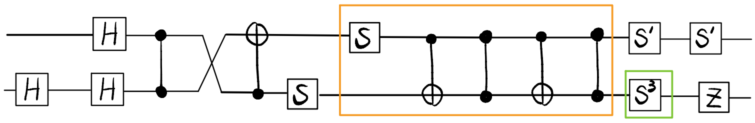
R_{27}^1:  $\cdot\, w$

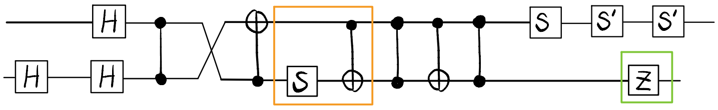
R_{27}^2:  $\cdot\, w$

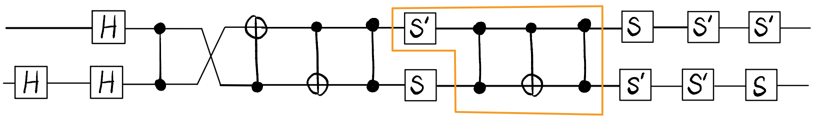
Proof: R_{27}. RHS :=  $\cdot\, w$

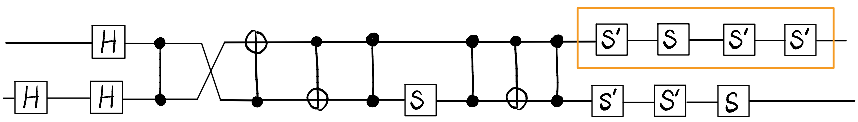
$\overset{R_{23}^4}{\underset{C_1}{=\!=}}$ 

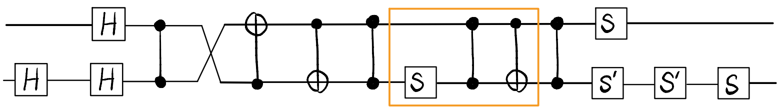
$\overset{R_{24}^3}{=\!=}$ 

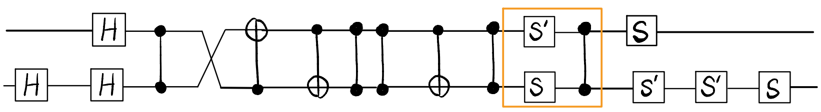
$\overset{R_{23}^4}{=\!=}$  $\cdot\, w^2$

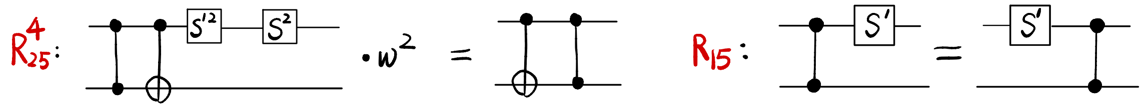
$\overset{R_{24}^3}{\underset{C_3,\, C_7}{=\!=}}$  $\cdot\, w^2$

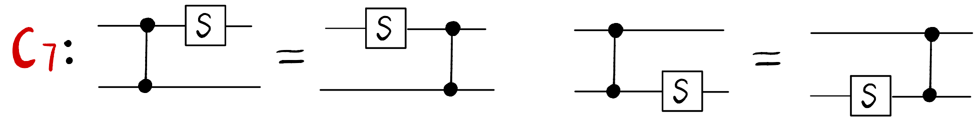
$\overset{R_{23}^4}{\underset{C_1,\, R_{10}}{=\!=}}$  $\cdot\, w$

$\overset{R_{23}^4}{\underset{R_{15}}{=\!=}}$  $\cdot\, w$

$\overset{R_2,\, R_3}{\underset{R_5}{=\!=}}$  $\cdot\, w$

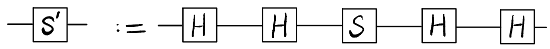
$\overset{R_{23}^4}{\underset{C_1,\, C_7}{=\!=}}$ 

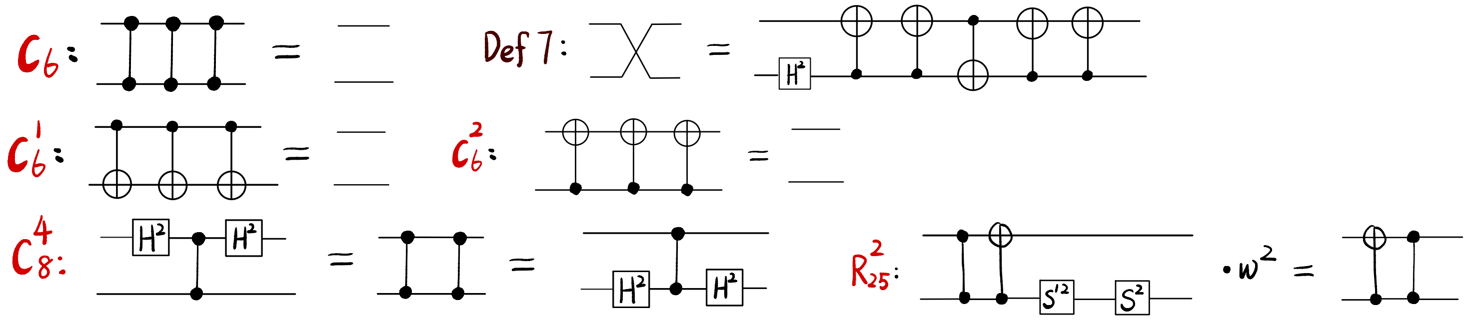




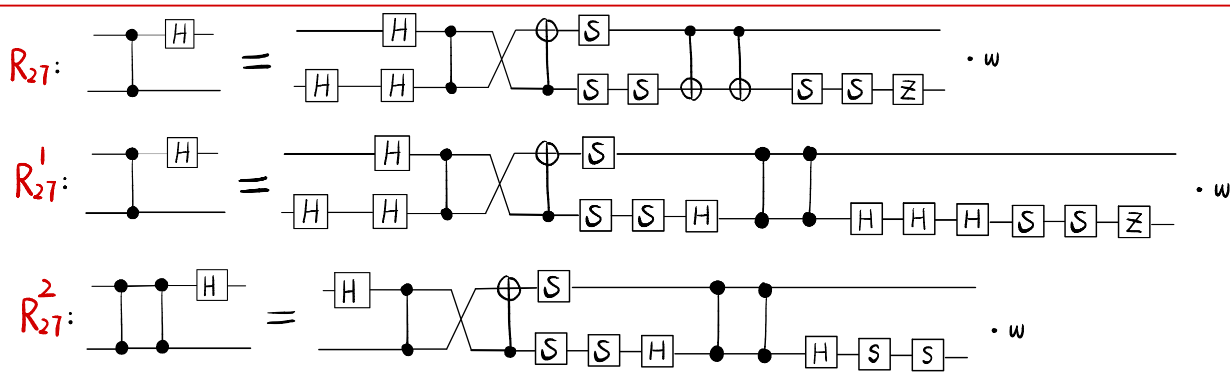
C1: $\omega^3 = 1$ C2: $H^4 = I$ C3: $S^3 = I$

C5: $SH^2SH^2 = H^2SH^2S$ $SS' = S'S$

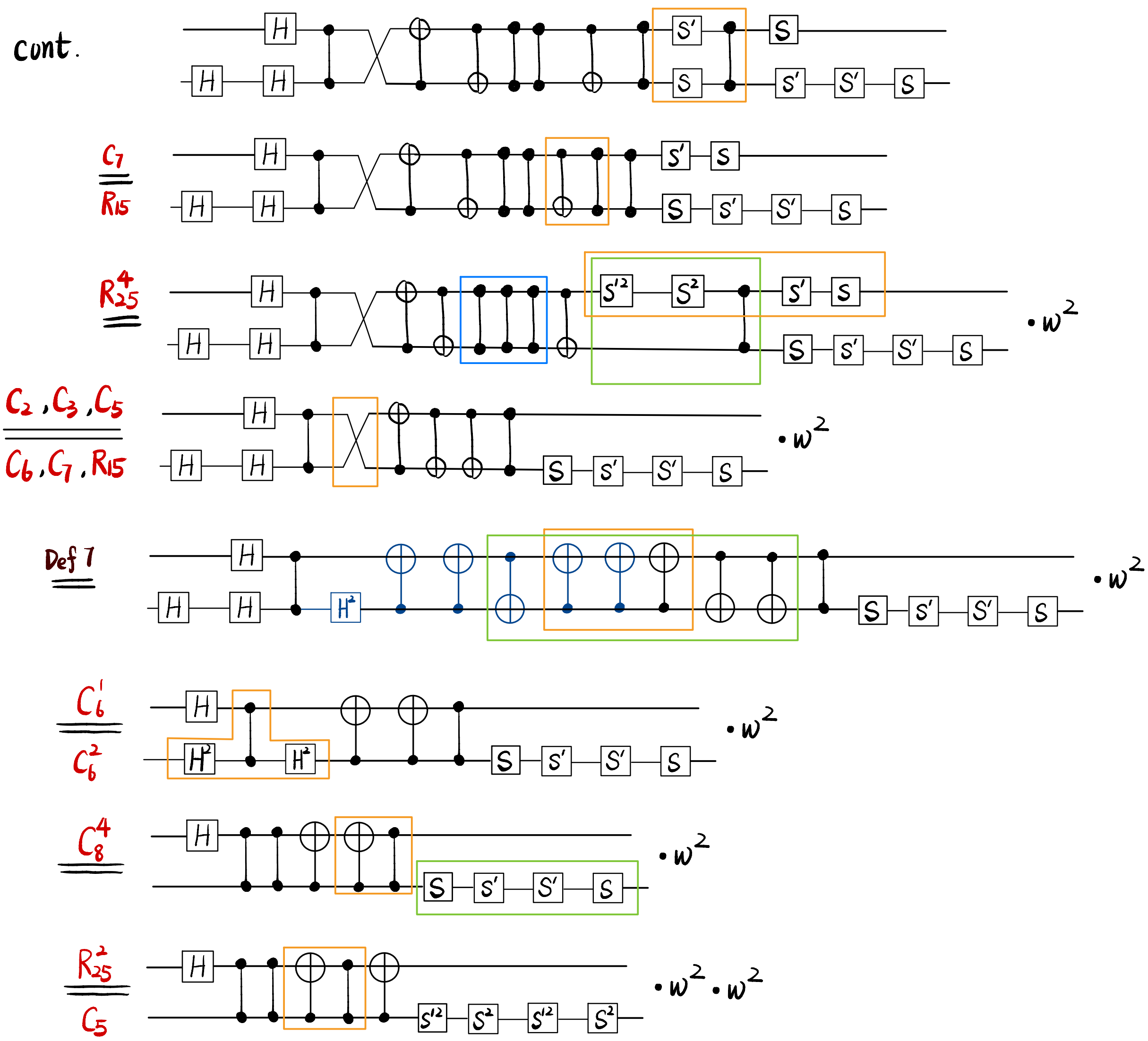
Def 1: 



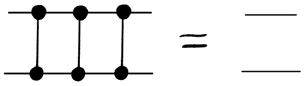
Lem Q

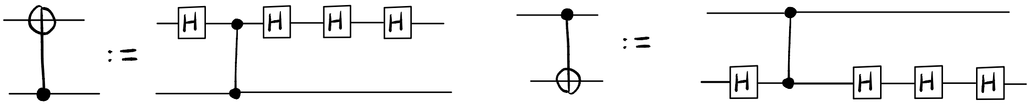


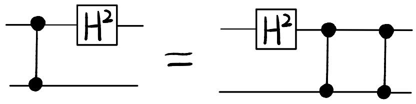
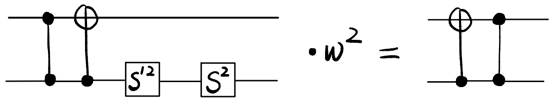
Proof cont.

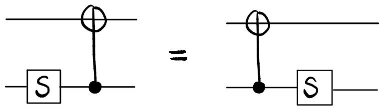
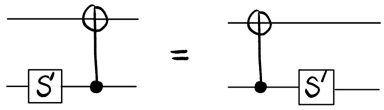


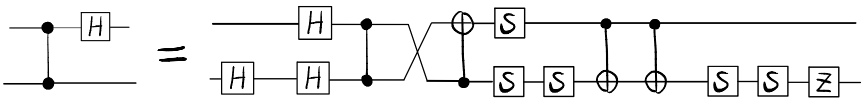
$C5$: $SH^2SH^2 = H^2SH^2S$ $\quad SS' = S'S$ $\quad$ Def 1: $-\boxed{S'}- \; := \; -\boxed{H}-\boxed{H}-\boxed{S}-\boxed{H}-\boxed{H}-$

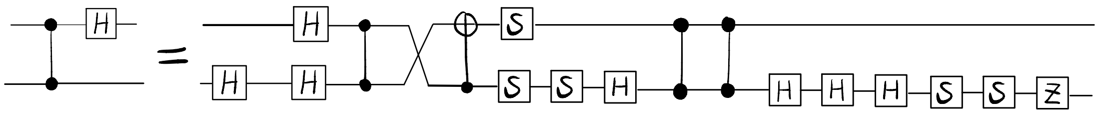
C_6:  $\quad C1: \omega^3 = 1 \quad C2: H^4 = I \quad C3: S^3 = I$

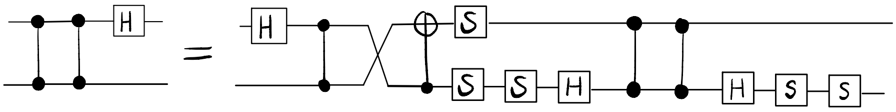
Def 2: 

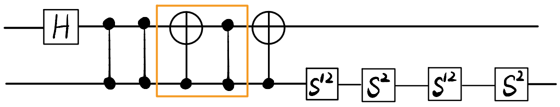
C_8:  $\quad R_{25}^2$: 

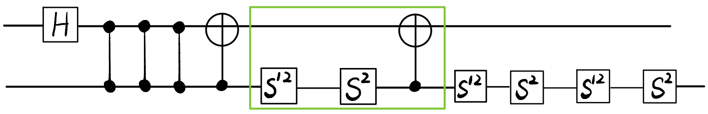
R_{24}:  $\quad R_{24}^1$: 

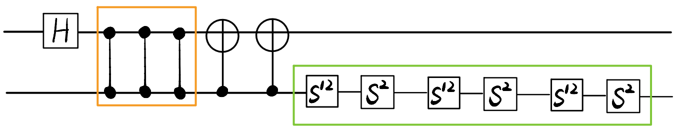
Lem Q $\quad R_{27}$:  $\cdot \omega$

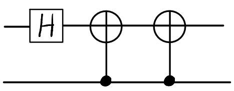
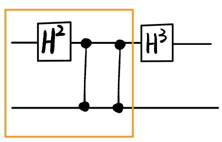
R_{27}^1:  $\cdot \omega$

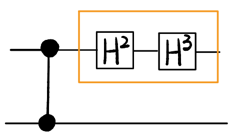
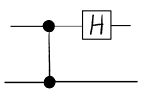
R_{27}^2:  $\cdot \omega$

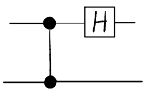
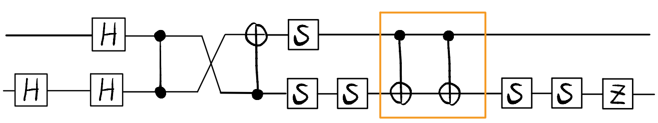
Proof cont.  $\cdot \omega^2 \cdot \omega^2$

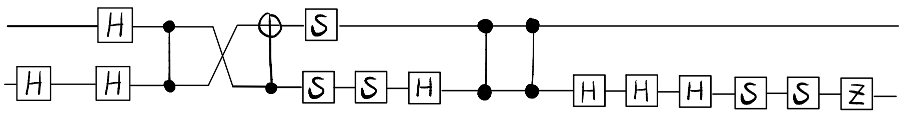
$\underset{=}{R_{25}^2}$  $\boxed{\cdot \, \omega^6}$

$\frac{C_1}{R_{24}, R_{24}^1}$ 

$\frac{C_2, C_3}{C_5, C_6}$  $\quad \frac{\text{Def } 2}{C_2}$ 

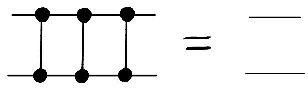
$\underset{=}{C_8^1}$  $\quad \underset{=}{C_2}$  $=: R_{27}$.LHS.

R_{27}^1.LHS $:=$  $\underset{=}{R_{27}}$  $\cdot \omega$

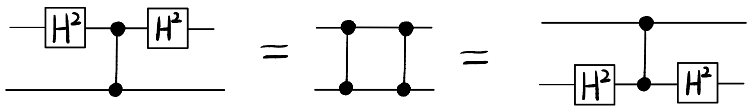
$\frac{\text{Def } 2}{C_2}$  $\cdot \omega \; =: R_{27}^1$.RHS.

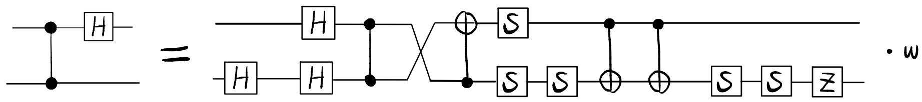
C5 : $SH^2SH^2 = H^2SH^2S$ $\quad SS' = S'S$

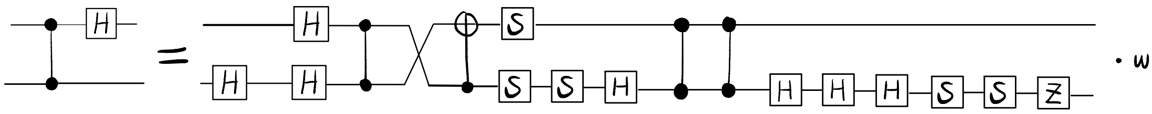
Def 1 : $-S'- \;=\; -H-H-S-H-H-$ $\qquad -S'-S'- \;=\; -H-H-S-S-H-H-$

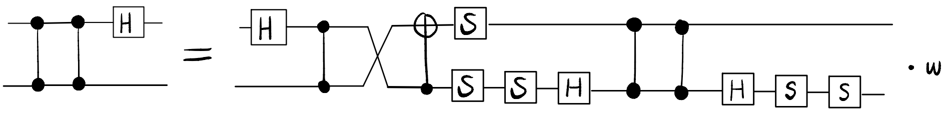
C6 : 

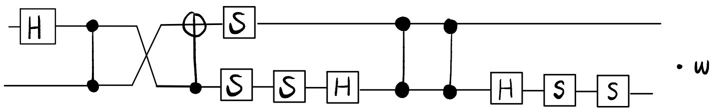
C1 : $\omega^3 = 1$ $\quad$ C2 : $H^4 = I$ $\quad$ C3 : $S^3 = I$ $\quad$ R10 : $-Z- \;=\; -S'-S'-S-$

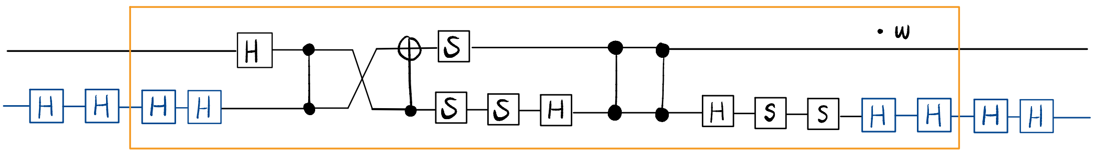
C_8^4 : 

Lem Q $\quad R_{27}$: 

R_{27}^1 : 

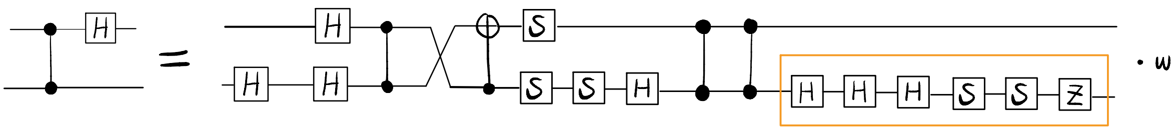
R_{27}^2 : 

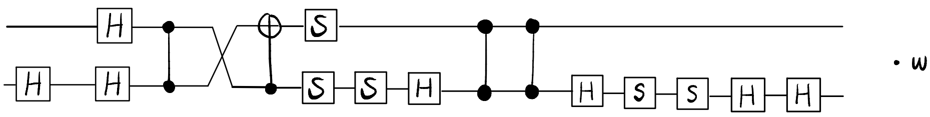
Proof cont. $\quad R_{27}^2.\mathrm{RHS} :=$  $\overset{C_2}{=}$

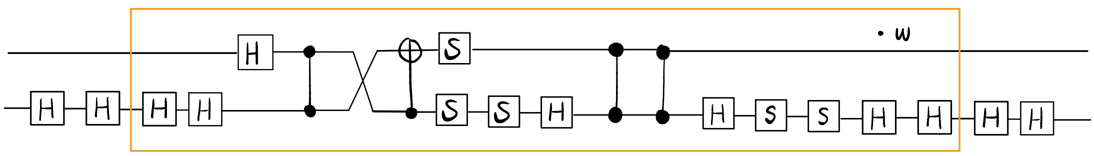


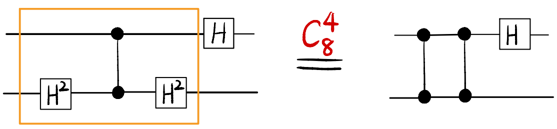
Composition in diagrammatic order:

$$H^3S^2\boxed{Z} \overset{R10}{=} H^3\boxed{S^2S'^2}S \overset{C_5}{=} H^3\boxed{S^2S}S'^2 \overset{C_3}{=} H^3S'^2 \overset{\text{Def 1}}{=} \boxed{H^3H^2}S^2H^2 \overset{C_2}{=} HS^2H^2 \quad (1)$$

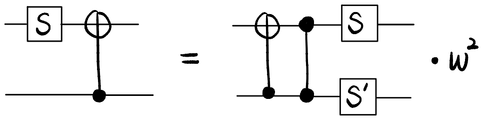
Hence R_{27}^1 : 

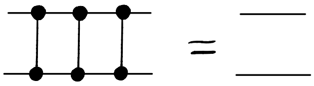
$\overset{(1)}{=}$ 

Thus, $R_{27}^2.\mathrm{RHS} =$ 

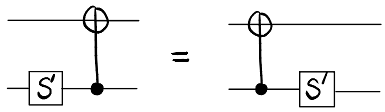
$\overset{R_{27}^1}{=}$  $=: R_{27}^2.\mathrm{LHS}$

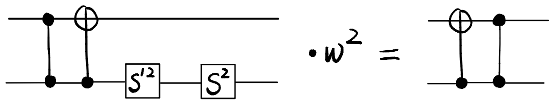
∎

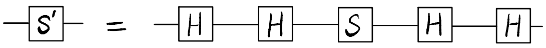
R_{23}^1: 

C_6: 

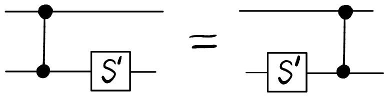
C_5: $SH^2SH^2 = H^2SH^2S$

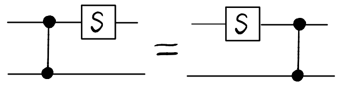
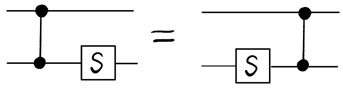
R_{24}^1: 

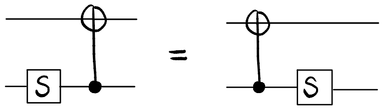
R_{25}^2: 

Def 1 : 

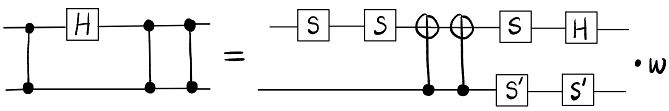
$C1: w^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$

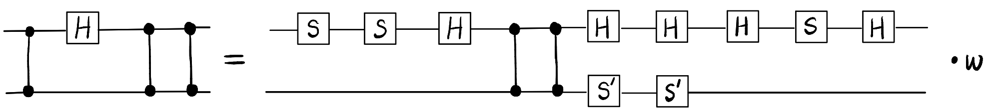
R_{15}: 

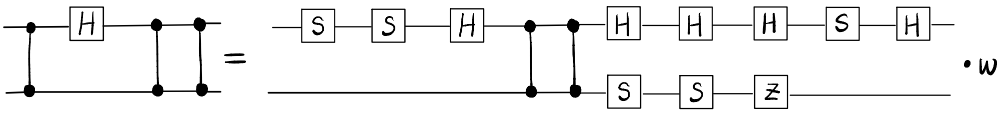
C_7: (1)  (2) 

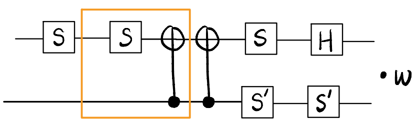
R_{24}: 

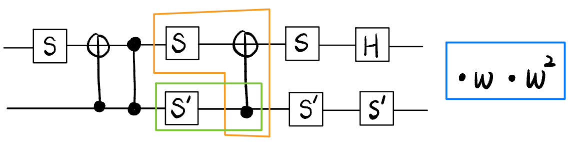
Lem R Def 1–3, $C1-7$, R_{10}, R_{15}, R_{23}, R_{24} & R_{25} imply

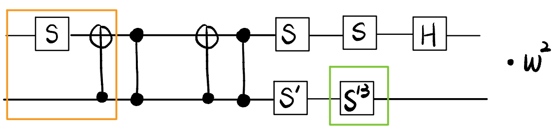
R_{28} : 

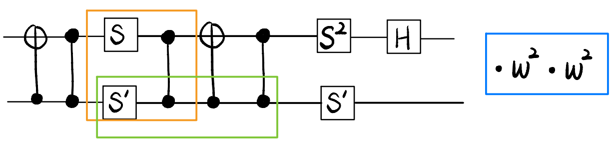
R_{28}^1 : 

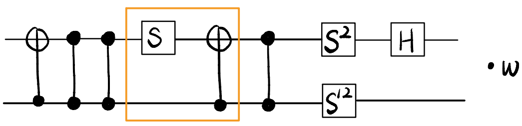
R_{28}^2: 

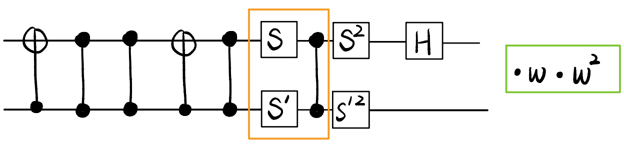
Proof: R_{28}.RHS := 

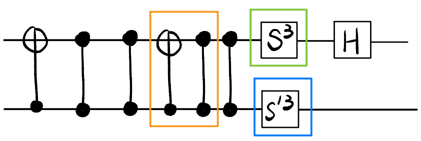
$\underline{\underline{R_{23}^1}}$ 

$\frac{C_1, R_{23}^1}{R_{24}^1}$ 

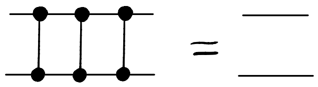
$\frac{C_2, C_3}{R_{23}^1}$ 

$\frac{C_1, C_7}{R_{15}, R_{24}^1}$ 

$\underline{\underline{R_{23}^1}}$ 

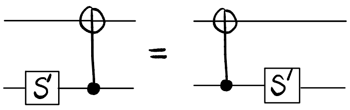
$\frac{C_1, C_7}{R_{15}}$ 

C_6:

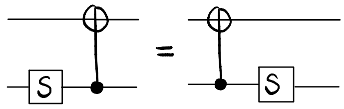


C_5: $SH^2SH^2 = H^2SH^2S$ C_1: $\omega^3 = 1$ C_2: $H^4 = I$ C_3: $S^3 = I$

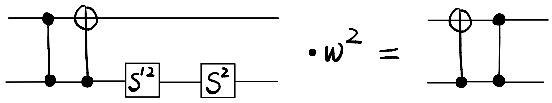
R_{24}^1:



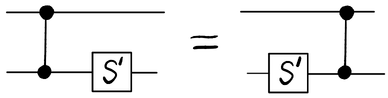
R_{24}:



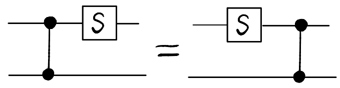
R_{25}^2:



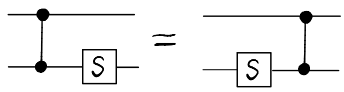
R_{15}:



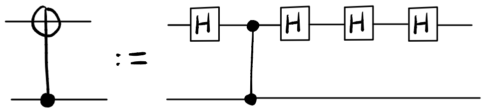
C_7: (1)

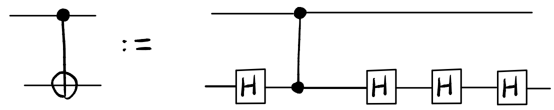


(2)



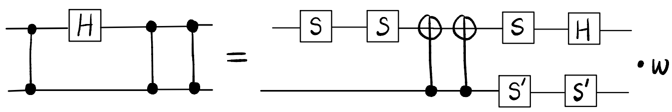
Def 2:



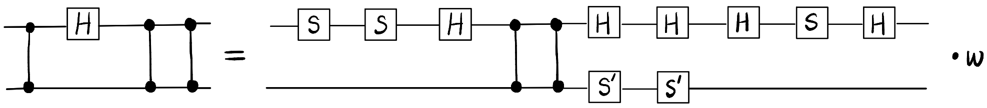


Lem R

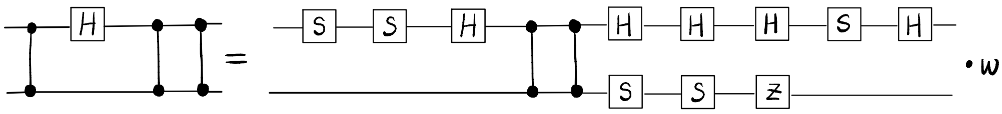
R_{28} :



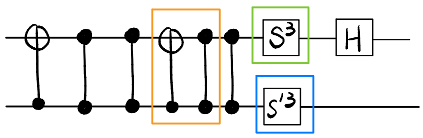
R_{28}^1 :



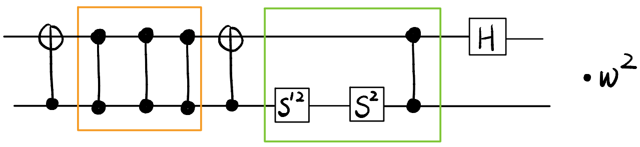
R_{28}^2:



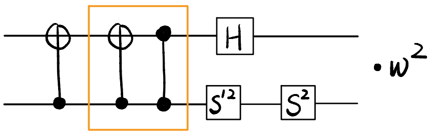
Proof cont.



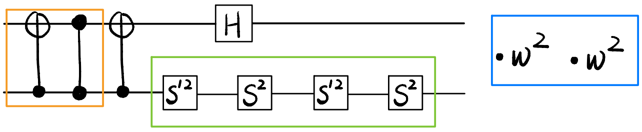
$\overset{C_2, C_3}{\underset{R_{25}^2}{=}}$



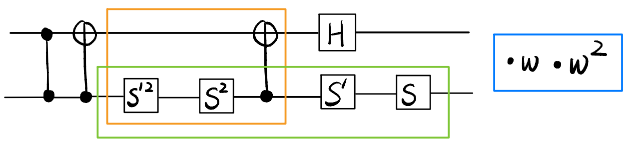
$\overset{C_6, C_7}{\underset{R_{15}}{=}}$



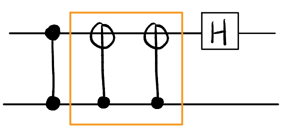
$\underset{R_{25}^2}{=}$



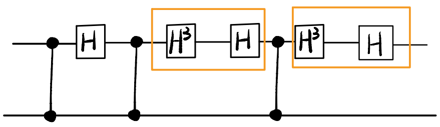
$\overset{C_1, C_2, C_3}{\underset{C_5, R_{25}^2}{=}}$



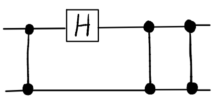
$\overset{C_1, C_2, C_3, C_5}{\underset{R_{24}^1, R_{24}}{=}}$



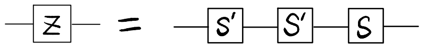
$\underset{}{\overset{\text{Def 2}}{=}}$

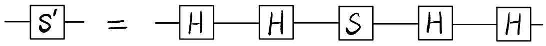


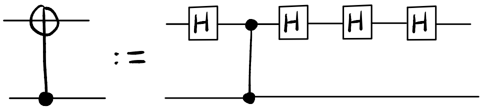
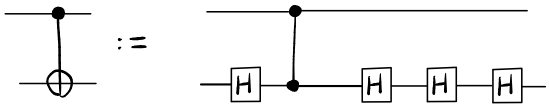
$\underset{}{\overset{C_2}{=}}$



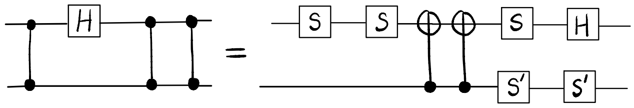
$=: R_{28}$.LHS

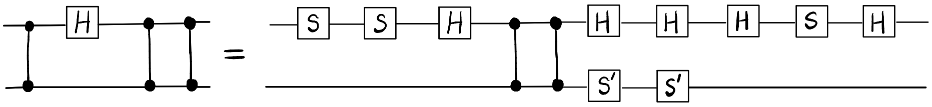
R_{10} :  $C_2 : H^4 = I$ $C_3 : S^3 = I$ $C_5 : SH^2SH^2 = H^2SH^2S$ $SS' = S'S$

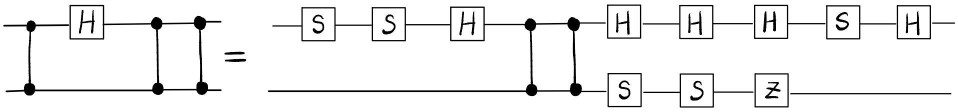
Def 1 : 

Def 2 :  

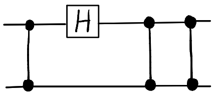
Lem R

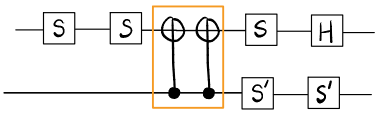
R_{28} :  $\cdot w$

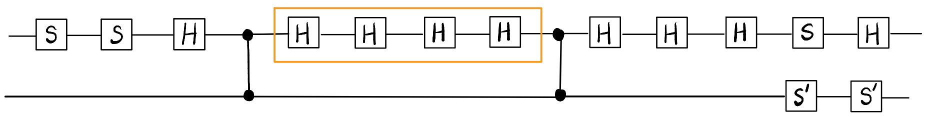
R_{28}' :  $\cdot w$

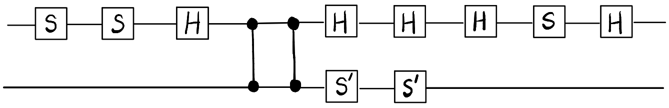
R_{28}^2 :  $\cdot w$

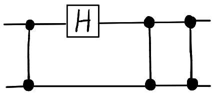
Proof cont.

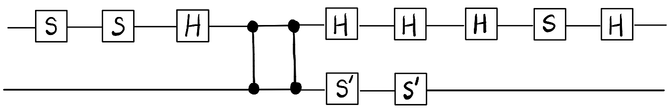
$R_{28}'.LHS :=$ 

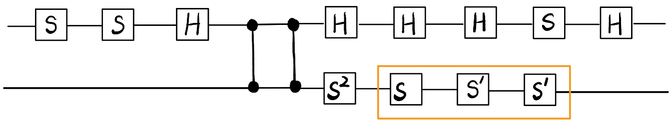
$\underline{\underline{R_{28}}}$  $\cdot w$

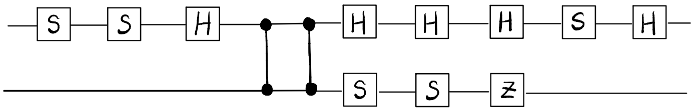
$\underline{\underline{\text{Def } 2}}$  $\cdot w$

$\underline{\underline{C_2}}$  $\cdot w$ $=: R_{28}'.RHS$

$R_{28}^2.LHS :=$ 

$\underline{\underline{R_{28}'}}$  $\cdot w$

$\underline{\underline{C_3}}$  $\cdot w$

$\underline{\underline{R_{10}}}$ C_5  $\cdot w$ $=: R_{28}^2.RHS$

$\boxtimes$