

Two - Qutrit Derived Relations : T₂

Def 1: $S' := \boxed{H} \quad H \quad \boxed{S} \quad H \quad H$ $S'^2 := \boxed{H} \quad H \quad \boxed{S^2} \quad H \quad H$

Def 2: $\bigoplus := \begin{array}{c} \oplus \\ \parallel \end{array}$ $\begin{array}{c} \bullet \\ \parallel \end{array} := \begin{array}{c} \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array}$

Def 3: $\bigtimes := \begin{array}{c} \times \\ \parallel \\ \parallel \\ \parallel \end{array}$ $C_6: \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} = \quad \quad \quad$

C₇: $\begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{S} = \quad \boxed{S} \quad \begin{array}{c} \bullet \\ \bullet \end{array}$

C₈: $\begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H^2} = \quad \boxed{H^2} \quad \begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array}$

R₁₆: $\begin{array}{c} \times \\ \times \\ \times \end{array} = \quad \quad \quad$

R₁₇: $\begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array} \quad \times = \quad \times \quad \begin{array}{c} \bullet \\ \bullet \end{array}$

R₁₈: $\begin{array}{c} \bullet \\ \bullet \end{array} \quad \times = \quad \times \quad \begin{array}{c} \bullet \\ \bullet \end{array}$

R₁₉: $\begin{array}{c} \bullet \\ \bullet \end{array} \quad \boxed{H} = \quad \times \quad \begin{array}{c} \bullet \\ \bullet \end{array} \quad \boxed{H}$

Def 2: $\bigoplus := \begin{array}{c} \oplus \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array}$ $\begin{array}{c} \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array} := \begin{array}{c} \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array}$

C₂: $H^4 = I$ **C₆:** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} = \quad \quad \quad$

Lem G By Def2, C₂ & C₆,

C₆: $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} = \quad \quad \quad$ implies **C₆¹:** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} = \quad \quad \quad$ & **C₆²:** $\begin{array}{c} \bigoplus \\ \bigoplus \\ \bigoplus \end{array} = \quad \quad \quad$

Proof: **C₆¹. LHS:** $= \begin{array}{c} \bigoplus \\ \bigoplus \\ \bigoplus \end{array}$ **Def 2** $\begin{array}{c} \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array} \quad \boxed{H} \quad \boxed{H \quad H \quad H \quad H} \quad \boxed{H \quad H \quad H \quad H} \quad \boxed{H \quad H \quad H \quad H} \quad \boxed{H \quad H \quad H \quad H}$

C₂ $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H} \quad H \quad H \quad H$ **C₆** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H \quad H \quad H \quad H}$ **C₂** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H \quad H \quad H \quad H}$ $=: C_{6}^1 \cdot \text{RHS}$

C₆². LHS: $= \begin{array}{c} \bigoplus \\ \bigoplus \\ \bigoplus \end{array}$ **Def 2** $\begin{array}{c} \bullet \\ \parallel \\ \bullet \\ \parallel \\ \bullet \\ \parallel \end{array} \quad \boxed{H \quad H \quad H \quad H} \quad \boxed{H \quad H \quad H \quad H} \quad \boxed{H \quad H \quad H \quad H}$

C₂ $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H} \quad H \quad H \quad H$ **C₆** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H \quad H \quad H \quad H}$ **C₂** $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad \boxed{H \quad H \quad H \quad H}$ $=: C_{6}^2 \cdot \text{RHS}$

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Def 1 : $\boxed{S'}$:= $\boxed{H} \quad \boxed{H} \quad \boxed{S} \quad \boxed{H} \quad \boxed{H}$ $\boxed{S'^2}$:= $\boxed{H} \quad \boxed{H} \quad \boxed{S^2} \quad \boxed{H} \quad \boxed{H}$

$$\text{Def 2: } \begin{array}{c} \textcircled{\text{+}} \\ \text{---} \\ \text{---} \end{array} := \begin{array}{c} \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} := \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array}$$

$$\text{Def 3: } \begin{array}{c} \text{Diagram of two wires crossing} \\ \times \end{array} := \begin{array}{c} \text{Diagram of two wires with a CNOT gate} \\ \text{between them} \end{array} = \text{C}_b:$$

$$C_7: \quad \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} \text{---} \boxed{S} \text{---} \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} = \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} \boxed{S} \text{---} \begin{array}{c} \text{---} \\ | \\ \bullet \end{array}$$

$$R_{16}: \text{[Diagram of a hexagon with two internal diagonals forming an X] } = \text{ [Blank box]} \quad R_{17}: \text{[Diagram of a hexagon with two vertical internal lines connecting opposite vertices] } = \text{ [Diagram of a hexagon with two vertical internal lines connecting opposite vertices, plus two small black dots at the top and bottom vertices]}$$

$$R_{18}: \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \times \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \times \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

$$\text{Def 3: } \text{Diagram} := \text{Circuit Diagram} \quad R_{16}: \text{Diagram} = \text{Diagram} \quad R_{17}: \text{Diagram} = \text{Diagram}$$

$$R_B : (1) \quad \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} \quad \boxed{\bar{z}} \quad = \quad \boxed{\bar{z}} \quad \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} \quad (2) \quad \begin{array}{c} \text{---} \\ | \\ \bullet \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \bullet \end{array}$$

$$R_{20}: (1) \quad \begin{array}{c} X \\ \square \end{array} \quad \begin{array}{c} \diagup \\ \diagdown \end{array} = \begin{array}{c} \diagdown \\ \diagup \end{array} \quad (2) \quad \begin{array}{c} \diagup \\ \diagdown \end{array} = \begin{array}{c} \diagdown \\ \diagup \end{array} \quad \begin{array}{c} X \\ \square \end{array}$$

$$R_{21} : (1) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad (2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \boxed{\text{---}}$$

$$R12: z^3 = I$$

Lem H By Def 3 $R_{12}, R_{13}, R_{16}, R_{17}, R_{20}$ & R_{21} ,

$$R_{14}: \quad (1) \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \boxed{X} \quad \text{---} = \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \boxed{X} \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \text{implies } (2) \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \boxed{Z} \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \boxed{Z} \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array}$$

$$(3) \quad \begin{array}{c} \text{---} \\ | \\ \boxed{X} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \bullet \\ | \\ \text{---} \\ | \\ \boxed{X} \\ | \\ \text{---} \\ | \\ \boxed{\bar{z}} \end{array}$$

$$\text{Proof: } R_{14} \cdot (2). \text{ RHS} := \begin{array}{c} \text{Diagram: } \begin{array}{ccccc} \boxed{z} & \boxed{z} & \bullet & & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ x & \bullet & & & \end{array} \end{array} = \begin{array}{c} \text{Diagram: } \begin{array}{ccccc} \boxed{z} & \boxed{z} & \bullet & & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ x & \bullet & & & \end{array} \end{array} \boxed{\text{Diagram: } \begin{array}{ccccc} \text{---} & \text{---} & \text{---} & \text{---} & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ \text{---} & \text{---} & \text{---} & \text{---} & \end{array}} \boxed{\frac{R_{17}, R_{20}}{R_{21}}} \quad \begin{array}{c} \text{Diagram: } \begin{array}{ccccc} \text{---} & \text{---} & \bullet & & \\ \text{---} & \text{---} & \text{---} & \text{---} & \\ \boxed{z} & \boxed{z} & \bullet & & \end{array} \end{array}$$

$$\underline{\underline{R_{14.(1)}}} \quad \text{Diagram: A bridge circuit with a central node connected to four resistors. The top-right resistor is labeled 'X'. An orange box encloses the top-left and top-right resistors.} \quad \underline{\underline{\frac{R_{17}}{R_{20}}}} \quad \text{Diagram: A bridge circuit with a central node connected to four resistors. The bottom-right resistor is labeled 'X'. An orange box encloses the top-left and top-right resistors.} \quad \underline{\underline{R_{16}}} \quad \text{Diagram: A simple series circuit with two resistors in series. The rightmost resistor is labeled 'X'.} \quad =: \underline{\underline{R_{14.(2)}.LHS}}$$

Def 3: := R₁₆: = R₁₇: =

R₁₃: (1) = (2) =

R₂₀: (1) = (2) =

R₂₁: (1) = (2) = R₁₂: $Z^3 = I$

R₁₄: (1) = implies (2) = (3) = (4) =

Proof cont. R₁₄.(3).LHS := R₁₂ R_{14.(1)}

R₁₃ =: R_{14.(3).RHS.}

R_{14.(4).RHS} := R₁₆ R_{17, R₂₀}

R_{14.(3)} R₂₀ R₁₆ =: R_{14.(4).LHS}

□

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Def 1 : $S' := \boxed{H} \boxed{H} \boxed{S} \boxed{H} \boxed{H}$ $S'^2 := \boxed{H} \boxed{H} \boxed{S^2} \boxed{H} \boxed{H}$

Def 2 : $\oplus := \boxed{H} \bullet \boxed{H} \boxed{H} \boxed{H}$ $\ominus := \bullet \boxed{H} \bullet \boxed{H} \boxed{H} \boxed{H}$

Def 3 : $\times := \boxed{H} \bullet \boxed{H} \bullet \boxed{H} \bullet \boxed{H}$ $C_6 := \bullet \bullet \bullet \bullet = \underline{\hspace{2cm}}$

C₇: $\bullet \boxed{S} = \boxed{S} \bullet$ **C₈:** $\bullet \boxed{H^2} = \boxed{H^2} \bullet$

R₁₄: $\bullet \boxed{X} = \boxed{Z} \boxed{Z}$ **R₁₆:** $\times \times \times = \underline{\hspace{2cm}}$ **R₁₇:** $\bullet \times \times = \times \times \bullet$

R₁₈: $\boxed{S} \times = \times \boxed{S}$ **R₁₉:** $\boxed{H} \times = \times \boxed{H}$

Def 2 : $\oplus := \boxed{H} \bullet \boxed{H} \boxed{H} \boxed{H}$ $\ominus := \bullet \boxed{H} \bullet \boxed{H} \boxed{H} \boxed{H}$

C₇: (1) $\bullet \boxed{S} = \boxed{S} \bullet$ **(2)** $\bullet \boxed{S} = \boxed{S}$

R₁₅: (1) $\bullet \boxed{S'} = \boxed{S'} \bullet$ **(2)** $\bullet \boxed{S'} = \boxed{S'}$

Lem I

R₂₄: $\begin{array}{c} \oplus \\ \boxed{S} \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \boxed{S} \end{array}$

R₂₄²: $\begin{array}{c} \boxed{S} \bullet \\ \ominus \end{array} = \begin{array}{c} \bullet \boxed{S} \\ \ominus \end{array}$

R₂₄¹: $\begin{array}{c} \oplus \\ \boxed{S} \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \boxed{S'} \end{array}$

R₂₄³: $\begin{array}{c} \boxed{S'} \bullet \\ \ominus \end{array} = \begin{array}{c} \bullet \boxed{S'} \\ \ominus \end{array}$

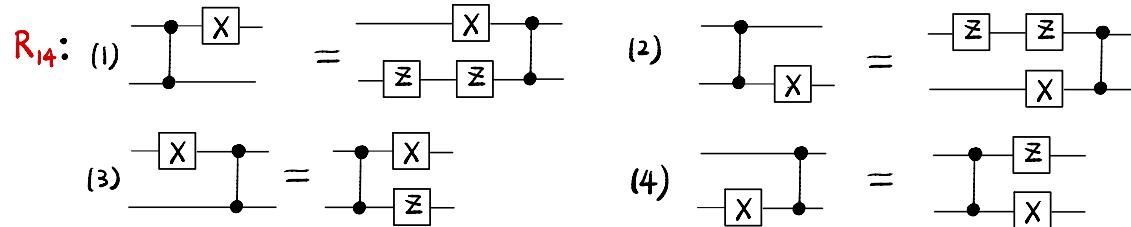
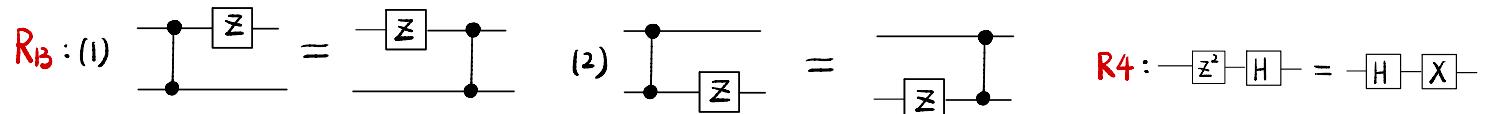
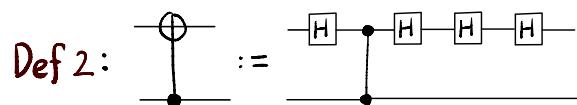
Proof: By Def 2 , C₇ & R₁₅,

R₂₄: $\begin{array}{c} I \oplus \\ S \bullet \end{array} := \begin{array}{c} I \boxed{H} I \bullet \boxed{H} \boxed{H} \boxed{H} I \\ S \quad \quad \quad S \end{array}$

R₂₄²: $\begin{array}{c} S \bullet \\ I \ominus \end{array} := \begin{array}{c} S \quad S \\ I \boxed{H} I \bullet \boxed{H} \boxed{H} \boxed{H} I \end{array}$

R₂₄¹: $\begin{array}{c} I \oplus \\ S' \bullet \end{array} := \begin{array}{c} I \boxed{H} I \bullet \boxed{H} \boxed{H} \boxed{H} I \\ S' \quad \quad \quad S' \end{array}$

R₂₄³: $\begin{array}{c} S' \bullet \\ I \ominus \end{array} := \begin{array}{c} S' \quad S' \\ I \boxed{H} I \bullet \boxed{H} \boxed{H} \boxed{H} I \end{array}$



Lem J

R_{26} : (1) $\begin{array}{c} \text{X} \oplus \\ \parallel \end{array} = \begin{array}{c} \oplus \text{X} \\ \parallel \end{array}$

(2) $\begin{array}{c} \text{z} \oplus \\ \parallel \end{array} = \begin{array}{c} \oplus \text{z} \\ \parallel \end{array}$

(3) $\begin{array}{c} \oplus \\ \text{X} \parallel \end{array} = \begin{array}{c} \oplus \text{X} \\ \parallel \end{array}$

(4) $\begin{array}{c} \oplus \\ \text{z} \parallel \end{array} = \begin{array}{c} \oplus \text{z} \\ \parallel \end{array}$

(5) $\begin{array}{c} \oplus \\ \bullet \text{X} \parallel \end{array} = \begin{array}{c} \text{X}^2 \oplus \\ \parallel \end{array}$

(6) $\begin{array}{c} \oplus \text{z} \\ \parallel \end{array} = \begin{array}{c} \text{z} \oplus \\ \parallel \end{array}$

(7) $\begin{array}{c} \text{X} \\ \parallel \end{array} \bullet = \begin{array}{c} \text{X} \\ \parallel \end{array}$

(8) $\begin{array}{c} \text{z} \\ \parallel \end{array} \bullet = \begin{array}{c} \text{z} \\ \parallel \end{array}$

(9) $\begin{array}{c} \text{X} \\ \parallel \end{array} \oplus = \begin{array}{c} \oplus \text{X} \\ \parallel \end{array}$

(10) $\begin{array}{c} \text{z} \\ \parallel \end{array} \oplus = \begin{array}{c} \oplus \text{z}^2 \\ \parallel \end{array}$

(11) $\begin{array}{c} \bullet \text{X} \\ \parallel \end{array} = \begin{array}{c} \text{X} \bullet \\ \parallel \end{array}$

(12) $\begin{array}{c} \bullet \text{z} \\ \parallel \end{array} = \begin{array}{c} \text{z} \bullet \\ \parallel \end{array}$

Proof cont.

$\begin{array}{c} \text{X} \\ \parallel \end{array} := \begin{array}{c} \text{X} \quad \text{X} \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{I} \quad \text{z} \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{X}} \text{X}$

$\begin{array}{c} \text{z} \\ \parallel \end{array} := \begin{array}{c} \text{z} \quad \text{z} \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{I} \quad \text{I} \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{I}} \text{z}$

$\begin{array}{c} \text{I} \\ \parallel \end{array} := \begin{array}{c} \text{I} \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{z} \quad \text{z} \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{X}} \text{I}$

$\begin{array}{c} \text{z} \\ \parallel \end{array} := \begin{array}{c} \text{z}^2 \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{x}^2 \quad \text{x}^2 \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{z}} \text{z}^2$

$\begin{array}{c} \text{I} \\ \parallel \end{array} := \begin{array}{c} \text{X} \quad \text{X} \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{z}^2 \quad \text{I} \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{I}} \text{I}$

$\begin{array}{c} \text{z} \\ \parallel \end{array} := \begin{array}{c} \text{z} \quad \text{I} \\ \parallel \end{array}$ $\begin{array}{c} \text{H} \quad \text{x}^2 \quad \text{x}^2 \quad \text{z}^2 \quad \text{z} \\ \parallel \end{array} \xrightarrow{\quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{H} \quad \text{z}} \text{z}$

$$\text{Def 2: } \begin{array}{c} \text{Circuit Diagram} \\ \oplus \end{array} := \begin{array}{c} \text{Circuit Diagram} \\ H \quad H \quad H \quad H \end{array}$$

$$\begin{array}{c} \text{Circuit Diagram} \\ \oplus \end{array} := \begin{array}{c} \text{Circuit Diagram} \\ H \quad H \quad H \quad H \end{array}$$

$$R_{17}: \begin{array}{c} \text{Circuit Diagram} \end{array} = \begin{array}{c} \text{Circuit Diagram} \end{array}$$

$$R_{19}: (1) \begin{array}{c} \text{Circuit Diagram} \end{array} = \begin{array}{c} \text{Circuit Diagram} \end{array} \quad (2) \begin{array}{c} \text{Circuit Diagram} \end{array} = \begin{array}{c} \text{Circuit Diagram} \end{array}$$

Lem K By Def 2, R_{17} & R_{19} ,

$$R_{31}: (1) \begin{array}{c} \text{Circuit Diagram} \end{array} = \begin{array}{c} \text{Circuit Diagram} \end{array} \quad (2) \begin{array}{c} \text{Circuit Diagram} \end{array} = \begin{array}{c} \text{Circuit Diagram} \end{array}$$

$$\text{Proof: } R_{31}.(1). \text{ LHS} := \begin{array}{c} \text{Circuit Diagram} \end{array} \stackrel{\text{Def 2}}{=} \begin{array}{c} \text{Circuit Diagram} \end{array}$$

$$\frac{R_{17}}{R_{19}} \begin{array}{c} \text{Circuit Diagram} \end{array} \stackrel{\text{Def 2}}{=} \begin{array}{c} \text{Circuit Diagram} \end{array} =: R_{31}.(1). \text{ RHS}$$

$$R_{31}.(2). \text{ LHS} := \begin{array}{c} \text{Circuit Diagram} \end{array} \stackrel{\text{Def 2}}{=} \begin{array}{c} \text{Circuit Diagram} \end{array}$$

$$\frac{R_{17}}{R_{19}} \begin{array}{c} \text{Circuit Diagram} \end{array} \stackrel{\text{Def 2}}{=} \begin{array}{c} \text{Circuit Diagram} \end{array} =: R_{31}.(2). \text{ RHS}$$

□

$$C1: w^3 = I$$

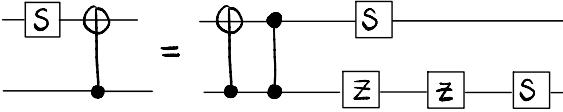
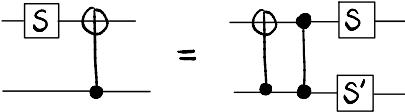
$$C2: H^4 = I$$

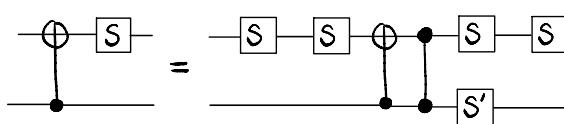
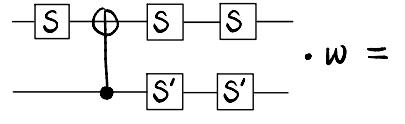
$$C3: S^3 = I$$

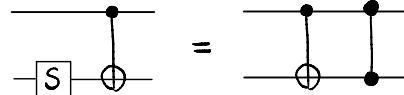
$$R11: \bar{z}^2 = S' S S$$

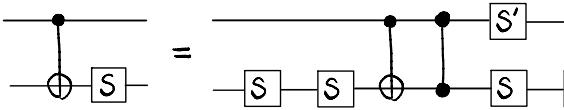
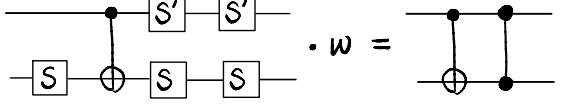
$$\text{Def 2: } \begin{array}{c} \oplus \\ \bullet \end{array} := \begin{array}{c} H \\ \bullet \end{array} \quad \begin{array}{c} H \\ \bullet \end{array} \quad \begin{array}{c} H \\ \bullet \end{array} \quad \begin{array}{c} H \\ \bullet \end{array}$$

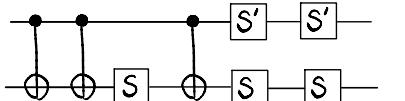
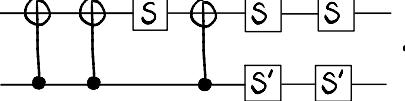
$$\begin{array}{c} \bullet \\ \oplus \end{array} := \begin{array}{c} \bullet \\ H \\ \bullet \end{array} \quad \begin{array}{c} \bullet \\ H \\ \bullet \end{array} \quad \begin{array}{c} \bullet \\ H \\ \bullet \end{array} \quad \begin{array}{c} \bullet \\ H \\ \bullet \end{array}$$

Lem L $R_{23}^1:$  $\cdot w^2 \equiv R_{23}^1:$ 

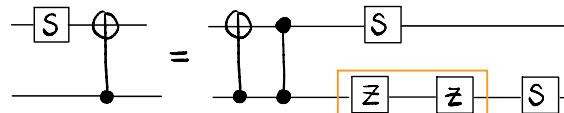
$$\equiv R_{23}^2:$$
  $\cdot w^2 \equiv R_{23}^3:$ 

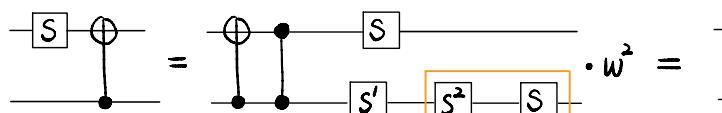
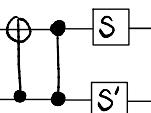
implies $R_{23}^4:$  $\cdot w^2 \equiv$

$$R_{23}^5:$$
  $\cdot w^2 \equiv R_{23}^6:$ 

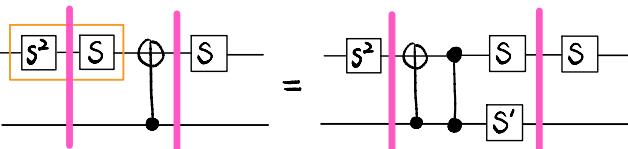
$$\equiv R_{23}^7:$$
  $\cdot w =$ 

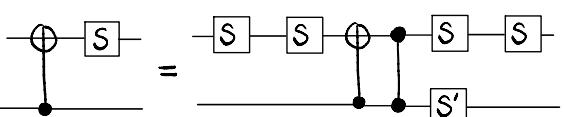
Proof:

$$R_{23}^1:$$
  $\cdot w^2$
 $\parallel R11 \& C3$

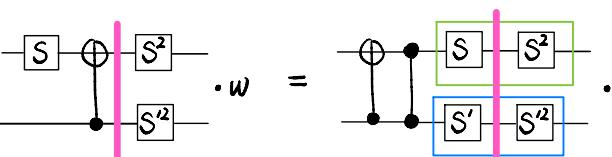
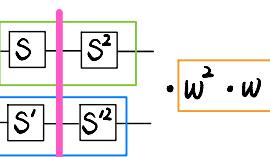
$$R_{23}^1:$$
  $\cdot w^2 =$  $\cdot w^2 : R_{23}^1$

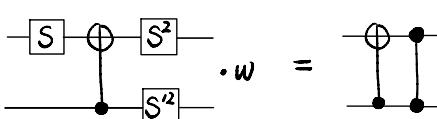
For both sides of R_{23}^1 , left-appending them by $S^2 \otimes I$ and right-appending them by $S \otimes I$ yields

$$R_{23}^1:$$
  $\cdot w^2$

$$\equiv$$
  $\cdot w^2 : R_{23}^2$

For both sides of R_{23}^1 , right-appending them by $S^2 \otimes S^2$ and multiplying them by w yields

$$R_{23}^1:$$
  $\cdot w =$ 
 $c_1 \parallel c_2 \& C3$

$$R_{23}^1:$$
  $\cdot w =$  $: R_{23}^3$

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