

$$\begin{aligned}
 -[C_0] - &= - \\
 -[C_1] - &= -[H][S][H][H][S][S][H] - \\
 -[C_2] - &= -[H][S][S][H][H][S][H] -
 \end{aligned}$$

$$R_5: -[X] - = -[H][S][H][H][S][S][H] - =: -[C_1] -$$

$$R_7: -[X^2] - = -[H][S][S][H][H][S][H] - =: -[C_2] -$$

$$R_{13}: (1) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z] = \begin{array}{c} [Z] \\ | \\ \bullet \end{array}$$

$$(2) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z] = \begin{array}{c} \bullet \\ | \\ [Z] \end{array}$$

$$R_{14}: (1) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [X] = \begin{array}{c} [X] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \begin{array}{c} [Z] \\ | \\ \bullet \end{array}$$

$$(2) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [X] = \begin{array}{c} [Z][Z] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [X]$$

$$(3) [X] \begin{array}{c} \bullet \\ | \\ \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [X] \begin{array}{c} [Z] \\ | \\ \bullet \end{array}$$

$$(4) [X] \begin{array}{c} \bullet \\ | \\ \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z] \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [X]$$

$$\text{Lem 54 } 18.(1) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_0] = \begin{array}{c} [C_0] \\ | \\ \bullet \end{array}, \quad (2) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_1] = \begin{array}{c} [C_1] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z][Z] \quad \&$$

$$(3) \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_2] = \begin{array}{c} [C_2] \\ | \\ \bullet \end{array} [Z] \quad \text{are the consequence of } R_5, R_7, R_{13} \& R_{14}.$$

$$\text{Proof: } 18.(1). \text{LHS} := \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_0] \stackrel{\text{def}}{=} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \stackrel{\text{def}}{=} \begin{array}{c} [C_0] \\ | \\ \bullet \end{array} =: 18.(1). \text{RHS}$$

$$18.(2). \text{LHS} := \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_1] \stackrel{\text{def}}{=} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [H][S][H][H][S][S][H]$$

$$\stackrel{R_5}{=} \begin{array}{c} [X] \\ | \\ \bullet \end{array} \stackrel{R_{14}}{=} \begin{array}{c} [X] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z][Z] \stackrel{R_5}{=} \begin{array}{c} [C_1] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z][Z]$$

$$\stackrel{R_{13}}{=} \begin{array}{c} [C_1] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z][Z] =: 18.(2). \text{RHS}$$

$$18.(3). \text{LHS} := \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [C_2] \stackrel{\text{def}}{=} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [H][S][S][H][H][S][H]$$

$$\stackrel{R_7}{=} \begin{array}{c} [X^2] \\ | \\ \bullet \end{array} \stackrel{R_{14}}{=} \begin{array}{c} [X^2] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z] \stackrel{R_7}{=} \begin{array}{c} [C_2] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z]$$

$$\stackrel{R_{13}}{=} \begin{array}{c} [C_2] \\ | \\ \bullet \end{array} \begin{array}{c} \bullet \\ | \\ \bullet \end{array} [Z] =: 18.(3). \text{RHS}$$