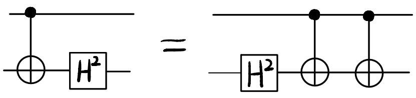
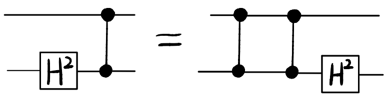
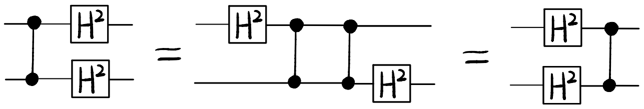
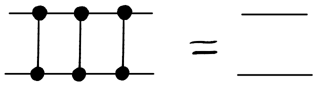
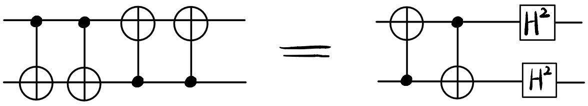
C_8^7: 

C_8^1: 

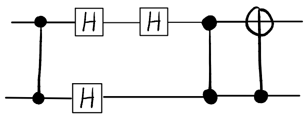
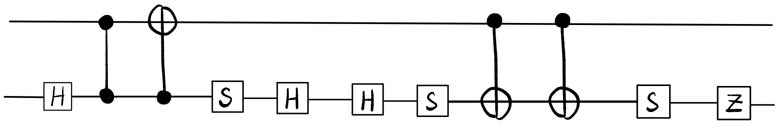
$C2: H^4 = I$ $C3: S^3 = I$

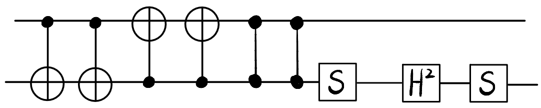
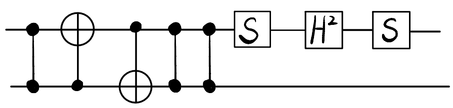
C_8^3: 

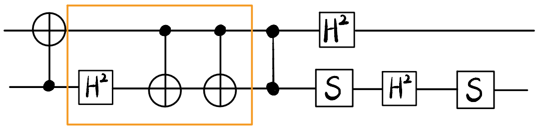
C_6: 

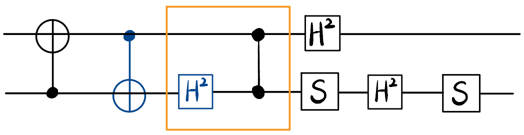
R_{36}: 

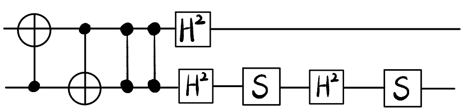
Lem V

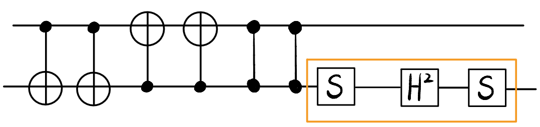
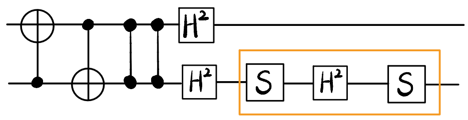
R_{30}:  = 

Proof cont. R_{30}:  $\underline{\underline{\text{WTS}}}$ 

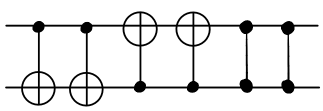
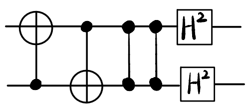
R_{30} RHS = 

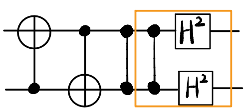
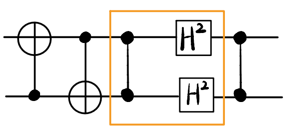
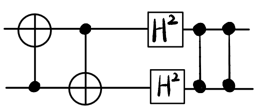
$\underline{\underline{C_8^7}}$ 

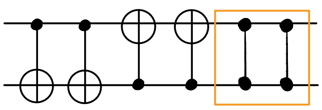
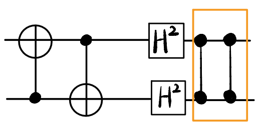
$\underline{\underline{C_8^1}}$ 

Hence R_{30}:  $\underline{\underline{\text{WTS}}}$ 

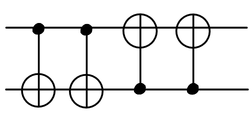
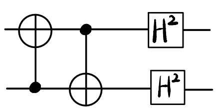
$C_2 \,|||\, C_3$

R_{30}:  $\underline{\underline{\text{WTS}}}$ 

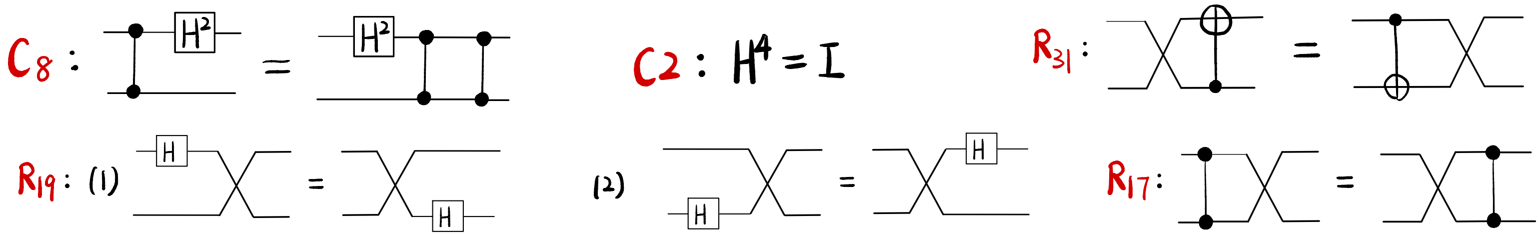
R_{30} RHS =  $\underline{\underline{C_8^3}}$  $\underline{\underline{C_8^3}}$ 

Hence R_{30}:  $\underline{\underline{\text{WTS}}}$ 

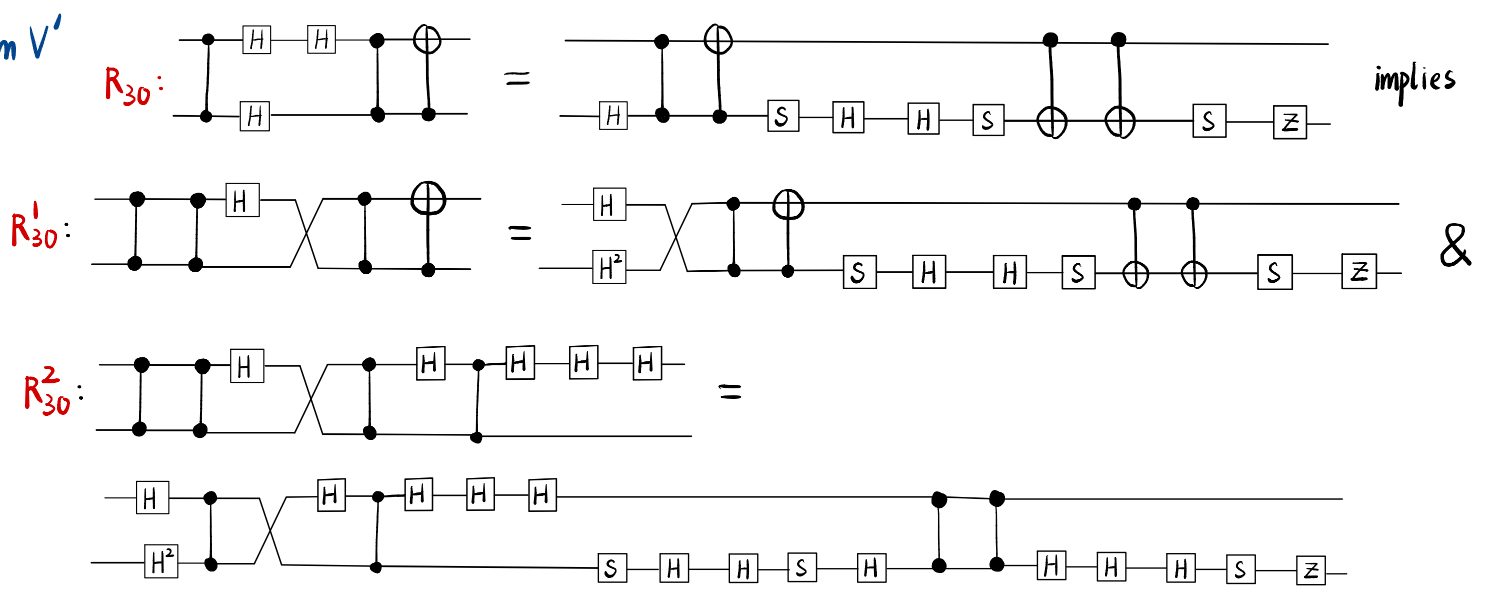
$|||\, C_6$

R_{30}:  $\underline{\underline{\text{WTS}}}$  : R_{36}

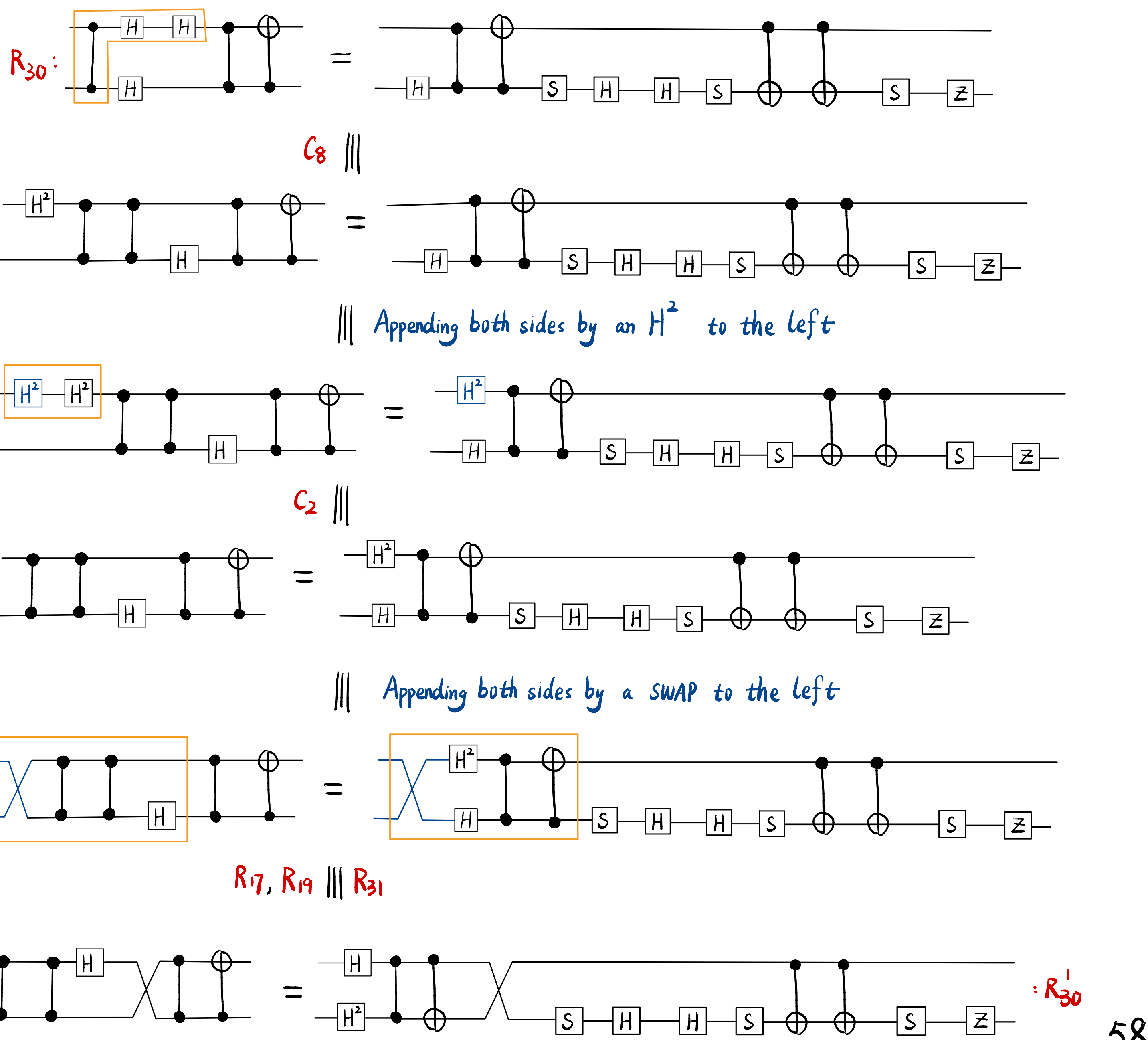
◪


C8 :
C2 : H⁴ = I
R31 :
R19 : (1)
(2)
R17 :

Lem V'

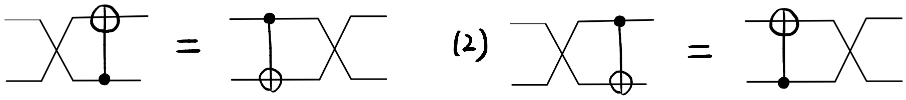

R30 :
implies
R'30 :
&
R²30 :

Proof:


R30 :
C8
R30 :
Appending both sides by an H² to the left
R30 :
C2
R30 :
Appending both sides by a SWAP to the left
R30 :
R17, R19 ||| R31
R30 :
: R'30

C2 : $H^4 = I$

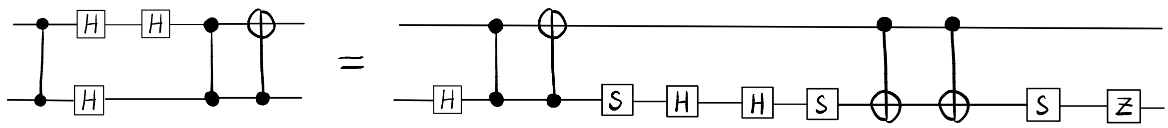
R_{31} : (1)



Def 2 :

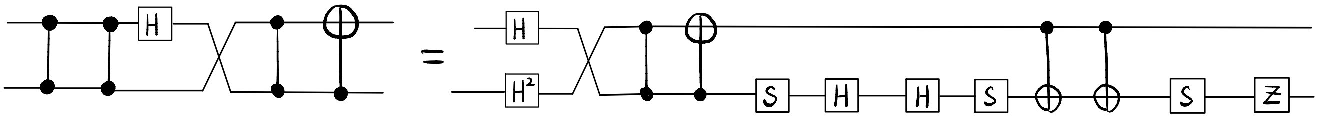
Lem V'

R_{30} :



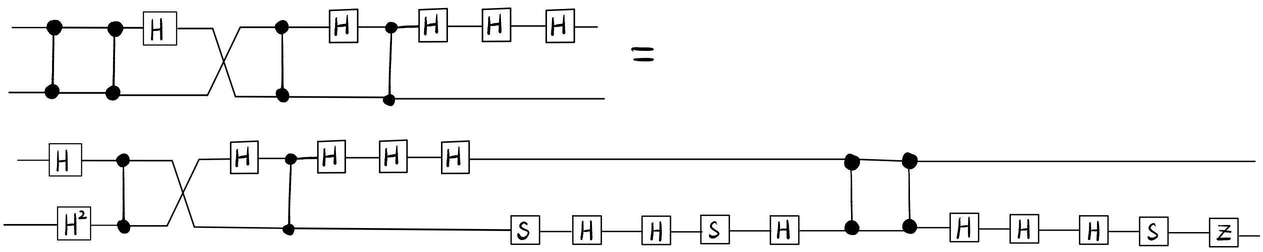
implies

R^1_{30} :



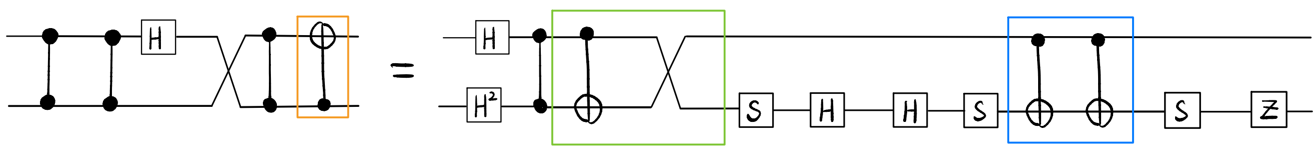
&

R^2_{30} :



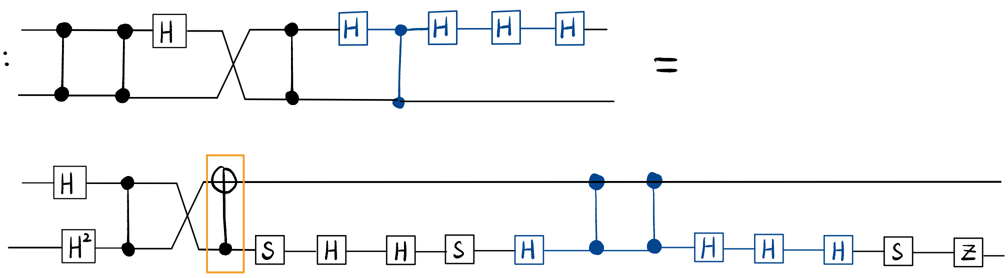
Proof cont.

R^1_{30} :



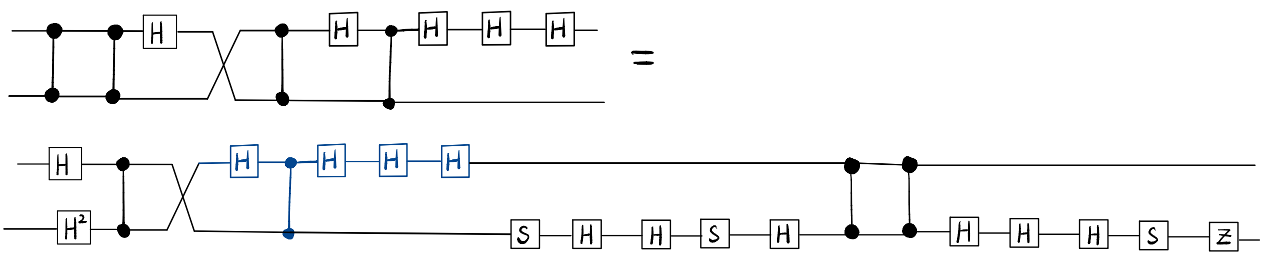
Def 2 ||| R_{31}, H_2

R^1_{30} :



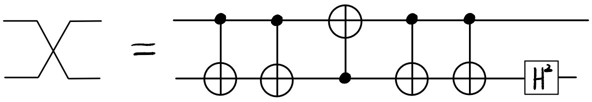
Def 2 |||

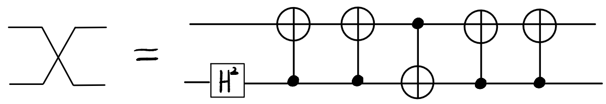
R^1_{30} :

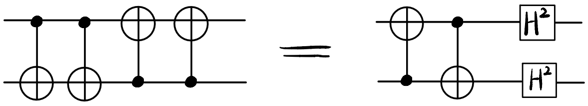


: R^2_{30}

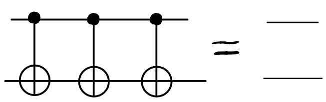
∎

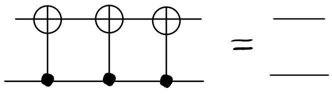
Def 4: 

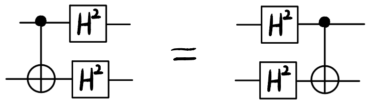
Def 7: 

R_{36}: 

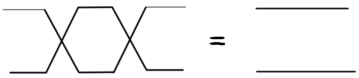
$C2$: $H^4 = I$

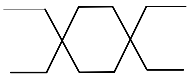
C_6^1: 

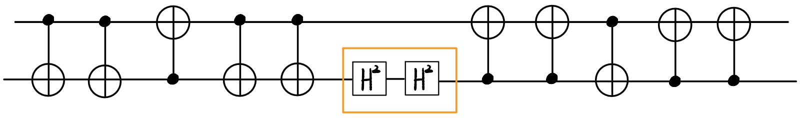
C_6^2: 

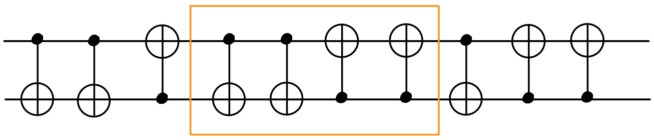
C_8^{14}: 

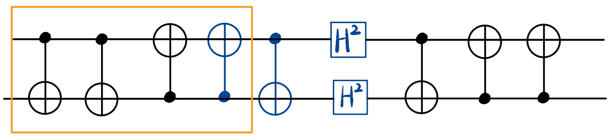
Lem W Def 4, Def 7, C_2, C_6, C_8 & R_{36} imply

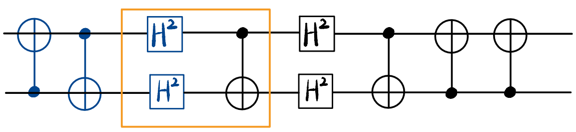
R_{16}: 

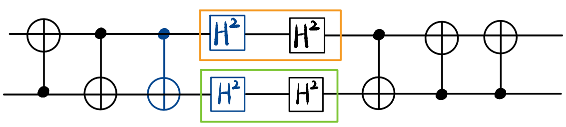
Proof: R_{16}.LHS := 

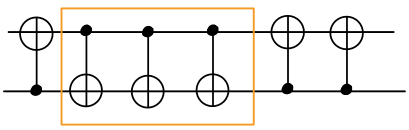
$\overset{\text{Def 4}}{\underset{\text{Def 7}}{=}}$ 

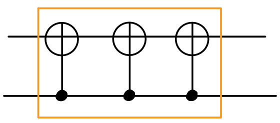
$\overset{C_2}{=}$ 

$\overset{R_{36}}{=}$ 

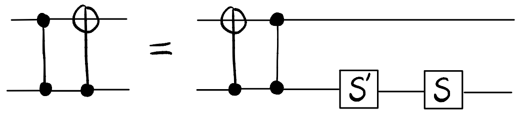
$\overset{R_{36}}{=}$ 

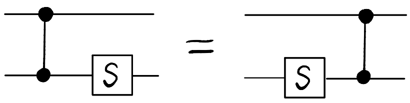
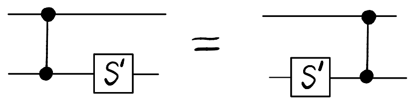
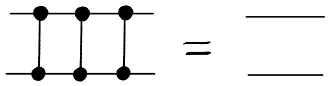
$\overset{C_8^{14}}{=}$ 

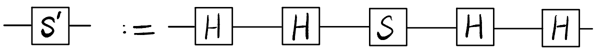
$\overset{C_2}{=}$ 

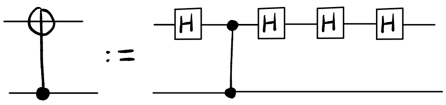
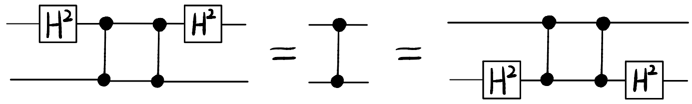
$\overset{C_6^2}{=}$ 

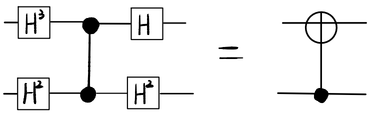
$\overset{C_6^1}{=}$ =: R_{16}.RHS

R_{25}^1:  $\cdot\, w$ $\quad C1: w^3 = 1 \quad C2: H^4 = I \quad C_3: S^3 = I \quad C5: SS' = S'S$

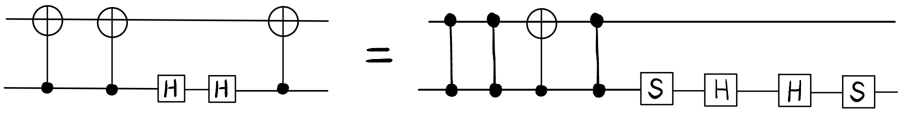
C_7:  $\quad R_{15}$:  $\quad C_6$: 

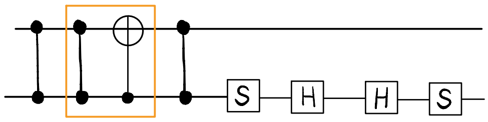
Def 1 : 

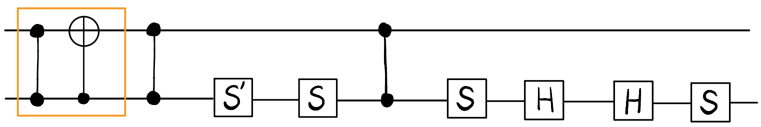
Def 2 :  $\quad C_8^2$: 

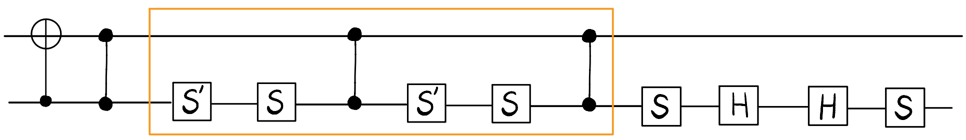
R_{32}^4 : 

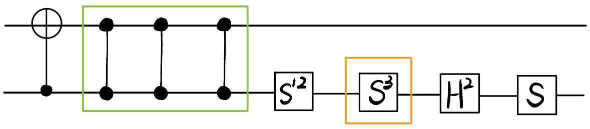
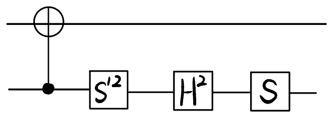
Lem X Def 1-2, $C1$-3, $C5$-8, R_{15}, R_{25} & R_{32} imply

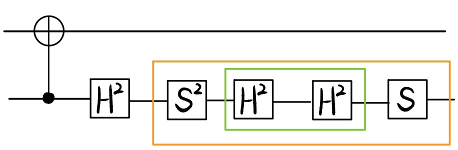
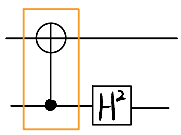
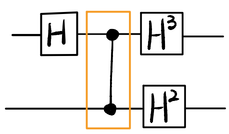
R_{33}:  $\cdot\, w$

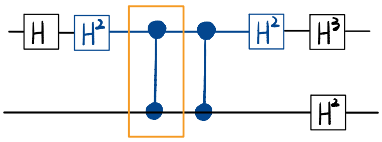
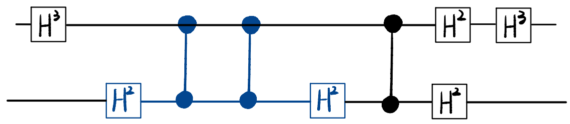
Proof: R_{33}. RHS :=  $\cdot\, w$

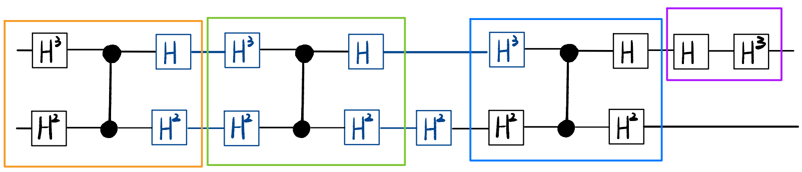
$\underline{\underline{R_{25}^1}}$  $\cdot\, w^2$

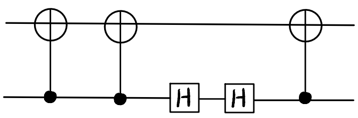
$\frac{R_{25}^1}{C_1}$ 

$\frac{C_5, C_7}{R_{15}}$  $\frac{C_3}{C_6}$ 

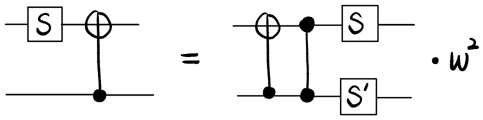
$\underline{\underline{\text{Def 1}}}$  $\frac{C_2}{C_3}$  $\underline{\underline{\text{Def2}}}$ 

$\underline{\underline{C_8^2}}$  $\underline{\underline{C_8^2}}$ 

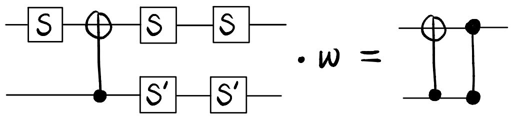
$\underline{\underline{C_2}}$ 

$\frac{R_{32}^4}{C_2}$  $=: R_{33}$. LHS. $\boxed{/\!/}$

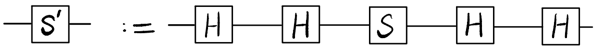
R^1_{23}:



R^3_{23}:

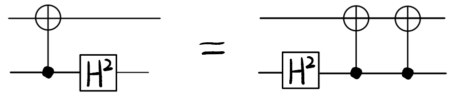


Def 1 :

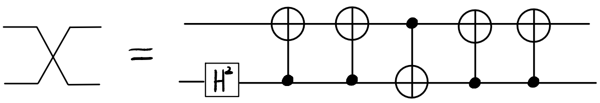


C2 : $H^4 = I$

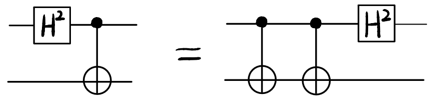
C^5_8:



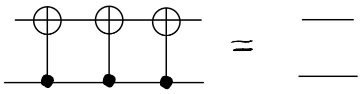
Def 7:



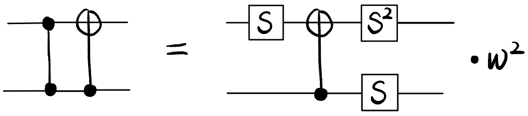
C^8_8:



C^2_6:

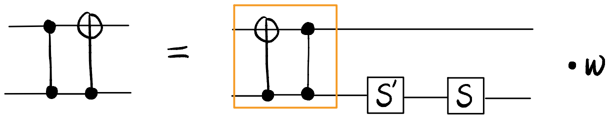


Lem A1 R_{23} & R_{25} imply R_{37}:



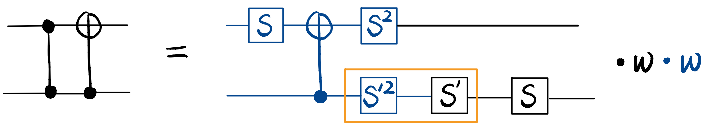
Proof:

R^1_{25}:



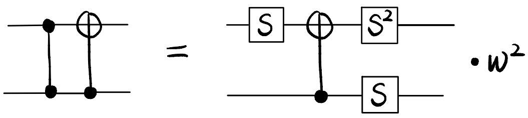
||| R^3_{23}

R_{37}:



Def 1 ||| C2

R_{37}:

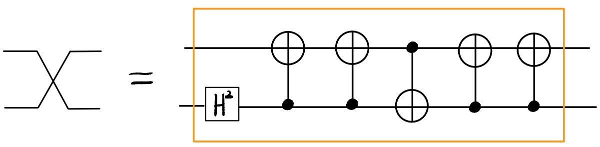


∎

Lem A2 C_6, C_8 & Def 7 imply R_{38} :

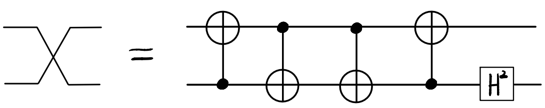
Proof:

Def 7:



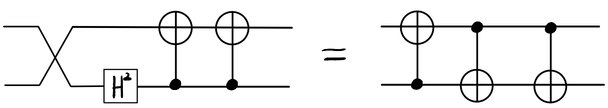
C^5_8 ||| C^8_8

Def 7:

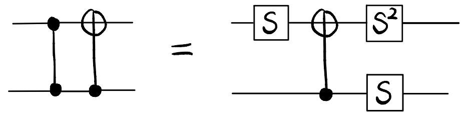


C^2_6 ||| C_2

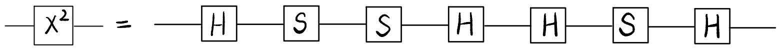
R_{38} :



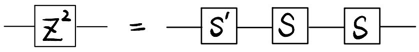
∎

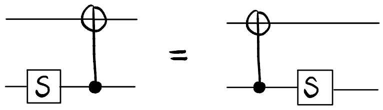
R37: 

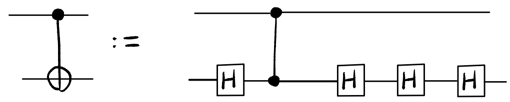
$\cdot\, \omega^2$

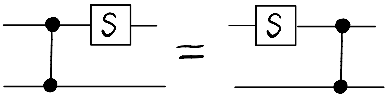
R7: 

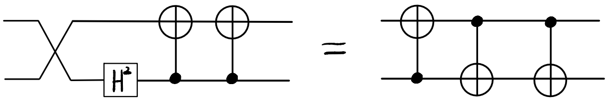
C1: $\omega^3 = 1$ C2: $H^4 = I$ C3: $S^3 = I$ C5: $SS' = S'S$

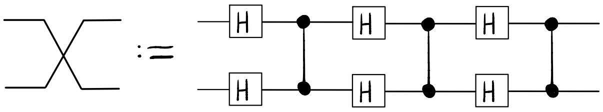
R11: 

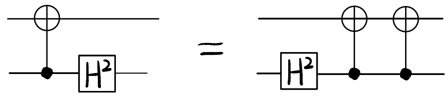
R24: 

Def 4: 

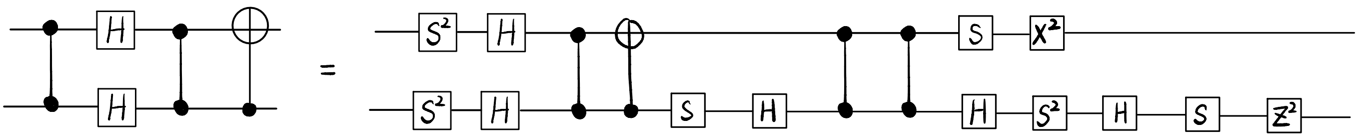
C7: 

R38: 

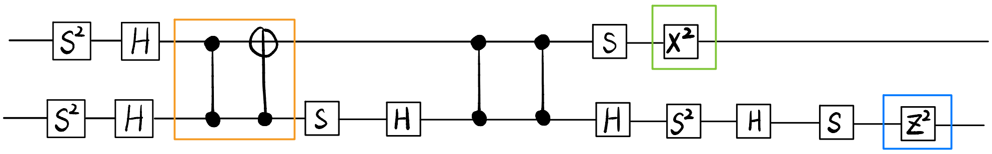
Def 3: 

C_8^5: 

Lem Y Def 1-4, C0-8, R7, R11, R15, R18, R19, R23, R24, R25, R32, R37 & R38 imply

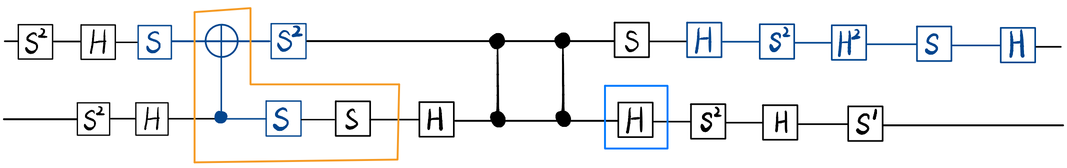
R34: 

$\cdot\,(-\omega^2)$

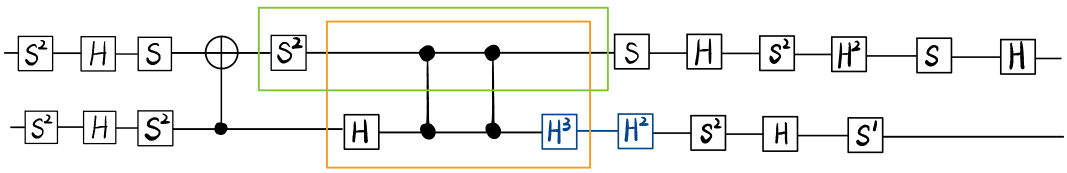
Proof: R34.RHS := 

$\cdot\,(-\omega^2)$

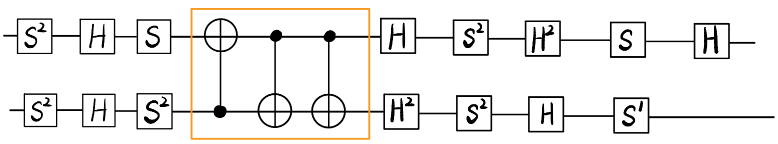
$HS^2HS\; S'S^2 \overset{C_3}{\underset{C_5}{=}} HS^2HS'$

$\dfrac{C_3, C_5, R_{37}}{R_7, R_{11}}$ 

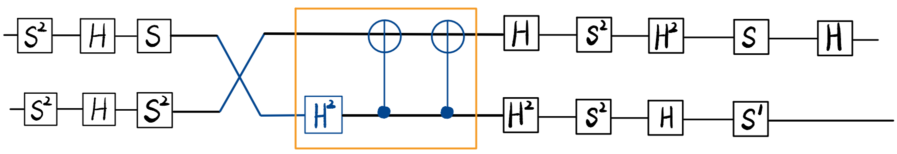
$\cdot\,(-\omega^2)\cdot\omega^2$

$\dfrac{C_1\ C_2}{R_{24}}$ 

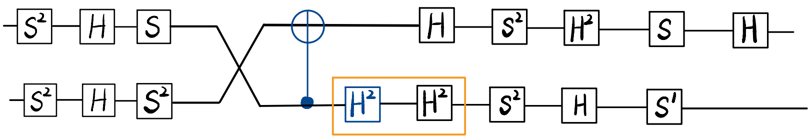
$\cdot\,(-\omega)$

$\dfrac{C_2, C_3, C_7}{\text{Def } 4}$ 

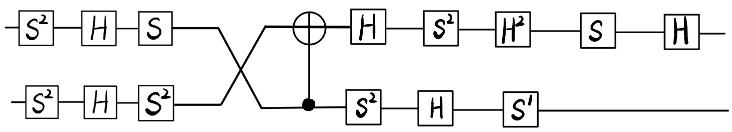
$\cdot\,(-\omega)$

$\dfrac{R_{38}}{\text{Def } 3}$ 

$\cdot\,(-\omega)$

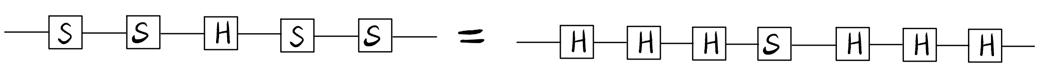
C_8^5 

$\cdot\,(-\omega)$

C_2 

$\cdot\,(-\omega)$

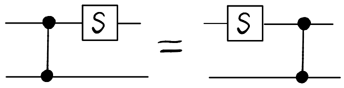
C_4' :



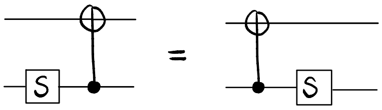
$\cdot(-\omega)$

C_0: $(-1)^2 = 1$

C_7 :

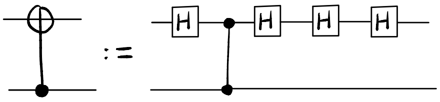


R_{24}:

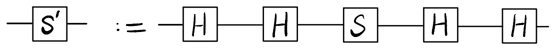


$C_2 : H^4 = I$ $C_3 : S^3 = I$

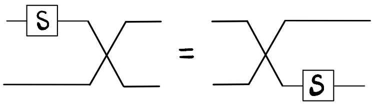
Def 2 :



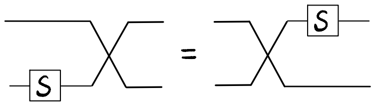
Def 1 :



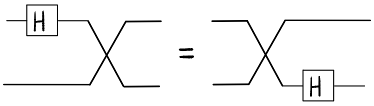
R_{18} : (1)



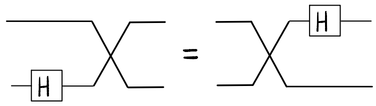
(2)



R_{19} : (1)

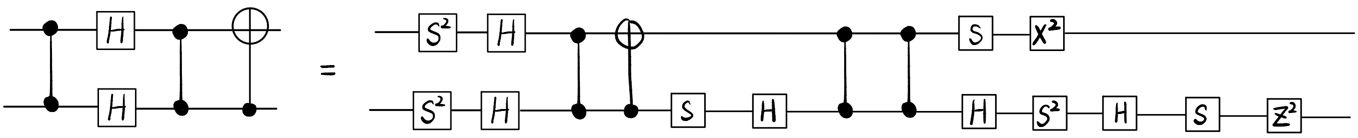


(2)



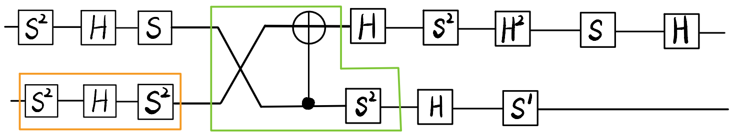
Lem Y

R_{34} :



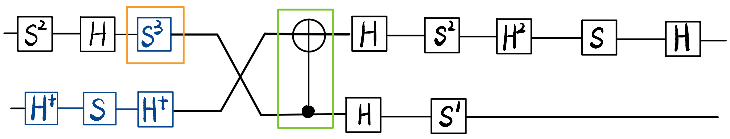
$\cdot(-\omega^2)$

Proof cont.



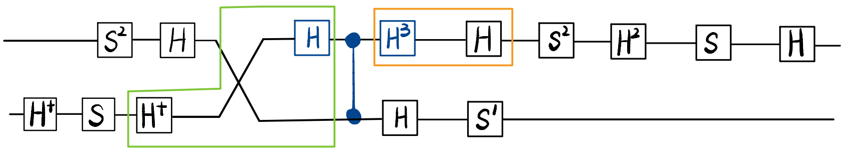
$\cdot(-\omega)$

$\overline{\overline{C_4', R_{18}}}$ R_{24}



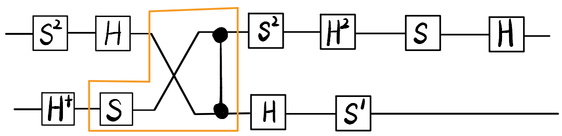
$\boxed{\cdot(-\omega) \cdot(-\omega)}$

$\overline{\overline{C_0, C_3}}$ Def 2



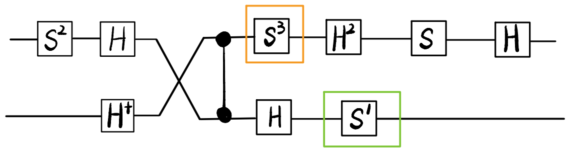
$\cdot\,\omega^2$

$\overline{\overline{C_2}}$ R_{19}



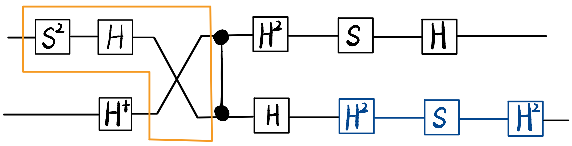
$\cdot\,\omega^2$

$\overline{\overline{C_7}}$ R_{18}



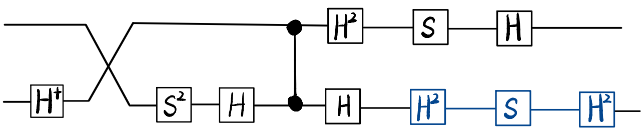
$\cdot\,\omega^2$

$\overline{\overline{C_1}}$ Def 1



$\cdot\,\omega^2$

$\overline{\overline{R_{18}}}$ R_{19}



$\cdot\,\omega^2$