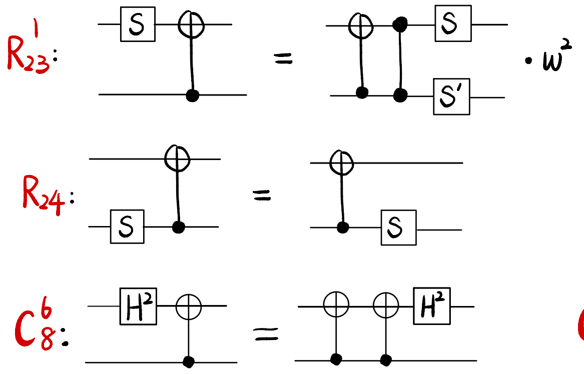
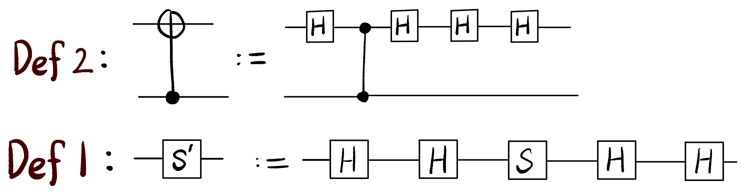
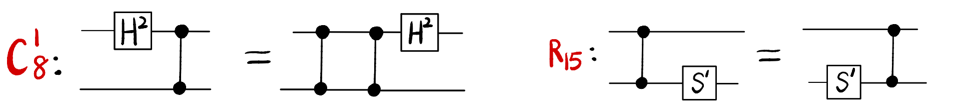
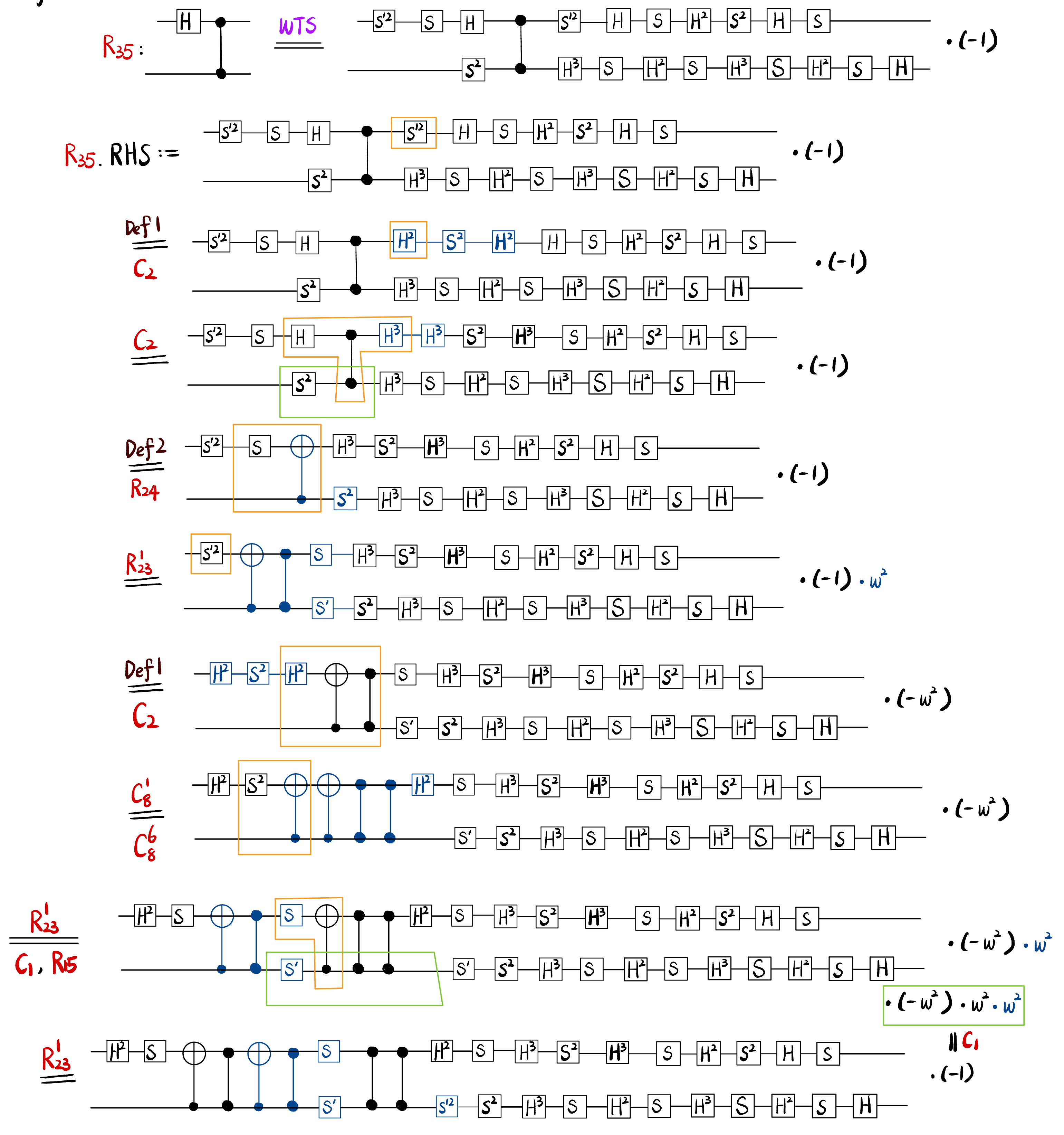


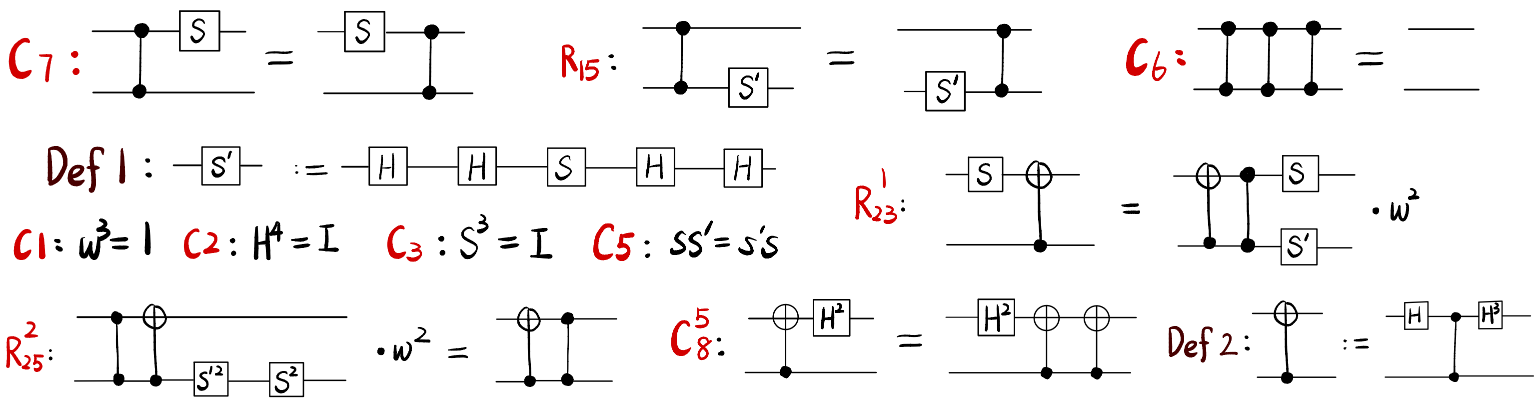


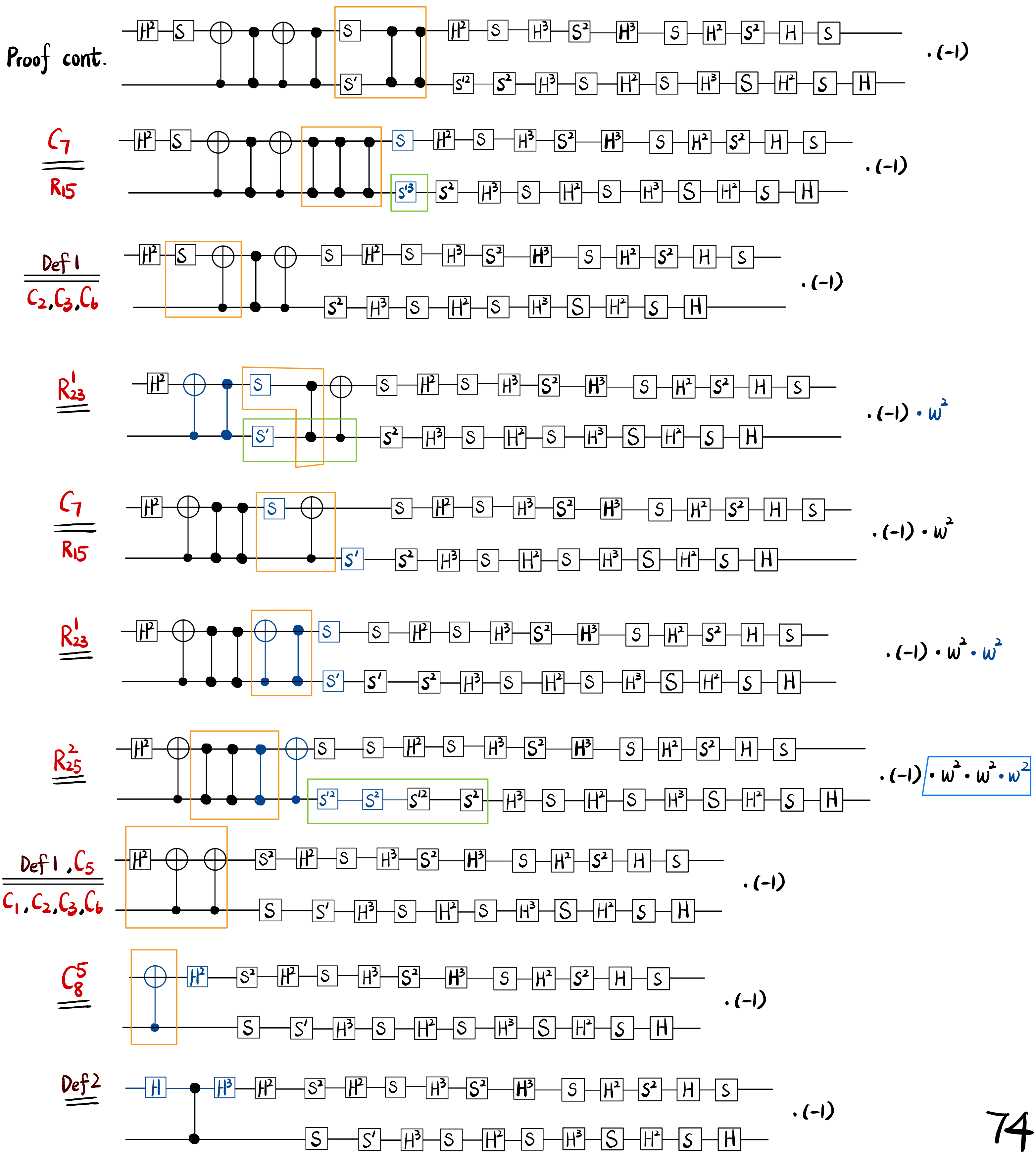
$C1: w^3 = 1$ $\quad C2: H^4 = I$ $\quad C_3: S^3 = I$ $\quad C5: SS' = S'S$



Proof cont.



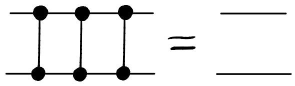

C_7:
=
R_{15}:
=
C_6:
=
Def 1:
:=
R^1_{23}:
=
$\cdot\, \omega^2$
C1: $\omega^3 = 1$
C2: $H^4 = I$
C_3: $S^3 = I$
$C5$: $SS' = S'S$
R^2_{25}:
$\cdot\, \omega^2$ =
C^5_8:
=
Def 2:
:=


Proof cont.
$\cdot(-1)$
$\underline{\underline{C_7}}$ R_{15}
$\cdot(-1)$
$\underline{\underline{\text{Def } 1}}$ C_2, C_3, C_6
$\cdot(-1)$
$\underline{\underline{R^1_{23}}}$
$\cdot(-1) \cdot \omega^2$
$\underline{\underline{C_7}}$ R_{15}
$\cdot(-1) \cdot \omega^2$
$\underline{\underline{R^1_{23}}}$
$\cdot(-1) \cdot \omega^2 \cdot \omega^2$
$\underline{\underline{R^2_{25}}}$
$\cdot(-1)$ $\boxed{\cdot\, \omega^2 \cdot \omega^2 \cdot \omega^2}$
$\underline{\underline{\text{Def } 1, C_5}}$ C_1, C_2, C_3, C_6
$\cdot(-1)$
$\underline{\underline{C^5_8}}$
$\cdot(-1)$
$\underline{\underline{\text{Def} 2}}$
$\cdot(-1)$

C_4^4 : $-\boxed{S}-\boxed{S}-\boxed{H}-\boxed{S}-\boxed{S}-\boxed{H}- \cdot(-\omega^2) = -\boxed{H}-\boxed{H}-\boxed{H}-\boxed{S}-$ $\quad C_0: (-1)^2 = 1$

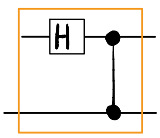
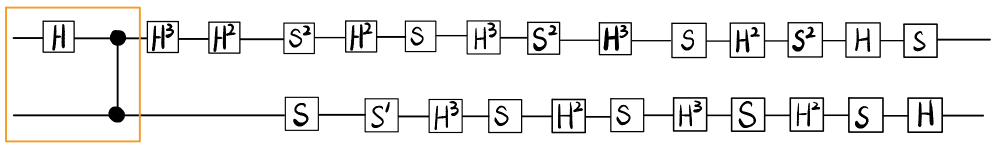
$C1: \omega^3 = 1 \quad C2: H^4 = I \quad C_3: S^3 = I \quad C5: SS' = S'S \quad$ Def 1 : $-\boxed{S'}- \;:=\; -\boxed{H}-\boxed{H}-\boxed{S}-\boxed{H}-\boxed{H}-$

C_4^1 : $-\boxed{S}-\boxed{S}-\boxed{H}-\boxed{S}-\boxed{S}- = -\boxed{H}-\boxed{H}-\boxed{H}-\boxed{S}-\boxed{H}-\boxed{H}-\boxed{H}- \cdot(-\omega)$

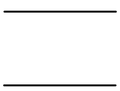
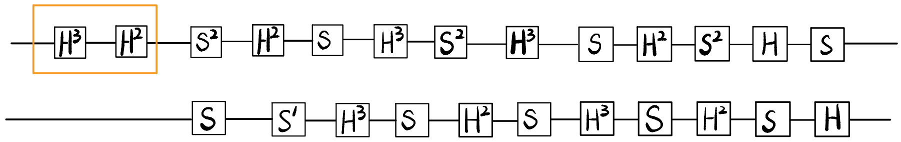
C_6: 

$S^2HS^2H\cdot(-\omega^2) = H^3S$

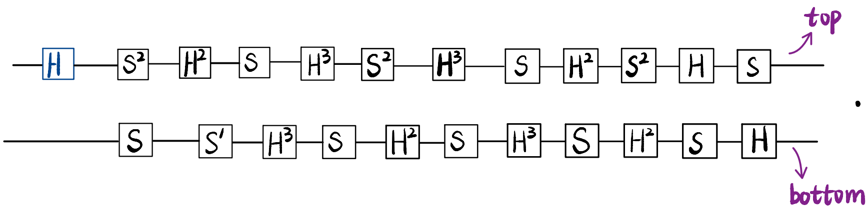
Proof cont.

Hence, R_{35}:  $\underline{\underline{\text{WTS}}}$  $\cdot(-1)$

$C_2 \;|||\; C_6$

R_{35}:  $\underline{\underline{\text{WTS}}}$  $\cdot(-1)$

$C_2 \;|||$

R_{35}:  $\underline{\underline{\text{WTS}}}$ 

$\cdot(-1)$

To show the above, we show that top $\otimes$ bottom $= I \otimes I$.

$$\text{top} := HS^2H^2SH^3S^2H^3SH^2S^2\boxed{H}S\cdot(-1) \overset{C_2}{=} HS^2H^2SH^3S^2H^3S\boxed{H^2S^2H^2}H^3S\cdot(-1)$$

$$\overset{\text{Def1}}{=} HS^2H^2SH^3S^2H^3SS'^2\boxed{H^3S}\cdot(-1) \overset{C_4^4}{=} HS^2H^2SH^3S^2H^3\boxed{SS'^2S^2}HS^2H\cdot\boxed{(-\omega^2)\cdot(-1)}$$

$$\underset{C_5}{\overset{C_0,C_3}{=}} HS^2H^2SH^3S^2H^3\boxed{S'^2}HS^2H\cdot\omega^2 \overset{\text{Def1}}{=} HS^2H^2SH^3S^2\boxed{H^3H^2}S^2H^2HS^2H\cdot\omega^2$$

$$\overset{C_2}{=} HS^2H^2SH^3\boxed{S^2HS^2}H^3S^2H\cdot\omega^2 \overset{C_4^1}{=} HS^2H^2S\boxed{H^3H^3}S\boxed{H^3H^3}S^2H\boxed{\cdot\omega^2\cdot(-\omega)}$$

$$\underset{C_1}{\overset{C_2}{=}} HS^2\boxed{H^2SH^2}SH^2S^2H\cdot(-1) \overset{\text{Def1}}{=} HS^2\boxed{S'S}H^2S^2H\cdot(-1) \overset{C_5}{=} H\boxed{S^3}S'H^2S^2H\cdot(-1)$$

$$\overset{C_3}{=} H\boxed{S'}H^2S^2H\cdot(-1) \overset{\text{Def1}}{=} HH^2S\boxed{H^2H^2}S^2H\cdot(-1) \overset{C_2}{=} HH^2\boxed{SS^2}H\cdot(-1) \overset{C_3}{=} H^4\cdot(-1)$$

$$\overset{C_2}{=} -I.$$

C_4^4: $-\boxed{S}-\boxed{S}-\boxed{H}-\boxed{S}-\boxed{S}-\boxed{H}- \cdot (-w^2) = -\boxed{H}-\boxed{H}-\boxed{H}-\boxed{S}-$ $\quad S^2HS^2H\cdot(-w^2) = H^3S$

$C1: w^3 = 1$ $\quad C2: H^4 = I$ $\quad C_3: S^3 = I$ $\quad C5: SS' = S'S$ $\quad$ Def 1: $-\boxed{S'}- := -\boxed{H}-\boxed{H}-\boxed{S}-\boxed{H}-\boxed{H}-$

C_4^1: $-\boxed{S}-\boxed{S}-\boxed{H}-\boxed{S}-\boxed{S}- = -\boxed{H}-\boxed{H}-\boxed{H}-\boxed{S}-\boxed{H}-\boxed{H}-\boxed{H}- \cdot (-w)$ $\quad C_0: (-1)^2 = 1$

$$S^2HS^2\cdot(-w^2) = H^3SH^3$$

Proof cont.

R_{35}: WTS

top: $-\boxed{H}-\boxed{S^2}-\boxed{H^2}-\boxed{S}-\boxed{H^3}-\boxed{S^2}-\boxed{H^3}-\boxed{S}-\boxed{H^2}-\boxed{S^2}-\boxed{H}-\boxed{S}- \cdot (-1)$

bottom: $-\boxed{S}-\boxed{S'}-\boxed{H^3}-\boxed{S}-\boxed{H^2}-\boxed{S}-\boxed{H^3}-\boxed{S}-\boxed{H^2}-\boxed{S}-\boxed{H}-$

$$\text{bottom} := SS'H^3SH^2S\boxed{H^3S}H^2SH \overset{C_4^4}{=} SS'H^3SH^2\boxed{SS^2}HS^2HH^2SH\cdot(-w^2)$$

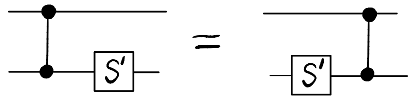
$$\overset{C_3}{=} SS'\boxed{H^3SH^2H}S^2HH^2SH\cdot(-w^2) \overset{C_4^1}{=} \boxed{SS'S^2H}\boxed{S^2S^2}HH^2SH\boxed{\cdot(-w^2)\cdot(-w^2)}$$

$$\overset{C_0,C_1}{\underset{C_3,C_5}{=}} \boxed{S'}HSH^3SH\cdot w \overset{\text{Def1}}{=} H^2S\boxed{H^2HSH^3}SH\cdot w$$

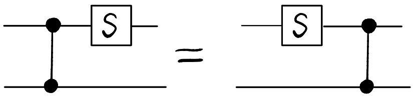
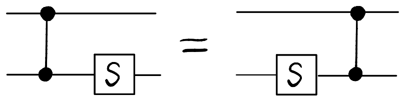
$$\overset{C_4^1}{=} H^2\boxed{S(S^2H}\boxed{S^2)S}H\boxed{\cdot w\cdot(-w^2)} \overset{C_0,C_1}{\underset{C_3}{=}} \boxed{H^2HH}\cdot(-1) \overset{C_2}{=} -I.$$

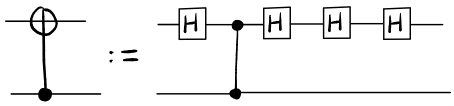
Hence, $\text{top}\otimes\text{bottom} := (-I)\otimes(-I) = (-1)^2\, I\otimes I \overset{C_0}{=} I\otimes I$.

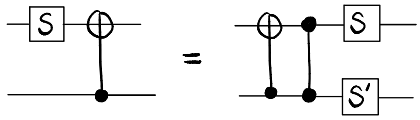
This completes the proof. $\blacksquare$

R_{15}: 

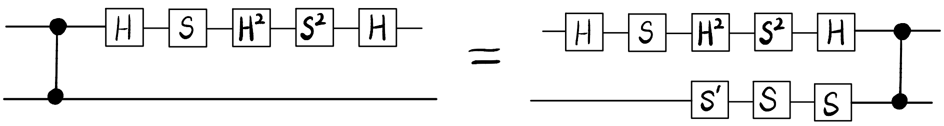
$C1: \omega^3 = 1$ $C2: H^4 = I$ $C_3: S^3 = I$

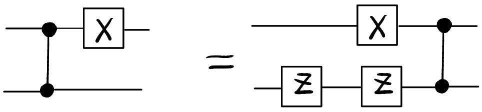
C_7: (1)  (2) 

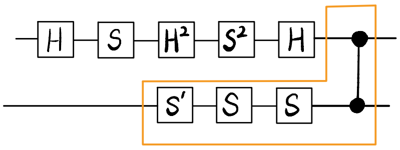
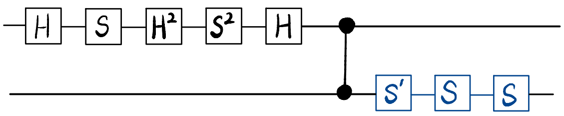
Def 2: 

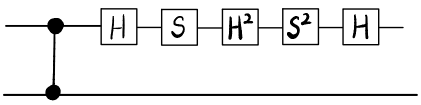
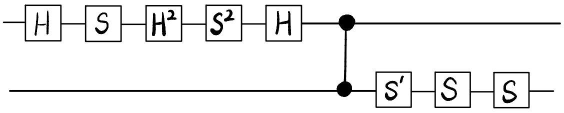
R'_{23}:  $\cdot\, \omega^2$

Lem Z Def 1-2, C1-8, R_5, R_{11}, R_{13}, R_{14}, R_{15}, R_{23}, R_{24}, R_{25} imply

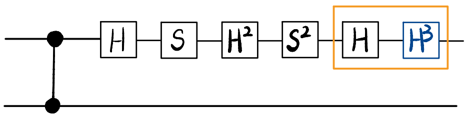
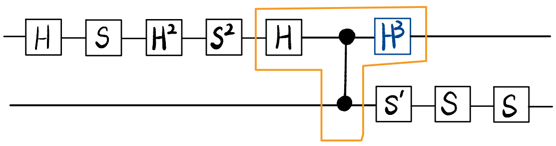
R_{14}: 

$\equiv R'_{14}$: 

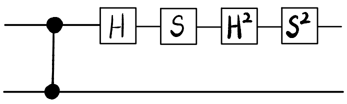
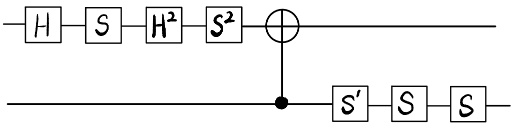
Proof: R_{14}. RHS :=  $\overset{C_7}{\underset{R_{15}}{=}}$ 

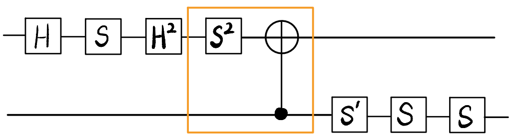
Hence, R_{14}:  $\overset{\text{WTS}}{=}$ 

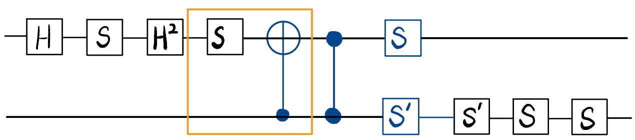
C_2 |||

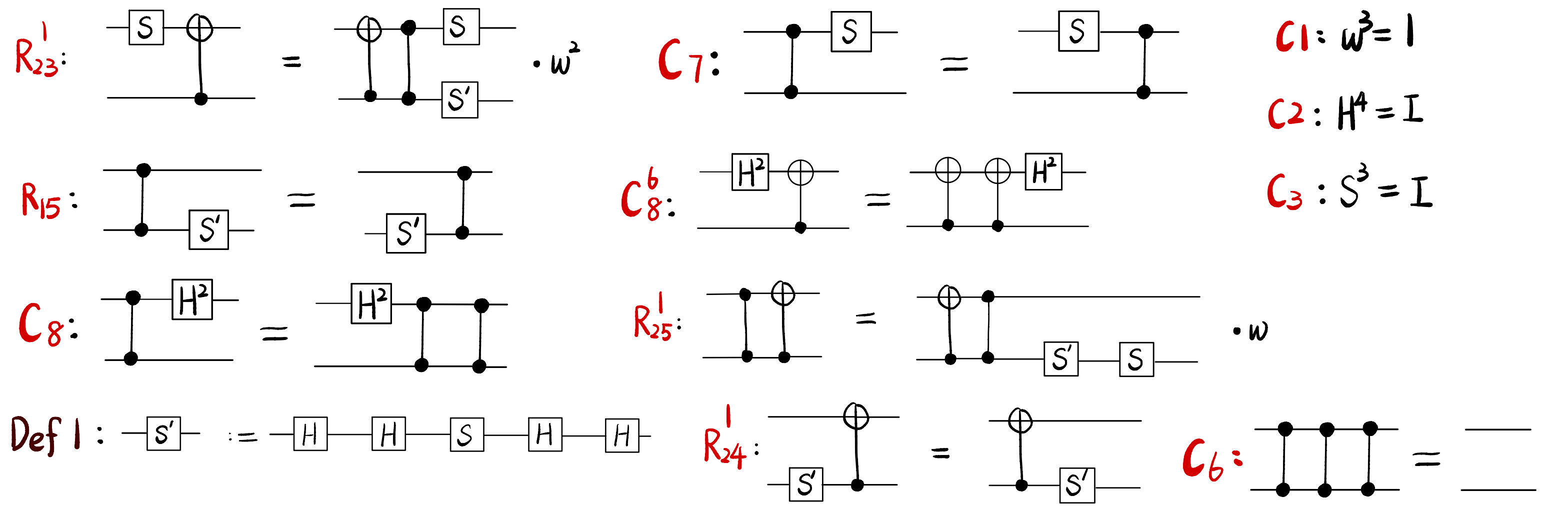
R_{14}:  $\overset{\text{WTS}}{=}$ 

C_2 ||| Def 2

R_{14}:  $\overset{\text{WTS}}{=}$ 

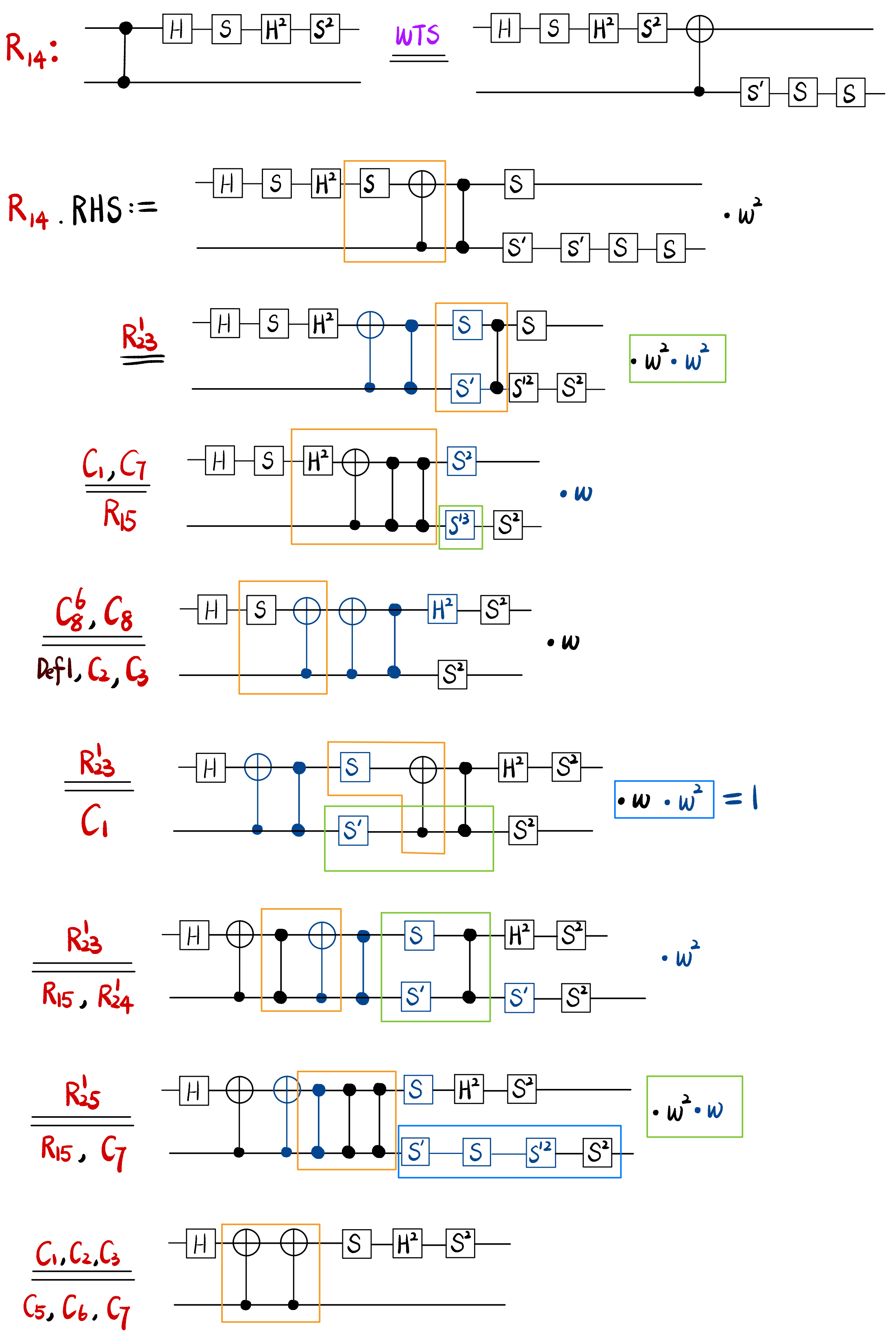
Then R_{14}. RHS := 

$\overset{R'_{23}}{=}$  $\cdot\, \omega^2$

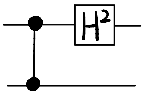
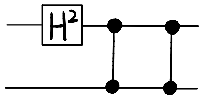


R'_{23}:
S
S
S'
$\cdot w^2$
C_7:
S
S
$C1: w^3 = 1$
$C2: H^4 = I$
$C3: S^3 = I$
R_{15}:
S'
S'
C_8^6:
H^2
H^2
C_8:
H^2
H^2
R'_{25}:
S'
S
$\cdot w$
Def 1 :
S'
:=
H
H
S
H
H
R'_{24}:
S'
S'
C_6:

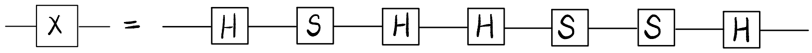
Proof cont.

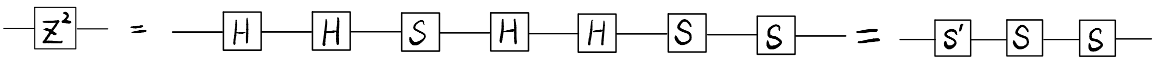


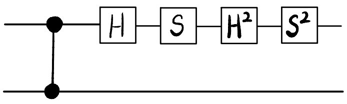
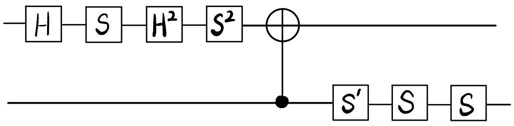
R_{14}:
H
S
H^2
S^2
WTS
H
S
H^2
S^2
S'
S
S
R_{14}. RHS :=
H
S
H^2
S
S
S'
S'
S
S
$\cdot w^2$
R'_{23}
H
S
H^2
S
S
S'
S'^2
S^2
$\cdot w^2 \cdot w^2$
C_1, C_7
R_{15}
H
S
H^2
S^2
S'^3
S^2
$\cdot w$
C_8^6, C_8
Def1, C_2, C_3
H
S
H^2
S^2
S^2
$\cdot w$
R'_{23}
C_1
H
S
S'
H^2
S^2
S^2
$\cdot w \cdot w^2$
$= 1$
R'_{23}
R_{15}, R'_{24}
H
S
S'
H^2
S^2
S'
S^2
$\cdot w^2$
R'_{25}
R_{15}, C_7
H
S
H^2
S^2
S'
S
S'^2
S^2
$\cdot w^2 \cdot w$
C_1, C_2, C_3
C_5, C_6, C_7
H
S
H^2
S^2

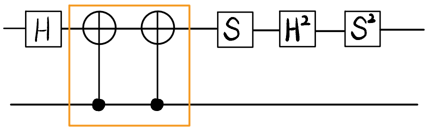
C_8 : (1)  = 

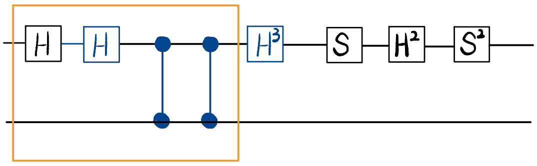
$C2 : H^4 = I$

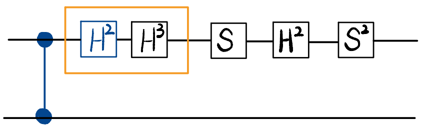
R5 : 

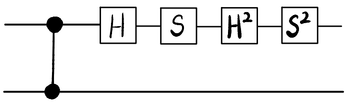
R11 : 

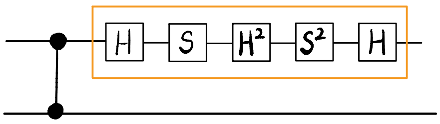
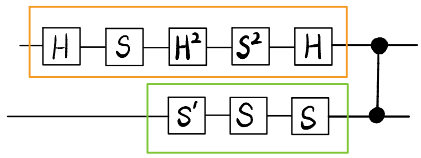
Proof cont. R_{14}:  $\underline{\underline{\text{WTS}}}$ 

R_{14} . RHS := 

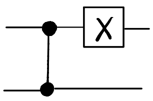
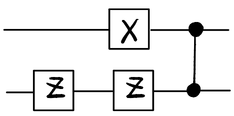
$\underline{\underline{\text{Def2}}}$ C_2 

$\underline{\underline{C_8}}$ 

$\underline{\underline{C_2}}$  = R_{14}. LHS.

Hence, R_{14}:  = 

$R5 \ ||| \ R11$

R'_{14}:  = 

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