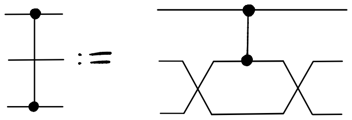
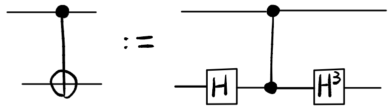
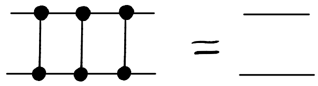
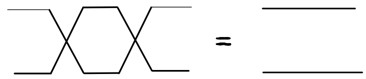
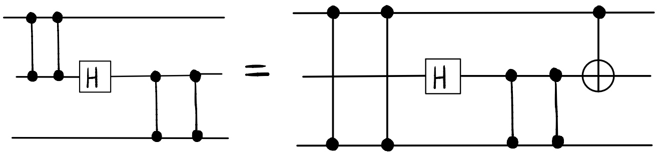
Def 5: 

Def 4: 

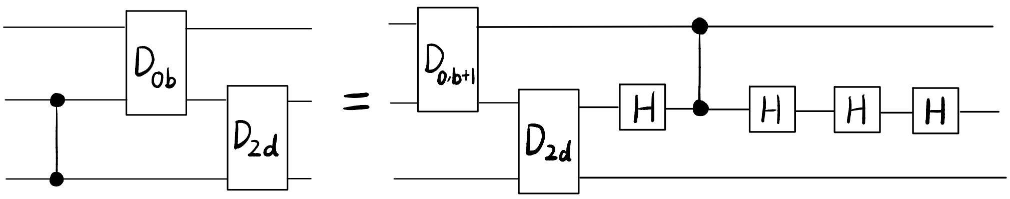
C_6: 

$C_2 : H^4 = I$ $C_3 : S^3 = I$

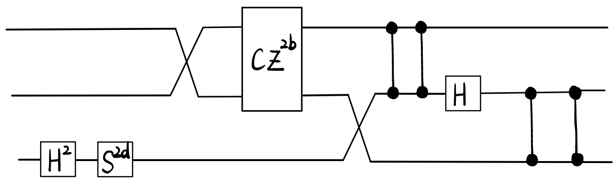
R_{16}: 

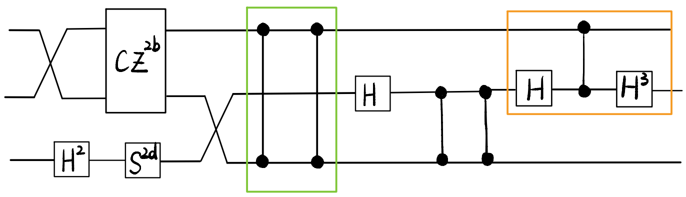
R_{40}: 

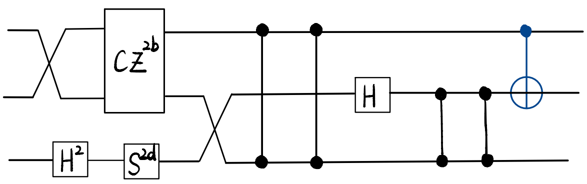
Lem3

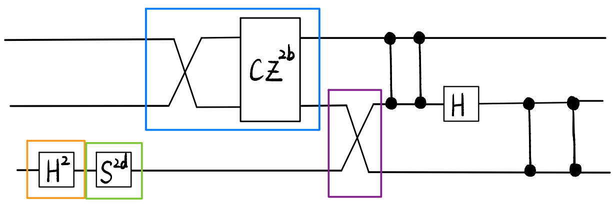
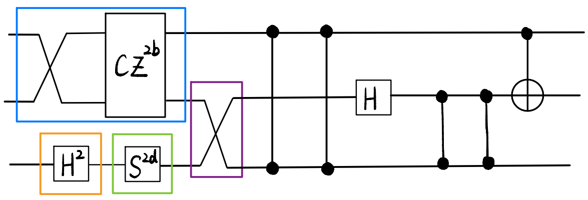
10. (c)  , $b, d \in \mathbb{Z}_3$.

Proof cont :

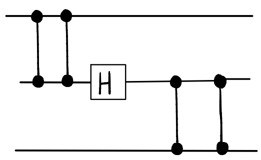
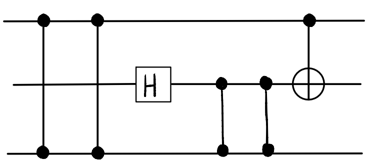
10.(c).LHS = 

10.(c).RHS = 

$\overset{\text{Def 4}}{\underset{\text{Def 5}}{=}}$ 

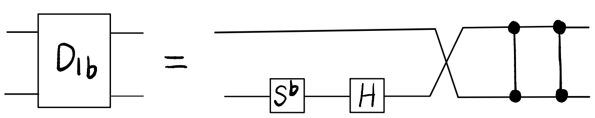
Hence  $\overset{\text{WTS}}{=}$ 

$C_3, C_6 \;|||\; R_{16}$

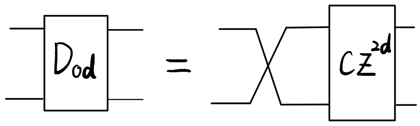
 $\overset{\text{WTS}}{=}$ 

By R_{40}, this completes the proof.

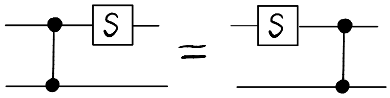
∎

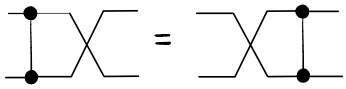


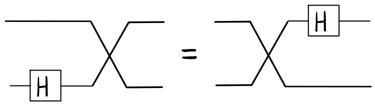
$b \in \mathbb{Z}_3$

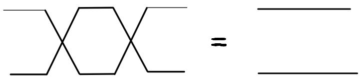


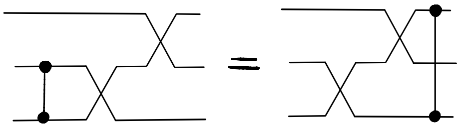
$d \in \mathbb{Z}_3$

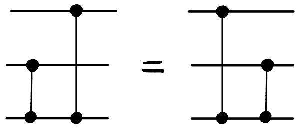
C_7: 

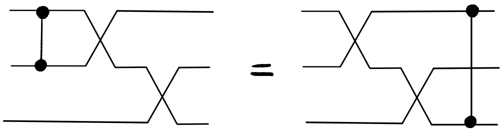
R_{17}: 

R_{19}: 

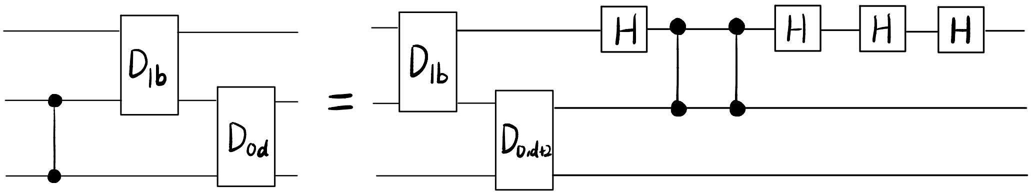
R_{16}: 

C_{15}^4: 

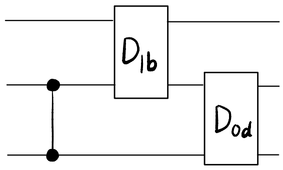
C_{13}^2: 

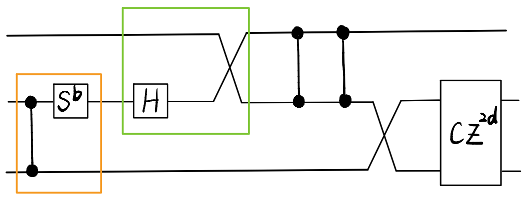
C_{15}^2: 

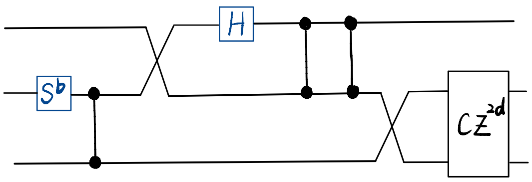
Lem 4 Def 2, Def 5, C_{2-3}, C_{6-7}, R_{16}, R_{17}, R_{19}, R_{41}, C_{13} & C_{15} imply

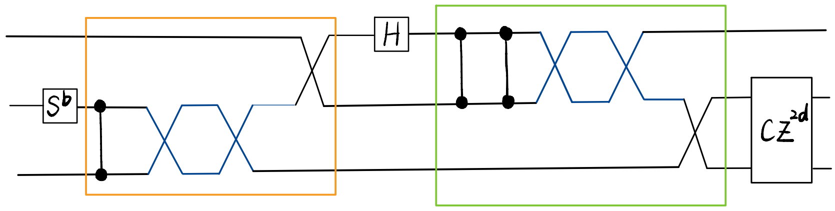
10. (d) 

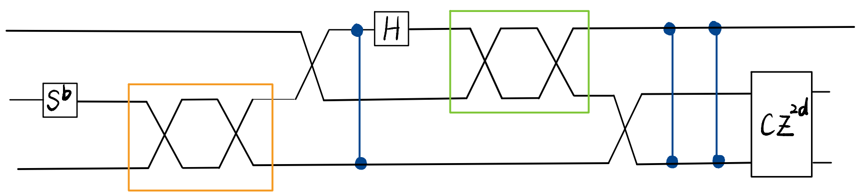
, $b, d \in \mathbb{Z}_3$.

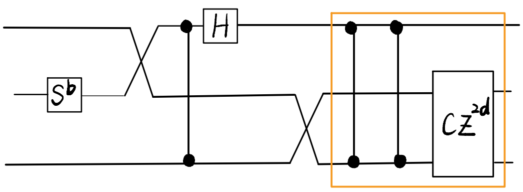
Proof: 10.(d).LHS := 

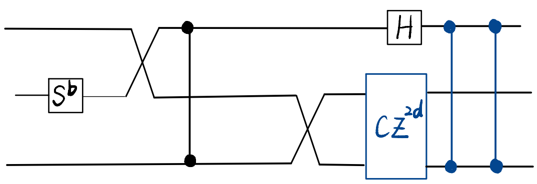
$\overset{\text{def}}{=}$ 

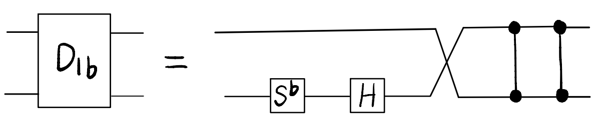
$\overset{C_7}{\underset{R_{19}}{=}}$ 

$\overset{R_{16}}{=}$ 

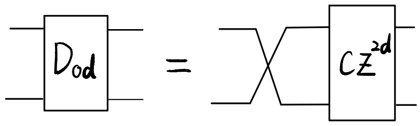
$\overset{C_{15}^2, C_{15}^4}{\underset{R_{17}}{=}}$ 

$\overset{R_{16}}{=}$ 

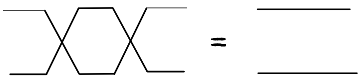
$\overset{C_{13}^2}{=}$ 

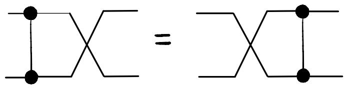


$b \in \mathbb{Z}_3$

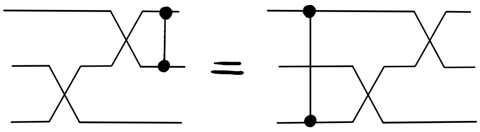


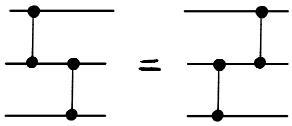
$d \in \mathbb{Z}_3$

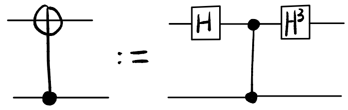
R_{16} : 

R_{17}: 

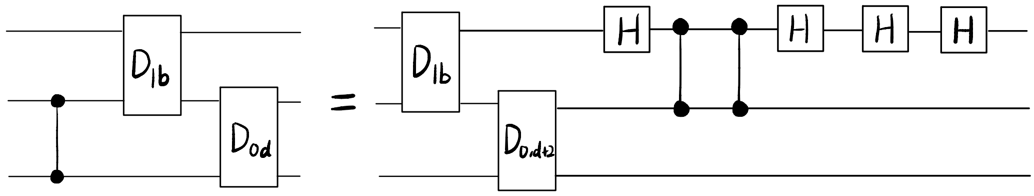
$C2 : H^4 = I$

C^5_{15} : 

C_{13} : 

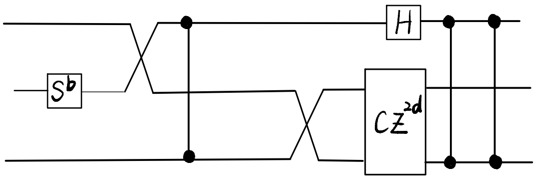
Def 2 : 

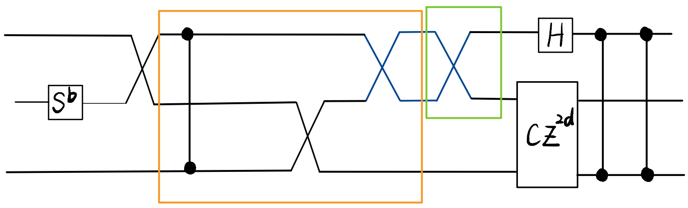
Lem 4

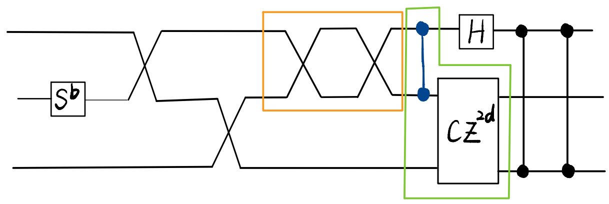
10. (d) 

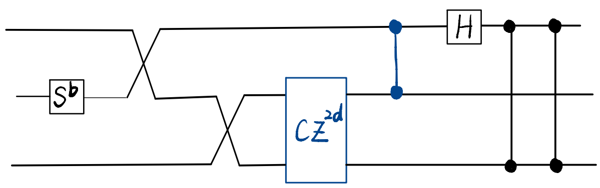
, $b, d \in \mathbb{Z}_3$.

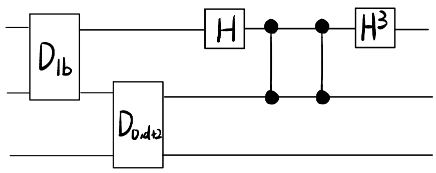
Proof cont :

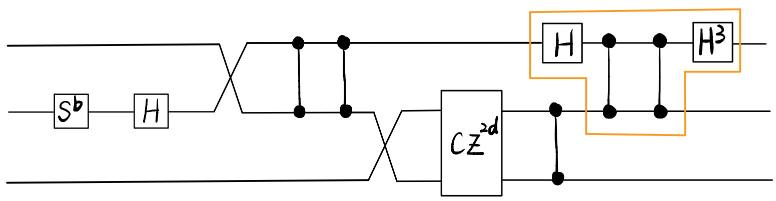
10. (d). LHS = 

$\underline{\underline{R_{16}}}$ 

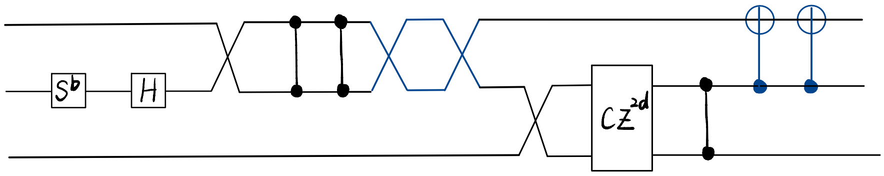
$\frac{C^5_{15}}{R_{17}}$ 

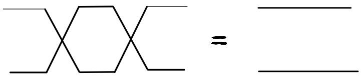
$\underline{\underline{C_{13}}}$ 

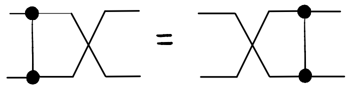
10. (d). RHS := 

$\underline{\underline{\text{def}}}$ 

$2(d+2) = 2d+1$

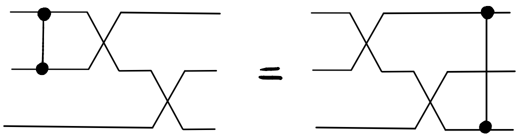
$\frac{\text{Def 2}}{R_{16}}$ 

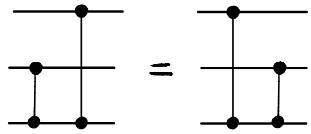
R_{16} : 

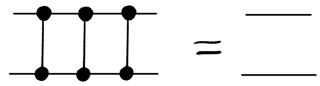
R_{17} : 

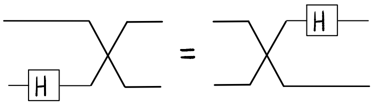
$C2 : H^4 = I$

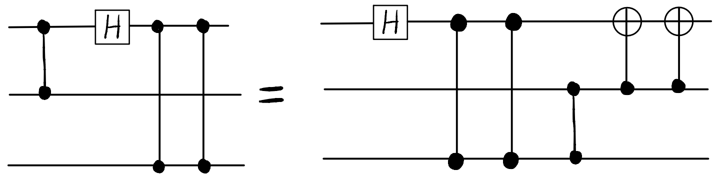
$C3 : S^3 = I$

C_{15}^2 : 

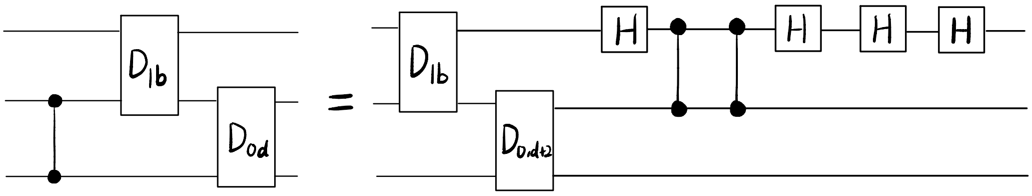
C_{13}^2 : 

C_6 : 

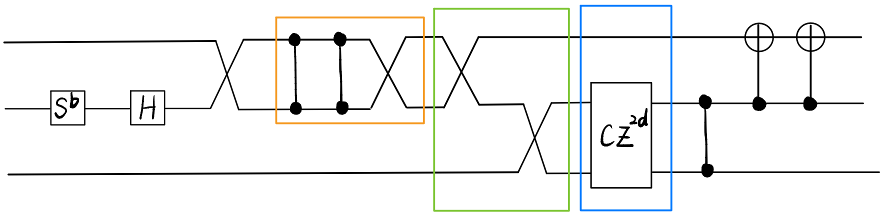
R_{19} : 

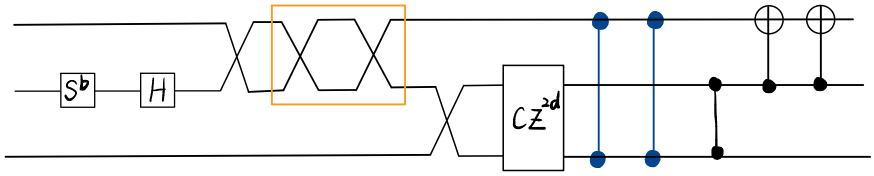
R_{41} : 

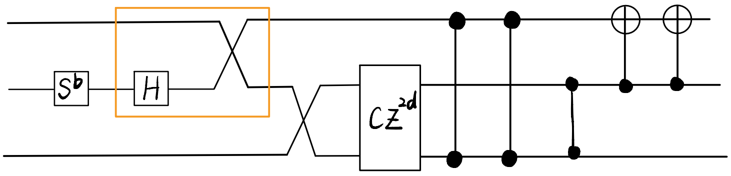
Lem 4

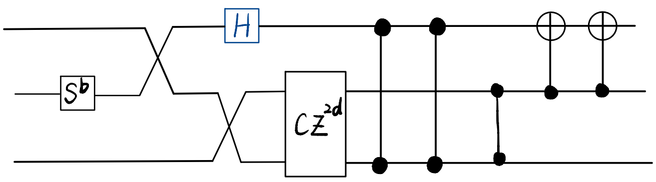
10. (d)  , $b, d \in \mathbb{Z}_3$.

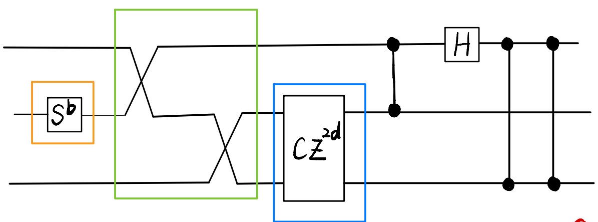
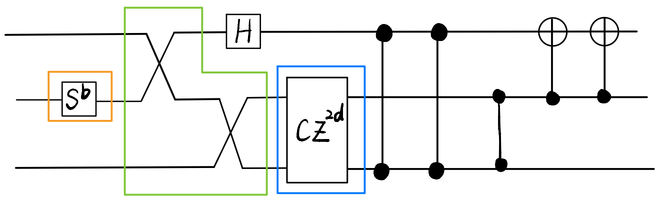
Proof cont :

10. (d). RHS = 

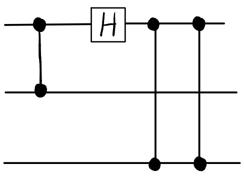
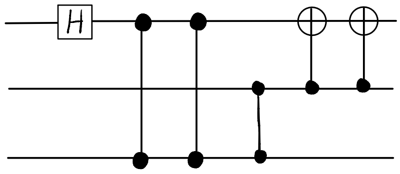
$\underset{C_{13}^2, R_{17}}{\overset{C_{15}^2}{=}}$ 

$\overset{R_{16}}{=}$ 

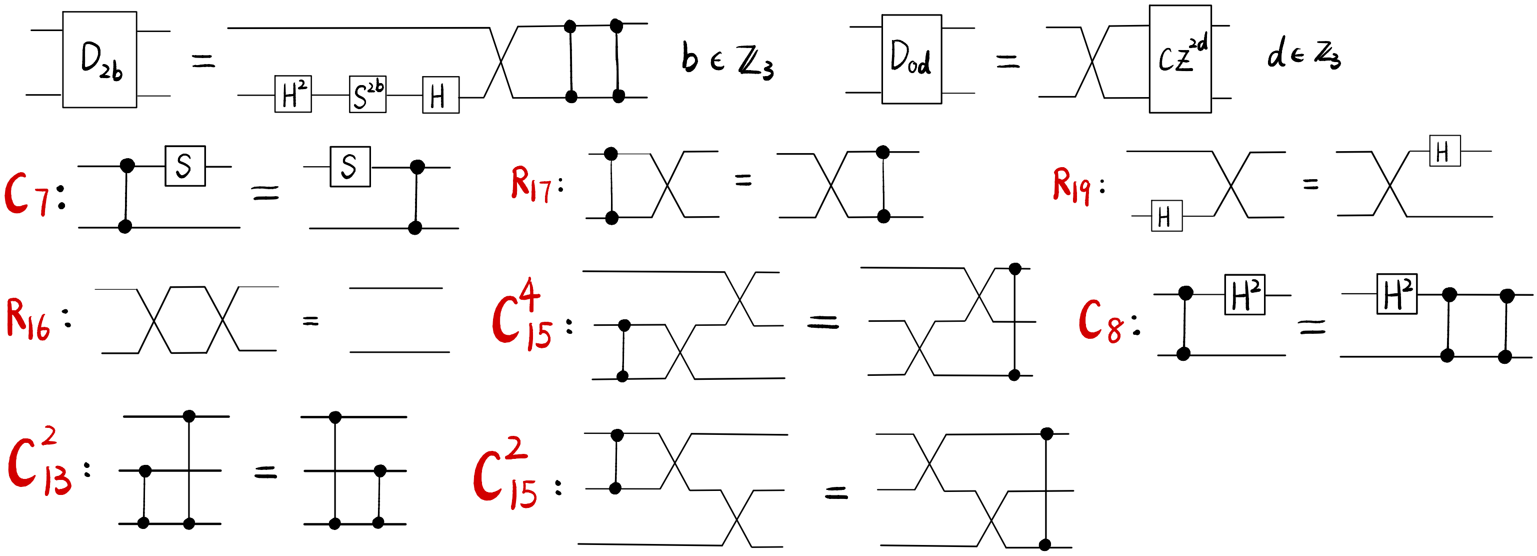
$\overset{R_{19}}{=}$ 

Hence  $\overset{\text{WTS}}{=}$ 

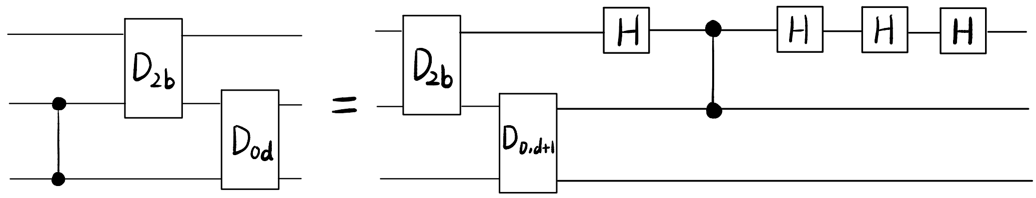
$C_3, C_6 \,|||\, R_{16}$

 $\overset{\text{WTS}}{=}$ 

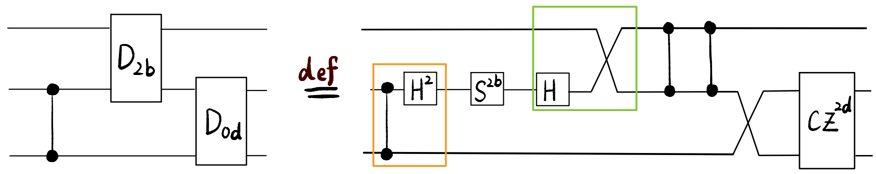
By R_{41}, this completes the proof. ∎

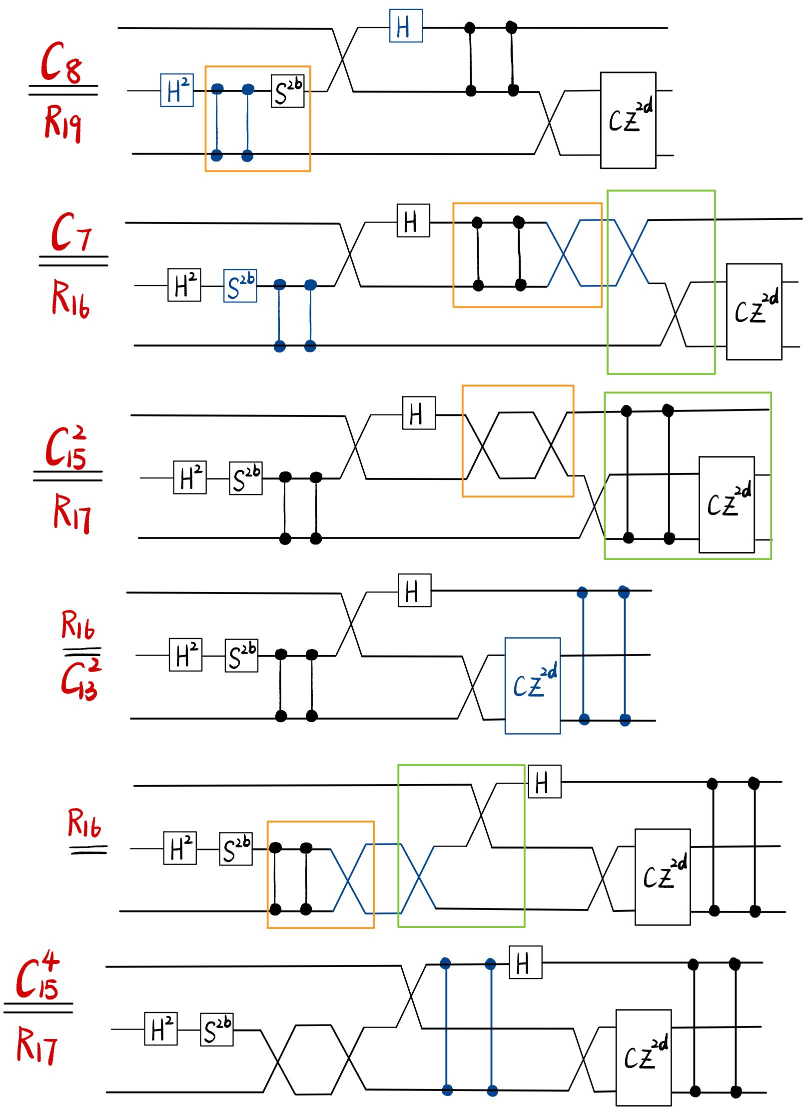


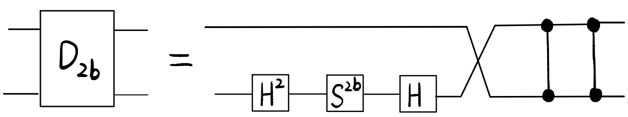
Lem 5 Def 2, Def 5, C_{2-3}, C_{6-8}, R_{16}, R_{17}, R_{19}, R_{42}, C_{13} & C_{15} imply

10. (e) 

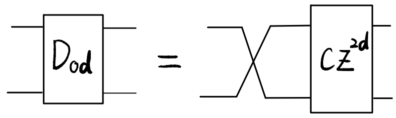
, $b, d \in \mathbb{Z}_3$.

Proof: 10.(e).LHS := 



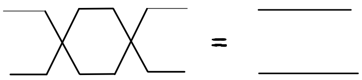


$b \in \mathbb{Z}_3$

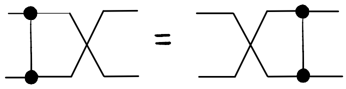


$d \in \mathbb{Z}_3$

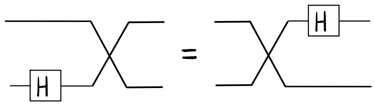
R_{16}:



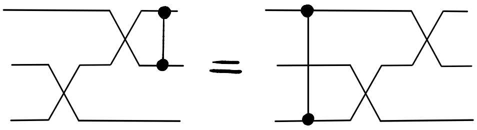
R_{17}:



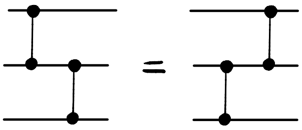
R_{19}:



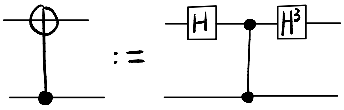
C_{15}^5:



C_{13}:

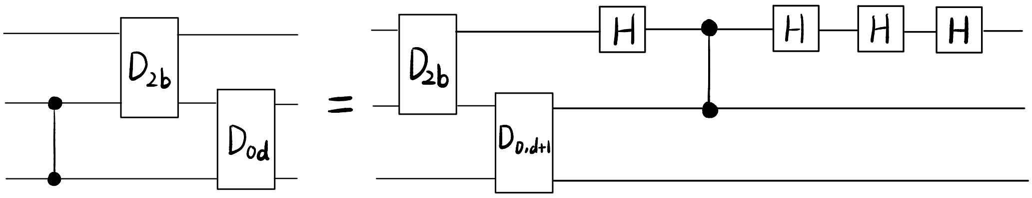


Def 2:



Lem 5

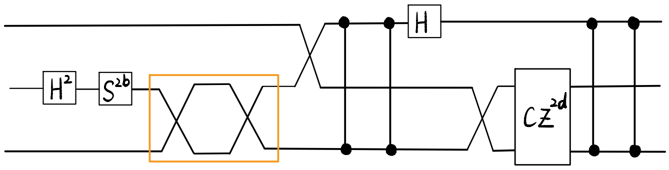
10. (e)



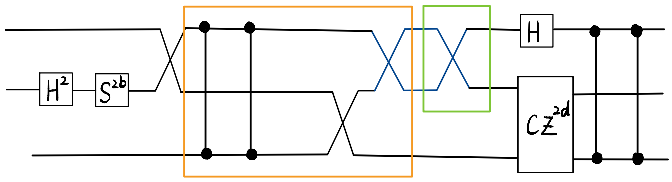
, $b, d \in \mathbb{Z}_3$.

Proof cont :

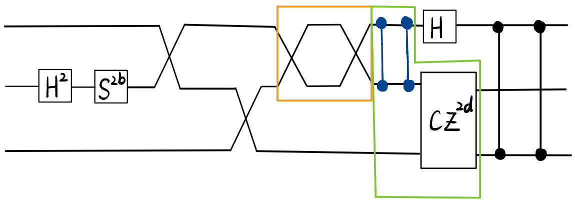
10.(e).LHS =



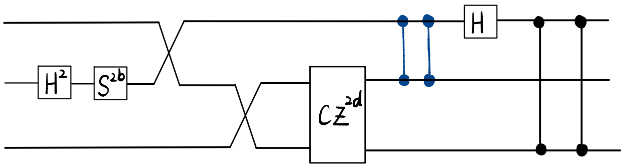
$\underline{\underline{R_{16}}}$



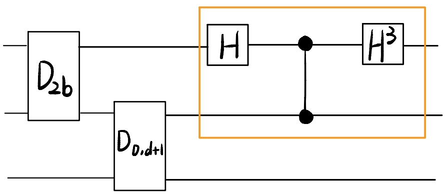
$\frac{C_{15}^5}{R_{17}}$



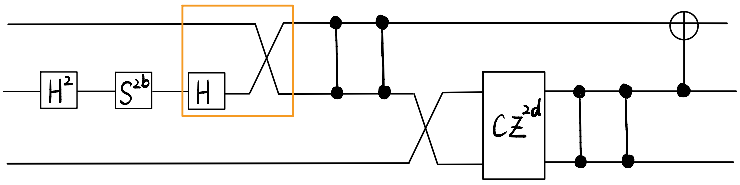
$\frac{C_{13}}{R_{16}}$



10.(e).RHS :=

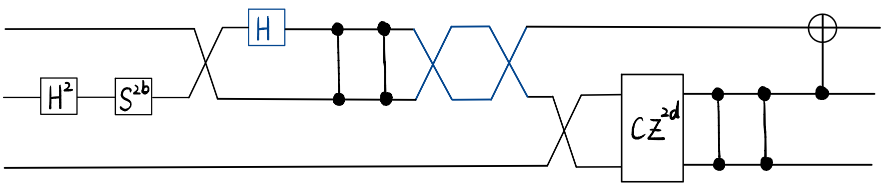


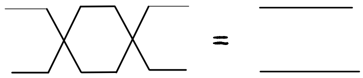
$\frac{\text{Def 2}}{\text{def}}$

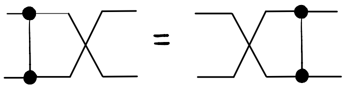


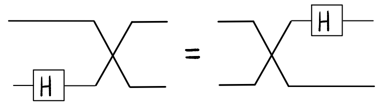
$2(d+1) = 2d+2$

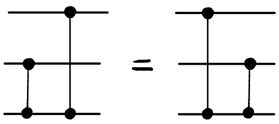
$\frac{R_{16}}{R_{19}}$

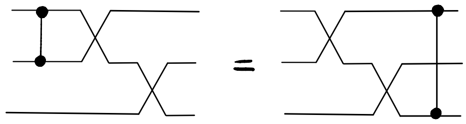


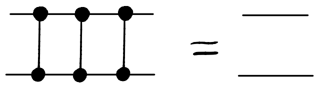
R_{16}: 

R_{17}: 

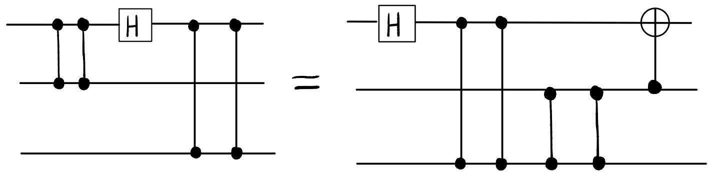
R_{19}: 

C_{13}^{2}: 

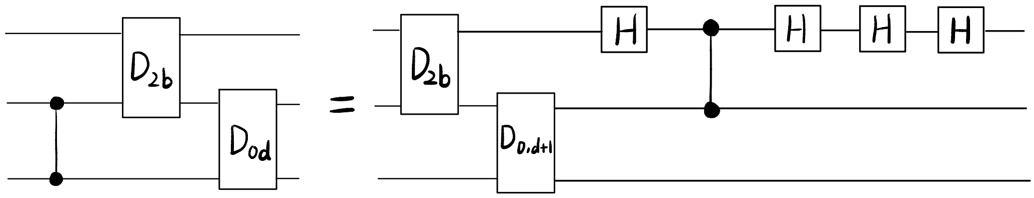
C_{15}^{2}: 

C_6: 

$C2: H^4 = I$ $\quad$ $C3: S^3 = I$

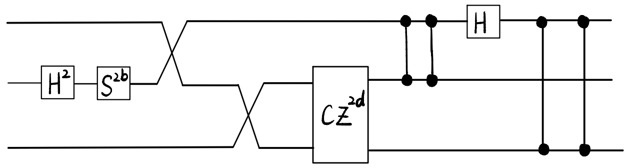
R_{42}: 

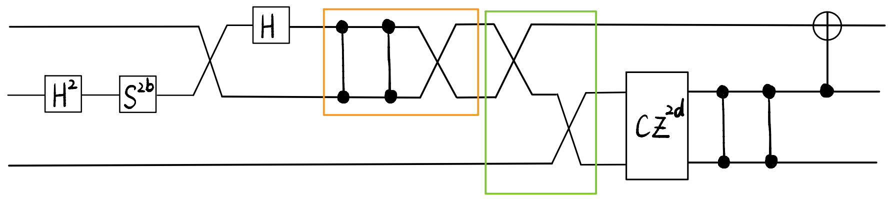
Lem 5

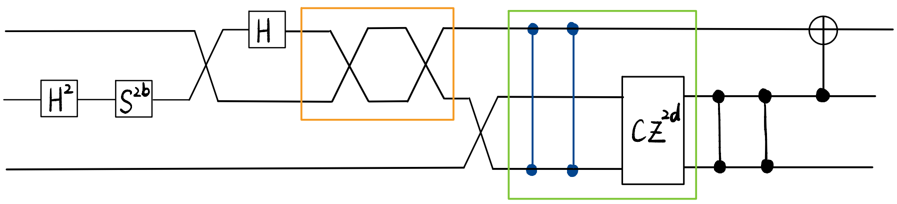
10. (e) 

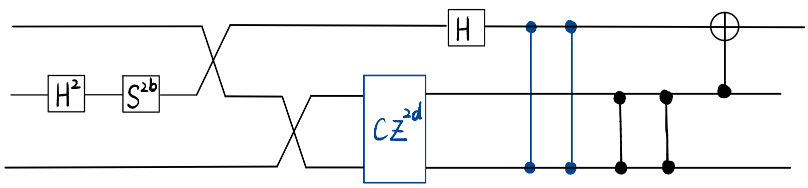
, $b, d \in \mathbb{Z}_3$.

Proof cont:

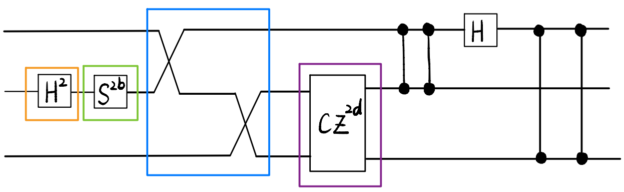
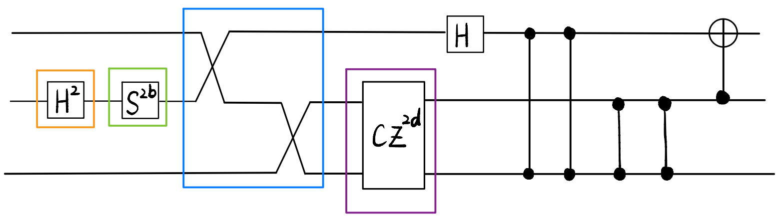
10.(e).LHS = 

10.(e).RHS = 

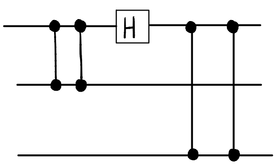
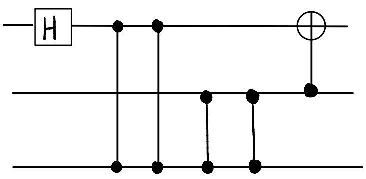
$\overset{C_{15}^{2}}{\underset{R_{17}}{=}}$ 

$\overset{C_{13}^{2}}{\underset{R_{16}}{=}}$ 

Hence

 WTS $=$ 

$C_3, C_6 \,|||\, R_{16}$

 WTS $=$ 

By R_{42}, this completes the proof. $\blacksquare$