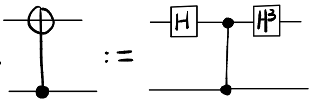
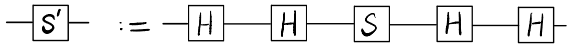
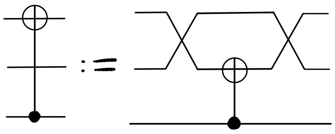
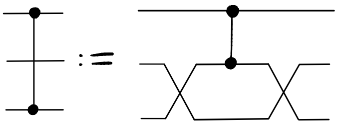
$C_0: (-1)^2 = 1$ $C1: \omega^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$

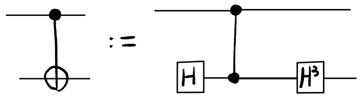
Def 2 : 

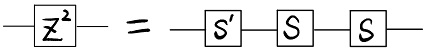
Def 1 : 

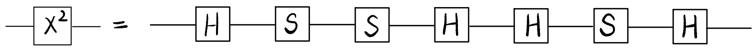
$C_5: SS' = S'S$

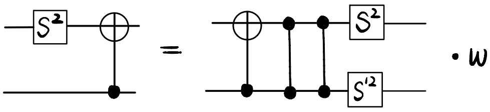
Def 7 : 

Def 5 : 

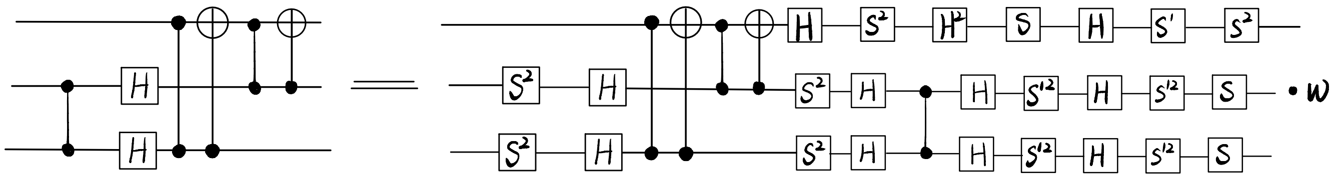
Def 4 : 

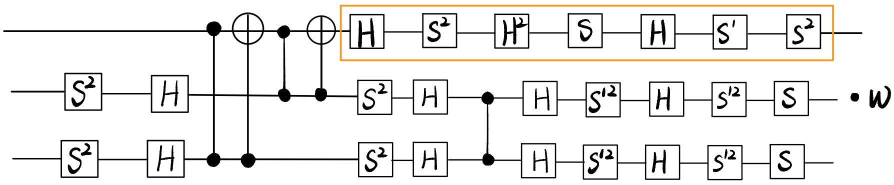
R11 : 

R7 : 

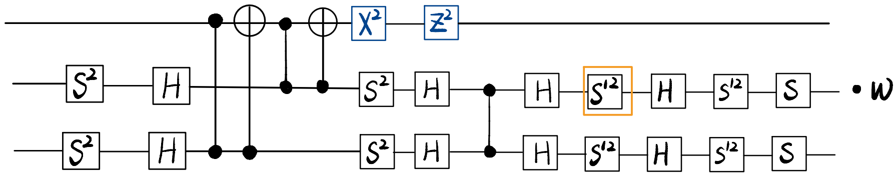
R_{23}^8: 

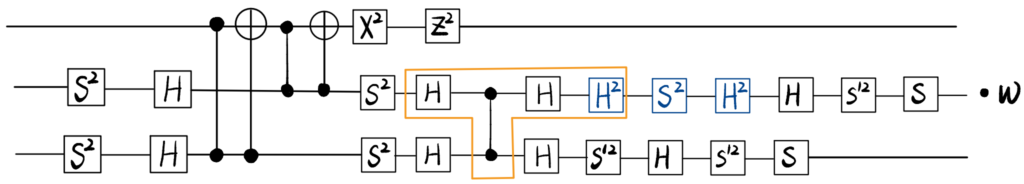
Lem D1 Def 0-2, Def 4-5, Def 7, C0-8, C13, C16, R15, R23, R24, R25 imply

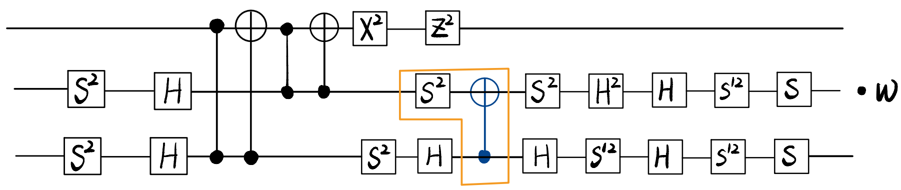
R_{59}: 

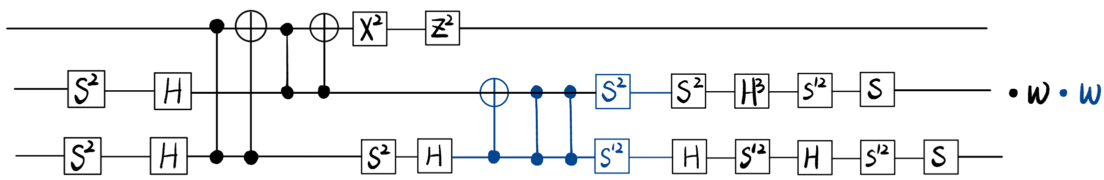
Proof: R_{59}.RHS := 

$HS^2H^2SH\,S'S^2 \overset{R_{11}}{=} HS^2H^2SH\,Z^2 \overset{R_7}{=} X^2Z^2$

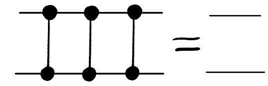
$\overset{R_1}{\underset{R_7}{=}}$ 

$\overset{\text{Def 1}}{=}$ 

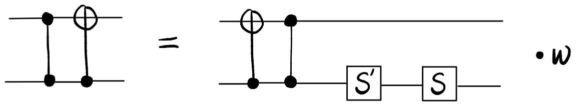
$\overset{\text{Def 2}}{=}$ 

$\overset{R_{23}^8}{=}$ 

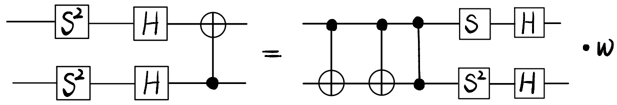
$C_0: (-1)^2 = 1$ $C_1: w^3 = 1$ $C_2: H^4 = I$ $C_3: S^3 = I$ $C_5: SS' = S'S$ C_6^*:



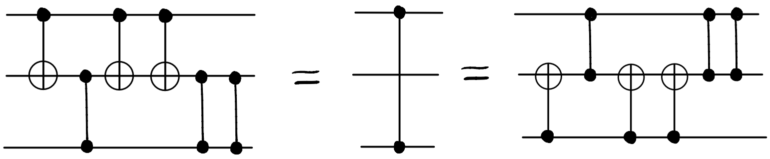
R_{25}^{1*}:



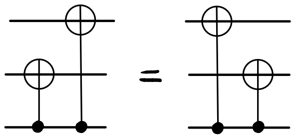
$R_{23\text{-}25}^{2}$:



C_{16}^{4}:



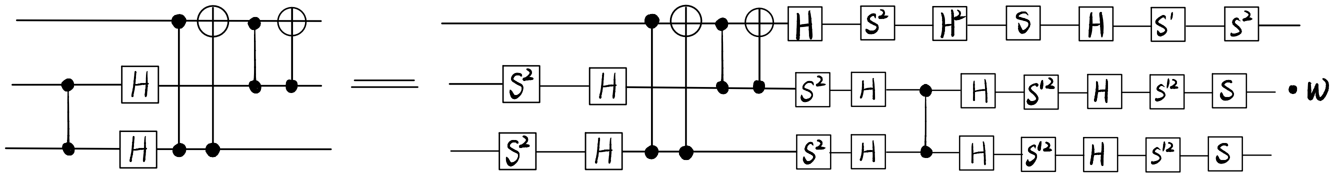
C_{13}^{11}:



C_4^3: $S' \, H^\dagger \, S' = H \, S' \, S' \, H \cdot (-w^2)$

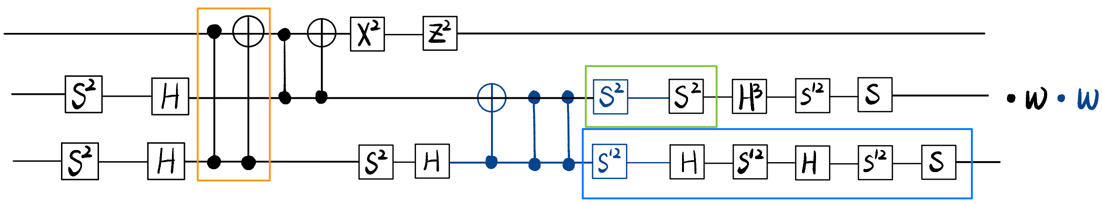
Lem D1

R_{59}:



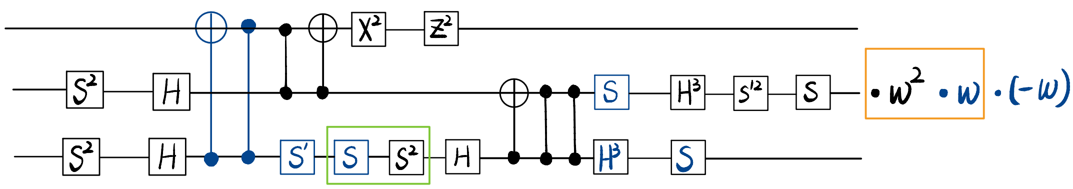
Proof cont.

$R_{59}.\mathrm{RHS} =$

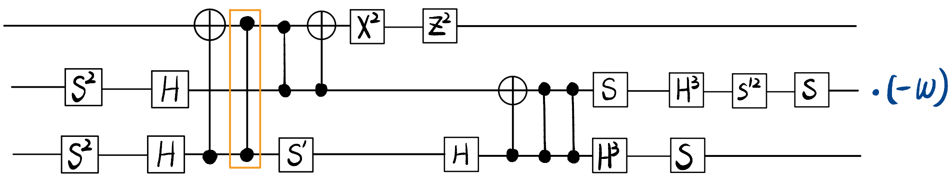


$$S'^2 \boxed{HS'^2H} \, S'^2 S \overset{C_4^3}{\underset{C_0, C_1}{=}} \boxed{S'^2 S'} \, H^\dagger \boxed{S' S'^2} \, S \cdot (-w) \overset{C_2}{\underset{C_3}{=}} H^\dagger S \cdot (-w)$$

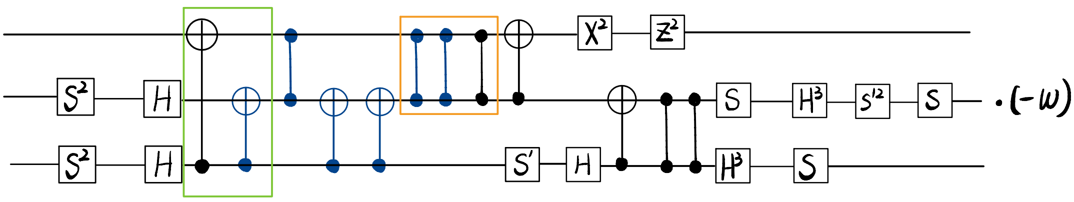
$\overset{R_{25}^{1*}, C_0, C_1}{\underset{C_2, C_3, C_4^3}{=}}$



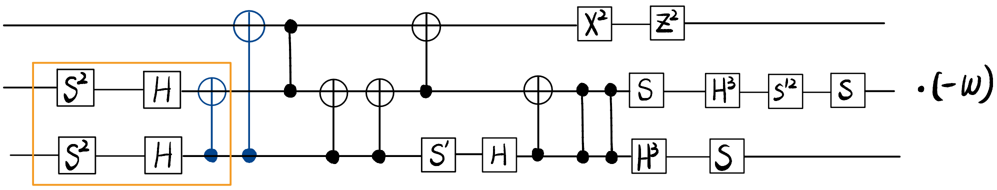
$\overset{C_1}{\underset{C_3}{=}}$



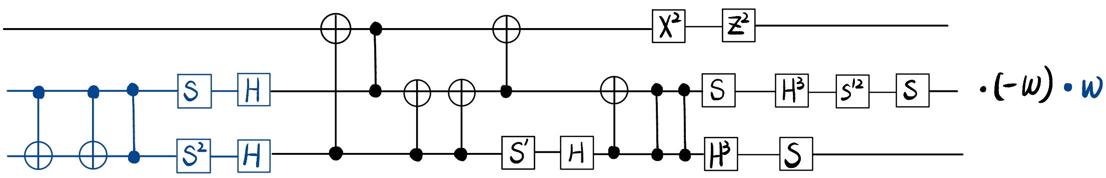
$\overset{C_{16}^4}{=}$



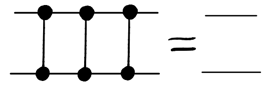
$\overset{C_6}{\underset{C_{13}^{11}}{=}}$



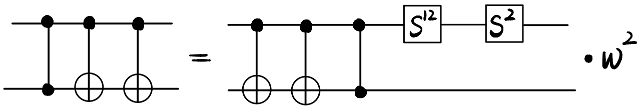
$\overset{R_{23\text{-}25}^{2}}{=}$



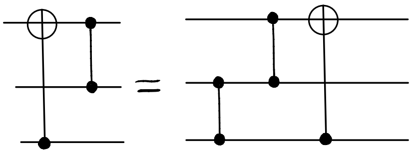
$C_0: (-1)^2 = 1$ $\quad C_1: \omega^3 = 1$ $\quad C_2: H^4 = I$ $\quad C_3: S^3 = I$ $\quad C_5: SS' = S'S$ $\quad C_6^*:$



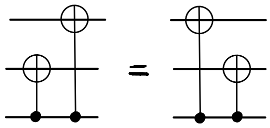
$R_{25}^{6'}:$



$C_{16}^{7}:$

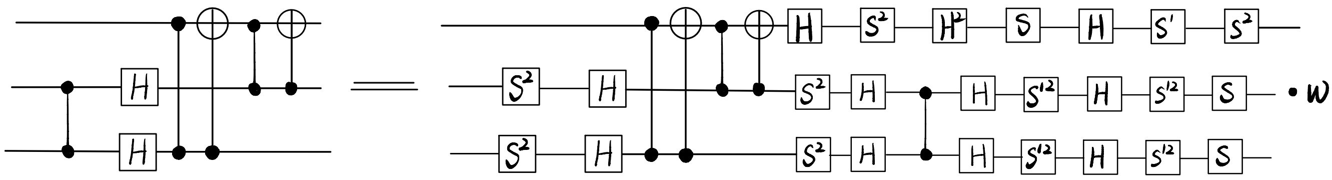


$C_{13}^{11}:$



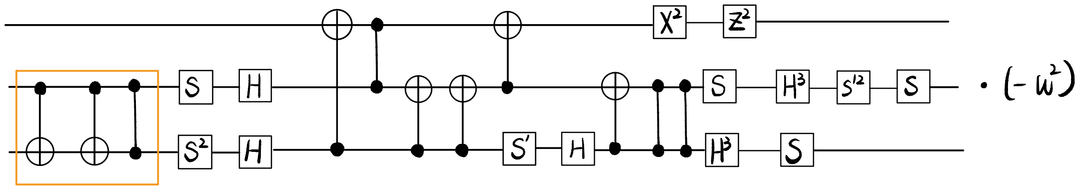
Lem D1

$R_{59}:$

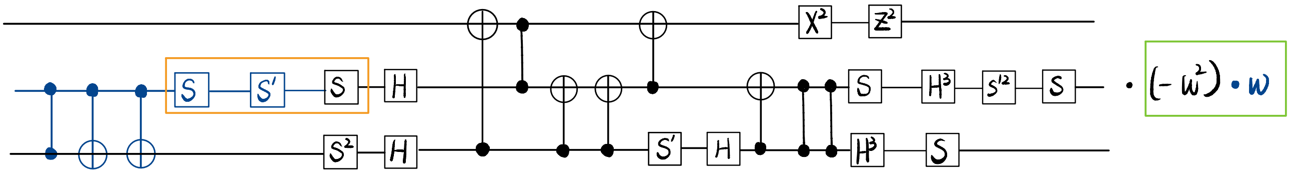


Proof cont.

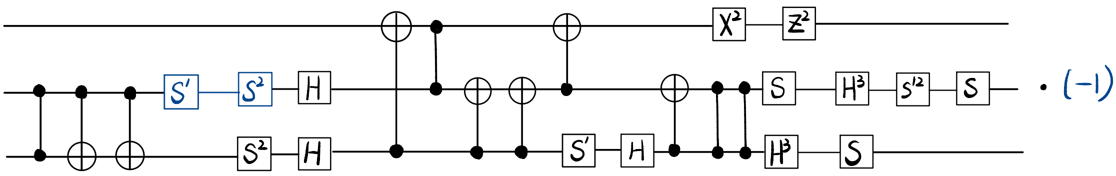
R_{59}.RHS =



$\overset{R_{25}^{6'}}{\underset{C_1, C_2, C_3}{=\!=}}$

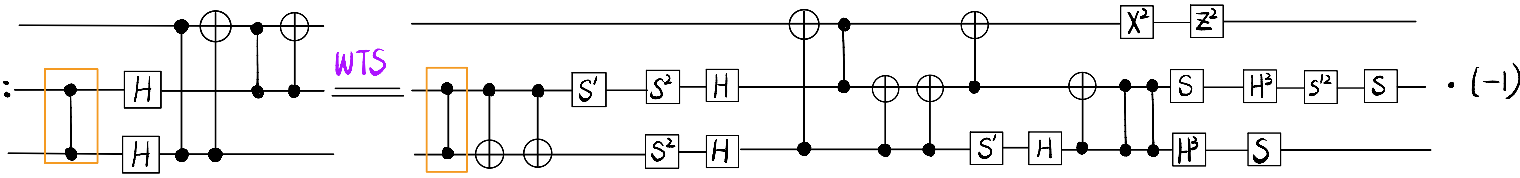


$\overset{C_5}{\underset{C_1}{=\!=}}$



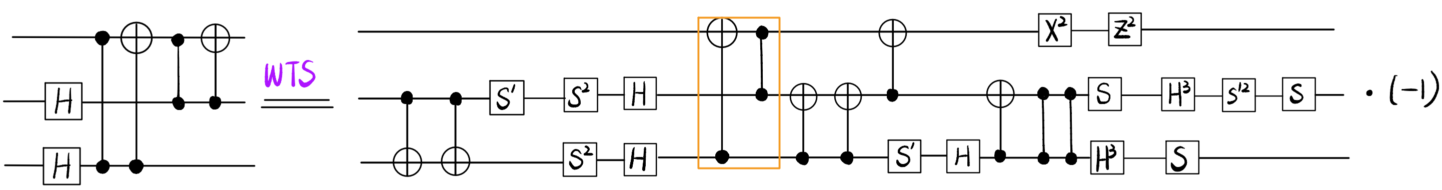
Hence

$R_{59}:$ WTS

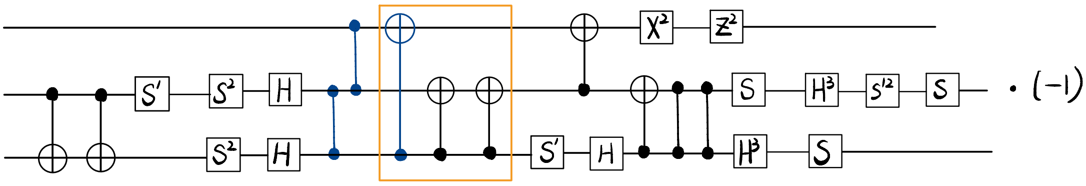


$\equiv C_6$

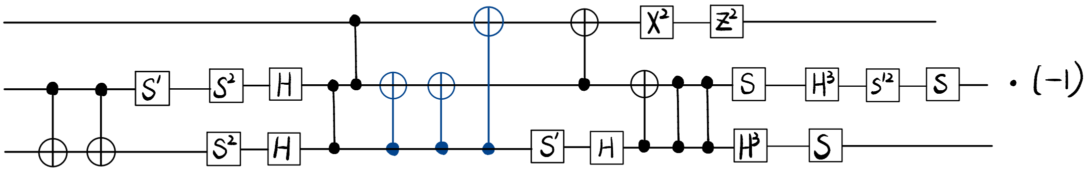
$R_{59}:$ WTS



Then R_{59}.RHS =

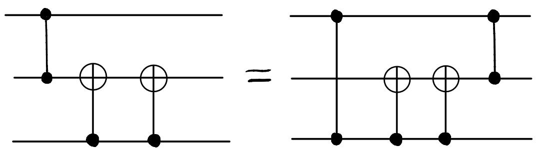


$\overset{C_{13}^{11}}{=\!=}$

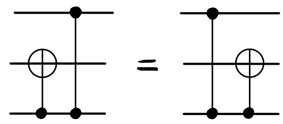


$C_0: (-1)^2 = 1$ $\quad$ $C_1: w^3 = 1$ $\quad$ $C_2: H^4 = I$ $\quad$ $C_3: S^3 = I$ $\quad$ $C_5: SS' = S'S$

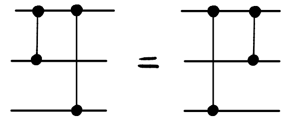
C_{16}^{27}:



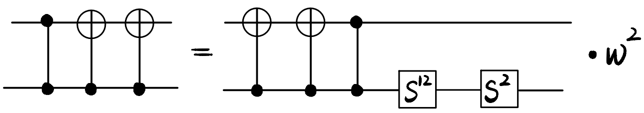
C_{13}^{7}:



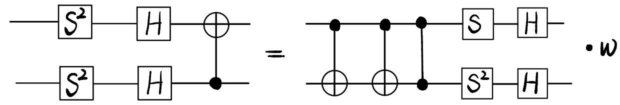
C_{13}^{1}:



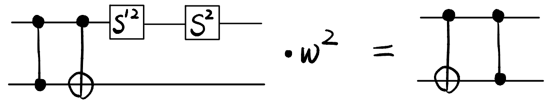
R_{25}^{6}:



R_{23-25}^{2}:

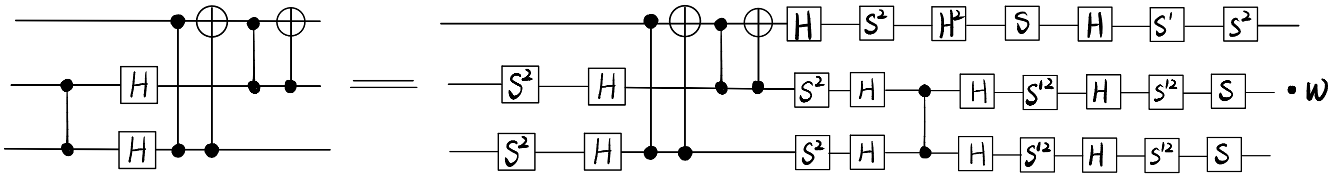


R_{25}^{4}:



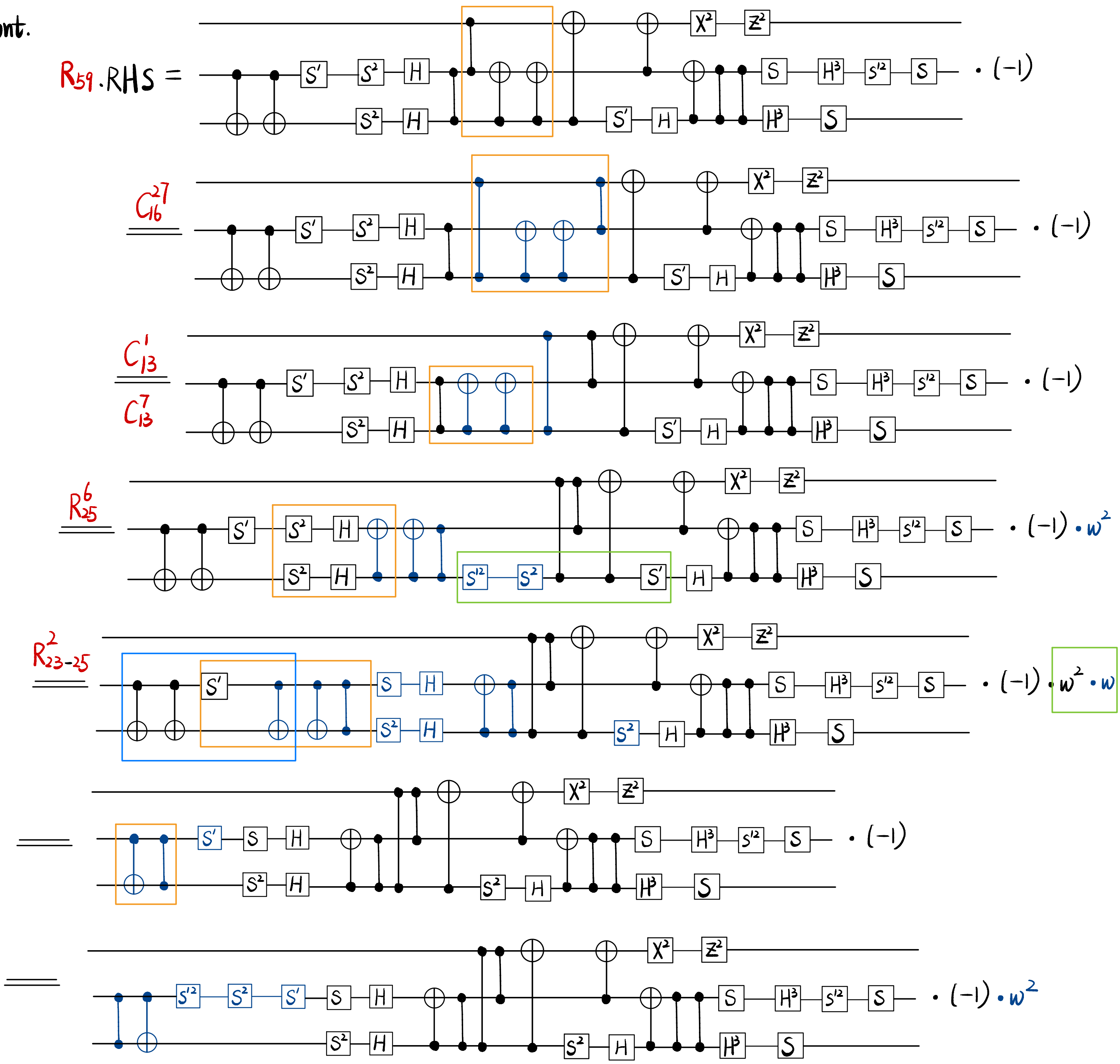
Lem D1

R_{59}:

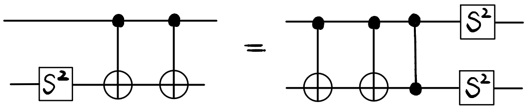
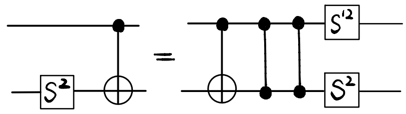


Proof cont.

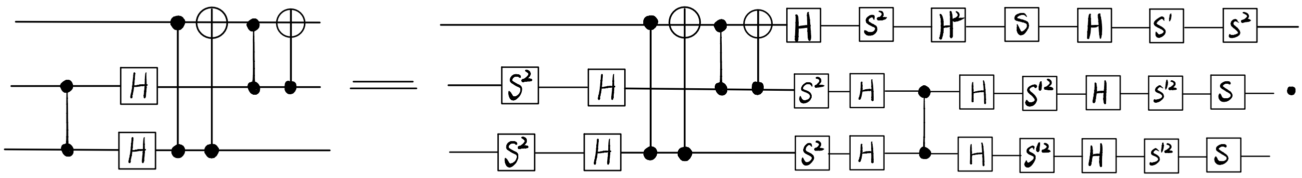
R_{59}.RHS =



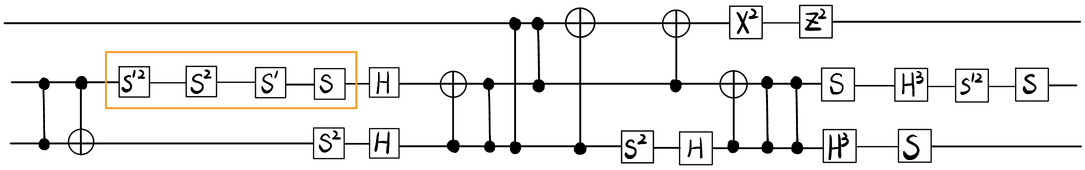
$C_0 : (-1)^2 = 1$ $C1 : w^3 = 1$ $C2 : H^4 = I$ $C3 : S^3 = I$ $C5 : SS' = S'S$

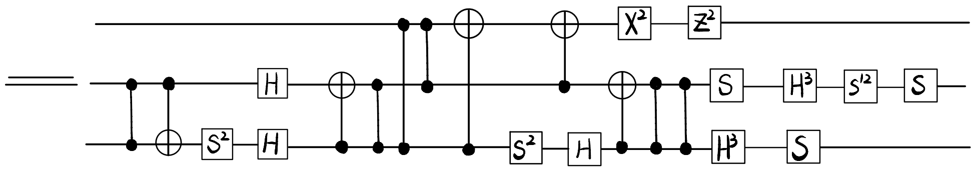
$R'_{23\text{-}25}$:  $\cdot w$ $R^{8'}_{23}$:  $\cdot w$

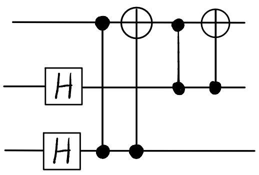
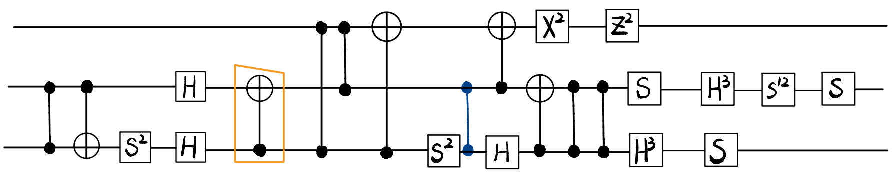
Lem D1

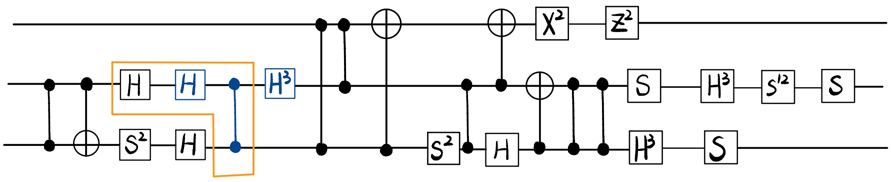
R_{59} :  $\cdot w$

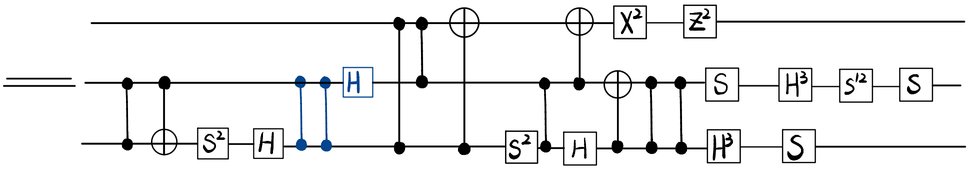
Proof cont.

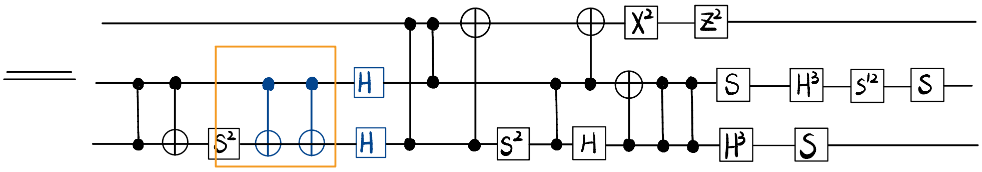
$R_{59}.\mathrm{RHS}$ =  $\cdot (-1) \cdot w^2$

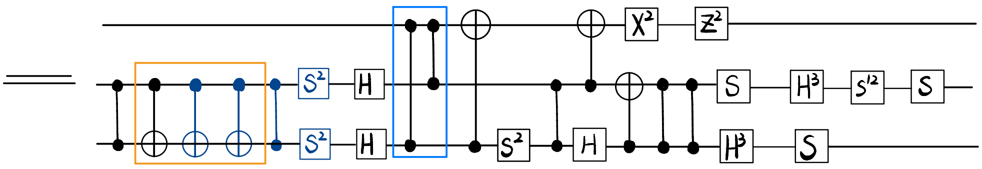
=  $\cdot (-1) \cdot w^2$

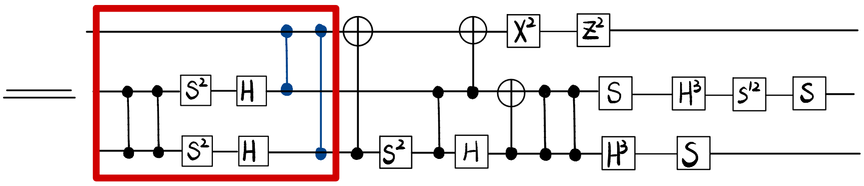
Hence R_{59} :  $\overset{\text{WTS}}{=}$  $\cdot (-1) \cdot w^2$

$R_{59}.\mathrm{RHS}$ =  $\cdot (-1) \cdot w^2$

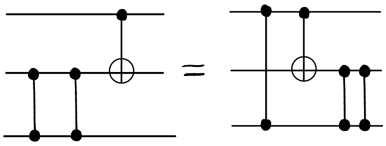
=  $\cdot (-1) \cdot w^2$

=  $\cdot (-1) \cdot w^2$

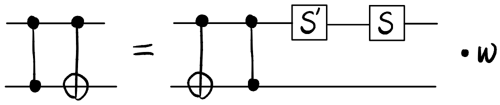
=  $\cdot (-1) \boxed{\cdot w^2 \cdot w}$

=  $\cdot (-1)$

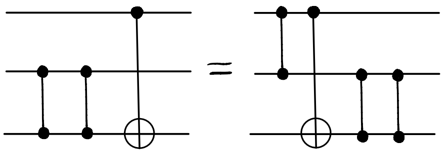
C_{16}^{27}:



R_{25}^{3*}:

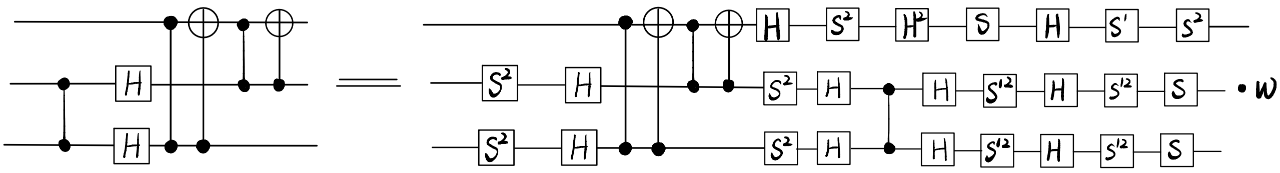


C_{16}^{28}:



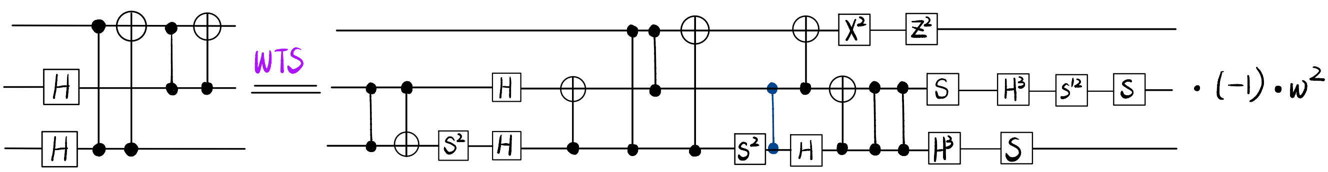
Lem D1

R_{59}:



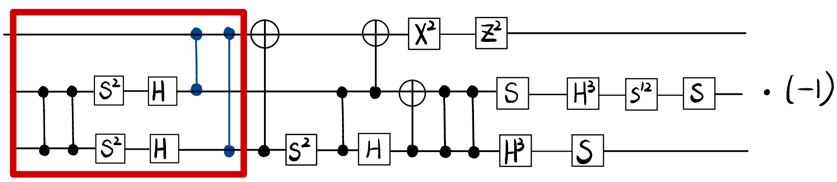
Proof cont.

R_{59}:

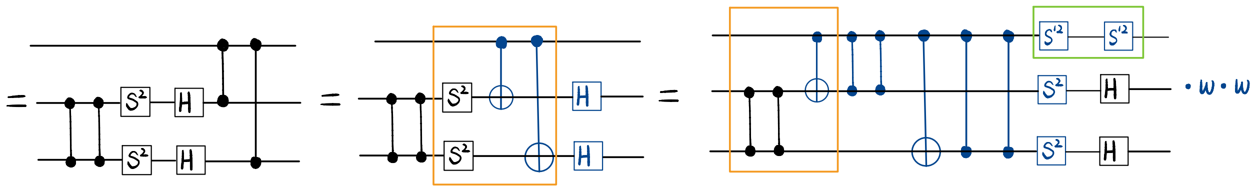


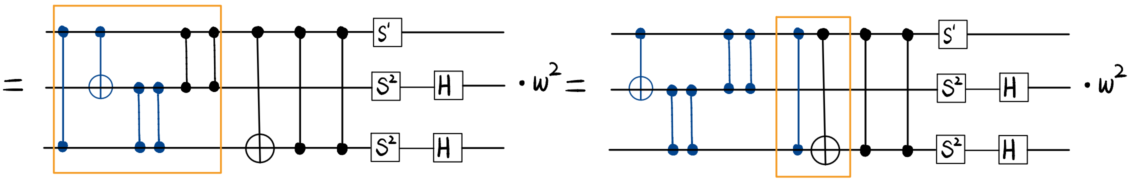
M

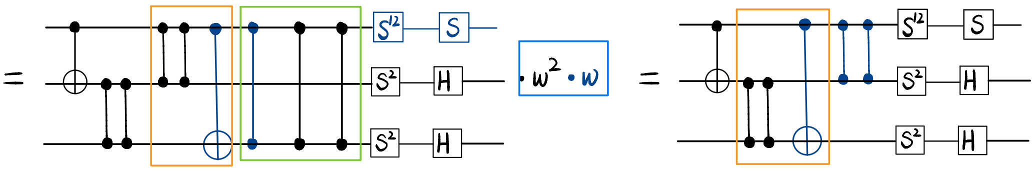
$R_{59}.RHS =$

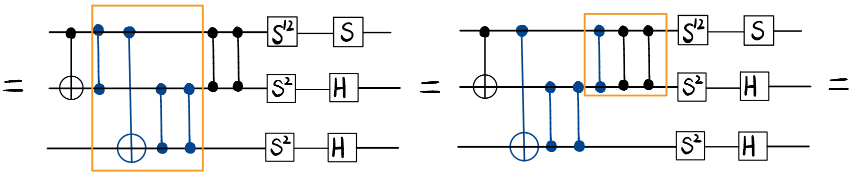


$M =$



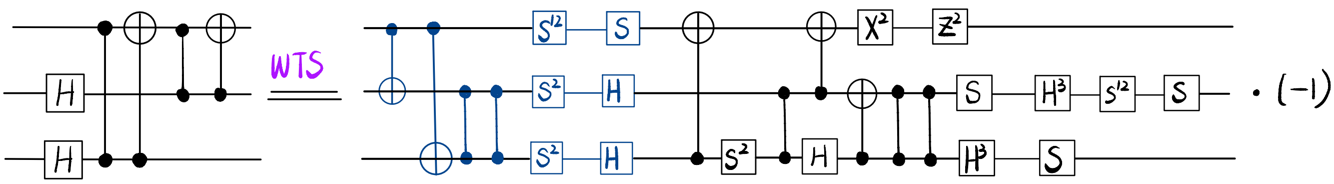




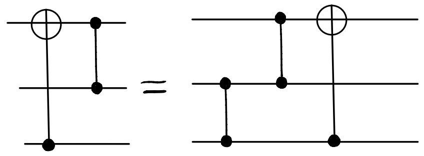


Hence

R_{59}:

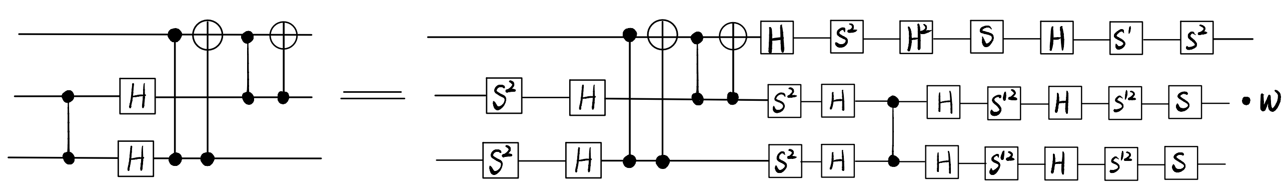


C^7_{16}:



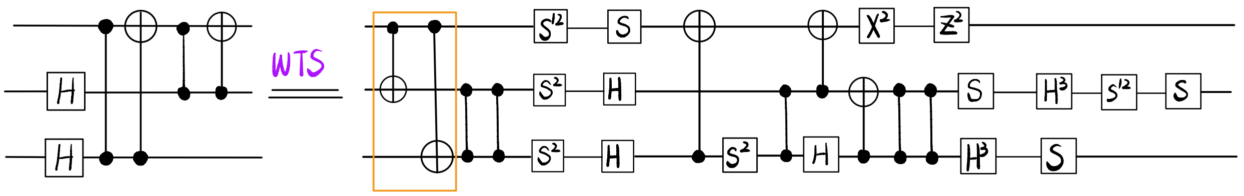
Lem D1

R_{59}:



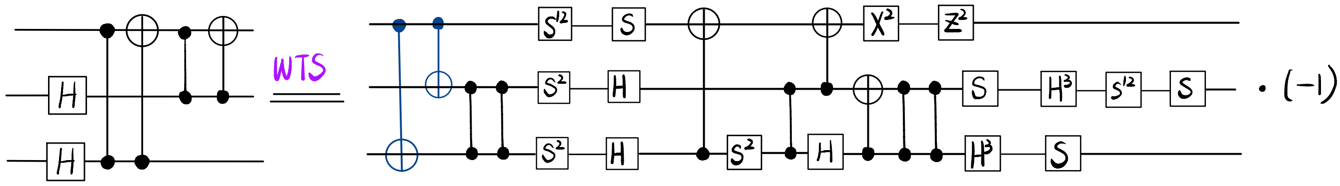
Proof cont.

R_{59}:



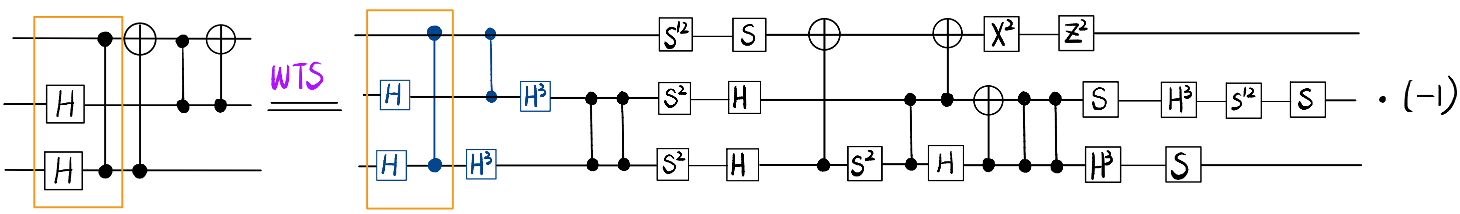
$\equiv$

R_{59}:



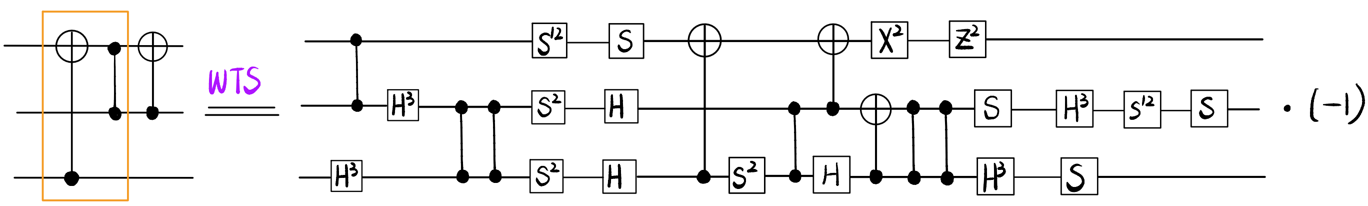
$\equiv$

R_{59}:



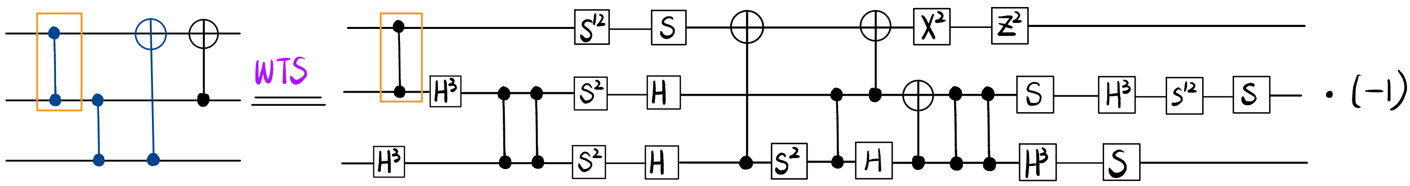
$\equiv$

R_{59}:



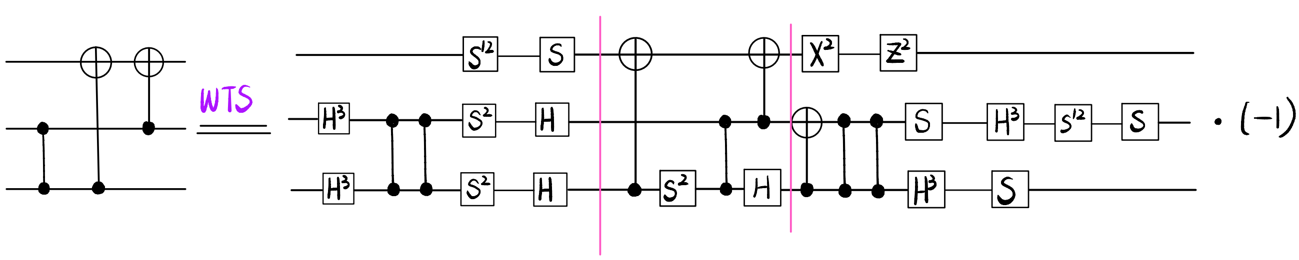
$\equiv$

R_{59}:

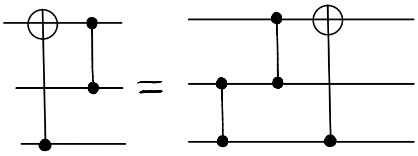


$\equiv$

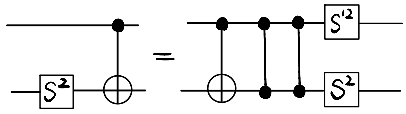
R_{59}:



C^{7}_{16} :

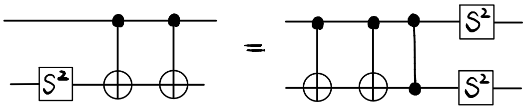


$R^{8'}_{23}$:



$\cdot\, w$

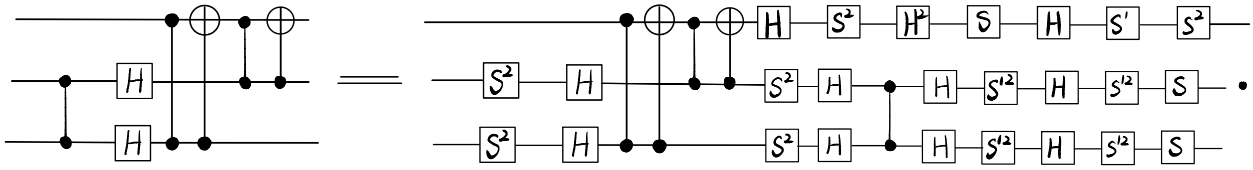
$R^{1}_{23\text{-}25}$:



$\cdot\, w$

Lem D1

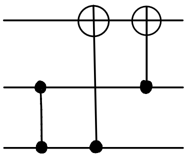
R_{59} :



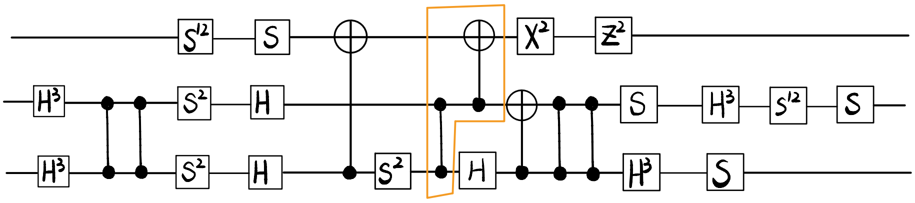
$\cdot\, w$

Proof cont.

R_{59} :



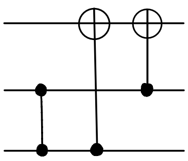
WTS



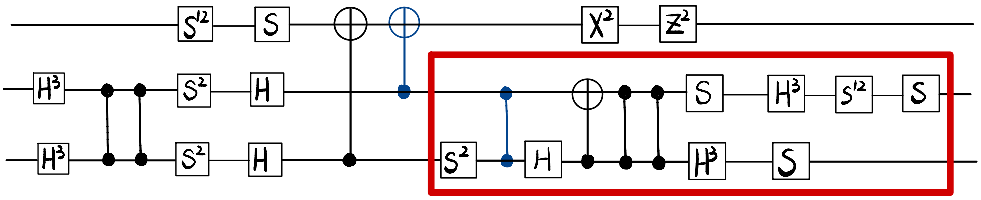
$\cdot\,(-1)$

$\equiv$

R_{59} :



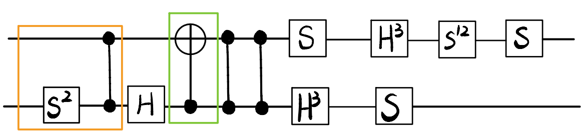
WTS



$\cdot\,(-1)$

N

N =



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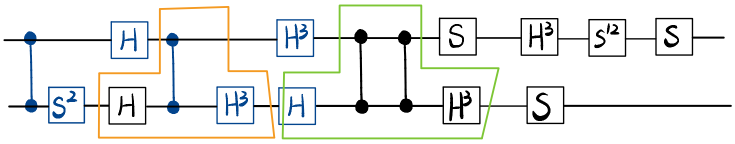

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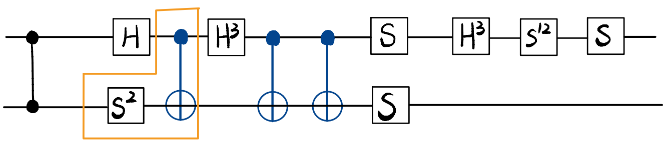

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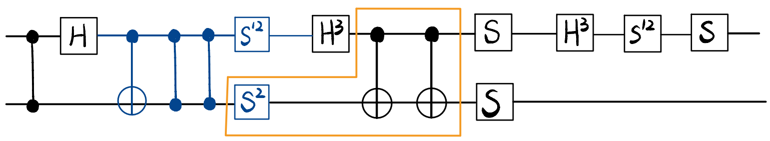
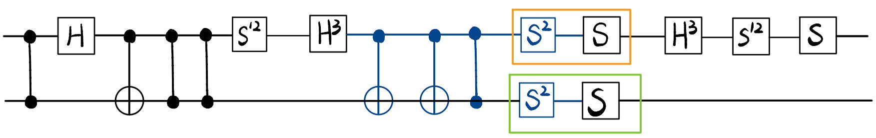


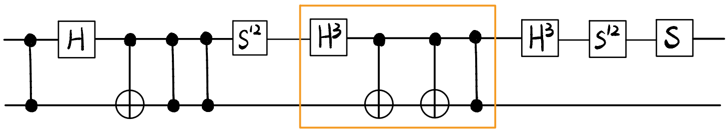
$\cdot\, w$

=



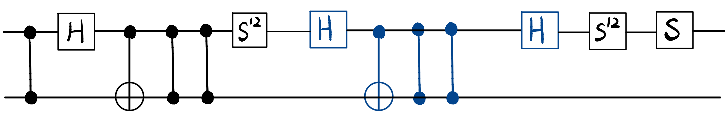
$\cdot\, w \cdot w$

=



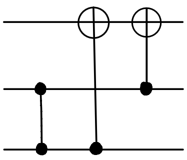
$\cdot\, w^2$

=

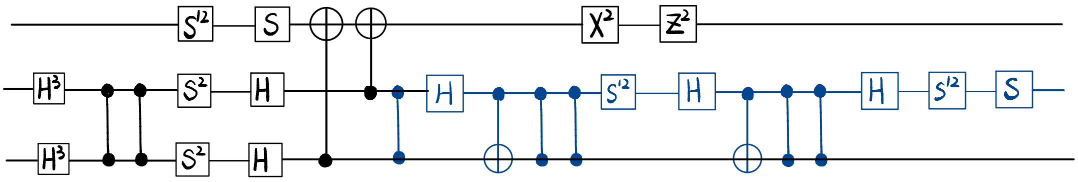


$\cdot\, w^2$

R_{59} :

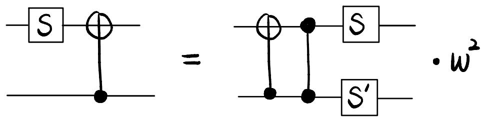


WTS

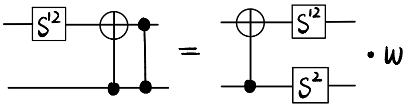


$\cdot\,(-w^2)$

R_{23}^{1*}:

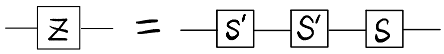


R_{23}^{14*}:

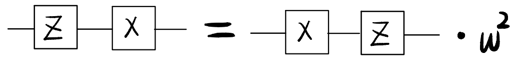


$R12: \; Z^3 = I$

$R10$:

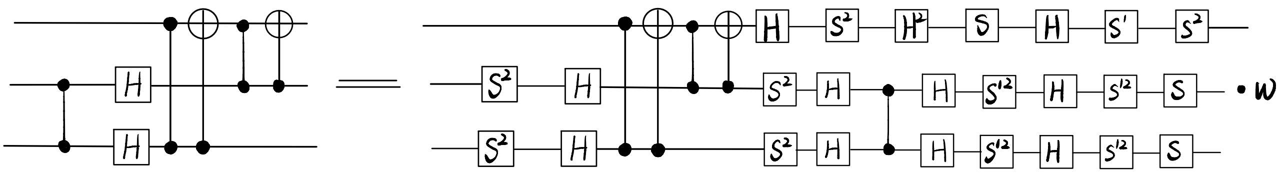


$R8: \; XZ = w^2 ZX$



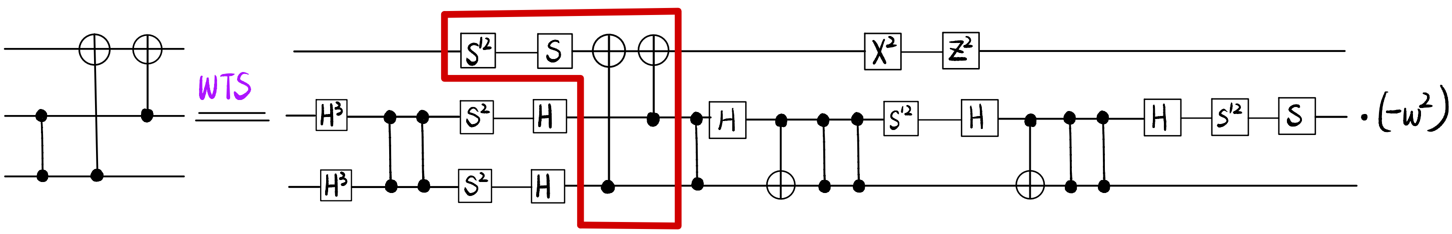
Lem D1

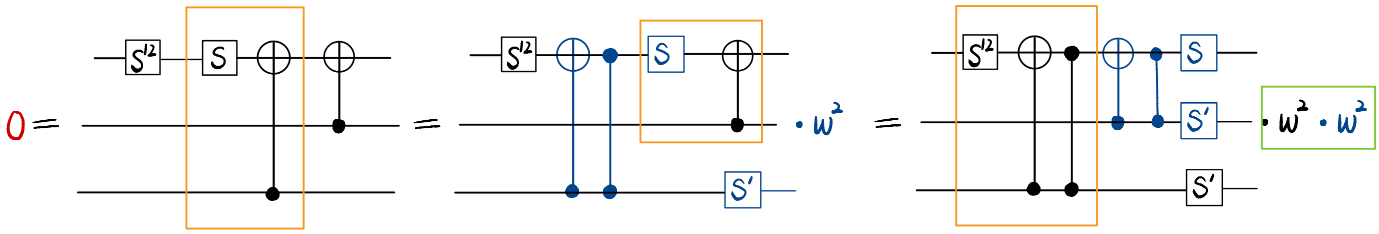
R_{59}:

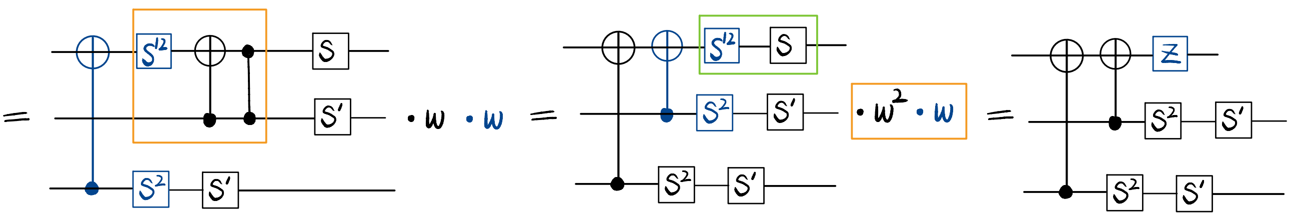


Proof cont.

R_{59}: WTS

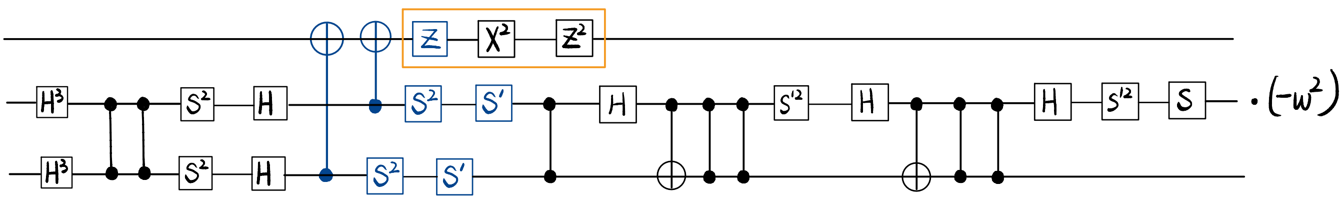






Hence

R_{59}.RHS =



$$\boxed{Z}X^2 Z^2 \overset{R_8}{=} X \boxed{ZX}Z^2 \cdot w^2 \overset{R_8}{=} X^2 \boxed{ZZ^2} \boxed{\cdot w^2 \cdot w^2} \underset{C_1}{\overset{R_{12}}{=}} X^2 \cdot w$$