

**Def 3:**  := 

$$R_{16} : \begin{array}{c} \text{Diagram of } R_{16} \text{ (a hexagon with internal lines)} \\ \text{Diagram of } R_{16} \text{ (a hexagon with internal lines)} \end{array} = \begin{array}{c} \text{Diagram of } R_{16} \text{ (a hexagon with internal lines)} \\ \text{Diagram of } R_{16} \text{ (a hexagon with internal lines)} \end{array}$$

$$R_{17}: \quad \text{Diagram} = \quad \text{Diagram}$$

$$R_{3|} : (1) \quad \text{Diagram} = \quad \text{Diagram}$$

$$(2) \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{R18: } (1) \quad \begin{array}{c} \text{S} \\ \diagup \quad \diagdown \\ \text{---} & \text{---} \end{array} = \begin{array}{c} \text{---} & \text{---} \\ \diagup \quad \diagdown \\ \text{S} \end{array}$$

$$(2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{S} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \boxed{S}$$

$$R_{22}: (1) \quad \begin{array}{c} S' \\ \diagdown \quad \diagup \\ \text{---} & \text{---} \end{array} = \begin{array}{c} \diagup \quad \diagdown \\ \text{---} & \text{---} \\ S' \end{array}$$

$$(2) \quad \begin{array}{c} \text{---} \\ | \\ S' \end{array} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \times \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ S' \end{array} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

## Lem L

$$R_{23}: \quad \text{Diagram} = \text{Diagram} \cdot w^2 \equiv R_{23}^1: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

$$\equiv R_{23}^2: \quad \text{Diagram} = \text{Diagram} \cdot w^2 \equiv R_{23}^3: \quad \text{Diagram} \cdot w = \text{Diagram}$$

implies

$$R_{23}^4: \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} = \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \bullet \\ | \\ \text{---} \end{array} S' \quad \cdot w^2 \quad \equiv$$

$$R_{23}^5 : \begin{array}{c} \bullet \\ \text{---} \\ \circ \oplus \text{---} \\ \text{---} \end{array} S = \begin{array}{c} \bullet \quad \bullet \quad \bullet \\ \text{---} \quad \text{---} \quad \text{---} \\ \circ \oplus \text{---} \\ \text{---} \end{array} S' \cdot w^2 \equiv R_{23}^6 : \begin{array}{c} \bullet \quad \bullet \quad \bullet \\ \text{---} \quad \text{---} \quad \text{---} \\ \circ \oplus \text{---} \\ \text{---} \end{array} S \cdot w = \begin{array}{c} \bullet \quad \bullet \\ \text{---} \quad \text{---} \\ \circ \oplus \text{---} \\ \text{---} \end{array}$$

$$\equiv \textcolor{red}{R_{23}^7}: \begin{array}{c} \bullet \quad \bullet \quad \bullet \\ \oplus \quad \oplus \quad S \\ \end{array} \quad \begin{array}{cc} S' & S' \end{array} \cdot w = \begin{array}{c} \bullet \\ \oplus \end{array} = \begin{array}{c} \oplus \quad \oplus \quad S \quad \oplus \quad S \quad S \\ \bullet \quad \bullet \quad \bullet \\ S' \quad S' \end{array} \cdot w$$

### Proof cont.

$$R_{23}^4 \cdot LHS := \text{Diagram A} \quad \underline{\underline{R_{16}}} \quad \text{Diagram B} \quad \frac{\underline{\underline{R_{18}}}}{\underline{\underline{R_{31}}}} \quad \text{Diagram C} \quad \underline{\underline{R_{23}}}$$

$$\cdot w^2 \frac{R_{17}, R_{18}}{R_{22}, R_{31}} \quad \text{Diagram: } \begin{array}{c} \text{A circuit diagram showing two parallel branches. The left branch contains a circle with a plus sign and a square labeled 'S'. The right branch contains a square labeled 'S' and a circle with a minus sign. The two branches are connected in parallel.} \end{array}$$

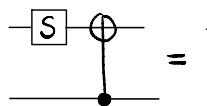
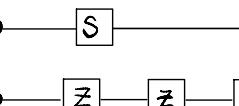
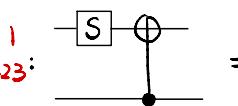
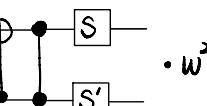
$$R_{23}^5 \cdot LHS := \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \oplus \boxed{S} \quad \underline{\underline{R_{16}}} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \quad \frac{\underline{\underline{R_{18}}}}{\underline{\underline{R_{31}}}}$$

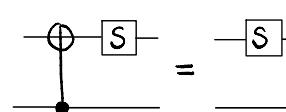
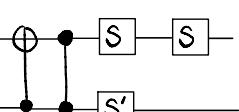
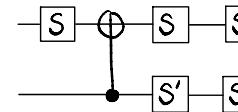
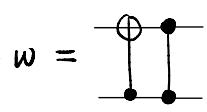
$$\begin{array}{c} \text{Diagram 1: } \\ \text{Left: } \text{Circuit diagram showing a sequence of operations: } S, S, \oplus, S, S, \text{ followed by } S' \text{ at the bottom. The first three operations are highlighted with an orange box.} \\ \text{Right: } \text{Equation } \cdot w^2 \frac{R_{17}, R_{18}}{R_{22}, R_{31}} \end{array}$$

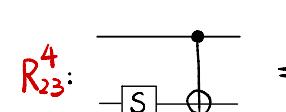
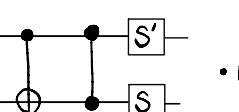
R<sub>16</sub>  •  $w^2 =: R_{23}^5 \cdot \text{RHS}$

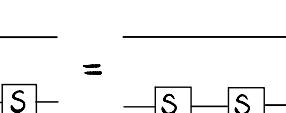
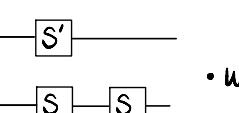
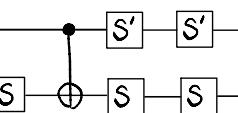
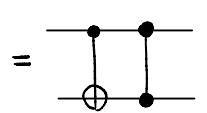
$$C_6^1: \quad \text{Diagram} = \text{Diagram}$$

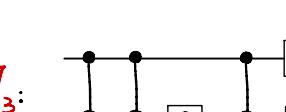
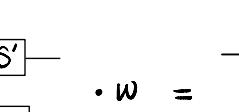
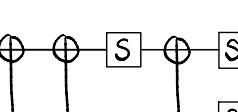
$$C_6^2: \quad \text{Diagram} = \text{Diagram}$$

Lem L  $R_{23}^1:$    $=$    $\cdot w^2 \equiv R_{23}^1:$    $=$    $\cdot w^2$

$\equiv R_{23}^2:$    $=$    $\cdot w^2 \equiv R_{23}^3:$    $=$    $\cdot w$

implies  $R_{23}^4:$    $=$    $\cdot w^2 \equiv$

$R_{23}^5:$    $=$    $\cdot w^2 \equiv R_{23}^6:$    $=$    $\cdot w$

$\equiv R_{23}^7:$    $=$    $\cdot w =$    $\cdot w$

Proof cont.

$$R_{23}^6 \cdot \text{LHS} := \text{Diagram} \cdot w \stackrel{R_{16}}{=} \text{Diagram} \cdot w$$

$$\begin{array}{c} R_{17}, R_{18} \\ \hline R_{22}, R_{31} \end{array} \quad \text{Diagram} \cdot w \stackrel{R_{23}^3}{=} \text{Diagram} \stackrel{R_{17} \equiv R_{31}}{=} \text{Diagram}$$

$$\stackrel{R_{16}}{=} \text{Diagram} =: R_{23}^6 \cdot \text{RHS}$$

$$R_{23}^7 \cdot \text{MID} := \text{Diagram} \stackrel{C_6^2}{=} \text{Diagram} \stackrel{R_{23}^3}{=} \text{Diagram} \cdot w =: R_{23}^7 \cdot \text{RHS}$$

$$R_{23}^7 \cdot \text{MID} := \text{Diagram} \stackrel{C_6^1}{=} \text{Diagram} \stackrel{R_{23}^6}{=} \text{Diagram} \cdot w =: R_{23}^7 \cdot \text{LHS}$$

□

$$C_1: w^3 = I$$

$$C_2: H^4 = I$$

$$C_3: S^3 = I$$

$$R_{11}: \boxed{z^2} = \boxed{s'} \boxed{s} \boxed{s}$$

$$\text{Def 2: } \begin{array}{c} \oplus \\ \bullet \end{array} := \begin{array}{c} H \quad H \quad H \quad H \\ \bullet \quad | \quad | \quad | \end{array}$$

$$\begin{array}{c} \bullet \\ \oplus \end{array} := \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \quad \begin{array}{c} \bullet \\ \oplus \end{array} := \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \quad \begin{array}{c} \bullet \\ \oplus \end{array} := \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array}$$

$$C_5: SH^2SH^2 = H^2SH^2S \quad SS' = S'S$$

$$\text{Def 3: } \begin{array}{c} \times \\ \times \end{array} := \begin{array}{c} H \quad H \quad H \quad H \\ \bullet \quad | \quad | \quad | \end{array}$$

$$R_{16}: \begin{array}{c} \times \quad \times \quad \times \\ \times \quad \times \quad \times \end{array} = \quad \quad \quad$$

$$R_{17}: \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} = \begin{array}{c} \times \quad \times \\ \times \quad \times \end{array}$$

$$R_{31}: (1) \quad \begin{array}{c} \times \quad \oplus \\ \bullet \quad | \end{array} = \begin{array}{c} \bullet \quad \times \\ | \quad \oplus \end{array} \quad (2) \quad \begin{array}{c} \times \quad \bullet \\ \bullet \quad | \end{array} = \begin{array}{c} \oplus \quad \times \\ \bullet \quad | \end{array}$$

$$R_{18}: (1) \quad \begin{array}{c} S \\ \times \end{array} = \begin{array}{c} \times \\ S \end{array} \quad (2) \quad \begin{array}{c} S \\ \times \end{array} = \begin{array}{c} \times \\ S \end{array}$$

$$R_{22}: (1) \quad \begin{array}{c} S' \\ \times \end{array} = \begin{array}{c} \times \\ S' \end{array} \quad (2) \quad \begin{array}{c} S' \\ \times \end{array} = \begin{array}{c} \times \\ S' \end{array}$$

$$\text{Lem M} \quad R_{25}: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S \quad S \quad z \quad z \cdot w$$

$$\equiv R_{25}^1: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S' \quad S \cdot w \equiv R_{25}^2: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S^2 \quad S^2 \cdot w^2 = \begin{array}{c} \oplus \\ \bullet \quad | \end{array}$$

$$R_{25}^3: \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} = \begin{array}{c} S' \quad S \\ | \quad | \end{array} \cdot w \equiv R_{25}^4: \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \quad S^2 \quad S^2 \cdot w^2 = \begin{array}{c} \oplus \\ \bullet \quad | \end{array}$$

$$\text{Proof: } R_{25}: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S \quad S \quad z \quad z \cdot w$$

$$C_5 \parallel R_{11} \& C_3 \quad S^2(S'S^2) = S'S$$

$$R_{25}^1: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S' \quad S \cdot w$$

For both sides of  $R_{25}^1$ , right-appending them by  $I \otimes S^2S^2$  and multiplying them by  $w^2$  yields

$$R_{25}^1: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S^2 \quad S^2 \cdot w^2 = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S \quad S \quad S^2 \quad S^2 \cdot w^2$$

$$C_1 \& C_2 \parallel C_3 \& C_5$$

$$R_{25}^1: \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S^2 \quad S^2 \cdot w^2 = \begin{array}{c} \oplus \\ \bullet \quad | \end{array} : R_{25}^2$$

$$R_{25}^3 \cdot \text{LHS} := \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \stackrel{R_{16}}{=} \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \quad \begin{array}{c} \times \quad \times \quad \times \\ \times \quad \times \quad \times \end{array} \stackrel{R_{17}}{=} \begin{array}{c} \bullet \quad \bullet \\ | \quad | \end{array} \quad \begin{array}{c} \times \quad \times \quad \times \\ \times \quad \times \quad \times \end{array} \stackrel{R_{25}^1}{=} \begin{array}{c} \oplus \\ \bullet \quad | \end{array} \quad S' \quad S \cdot w$$

$$C1: \omega^3 = I$$

$$C2: H^4 = I$$

$$C3: S^3 = I$$

$$R11: \boxed{z^2} = \boxed{s'} \boxed{s} \boxed{s}$$

$$\text{Def 2: } \begin{array}{c} \oplus \\ \bullet \end{array} := \begin{array}{c} \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \end{array}$$

$$\begin{array}{c} \bullet \\ \oplus \end{array} := \begin{array}{c} \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \end{array}$$

$$C5: SH^2SH^2 = H^2SH^2S \quad SS' = S'S$$

$$\text{Def 3: } \begin{array}{c} \times \\ \times \end{array} := \begin{array}{c} \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \quad \cdot \quad \text{H} \end{array}$$

$$R16: \begin{array}{c} \times \\ \times \\ \times \end{array} = \begin{array}{c} \text{---} \end{array}$$

$$R17: \begin{array}{c} \bullet \\ \times \\ \times \end{array} = \begin{array}{c} \text{---} \end{array}$$

$$R_{31}: (1) \begin{array}{c} \oplus \\ \times \\ \bullet \end{array} = \begin{array}{c} \bullet \\ \oplus \\ \times \end{array} \quad (2) \begin{array}{c} \times \\ \times \\ \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \\ \times \end{array}$$

$$R18: (1) \begin{array}{c} S \\ \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \\ S \end{array} \quad (2) \begin{array}{c} S \\ \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \\ S \end{array}$$

$$R_{22}: (1) \begin{array}{c} S' \\ \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \\ S' \end{array} \quad (2) \begin{array}{c} S' \\ \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \\ S' \end{array}$$

$$\text{Lem M} \quad R_{25}: \begin{array}{c} \bullet \\ \oplus \\ \bullet \\ \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \\ \bullet \\ \text{S} \quad \text{S} \quad z \quad z \end{array} \cdot w$$

$$\equiv R_{25}^1: \begin{array}{c} \bullet \\ \oplus \\ \bullet \\ \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \\ \bullet \\ \text{S}' \quad \text{S} \end{array} \cdot w \quad \equiv R_{25}^2: \begin{array}{c} \bullet \\ \oplus \\ \bullet \\ \bullet \end{array} = \begin{array}{c} \oplus \\ \bullet \\ \bullet \\ \text{S}'^2 \quad \text{S}^2 \end{array} \cdot w^2 = \begin{array}{c} \bullet \\ \oplus \\ \bullet \\ \bullet \end{array}$$

$$R_{25}^3: \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \end{array} = \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \text{S} \quad \text{S} \end{array} \cdot w \quad \equiv R_{25}^4: \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \end{array} = \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \text{S}'^2 \quad \text{S}^2 \end{array} \cdot w^2 = \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \end{array}$$

Proof cont.

$$\begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array} = \boxed{\begin{array}{c} \oplus \\ \bullet \\ \bullet \\ \text{S}' \quad \text{S} \end{array}} \cdot w \xrightarrow[R_{22}, R_{31}]{R_{17}, R_{18}} \begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array} = \begin{array}{c} \bullet \\ \oplus \\ \bullet \\ \bullet \\ \text{S}' \quad \text{S} \end{array} \cdot w$$

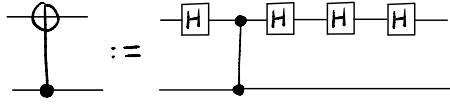
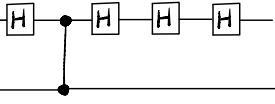
$$\xrightarrow{R_{16}} \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \end{array} = R_{25}^3 \cdot \text{RHS.}$$

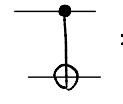
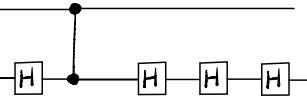
$$R_{25}^4 \cdot \text{LHS.} = \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \\ \bullet \end{array} \cdot w^2 \xrightarrow{R_{16}} \boxed{\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \\ \bullet \end{array}} = \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \\ \bullet \end{array} \cdot w^2 \xrightarrow[R_{22}, R_{31}]{R_{17}, R_{18}} \begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array}$$

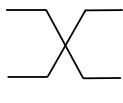
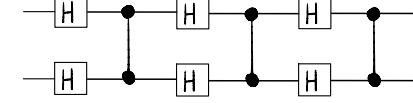
$$\begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array} = \boxed{\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \\ \bullet \end{array}} \xrightarrow{R_{25}^2} \begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array} = \boxed{\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \\ \bullet \end{array}} \xrightarrow[R_{31}]{R_{17}} \begin{array}{c} \times \\ \times \\ \times \\ \times \\ \oplus \\ \bullet \end{array} \xrightarrow{R_{16}}$$

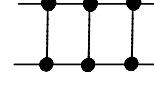
$$\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \oplus \end{array} = R_{25}^4 \cdot \text{RHS.}$$

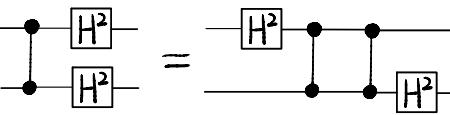
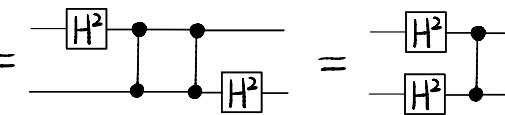


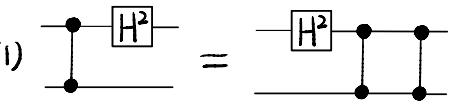
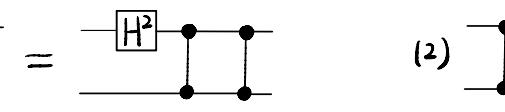
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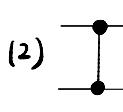
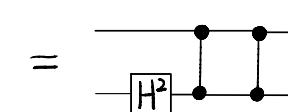
 := 

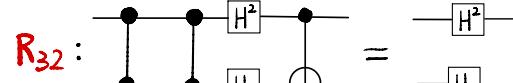
Def 3:  := 

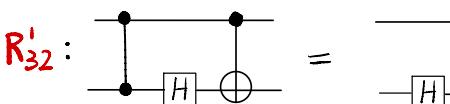
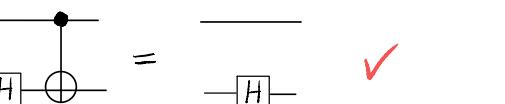
$C_2 : H^4 = I$   $C_6 :$   = 

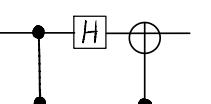
$C_8^3 :$   = 

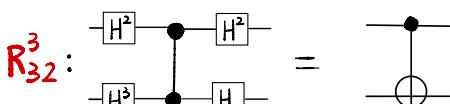
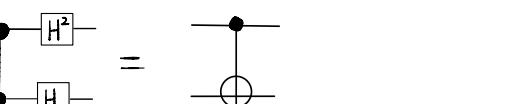
$C_8 :$  (1)  = 

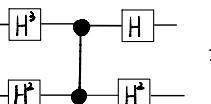
(2)  = 

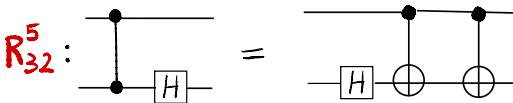
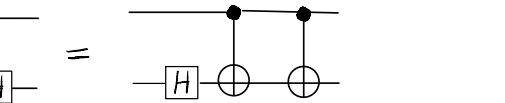
Lem N  $R_{32}^1 :$   =  ✓

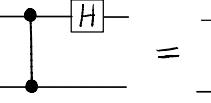
$R_{32}^1 :$   =  ✓

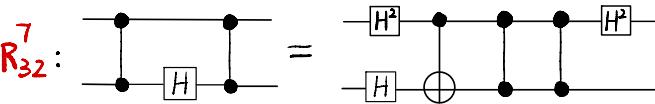
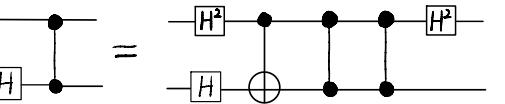
$R_{32}^2 :$   = 

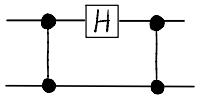
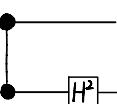
$R_{32}^3 :$   = 

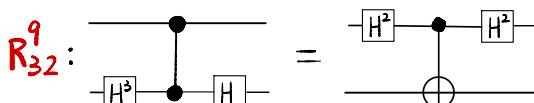
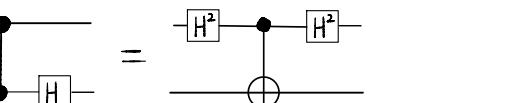
$R_{32}^4 :$   = 

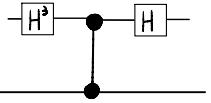
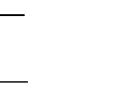
$R_{32}^5 :$   = 

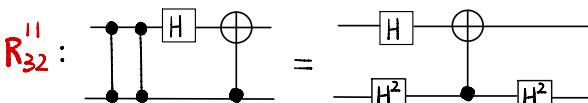
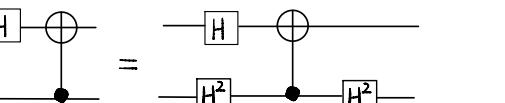
$R_{32}^6 :$   = 

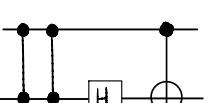
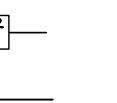
$R_{32}^7 :$   = 

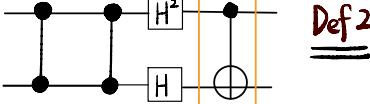
$R_{32}^8 :$   = 

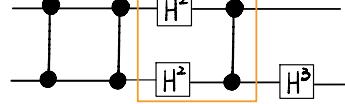
$R_{32}^9 :$   = 

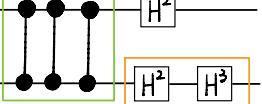
$R_{32}^{10} :$   = 

$R_{32}^{11} :$   = 

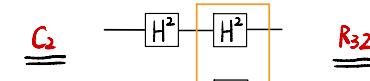
$R_{32}^{12} :$   = 

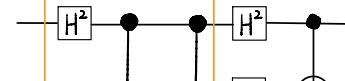
Proof:  $R_{32} . LHS :=$  



$C_8^3 :$  

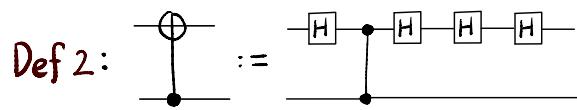
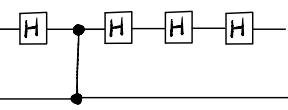
$\underline{C_2} \quad \underline{\underline{H^2}} \quad =: R_{32} . RHS.$

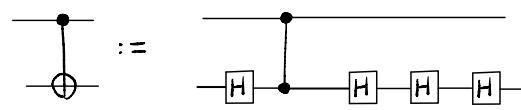
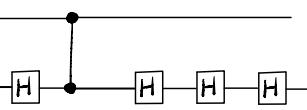
$R_{32}^1 . RHS :=$  

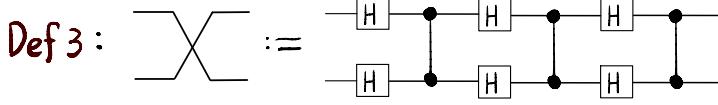
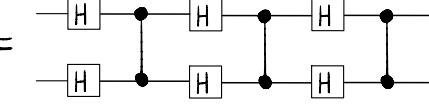


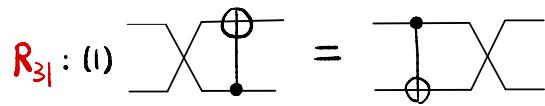
$\underline{C_8} \quad \underline{\underline{H^2 H^2}}$

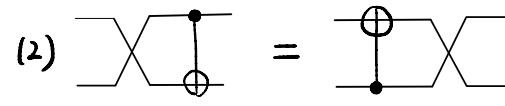
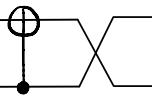
$\underline{C_2} \quad \underline{\underline{H}}$  =:  $R_{32}^1 . LHS$

Def 2:  := 

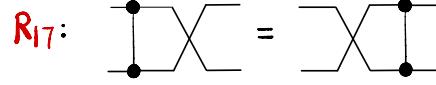
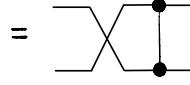
 := 

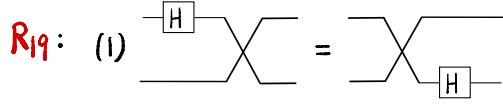
Def 3:  := 

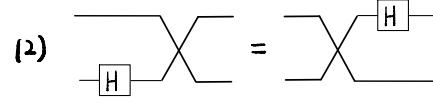
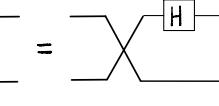
$R_{31}$ : (1)  = 

(2)  = 

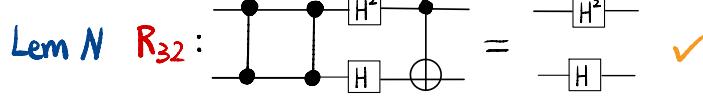
$R_{16}$ :  = 

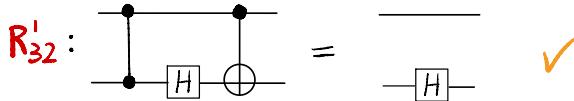
$R_{17}$ :  = 

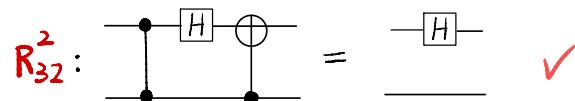
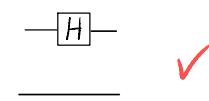
$R_{19}$ : (1)  = 

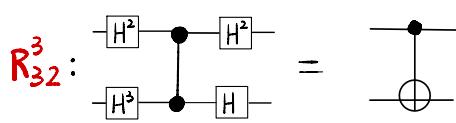
(2)  = 

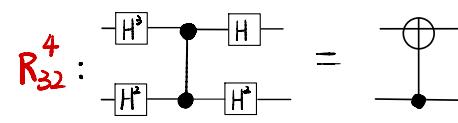
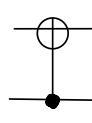
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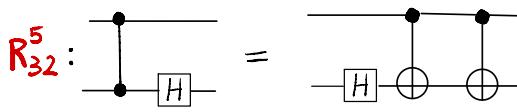
Lem N  $R_{32}$ :  =  ✓

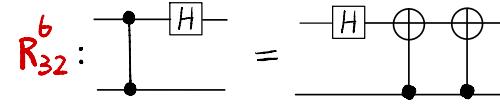
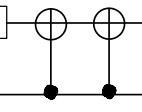
$R_{32}^1$ :  =  ✓

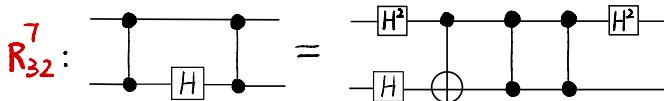
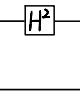
$R_{32}^2$ :  =  ✓

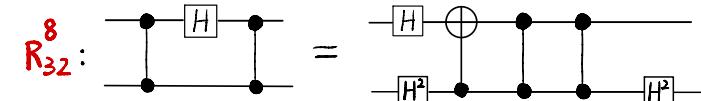
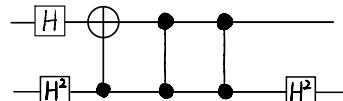
$R_{32}^3$ :  = 

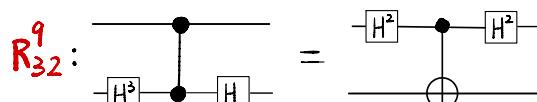
$R_{32}^4$ :  = 

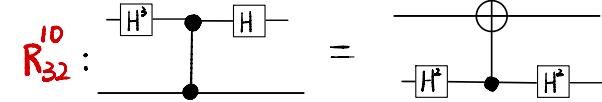
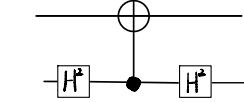
$R_{32}^5$ :  = 

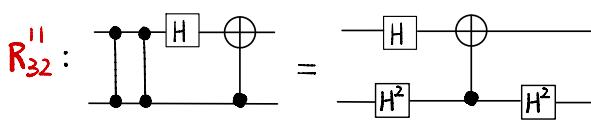
$R_{32}^6$ :  = 

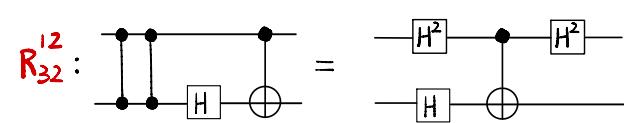
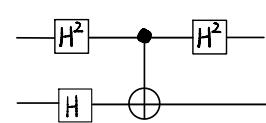
$R_{32}^7$ :  = 

$R_{32}^8$ :  = 

$R_{32}^9$ :  = 

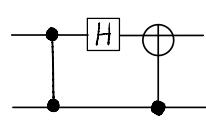
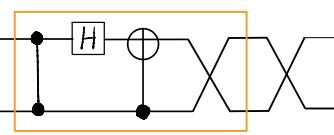
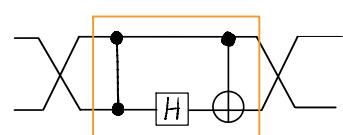
$R_{32}^{10}$ :  = 

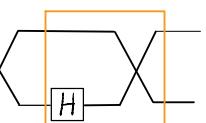
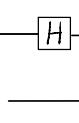
$R_{32}^{11}$ :  = 

$R_{32}^{12}$ :  = 

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Proof cont.

$R_{32}^2 \cdot LHS :=$    $\xrightarrow{R_{16}}$    $\xrightarrow{R_{17}, R_{19}}$  

$\xrightarrow{R_{32}^1}$    $\xrightarrow{R_{19}}$    $\xrightarrow{R_{16}}$    $=: R_{32}^2 \cdot RHS$

$$\begin{array}{ll}
 R_{31}: (1) \quad \text{Diagram} = \text{Diagram} & (2) \quad \text{Diagram} = \text{Diagram} \\
 R_{16}: \quad \text{Diagram} = \text{Diagram} & R_{17}: \quad \text{Diagram} = \text{Diagram} \quad C_2: H^4 = I \\
 R_{19}: (1) \quad \text{Diagram} = \text{Diagram} & (2) \quad \text{Diagram} = \text{Diagram} \quad C_6: \quad \text{Diagram} = \text{Diagram}
 \end{array}$$

Lem N  $R_{32}$ :

$$\begin{array}{ll}
 R_{32}: & \text{Diagram} = \text{Diagram} \quad \checkmark \\
 R_{32}^1: & \text{Diagram} = \text{Diagram} \quad \checkmark \\
 R_{32}^2: & \text{Diagram} = \text{Diagram} \quad \checkmark \\
 R_{32}^3: & \text{Diagram} = \text{Diagram} \quad \checkmark \\
 R_{32}^4: & \text{Diagram} = \text{Diagram} \quad \checkmark \\
 R_{32}^5: & \text{Diagram} = \text{Diagram} \\
 R_{32}^6: & \text{Diagram} = \text{Diagram} \\
 R_{32}^7: & \text{Diagram} = \text{Diagram} \\
 R_{32}^8: & \text{Diagram} = \text{Diagram} \\
 R_{32}^9: & \text{Diagram} = \text{Diagram} \\
 R_{32}^{10}: & \text{Diagram} = \text{Diagram} \\
 R_{32}^{11}: & \text{Diagram} = \text{Diagram} \\
 R_{32}^{12}: & \text{Diagram} = \text{Diagram}
 \end{array}$$

Proof cont. Left-appending both sides of  $R_{32}$  by

$$\begin{array}{c}
 \text{Diagram} \\
 \text{yields}
 \end{array}$$

$$\begin{array}{ll}
 \text{Diagram} = \text{Diagram} & C_6: \quad \text{Diagram} = \text{Diagram} \\
 \text{Diagram} = \text{Diagram} & \text{Diagram} = \text{Diagram} \\
 \text{Diagram} = \text{Diagram} & C_2: \quad \text{Diagram} = \text{Diagram} \quad : R_{32}^3
 \end{array}$$

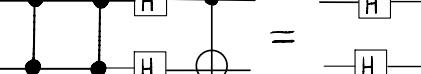
$$\begin{array}{llll}
 R_{32}^4 \cdot \text{LHS} := & \text{Diagram} & \xrightarrow{\text{R}_{16}} & \text{Diagram} \\
 & \xrightarrow{\text{R}_{19}} & \xrightarrow{\text{R}_{17}} & \text{Diagram} \\
 & \xrightarrow{\text{R}_{32}^3} & \xrightarrow{\text{R}_{31}} & \xrightarrow{\text{R}_{16}} & =: R_{32}^4 \cdot \text{RHS}
 \end{array}$$

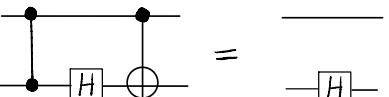
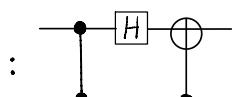
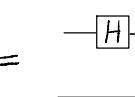
$C_2: H^4 = I$

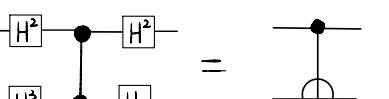
$C_6^1: \quad \text{Diagram} = \text{Diagram}$

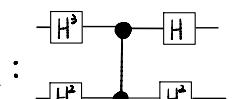
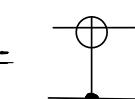
$C_6^2: \quad \text{Diagram} = \text{Diagram}$

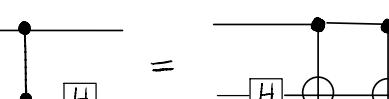
$C_8^2: \quad \text{Diagram} = \text{Diagram} = \text{Diagram}$

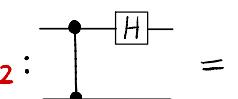
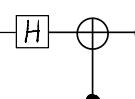
Lem N  $R_{32}^1:$    $=$   ✓

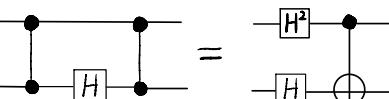
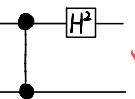
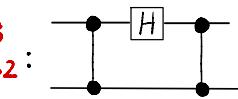
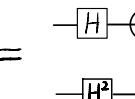
$R_{32}^1:$    $=$   ✓  $R_{32}^2:$    $=$   ✓

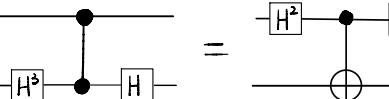
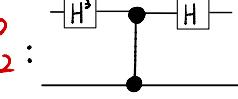
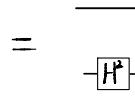
$R_{32}^3:$    $=$   ✓

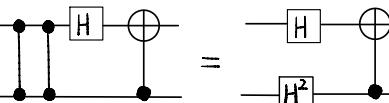
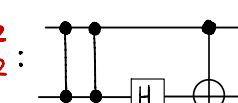
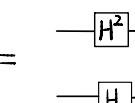
$R_{32}^4:$    $=$   ✓

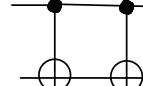
$R_{32}^5:$    $=$   ✓

$R_{32}^6:$    $=$   ✓

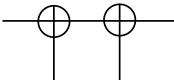
$R_{32}^7:$    $=$   ✓  $R_{32}^8:$    $=$  

$R_{32}^9:$    $=$   ✓  $R_{32}^{10}:$    $=$  

$R_{32}^{11}:$    $=$   ✓  $R_{32}^{12}:$    $=$  

Proof cont. Right-appending both sides of  $R_{32}^1$  by  yields

$\text{Diagram} = \text{Diagram} \equiv \text{Diagram} = \text{Diagram} : R_{32}^5$

Right-appending both sides of  $R_{32}^2$  by  yields

$\text{Diagram} = \text{Diagram} \equiv \text{Diagram} = \text{Diagram} : R_{32}^6$

$R_{32}^7 \cdot \text{RHS} := \text{Diagram} \stackrel{\text{C}_3}{=} \text{Diagram} \stackrel{\text{C}_2}{=} \text{Diagram}$

$\text{Diagram} \stackrel{\text{C}_8}{=} \text{Diagram} =: R_{32}^7 \cdot \text{LHS}$