

$$\text{Def 3: } \begin{array}{c} \text{Diagram of two wires crossing} \end{array} := \begin{array}{c} \text{Diagram of two wires with H-gates between them} \end{array} \quad R_B : \begin{array}{c} (1) \text{ Diagram with } Z \text{-gate} \\ (2) \text{ Diagram with } Z \text{-gate} \end{array} = \begin{array}{c} \text{Diagram with } Z \text{-gate} \\ \text{Diagram with } Z \text{-gate} \end{array}$$

$$R_{20}: (1) \quad \begin{array}{c} X \\ \diagup \quad \diagdown \\ \text{---} & \text{---} \end{array} = \begin{array}{c} \diagup \quad \diagdown \\ \text{---} & \text{---} \\ X \end{array} \quad (2) \quad \begin{array}{c} \diagup \quad \diagdown \\ \text{---} & \text{---} \\ X \end{array} = \begin{array}{c} \diagup \quad \diagdown \\ \text{---} & \text{---} \\ X \end{array}$$

$$R_{14}: (3) \quad \begin{array}{c} X \\ \square \end{array} \quad = \quad \begin{array}{c} \bullet \\ \bullet \\ \square \end{array} \quad \begin{array}{c} X \\ \square \\ \square \end{array}$$

$$R_{26}: \quad (1) \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \boxed{X} \quad \bigoplus = \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \bigoplus \quad \boxed{X} \quad (4) \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \bigoplus = \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \bigoplus \quad \boxed{\Xi} \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array}$$

Lem 29 By Def 3, R_{13}, R_{14}, R_{20} & R_{26} ,

$$12.(1) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{00}} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{00}} \boxed{X} \quad (2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{01}} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{01}} \boxed{X} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{z} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{z} \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{z}$$

$$(3) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{02}} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{B_{02}} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{X}$$

Proof: 12. (1). LHS :=

$$\underline{R_{14}} \quad \begin{array}{c} \text{Diagram of } R_{14} \\ \text{def} \end{array} \quad \begin{array}{c} \text{Diagram of } B_{00} \\ \text{def} \end{array} \quad =: 12.(1).\text{RHS}$$

12.(2). LHS :=

The diagram illustrates the definition of R_{20} and its equivalence to R_{14} and R_B . It consists of two parts separated by an equals sign ($=$). The left part shows a circuit with a dependent voltage source B_{01} (represented by a box with X) connected between the top wire and the common ground rail. The right part shows the equivalent circuit where the dependent source is removed, resulting in a bridge-like network. A blue box highlights the top-left branch of the bridge. The label def is written above the first part, and the labels R_{20} , \equiv , R_{14} , and $\equiv R_B$ are written above the second part.

$$\text{Diagram 12.12. RHS} = \text{Diagram 12.12. LHS}$$

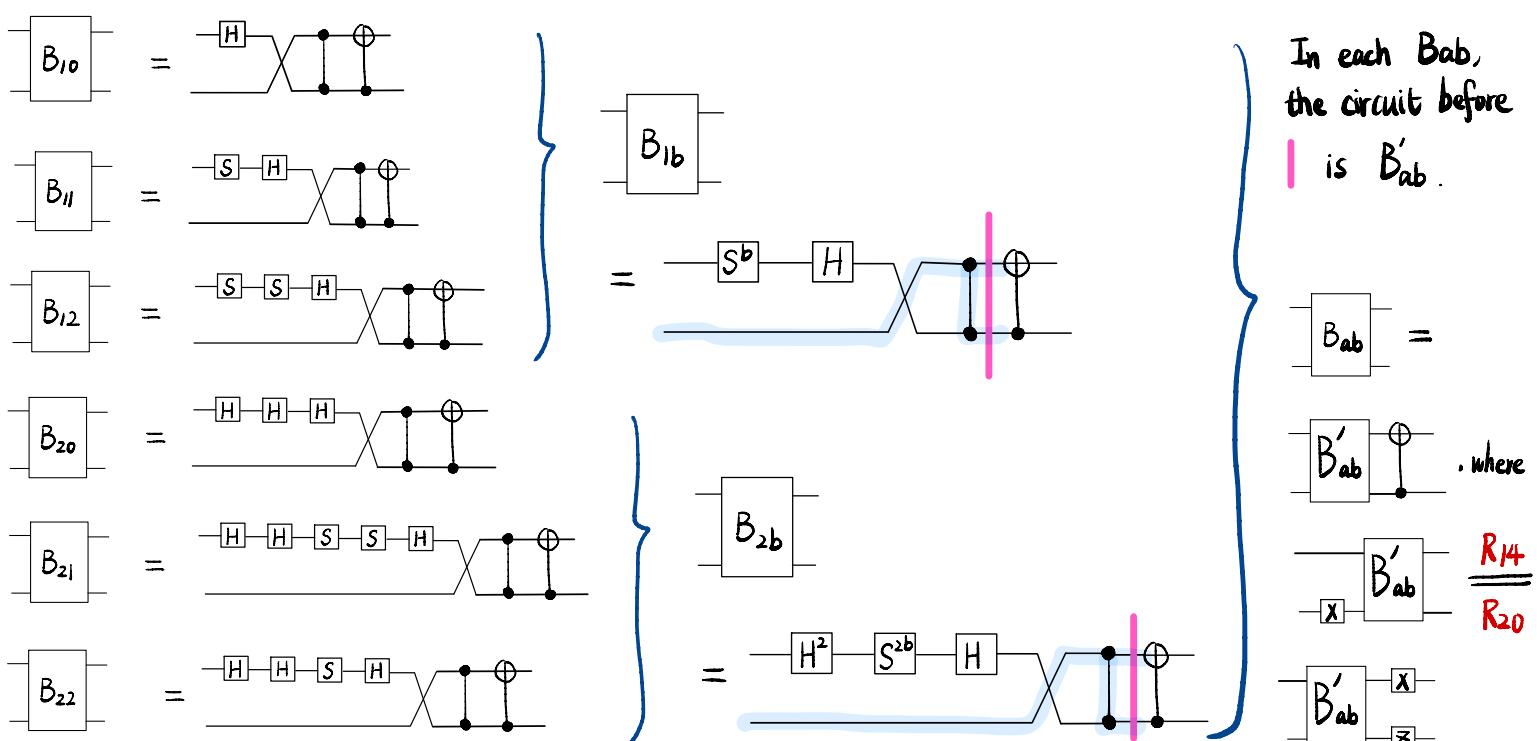
12.(3). LHS :=

The diagram illustrates the decomposition of the operator B_{02} into a sequence of quantum gates. The circuit starts with an initial state, followed by a box labeled B_{02} , which is then decomposed into three Hadamard (H) gates, one CNOT gate, and one controlled-X (X) gate. The circuit concludes with a measurement operation.

$$\underline{\underline{R_{26}}} \quad \text{def} \quad B_{02} =: 12.(3).RHS$$


11

In each Bab,
the circuit before
| is B'_{ab} .



Def 3: := $R_B : (1)$ = (2) =

$R_{20} : (1)$ = X (2) =

$R_{14} : (3)$ =

$R_{26} : (4)$ = (4) =

Lem 30 By Def 3, R_{13} , R_{14} , R_{20} & R_{26} ,

12. (4) =

(7) =

(5) =

(8) =

(6) =

(9) =

Prof: 12.(4)-(9).LHS := $\stackrel{\text{def}}{=}$ $\stackrel{R_{14}}{=} \stackrel{R_{20}}{=}$ $\stackrel{R_{26}}{=}$

$\stackrel{\text{def}}{=}$ =: 12.(4)-(9).RHS