

$$A_{01} := \dots$$

$$B_{00} = \dots$$

$$B_{01} = \dots$$

$$R_{17}: \quad \dots = \dots$$

$$\text{Def 3: } \dots := \dots$$

$$R_{25}: \quad \dots = \dots \cdot w$$

$$R_{25}^I: \quad \dots = \dots \cdot w$$

$$\text{Def: } S' = \dots$$

$S' := H^2 SH^2$

Lem 36 By Def 3 & R₁₇, 4.(1)

$$\begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{00}} = \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{00}}$$

$$\text{Proof: } 4.(1).LHS := \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{00}} \stackrel{\text{def}}{=} \boxed{\dots}$$

$$\stackrel{R_{17}}{=} \boxed{\dots} \stackrel{\text{def}}{=} \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{00}} = 4.(1).RHS$$

□

Lem 37 By Def 3, R₁₇ & R₂₅,

$$4.(2) \quad \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{01}} = \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{01}} \cdot w$$

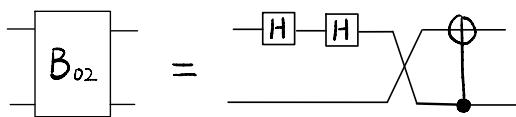
$$\text{Proof: } 4.(2).LHS := \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{01}} \stackrel{\text{def}}{=} \boxed{\dots} \stackrel{R_{17}}{=} \boxed{\dots}$$

$$\stackrel{R_{25}}{=} \boxed{\dots} \cdot w$$

$$\stackrel{\text{def}}{=} \begin{array}{c} \dots \\ | \\ A_{01} \end{array} \boxed{B_{01}} \cdot w =: 4.(2).RHS.$$

□

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R_{17} : = $\text{Def 3}:$:=

R_{25} : = $\cdot w$ R_3 : $SZ = ZS$

$\text{Def}:$ $S' := H^2 SH^2$ C_3 : $S^3 = I$ R_{12} : $Z^3 = I$

C_8 : (1) = (2) =

C_7 : (1) = (2) =

R_B : (1) = (2) =

Lem 38 By Def 3, C_3 , C_7 , C_8 , R_3 , R_{12} , R_{13} , R_{17} & R_{25} ,

4. (3) = $\cdot w^2$

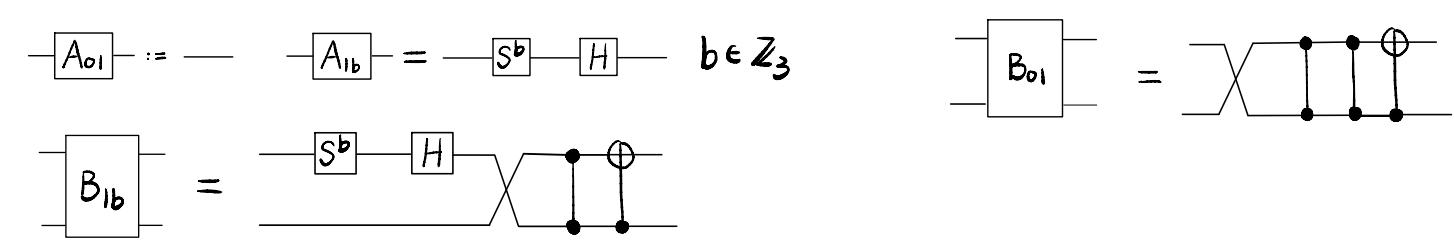
Proof: 4.(3). LHS := $\stackrel{\text{def}}{=}$ $\stackrel{C_8}{=}$

$\stackrel{R_{17}}{=}$ $\stackrel{R_{25}}{=}$ $\cdot w$

$\stackrel{R_{25}}{=}$ $\cdot w \cdot w$

$\stackrel{C_7}{=} \stackrel{R_{13}}{=}$ $\cdot w^2$ $\frac{R_3, C_3}{R_{12}}$

$\cdot w^2$ $\stackrel{\text{def}}{=}$ $\cdot w^2 =: 4.(3). \text{RHS.}$



Def 3: :=

R₂₉: = • w²

C₇: (1) = (2) =

Lem 39 By Def 3, C₇ & R₂₉,

4. (4) = • w²

(5) = • w²

(6) = • w²

Proof: 4.(4)/(5)/(6). LHS := $\stackrel{\text{def}}{=}$

$\stackrel{C_7}{=}$

$\stackrel{R_{29}}{=}$ • w²

$\stackrel{\text{def}}{=}$ • w² =: 4.(4)/(5)/(6). RHS.

$$A_{01} := \dots \quad A_{2b} := H^2 S^{2b} H \quad A_{02} := H H$$

$$B_{2b} = H^2 S^{2b} H \quad \text{with a CNOT gate between the second and third wires}$$

$$C_7 : (1) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} S = \begin{array}{c} S \\ \text{---} \\ \bullet \end{array} \quad (2) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} S = \begin{array}{c} S \\ \text{---} \\ \bullet \end{array} \quad C_2 : H^4 = I$$

$$C_8 : (1) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} H^2 = \begin{array}{c} H^2 \\ \text{---} \\ \bullet \end{array} \quad (2) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} H^2 = \begin{array}{c} H^2 \\ \text{---} \\ \bullet \end{array}$$

$$R_{30} : \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} H \quad \text{with a CNOT gate between the second and third wires} = \begin{array}{c} H \\ \text{---} \\ H^2 \end{array} \quad \text{with a CNOT gate between the second and third wires} \quad S H H S H \quad \text{with a CNOT gate between the second and third wires} \quad S \quad S \quad Z$$

$$\text{Def 2: } \begin{array}{c} \oplus \\ \text{---} \\ \bullet \end{array} := H H H H \quad \begin{array}{c} \bullet \\ \text{---} \\ \oplus \end{array} := H H H H$$

Lem 40 By Def 2, Def 3, C₂, C₇, C₈ & R₃₀,

$$4.(7) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} A_{01} B_{20} = \begin{array}{c} B_{20} \\ \text{---} \\ A_{02} S H H S H H H S Z \end{array}$$

$$(8) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} A_{01} B_{21} = \begin{array}{c} B_{21} \\ \text{---} \\ A_{02} S H H S H H H S Z \end{array}$$

$$(9) \quad \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} A_{01} B_{22} = \begin{array}{c} B_{22} \\ \text{---} \\ A_{02} S H H S H H H S Z \end{array}$$

$$\text{Proof: } 4.(7)/(8)/(9). \text{LHS} := \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} A_{01} B_{2b} \stackrel{\text{def}}{=} \begin{array}{c} H^2 S^{2b} H \\ \text{---} \\ \bullet \end{array} \quad \text{with a CNOT gate between the second and third wires}$$

$$C_8 \quad \begin{array}{c} H^2 S^{2b} H \\ \text{---} \\ \bullet \end{array} \quad C_7 \quad \begin{array}{c} H^2 S^{2b} H \\ \text{---} \\ \bullet \end{array} \quad \text{with a CNOT gate between the second and third wires}$$

$$R_{30} \quad \begin{array}{c} H^2 S^{2b} H \\ \text{---} \\ H^2 \end{array} \quad \text{with a CNOT gate between the second and third wires} \quad S H H S H \quad \text{with a CNOT gate between the second and third wires} \quad S \quad S \quad Z$$

$$\text{def} \quad \begin{array}{c} B_{2b} \\ \text{---} \\ A_{02} S H H S \oplus S Z \end{array}$$

$$= \begin{array}{c} B_{2b} \\ \text{---} \\ A_{02} S H H S H H H S Z \end{array} = 4.(7)/(8)/(9). \text{RHS}$$