

$C_{13}^5: \quad \text{Diagram} = \text{Diagram}$

$C_{13}: \quad \text{Diagram} = \text{Diagram}$

$R_8: \quad \text{Diagram} = \text{Diagram}$

$R_{17}: \quad \text{Diagram} = \text{Diagram}$

$R_{16}: \quad \text{Diagram} = \text{Diagram}$

$C_{15}^2: \quad \text{Diagram} = \text{Diagram}$

$C_{15}^3: \quad \text{Diagram} = \text{Diagram}$

Lem 13

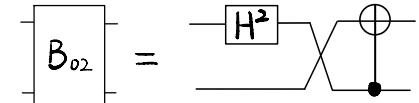
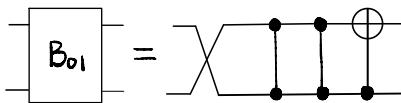
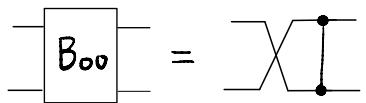
$7.3. (2) \quad \text{Diagram} = \text{Diagram}$

$\text{Proof cont: } 7.3. (2). \text{ LHS} := \text{Diagram} \stackrel{\text{def}}{=} \text{Diagram}$

$\text{C}_8 \quad \text{Diagram} \quad \dots \quad \text{Diagram}$

$\text{C}_{13}^5 \quad \dots \quad \text{Diagram} \quad \text{C}_{13} \quad \text{Diagram}$

$\text{def} \quad \text{Diagram} =: 7.3. (2). \text{ RHS.}$



$C_{13}^5: \quad \text{Diagram} = \text{Diagram}$

$C_{13}: \quad \text{Diagram} = \text{Diagram}$

$R_{17}: \quad \text{Diagram} = \text{Diagram}$

$R_{16}: \quad \text{Diagram} = \text{Diagram}$

$C_8^3: \quad \text{Diagram} = \text{Diagram}$

$C_{15}^2: \quad \text{Diagram} = \text{Diagram}$

$C_{15}^3: \quad \text{Diagram} = \text{Diagram}$

Lem 13

$7.3.(3) \quad \text{Diagram} = \text{Diagram}$

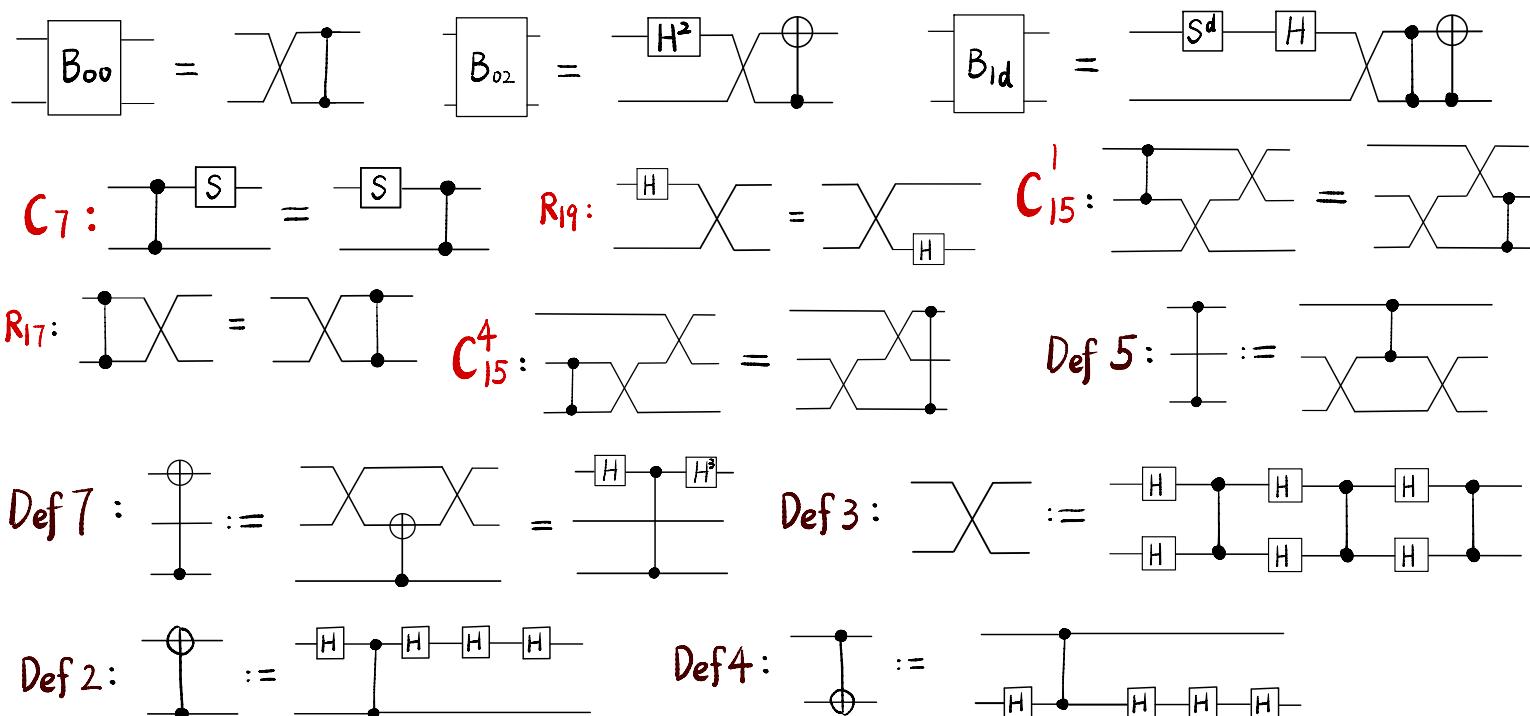
$\text{Proof cont: } 7.3.(3). \text{ LHS} := \text{Diagram} \stackrel{\text{def}}{=} \text{Diagram}$

$\stackrel{C_8^3}{=} \text{Diagram} \dots \stackrel{C_8^3}{=} \text{Diagram}$

$\stackrel{C_{13}^5}{=} \dots \stackrel{C_{13}}{=} \text{Diagram}$

$\stackrel{\text{def}}{=} \text{Diagram} =: 7.3.(3). \text{ RHS.}$





Lem 14 Def 1-5, Def 7, $C_2, C_3, C_5, C_7, C_8^*, C_5, R_7, R_{16}, R_{17}, R_{19}, R_{31}$ & R_{47} imply

7.1.(4)-(6)

$$B_{00} = B_{02} \cdot H \cdot S \cdot H \cdot H \cdot S \cdot S \cdot H \cdot S \cdot S \cdot H \cdot X \cdot X \cdot w$$

Proof: 7.1.(4)-(6). LHS :=

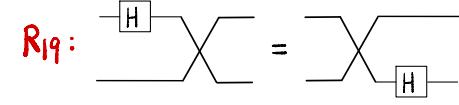
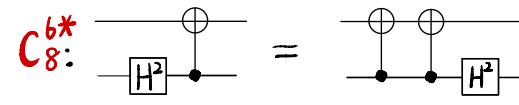
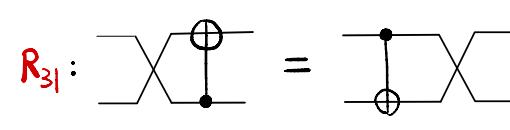
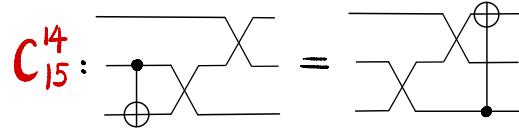
G

R_{19}

7.1.(4)-(6). RHS :=

$$B_{02} \cdot H \cdot S \cdot H \cdot H \cdot S \cdot S \cdot H \cdot S \cdot S \cdot H \cdot X \cdot X \cdot w$$

def



Lem 14

$$7.1.(4)-(6) \quad \text{LHS} = \text{RHS} \cdot w$$

Quantum circuit diagram showing the equality between the left-hand side (LHS) and right-hand side (RHS) of equation 7.1.(4)-(6), multiplied by a global phase w . The LHS consists of a sequence of operations: B_{00} , B_{02} , B_{1d} , followed by a sequence of H , S , and H gates. The RHS consists of a sequence of H , S , S , H , X , and X gates.

Proof cont:

$$7.1.(4)-(6). \text{LHS} = \text{RHS} \cdot w$$

Quantum circuit diagram showing the left-hand side (LHS) of equation 7.1.(4)-(6). It starts with an S^d gate, followed by a sequence of operations: H^2 , X , \oplus , H , S , H , H , H , S , S , H , X , X .

$$7.1.(4)-(6). \text{RHS} = \text{RHS} \cdot w$$

Quantum circuit diagram showing the right-hand side (RHS) of equation 7.1.(4)-(6). It starts with an S^d gate, followed by a sequence of operations: H^2 , X , \oplus , H , S , H , H , H , S , S , H , X , X .

$$\underline{\underline{R_{31}}} \quad \underline{\underline{C_{14}^{14}}}$$

Quantum circuit diagram showing the application of R_{31} and C_{14}^{14} to the RHS circuit. A green box highlights the H^2 gate, and an orange box highlights the X gate. The resulting circuit is identical to the RHS circuit shown above.

$$\underline{\underline{R_{19}}}$$

Quantum circuit diagram showing the application of R_{19} to the RHS circuit. An orange box highlights the H^2 gate. The resulting circuit is identical to the RHS circuit shown above.

$$\underline{\underline{C_8^{6*}}}$$

Quantum circuit diagram showing the application of C_8^{6*} to the RHS circuit. An orange box highlights the H^2 gate. The resulting circuit is identical to the RHS circuit shown above.

$$\underline{\underline{C_2}}$$

Quantum circuit diagram showing the application of C_2 to the RHS circuit. A blue box highlights the S gate. The resulting circuit is identical to the RHS circuit shown above.

$$C_2: H^4 = I \quad C_3: S^3 = I$$

$$R_{16}: \text{Diagram} = \text{Diagram}$$

$$C_5: SS' = S'S$$

$$R_7: \text{Diagram} = \text{Diagram}$$

$$\text{Def 1: } S' := \text{Diagram}$$

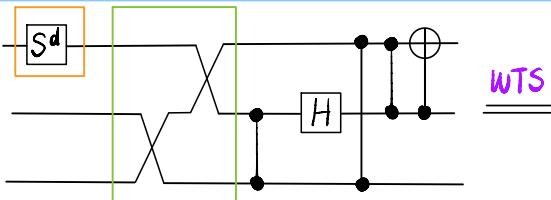
$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$R_{47}: \text{Diagram} = \text{Diagram} \cdot w$$

Lem 14

$$7.1.(4)-(6) \text{Diagram} = \text{Diagram} \cdot w$$

Proof cont: Hence



$$\text{Diagram} \cdot w$$

$C_2, C_3 \parallel R_{16}$

$$\text{Diagram} \cdot w$$

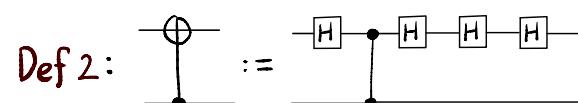
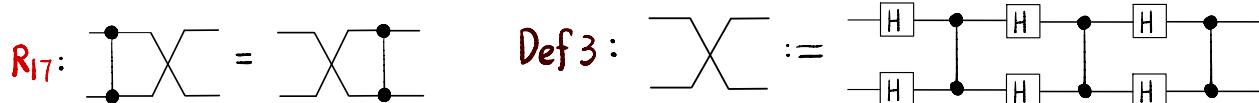
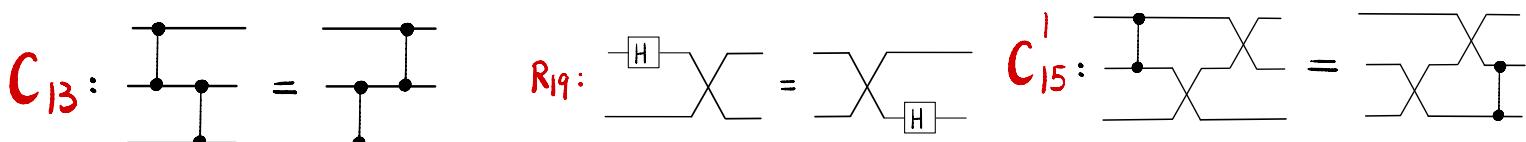
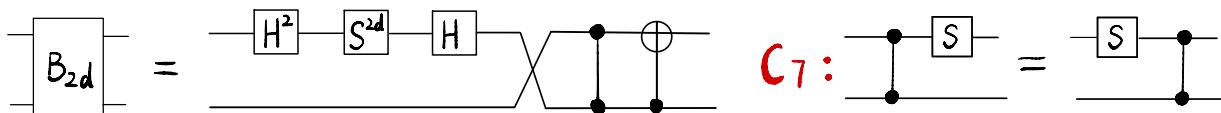
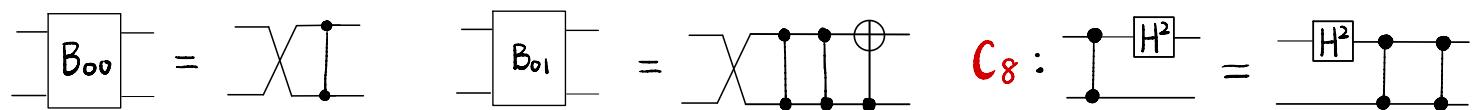
$$\text{III } R_7 \quad HS^2Hx^2 = HS^2HHS^2H^2SH = HS^2 \boxed{H^2S^2H^2} SH \xrightarrow{\text{Def 1}} HS^2 \boxed{S'^2S} H \xrightarrow{\substack{C_5 \\ C_3}} HS^2H \xrightarrow{\text{Def 1}} HH^2S^2H^2H = H^3S^2H^3$$

$$\text{Diagram} \cdot w$$

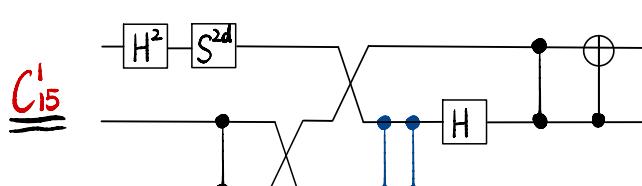
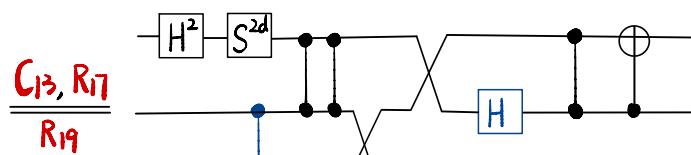
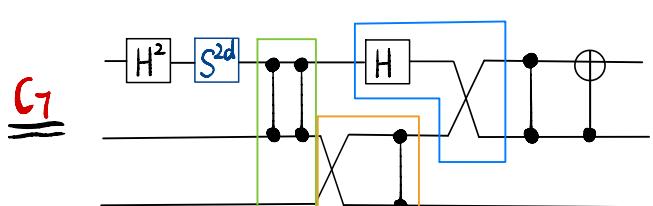
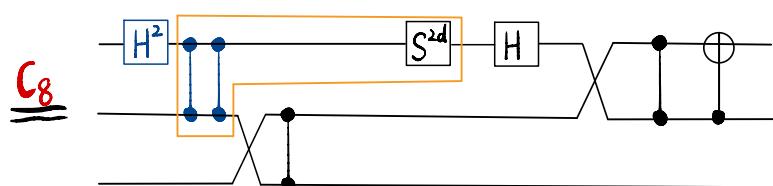
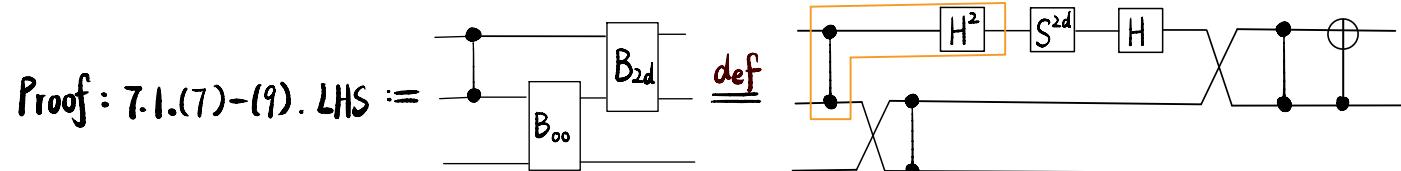
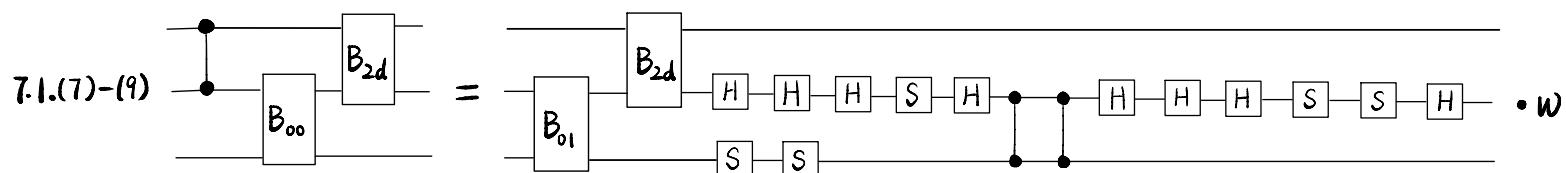
Def 2

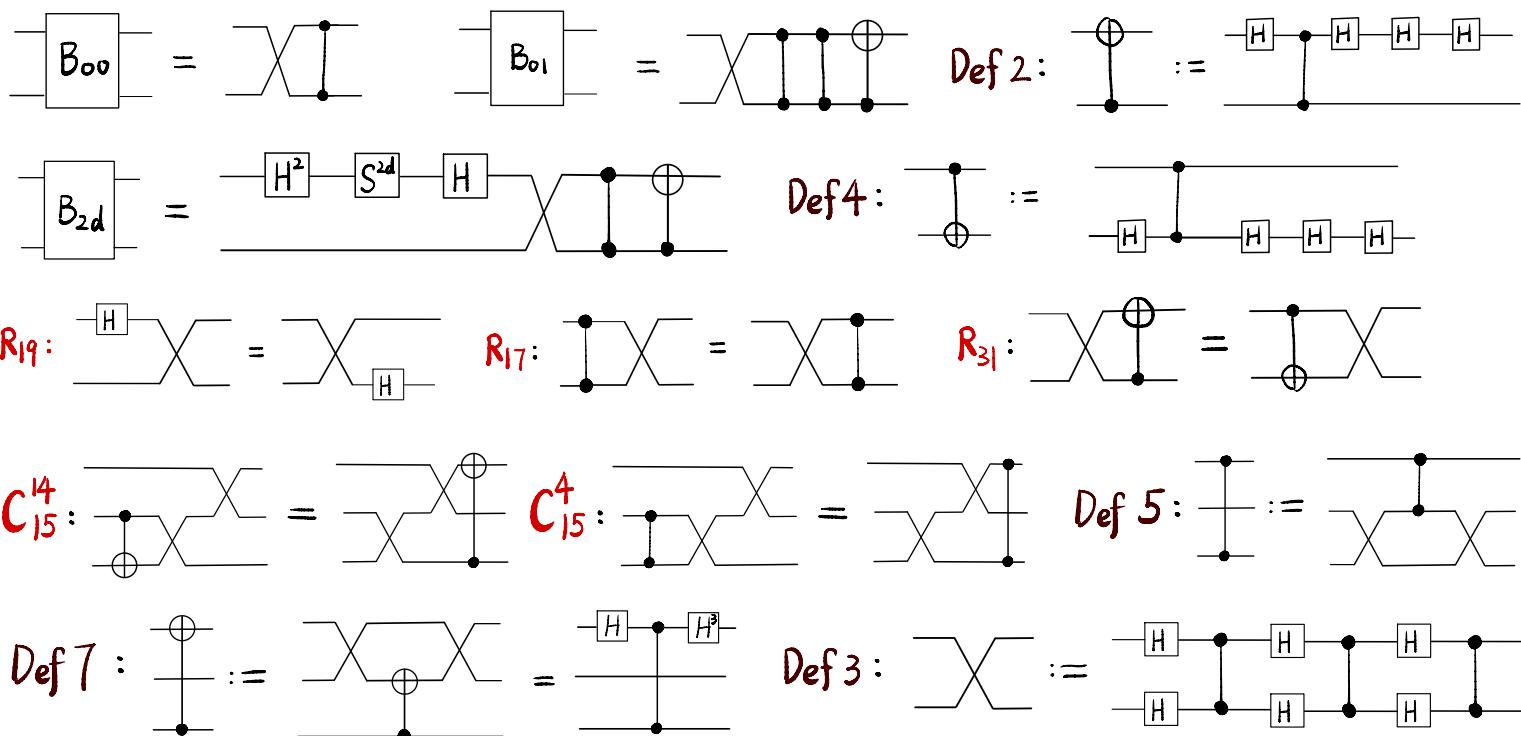
$$\text{Diagram} \cdot w$$

By R_{47} , this completes the proof.



Lem 15 Def 2-5, Def 7, $C_2, C_3, C_6, C_7, C_8, C_{13}, C_{15}, R_{16}, R_{17}, R_{19}, R_{31}$ & R_{48} imply

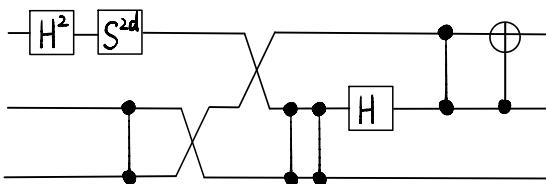




Lem 15

7.1.(7)-(9)

Proof cont : 7.1.(7)-(9). LHS =



7.1.(7)-(9). RHS :=

def

R₁₉
R₁₇

R₁₇, R₃₁
C₁₅⁴, C₁₅¹⁴