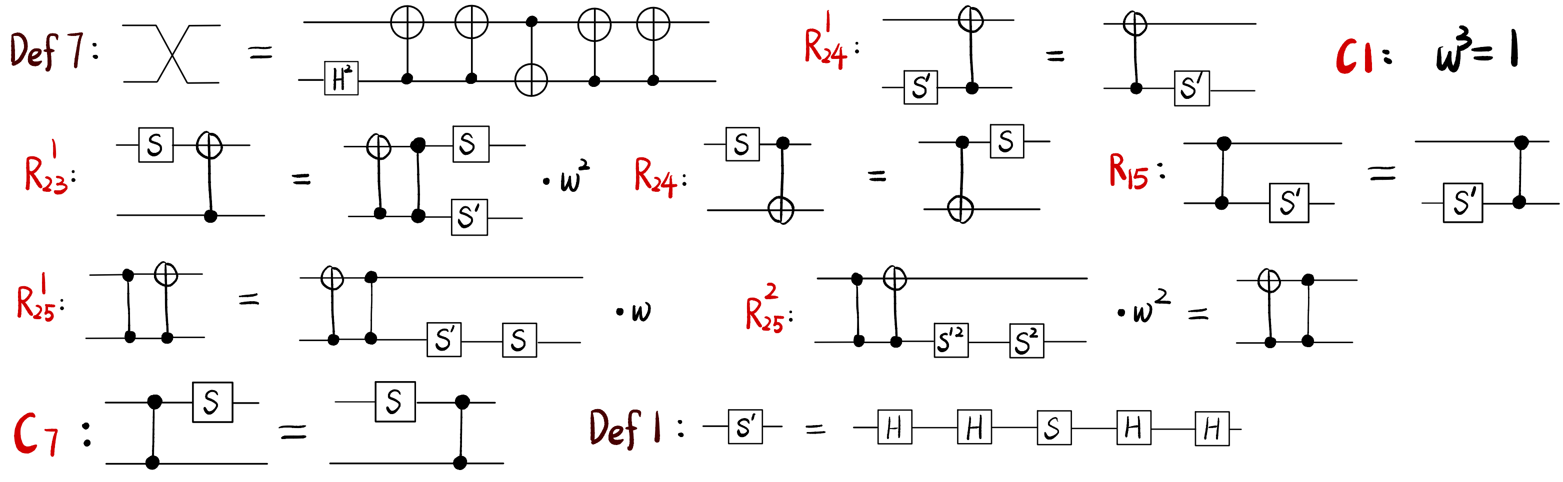
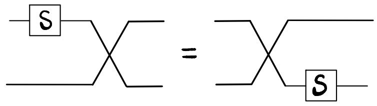
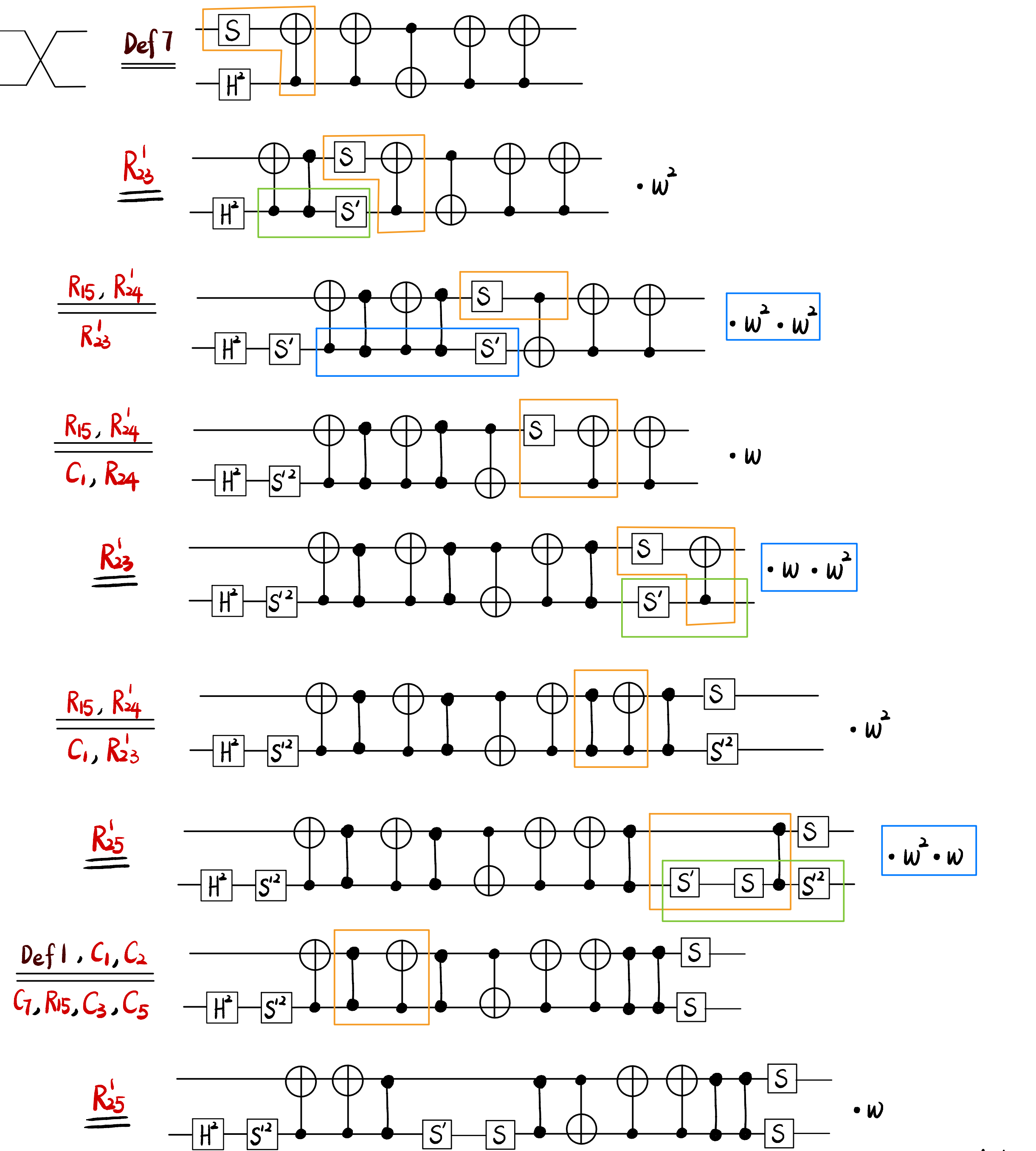


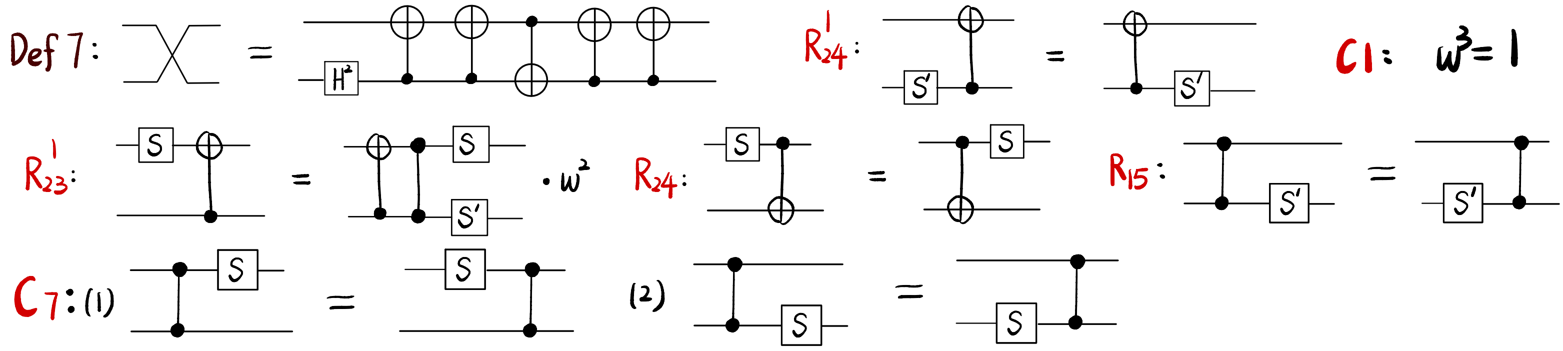
C2 : $H^4 = I$ C3 : $S^3 = I$ C5 : $SH^2SH^2 = H^2SH^2S$ $SS' = S'S$

Lem S Def 1-3, C1-7, R_{15}, R_{23}, R_{24} & R_{25} imply

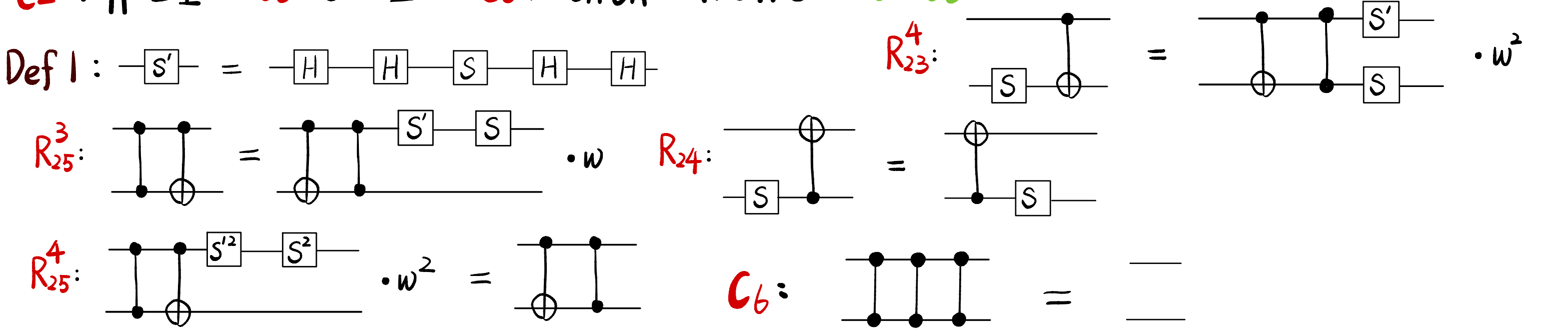
R_{18}: (1) 

Proof: R_{18}.(1). LHS :=

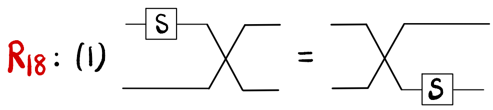




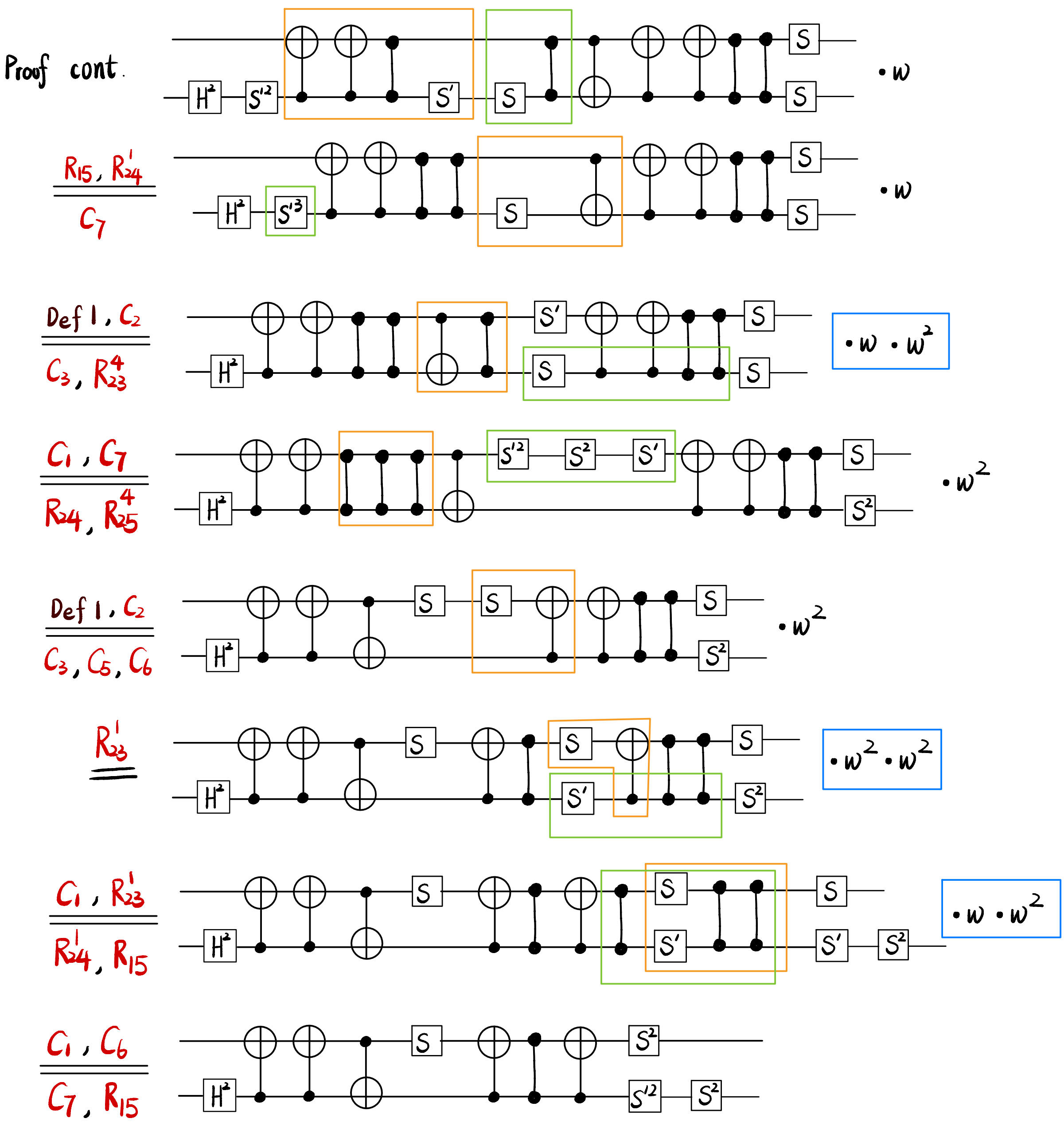
C2 : $H^4 = I$ C3 : $S^3 = I$ C5 : $SH^2SH^2 = H^2SH^2S$ $SS' = S'S$

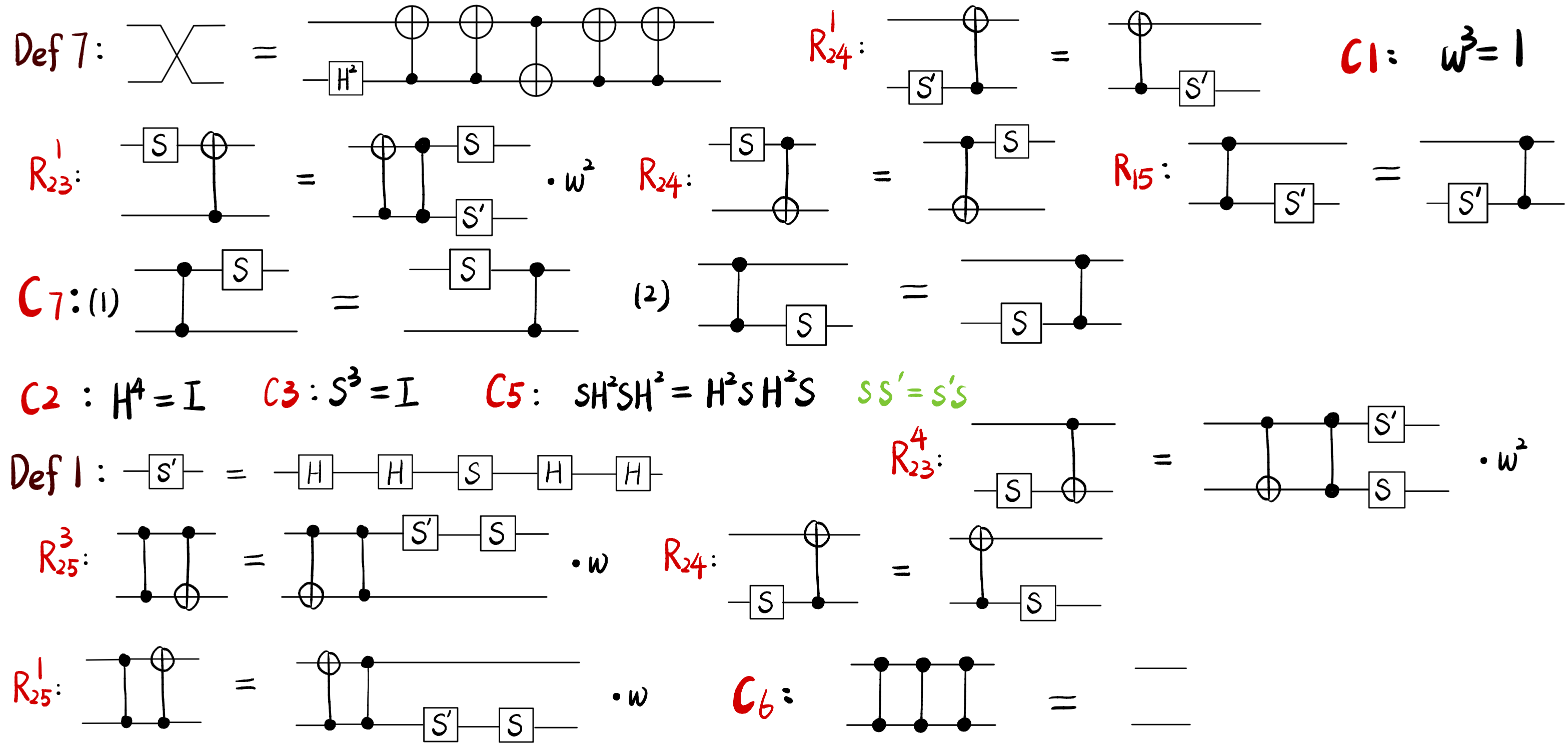


Lem S



Proof cont.

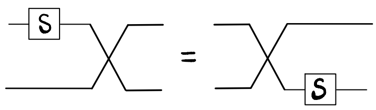


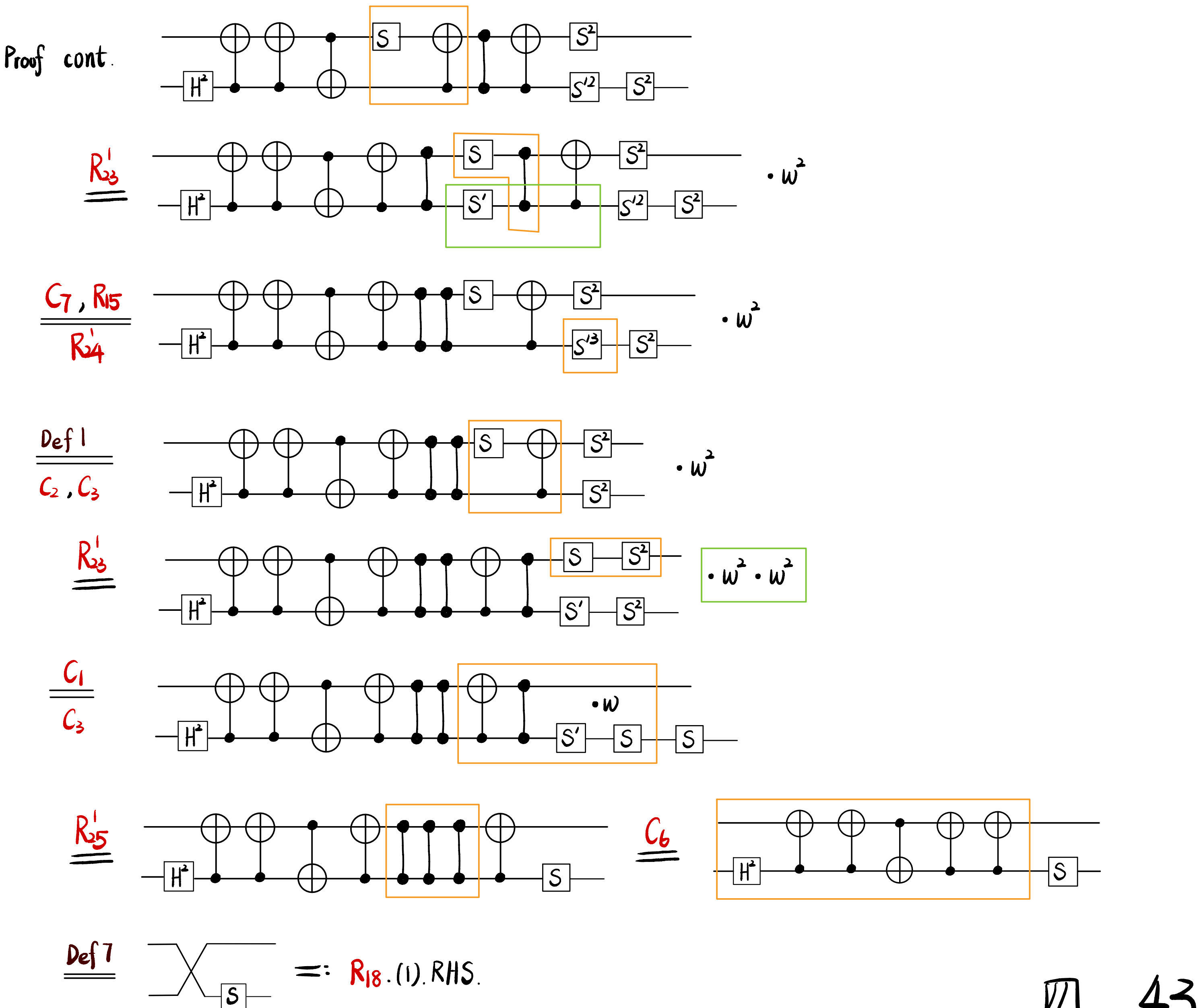


Def 7:
R'_{24}:
$C1: w^3 = 1$
R'_{23}:
$\cdot w^2$
R_{24}:
R_{15}:
C_7: (1)
(2)
$C2: H^4 = I$
$C3: S^3 = I$
$C5: SH^2SH^2 = H^2SH^2S$
$SS' = S'S$
Def 1:
R^4_{23}:
$\cdot w^2$
R^3_{25}:
$\cdot w$
R_{24}:
R^1_{25}:
$\cdot w$
C_6:

Lem S

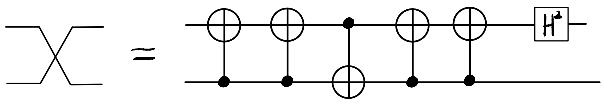
R_{18}: (1)

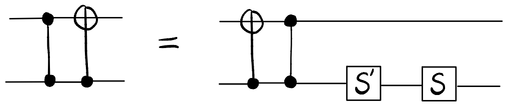


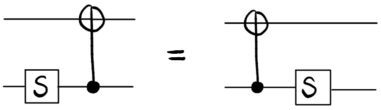


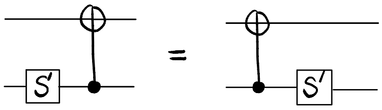
Proof cont.
R'_{23}
$\cdot w^2$
C_7, R_{15}
R'_{24}
$\cdot w^2$
Def 1
C_2, C_3
$\cdot w^2$
R'_{23}
$\cdot w^2 \cdot w^2$
C_1
C_3
$\cdot w$
R'_{25}
C_6
Def 7
=: R_{18}.(1).RHS.

□

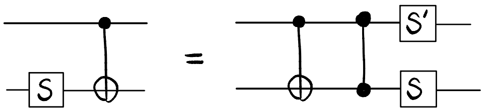
Def 5: 

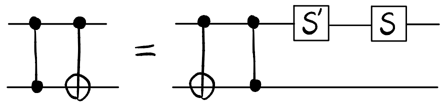
R^1_{25}:  $\cdot w$

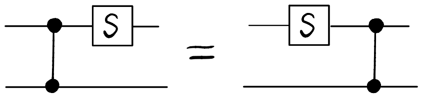
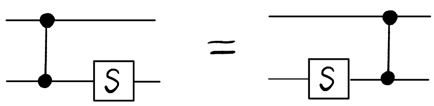
R_{24}: 

R'_{24}: 

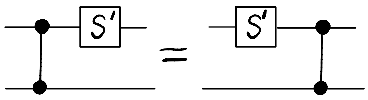
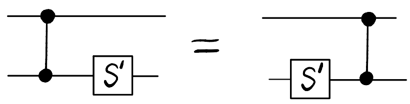
C1: $w^3 = 1$

R^4_{23}:  $\cdot w^2$

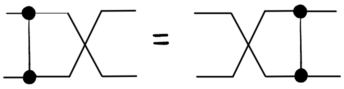
R^3_{25}:  $\cdot w$

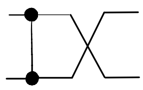
C7: (1)  (2) 

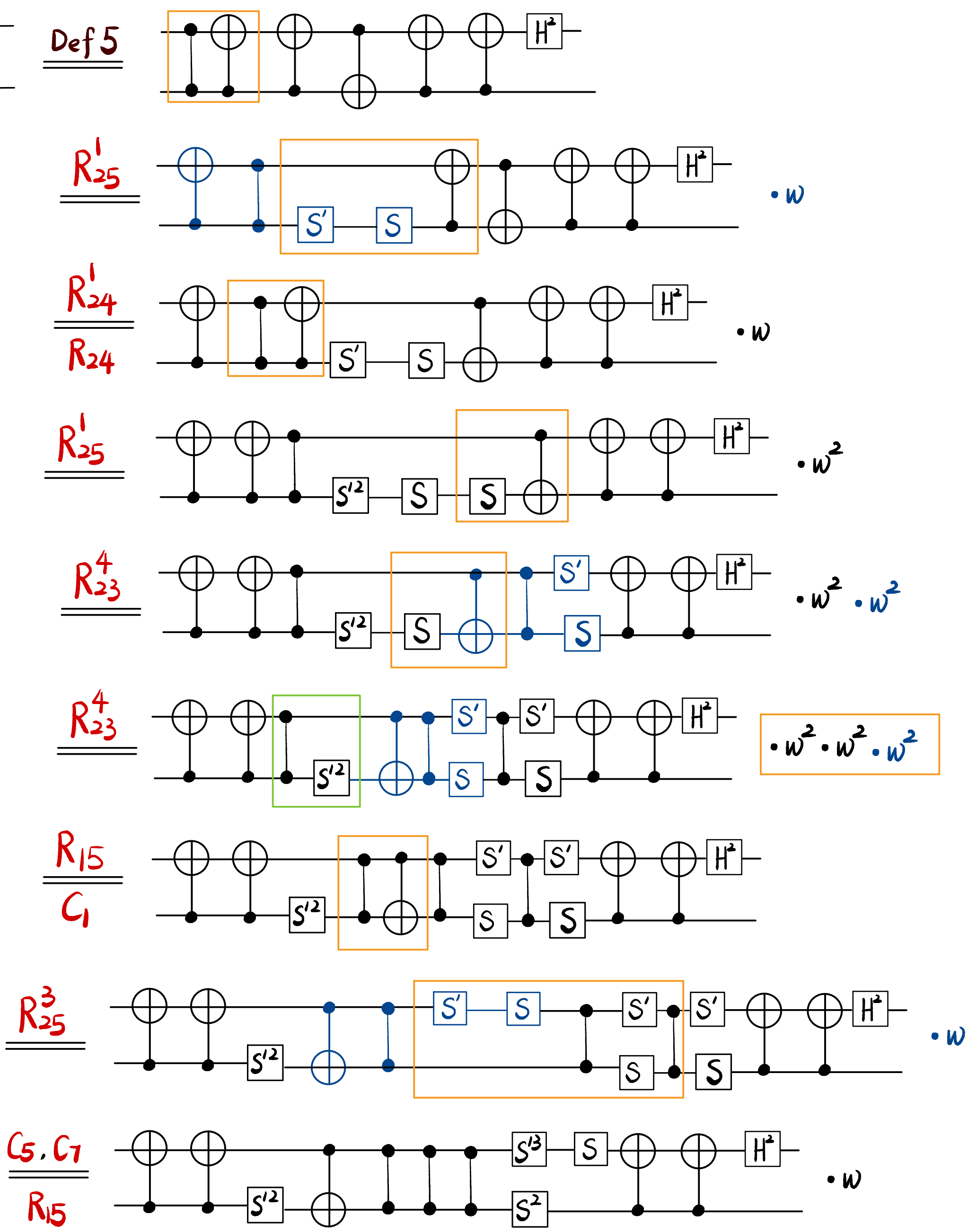
C5: $SH^2SH^2 = H^2SH^2S$ $\quad SS' = S'S$

R_{15}: (1)  (2) 

Lem T Def 1-3, C1-8, R_{15}, R_{23}, R_{24} & R_{25} imply

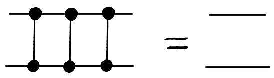
R_{17}: 

Proof: R_{17}. LHS := 

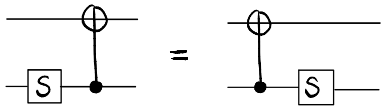


$C1: w^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$ $C5: SH^2SH^2 = H^2SH^2S$ $SS' = S'S$

C_6:

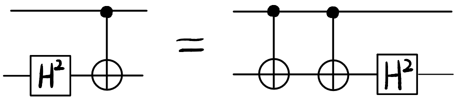


R_{24}:

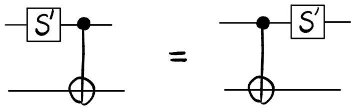


Def 1 : $S' := H\,H\,S\,H\,H$ $S'^2 := H\,H\,S^2\,H\,H$

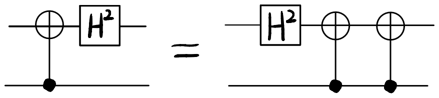
C_8^8:



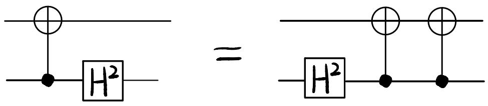
R_{24}^3:



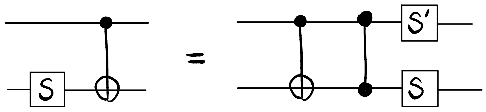
C_8^5: (1)



(2)



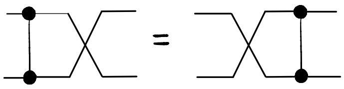
R_{23}^4:



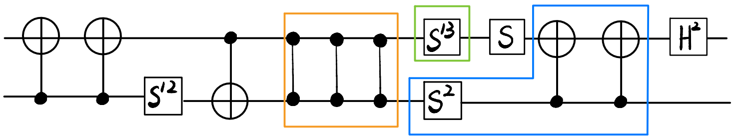
$\cdot w^2$

Lem T

R_{17}:

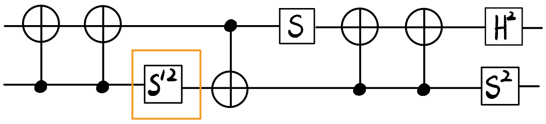


Proof cont.



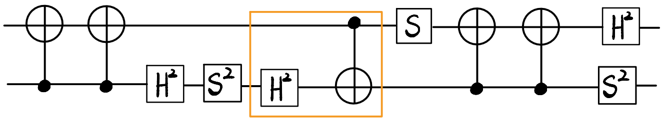
$\cdot w$

$\underset{C_6, R_{24}}{\overset{C_2, C_3}{=\!=}}$



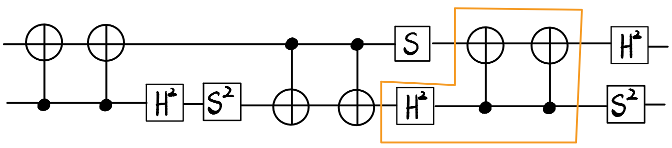
$\cdot w$

$\overset{\text{Def } 1}{=\!=}$



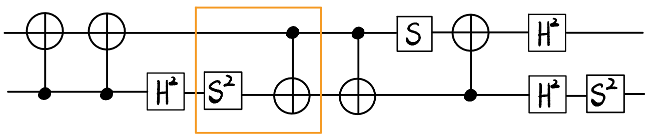
$\cdot w$

$\overset{C_8^8}{=\!=}$



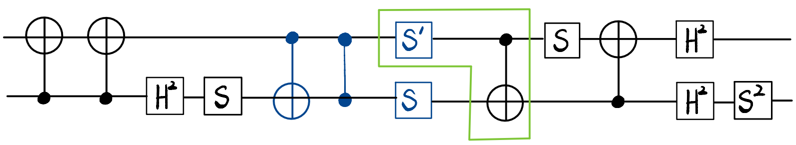
$\cdot w$

$\overset{C_8^5}{=\!=}$



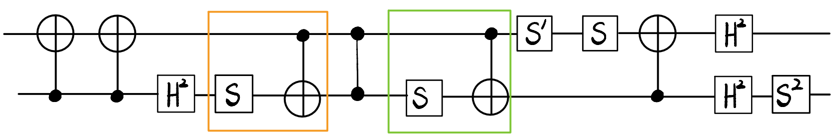
$\cdot w$

$\overset{R_{23}^4}{=\!=}$

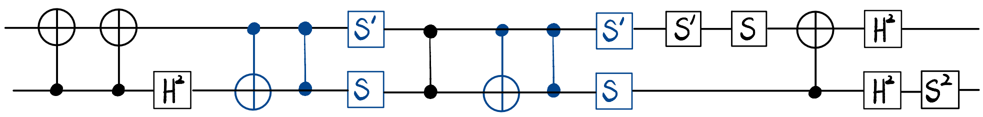


$\boxed{\cdot w \cdot w^2}$

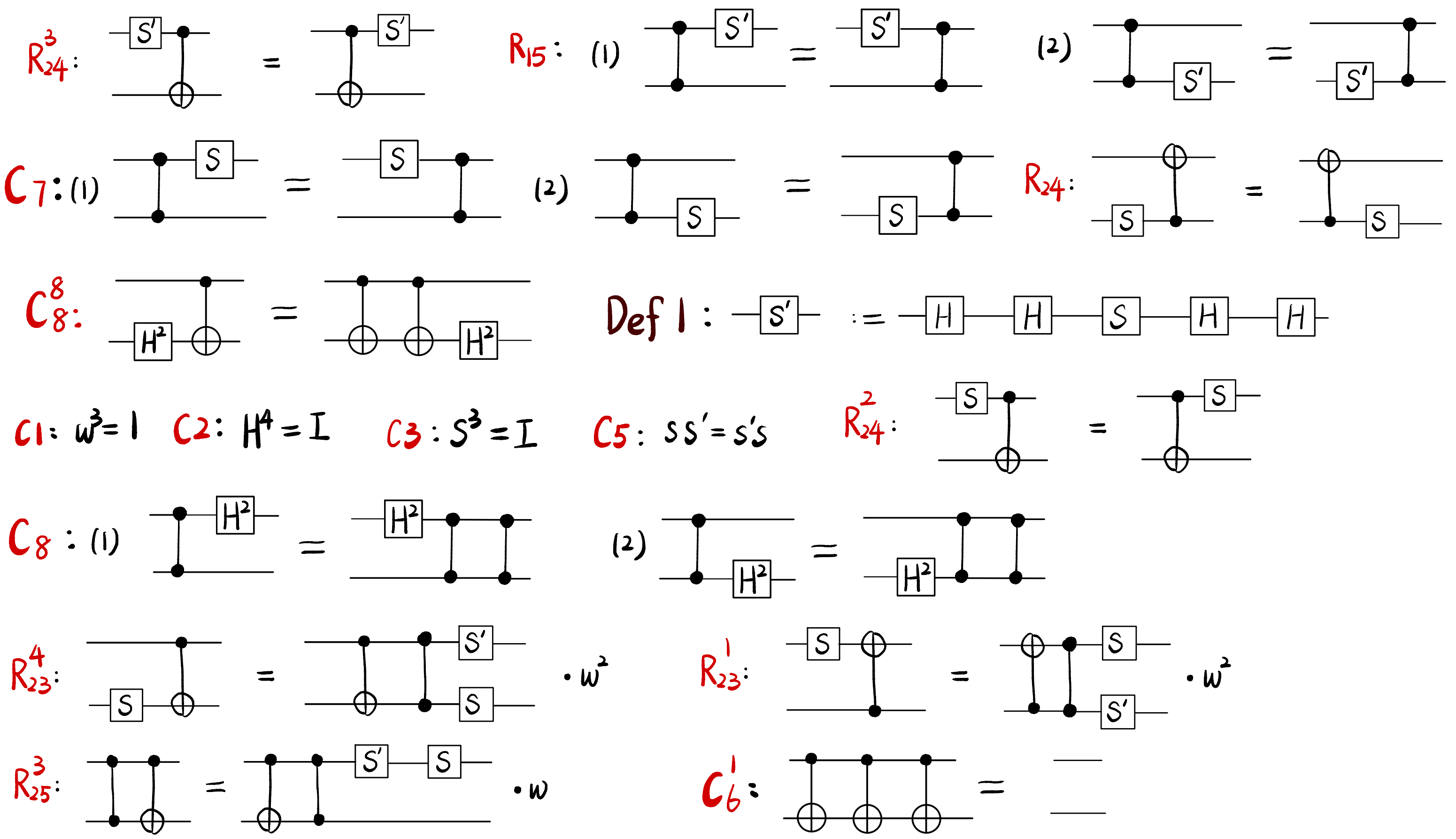
$\underset{C_1}{\overset{R_{24}^3}{=\!=}}$



$\overset{R_{23}^4}{=\!=}$

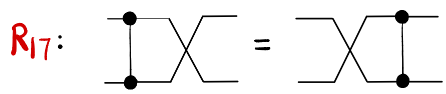


$\cdot w^2 \cdot w^2$

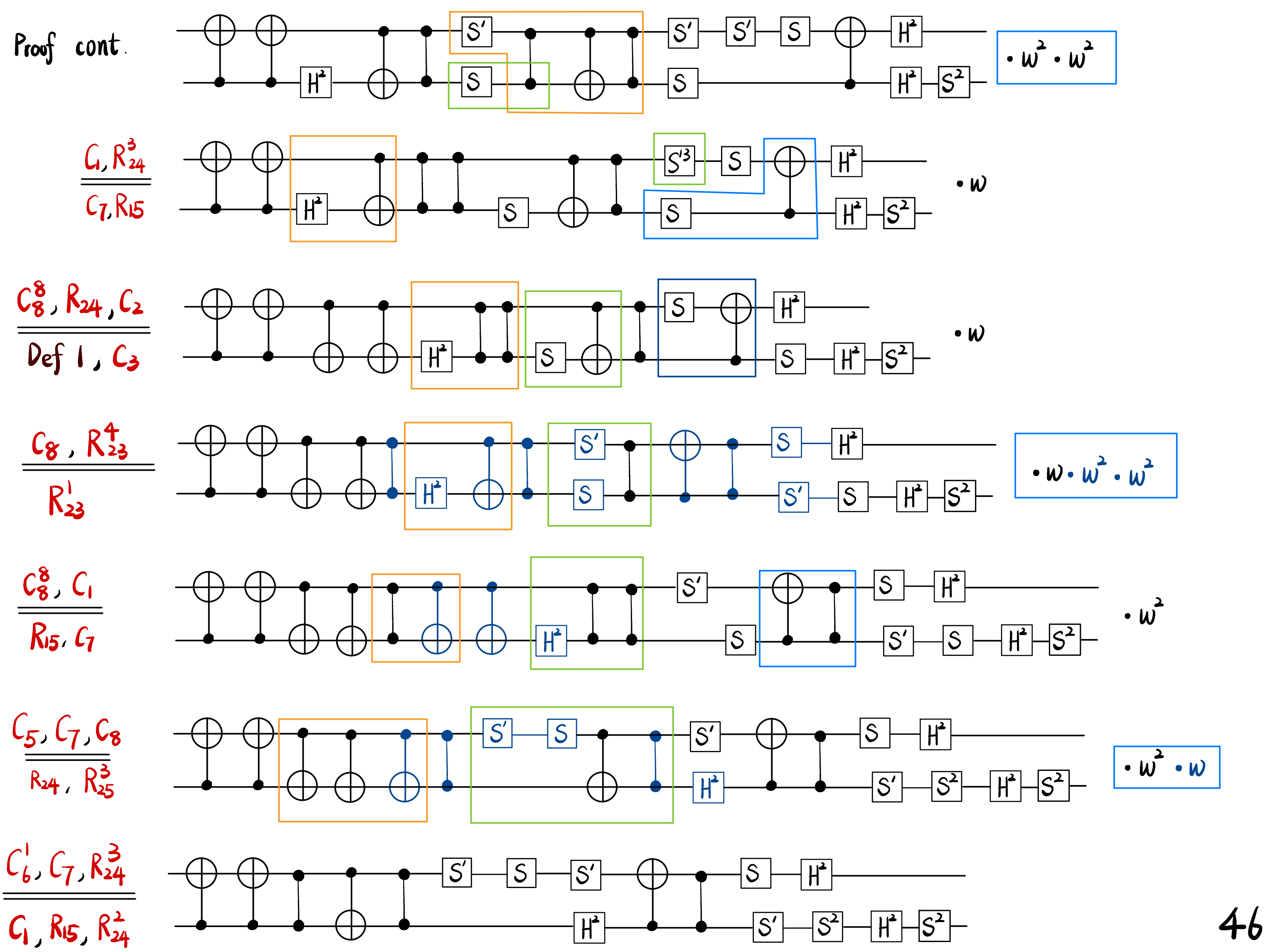


R^3_{24}:
R_{15}: (1)
(2)
C_7: (1)
(2)
R_{24}:
C_8^8:
Def 1 : S′ := H H S H H
C1: $\omega^3 = 1$
C2: $H^4 = I$
C3: $S^3 = I$
C5: $SS' = S'S$
R^2_{24}:
C_8 : (1)
(2)
R^4_{23}: $\cdot\, \omega^2$
R^1_{23}: $\cdot\, \omega^2$
R^3_{25}: $\cdot\, \omega$
C_6':

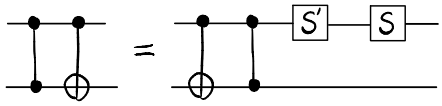
Lem T

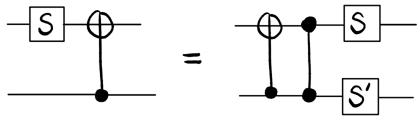


R_{17}:

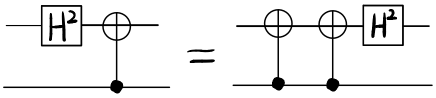
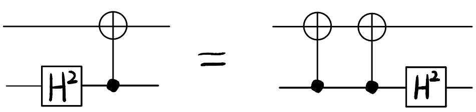


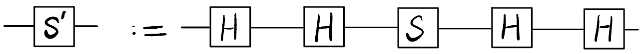
Proof cont.
$\cdot\, \omega^2 \cdot \omega^2$
C_1, R^3_{24}
C_7, R_{15}
$\cdot\, \omega$
C_8^8, R_{24}, C_2
Def 1, C_3
$\cdot\, \omega$
C_8, R^4_{23}
R^1_{23}
$\cdot\, \omega \cdot \omega^2 \cdot \omega^2$
C_8^8, C_1
R_{15}, C_7
$\cdot\, \omega^2$
C_5, C_7, C_8
R_{24}, R^3_{25}
$\cdot\, \omega^2 \cdot \omega$
C_6', C_7, R^3_{24}
C_1, R_{15}, R^2_{24}

R_{25}^3:

$\cdot w$

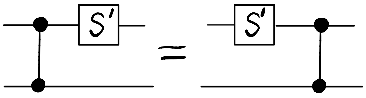
R_{23}^1:

$\cdot w^2$

$C1$: $w^3 = 1$

C_8^6: (1)

(2)


Def 1 :


$C5$: $SS' = S'S$ $\quad$ $C2$: $H^4 = I$ $\quad$ $C3$: $S^3 = I$

R_{15}: (1)

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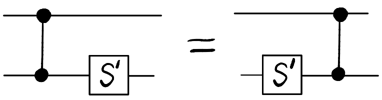
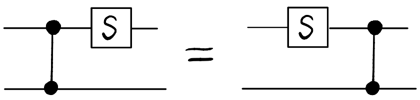


C_7: (1)

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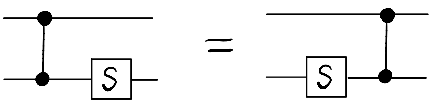
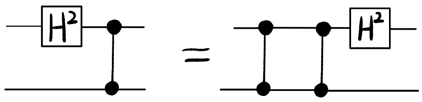
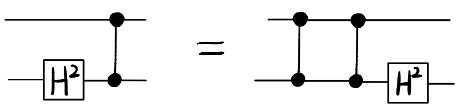
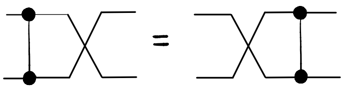
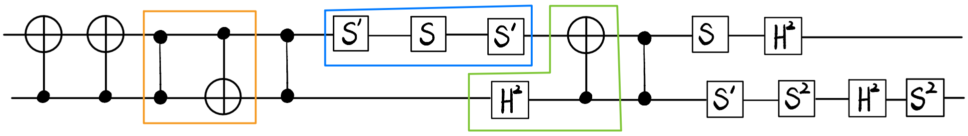
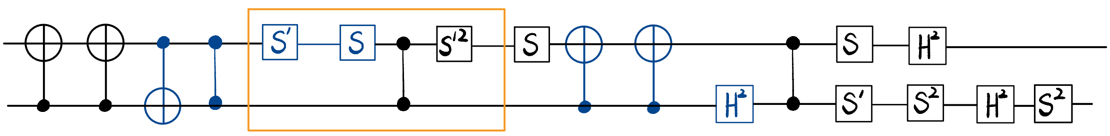


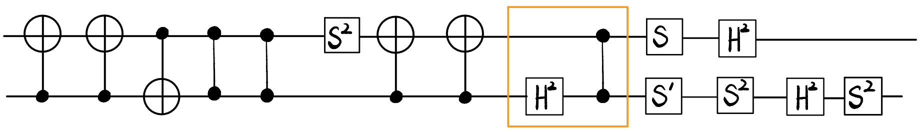
C_8^1: (1)

(2)


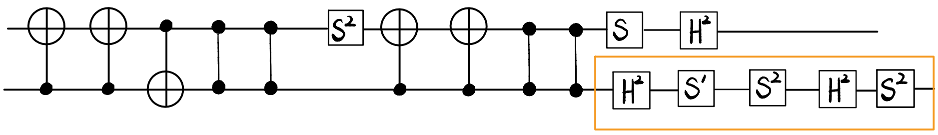
Lem T

R_{17}:


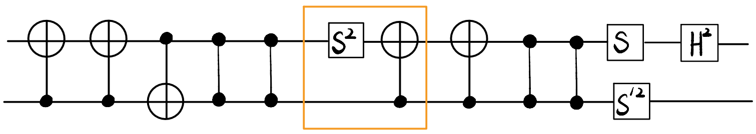
Proof cont.


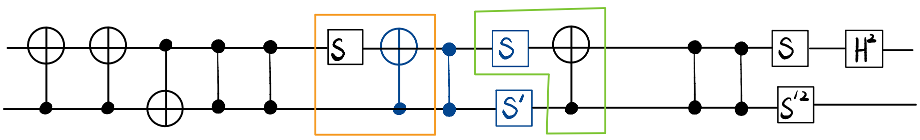
$\dfrac{R_{25}^3}{C_5, C_8^6}$

$\cdot w$

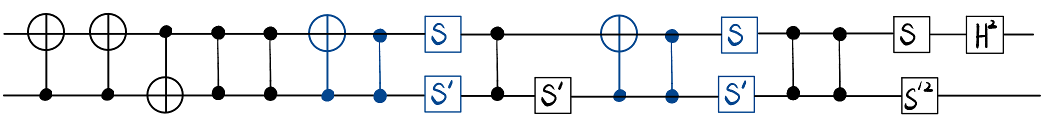
$\dfrac{C_7, R_{15}, C_2}{\text{Def } 1, C_3}$

$\cdot w$

$\overline{\overline{C_8^1}}$

$\cdot w$

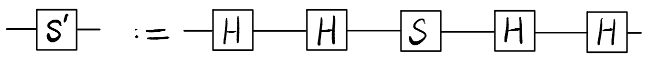
$$\boxed{H^2(H^2}SH^2)S^2H^2S^2 \overset{C_2}{=} S\,\boxed{H^2S^2H^2}\,S^2 \overset{\text{Def } 1}{=} \boxed{SS'^2}\,S^2 \overset{C_5}{=} S'^2\,\boxed{SS^2} \overset{C_3}{=} S'^2$$

$\dfrac{C_2, C_3}{\text{Def } 1, C_5}$

$\cdot w$

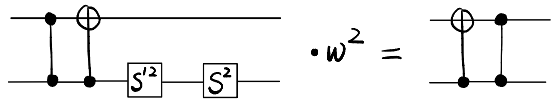
$\overline{\overline{R_{23}^1}}$

$\boxed{\cdot w \cdot w^2}$

$\dfrac{R_{23}^1}{C_1}$

$\cdot w^2 \cdot w^2$

Def 1 :

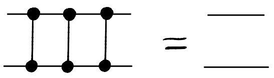


R^2_{25} :

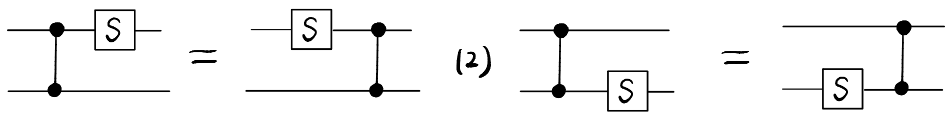


C1 : $w^3 = 1$ C2 : $H^4 = I$ C3 : $S^3 = I$ C5 : $SS' = S'S$

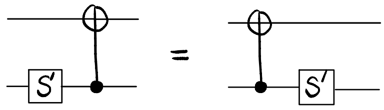
C6 :



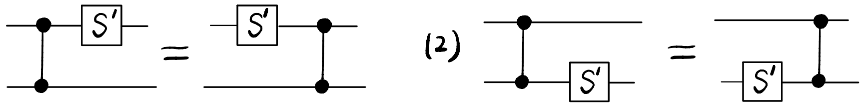
C7 : (1) (2)



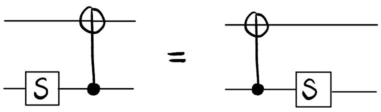
R^1_{24} :



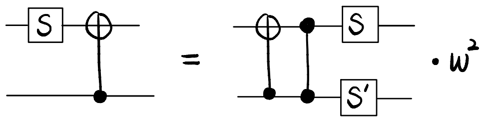
R_{15} : (1) (2)



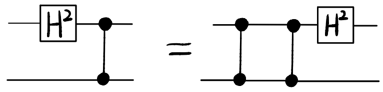
R_{24} :



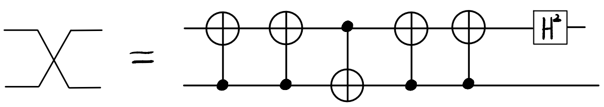
R^1_{23} :



C^1_8 :



Def 5 :



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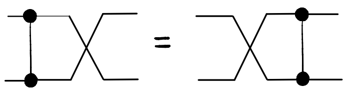
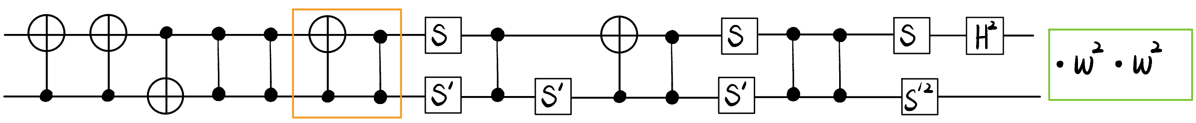
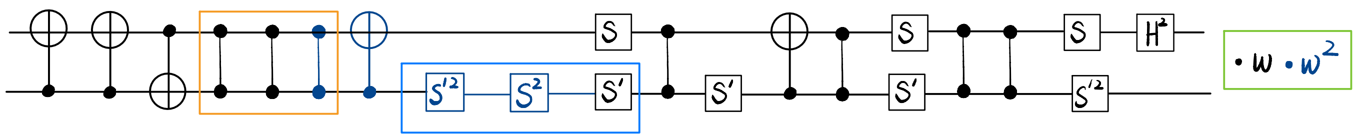
R_{17} :



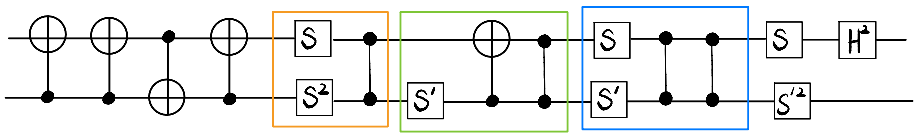
Proof cont.



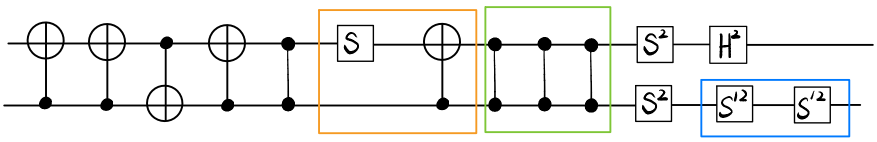
$\underline{\underline{R^2_{25}}}$ / C_1



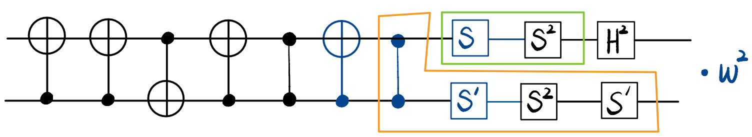
C_1, C_2, C_3 / C_5, C_6



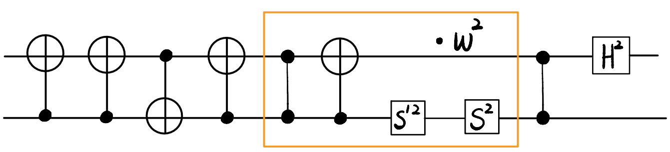
C_7, R_{24} / R_{15}, R^1_{24}



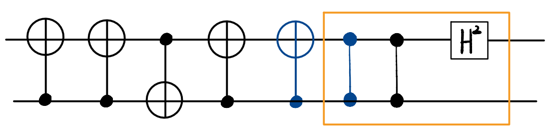
C_2, C_3, C_6 / Def 1, R^1_{23}



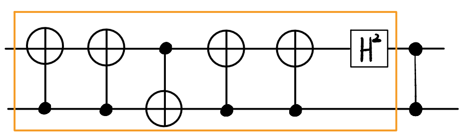
C_3, C_7 / C_5, R_{15}



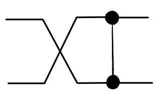
$\underline{\underline{R^2_{25}}}$



$\underline{\underline{C^1_8}}$



$\underline{\underline{\text{Def 5}}}$



$=:$ R_{17}.RHS

$\square$