

$$C_{16}^3 : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{16}^8 : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^7 : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_6^*: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_7^*: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{25}^6 : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \cdot w^2$$

$$R_{15}^*: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Lem M

$$R_{43} : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{43} \cdot \text{RHS} = \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w^2$$

$$R_{43} \cdot \text{LHS} := \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \stackrel{C_{16}^8}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\stackrel{C_{16}^8}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \quad \stackrel{C_{13}^7, C_6}{\stackrel{C_{16}^3}{=}} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\stackrel{C_{16}^3}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \quad \stackrel{C_6^*}{\stackrel{C_{13}^7}{=}} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\stackrel{C_7^7}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \quad \stackrel{R_{25}^6}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w^2$$

$$\stackrel{C_7^*}{\stackrel{R_{15}}{=}} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w^2 \quad \stackrel{C_{13}^2}{=} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w^2 =: R_{43} \cdot \text{RHS}.$$

$$\begin{aligned}
R_{23}^{q*}: \quad & \text{Diagram showing } R_{23}^{q*} = \text{Diagram with } S \text{ boxes} \cdot w^2 \\
& \text{Def 2: } \text{Diagram with } \oplus \text{ and } H \text{ boxes} := \text{Diagram with } H \text{ and } H^\dagger \text{ boxes} \\
& \text{Def 7: } \text{Diagram with } \oplus \text{ and } X \text{ boxes} := \text{Diagram with } X \text{ and } X \text{ boxes} \\
& \text{Def 5: } \text{Diagram with } \oplus \text{ and } Z \text{ boxes} := \text{Diagram with } Z \text{ and } Z \text{ boxes} \\
C_7: \quad & \text{Diagram with } S \text{ box} = \text{Diagram with } S \text{ box} \\
R_{24}: \quad & \text{Diagram with } S \text{ box} = \text{Diagram with } \oplus \text{ box} \\
C_{13}: \quad & \text{Diagram with } \oplus \text{ box} = \text{Diagram with } \oplus \text{ box}
\end{aligned}$$

$$C_1: w^3 = I$$

$$C_3: S^3 = I$$

$$C_5: SS' = S'S$$

Lem N Def 2, Def 5, Def 7, $C_1, C_3, C_5, G_1, G_3, G_5, C_7, C_{13}, C_{16}, R_{23}^{q*}, R_{25}^{q*}$ & R_{24} imply

$$\begin{aligned}
R_{44}: \quad & \text{Diagram with } S \text{ boxes} = \text{Diagram with } S \text{ boxes} \cdot S^3 \cdot S^3 \cdot S' \cdot S^2
\end{aligned}$$

$$\begin{aligned}
\text{Proof: } R_{44} \cdot \text{RHS} := & \text{Diagram with } S \text{ boxes} \cdot S^2 \cdot S^2 \cdot S' \cdot S^2
\end{aligned}$$

$$\begin{aligned}
R_{23}^{q*} & \text{Diagram with } S \text{ boxes} \cdot S^3 \cdot S^3 \cdot S' \cdot S^2 \cdot w^2
\end{aligned}$$

$$\begin{aligned}
\frac{R_{23}^{q*}}{C_3} & \text{Diagram with } S \text{ boxes} \cdot S^2 \cdot S^2 \cdot S' \cdot S^2 \cdot w^2 \cdot w^2
\end{aligned}$$

$$\begin{aligned}
\frac{C_1, C_3}{C_5} & \text{Diagram with } S \text{ boxes} \cdot S' \cdot S^2 \cdot w
\end{aligned}$$

$$\begin{aligned}
\frac{C_7}{R_{24}} & \text{Diagram with } S \text{ boxes} \cdot S' \cdot S^2 \cdot w
\end{aligned}$$

$$\begin{aligned}
\frac{C_{13}}{} & \text{Diagram with } S \text{ boxes} \cdot S' \cdot S^2 \cdot w
\end{aligned}$$

$$C_{16}^{7'}: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{16}^{2'}: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{25}^7: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \cdot w$$

$$C_{13}: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_5: SS' = S'S$$

$$C_{13}^1: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Lem N

$$R_{44}: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \end{array}$$

Proof cont. $R_{44} \cdot \text{RHS} = \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w$

$$R_{44} \cdot \text{LHS} := \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\underline{C_{16}^{7'}}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\underline{R_{25}^7} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w$$

$$\underline{C_{16}^{2'}} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w$$

$$\underline{C_{13}, C_5} \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \cdot w =: R_{44} \cdot \text{RHS}.$$

III

11

R_{23}^{q*} :
 $=$
 $\cdot w^2$ Def 2: **C1:** $w^3 = I$
C2: $H^4 = I$
C3: $S^3 = I$
C5: $SS' = S'S$
Def 7: Def 5:
C7:
 $=$
R24:
 $=$
C13:
 $=$
C7, C2, C3, C5, G1, G8, C16, C1b, R23*, R25*, R24 imply

R47:
 $=$
 $\cdot w$

Proof: $R_{47}.RHS :=$
 $\cdot w$

R_{23}^{q*}
 $\cdot w \cdot w^2$

C_7, R_{24}

Def 2

G8
Def 2

$$C_{13}^2 : \text{Diagram} = \text{Diagram} \quad \text{Def 2: } \text{Diagram} := \text{Diagram} \quad C_1: w^3=I \quad C_2: H^4=I \quad C_3: S^3=I$$

$$C_5: SS'=S'S$$

$$C_{13}^3 : \text{Diagram} = \text{Diagram} \quad C_{16}^{14} : \text{Diagram} = \text{Diagram}$$

Lem 0

$$R_{47}: \text{Diagram} = \text{Diagram} \cdot w$$

Proof cont. Hence R_{47} :

$$C_{13}^2 \parallel C_2$$

$$R_{47}: \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$\text{Def 2} \parallel C_8$

$$R_{47}: \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$\parallel C_2$

$$R_{47}: \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$R_{47}. \text{RHS} := \text{Diagram} \xrightarrow{\text{C13}^3} \text{Diagram}$$

$$C_{16}^{14} : \text{Diagram}$$

$$C_{16}^{15}: \quad \begin{array}{c} \text{Diagram of } C_{16}^{15} \text{ on } 4 \times 4 \text{ grid} \\ \oplus \end{array} = \quad \begin{array}{c} \text{Diagram of } C_{16}^{15} \text{ on } 4 \times 4 \text{ grid} \\ \oplus \end{array}$$

$$C_{16}^{8'} : \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \quad \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_6^*: \quad \begin{array}{c} \bullet \\ \text{---} \\ | \\ \bullet \\ \text{---} \\ | \\ \bullet \end{array} \quad = \quad \underline{\hspace{2cm}} \quad \underline{\hspace{2cm}}$$

$$C_6^{2*} = \begin{array}{c} \text{Diagram of } C_6^{2*} \\ \text{A hexagon with vertices at } (-1, 0) \text{ and } (1, 0), \text{ and midpoints at } (\pm 1, \pm 1). \\ \text{Each edge has length } \sqrt{2}. \end{array}$$

Lem 0

$$R_{47}: \quad \text{Circuit Diagram} = \quad \text{Circuit Diagram} \cdot w$$

Proof cont. Hence R_{47} :

The diagram consists of two parts. On the left, labeled R_{47} , there is a graph with four horizontal rows of vertices. A central column of three vertices is highlighted with an orange box. The top vertex of this column is connected to the top vertex of the second column from the left. The bottom vertex of the third column is connected to the bottom vertex of the second column from the left. The middle row has two vertices in the first and third columns, and one vertex in the second column. The bottom row has two vertices in the first and second columns, and one vertex in the third column. On the right, labeled \equiv C_6 , there is a graph with four horizontal rows of vertices. A central column of three vertices is highlighted with a green box. The top vertex of this column is connected to the top vertex of the second column from the left. The bottom vertex of the third column is connected to the bottom vertex of the second column from the left. The middle row has two vertices in the first and third columns, and one vertex in the second column. The bottom row has two vertices in the first and second columns, and one vertex in the third column. The label WTS is written between the two graphs.

$R_{47}:$

$\text{R}_{47}:$

WTS

$\equiv C_6^2$

R_{47} :

This completes the proof.

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$$R_{23}^{q*} = \text{Diagram} = \text{Diagram} \cdot w^2 \quad R_{24}: \quad C_1: w^3 = I \quad C_2: H^4 = I$$

$$C_3: S^3 = I$$

Def 7: :=

Def 2: :=

Def 5:

Lem P Def 2, Def 5, Def 7, $C_1, C_2, C_3, C_5, G_7, G_8, C_{13}, C_{16}$, R_{23}^{*} & R_{24} imply

$$R_{48}: \quad \text{Diagram} = \text{Diagram} \cdot w$$

Proof: $R_{48} \cdot \text{RHS} :=$

$$\stackrel{R_{23}^{q*}}{\equiv} \stackrel{R_{24}}{\equiv} \text{Diagram} \cdot w \cdot w^2$$

$$\stackrel{C_1}{\equiv} \stackrel{C_3}{\equiv} \text{Diagram}$$

Hence $R_{48}:$

$$R_{48}: \quad \text{Diagram} \stackrel{WTS}{=} \text{Diagram}$$

||| Def 2

$$R_{48}: \quad \text{Diagram} \stackrel{WTS}{=} \text{Diagram}$$

$$C_8^1: \quad \text{Diagram} = \text{Diagram}$$

$$C_8^6: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$C_8: \quad \text{Diagram} = \text{Diagram}$$

$$C_2: H^4 = I$$

Lem P

$$R_{48}: \quad \text{Diagram} = \text{Diagram} \cdot w$$

Proof cont.

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$C_8^1 \parallel C_8^6$$

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$\parallel \text{Def 2}$$

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$C_8 \parallel C_2$$

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$\parallel C_2$$

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$\parallel \text{Def 2}$$

$$R_{48}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$