

$$B_{00} = \text{X}$$

$$B_{2b} = H^2 S^{2b} H \quad R_{17}: \text{X} = \text{X}$$

$$B_{02} = H^2 \quad \text{Def 4: } \oplus := H \cdot H^3$$

$$\text{Def 4: } \oplus := H \cdot H^3$$

$$\text{Def 2: } \oplus := H \cdot H^3$$

$$\text{Def 7: } \oplus := H \cdot H^3$$

$$\text{Def 5: } \text{X} := \text{X}$$

$$C_{15}^1: \text{X} = \text{X}$$

$$R_{31}: \text{X} = \text{X}$$

$$C_{15}^{14}: \text{X} = \text{X}$$

$$\text{Def 3: } \text{X} := H \cdot H \cdot H \cdot H \cdot H$$

$$C_{15}^4: \text{X} = \text{X}$$

$$C_8: \text{X} = H^2 \cdot \text{X}$$

$$C_7: \text{X} = S \cdot \text{X}$$

$$R_{19}: H \cdot \text{X} = \text{X} \cdot H$$

Lem 25 Def 1-5, Def 7, C₂, C₃, C₇, C₈, C₅, R₁₆, R₁₇, R₁₉, R₃₁ & R₅₈ imply

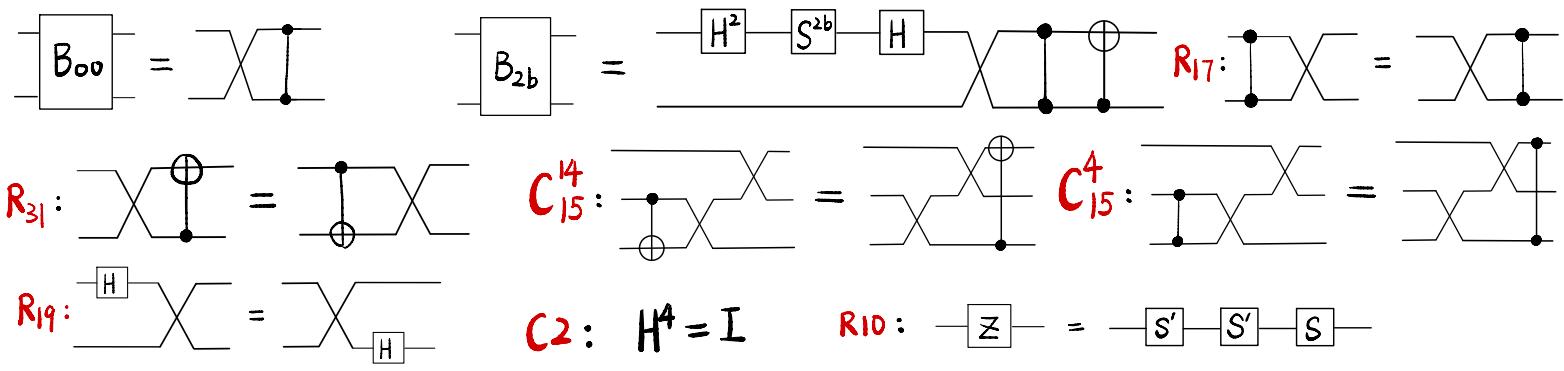
$$7.7-9.(3) \quad B_{2b} B_{02} = B_{2b} B_{00} H H S S Z \cdot S S H H H S \cdot w$$

$$\text{Proof: } 7.7-9.(3). \text{LHS} := B_{2b} B_{02} \stackrel{\text{def}}{=} H^2 S^{2b} H \cdot \text{X}$$

$$\frac{R_{17}, R_{31}}{C_{15}^4, C_{15}^{14}} \quad \text{RHS} = H^2 S^{2b} H \cdot \text{X}$$

$$\frac{C_8}{\text{RHS}} = H^2 \cdot \text{X}$$

$$\frac{C_7}{C_5} = H^2 \cdot \text{X}$$



Lem 25

$$7.7-9.(3) \quad B_{02} = B_{00} H H S S \cdot w$$

Proof cont:

$$\begin{aligned} 7.7-9.(3).\text{RHS} &:= B_{00} H H S S \cdot w \\ &= B_{2b} S S H H H H S \cdot w \\ &\stackrel{\text{def}}{=} H^2 S^{2b} H \text{ (orange box)} \text{ } \text{ (green box)} H^2 S^2 \text{ (blue box)} H^3 S \cdot w \end{aligned}$$

$$\begin{aligned} &\frac{R_{17}, R_{31}, \text{Def 4}}{C_{15}^4, C_{15}^4, C_2} \\ &H^2 S^{2b} H \cdot w \end{aligned}$$

$$\begin{aligned} &\stackrel{R_{19}}{=} \text{ (orange box)} H^2 S^2 \text{ (blue box)} S^{12} S \cdot w \\ &H^2 S^{2b} \text{ (orange box)} H^2 S^2 \text{ (blue box)} S^{12} S \cdot w \end{aligned}$$

Hence,

$$\begin{aligned} &H^2 S^{2b} \text{ (orange box)} H^2 \text{ (blue box)} \text{ (green box)} \text{ (purple box)} \cdot w \\ &H^2 S^{2b} \text{ (orange box)} H^2 S^2 S^{12} S \cdot w \end{aligned}$$

$$C_2 : H^4 = I \quad C_3 : S^3 = I \quad R_{16} : \text{Diagram} = \text{Diagram}$$

$R_{58}:$

Lem 25

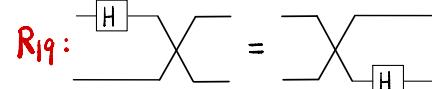
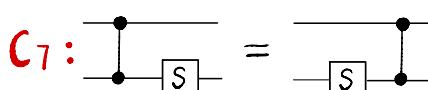
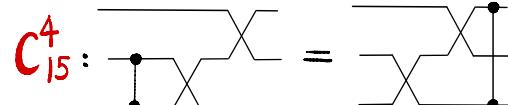
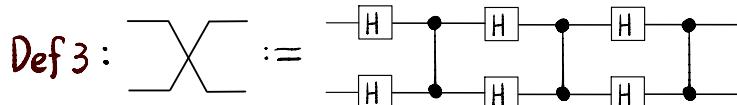
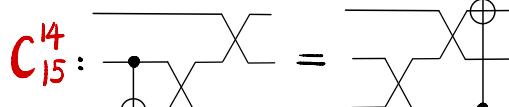
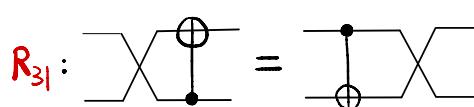
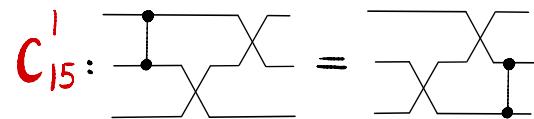
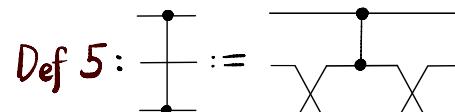
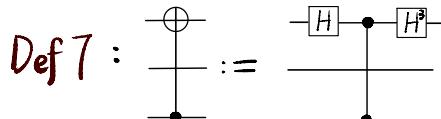
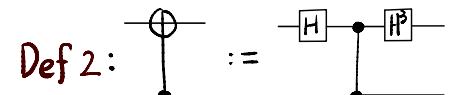
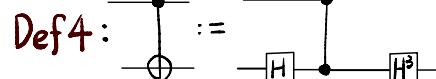
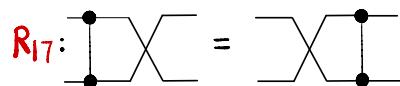
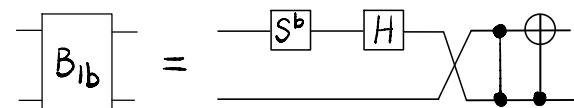
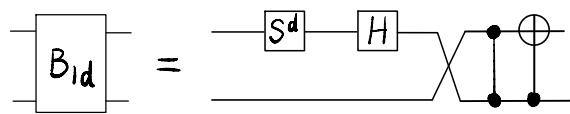
7.7-9.(3)

Proof cont:

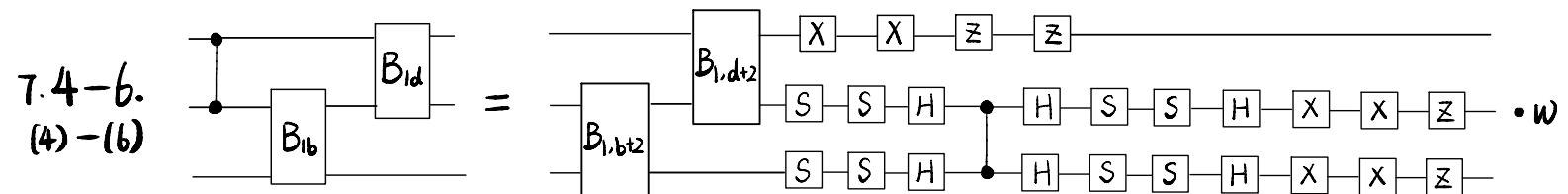
$C_2, G_3 \parallel R_{16}$

By R_{58} , this completes the proof.

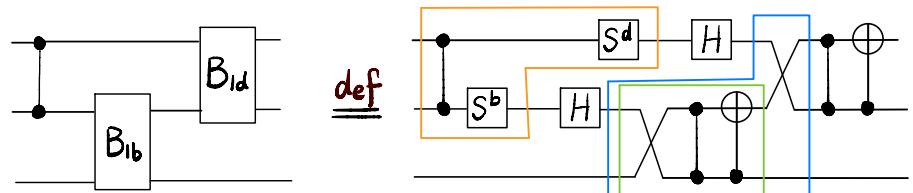




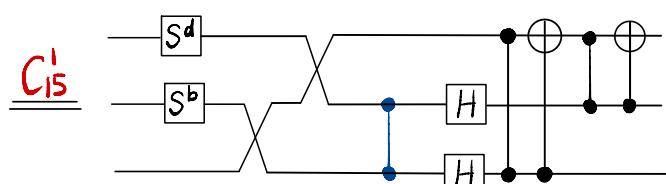
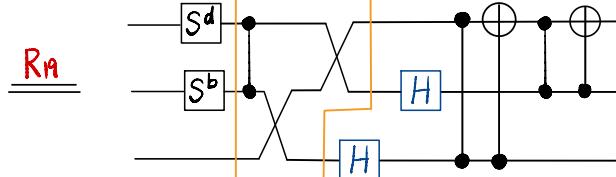
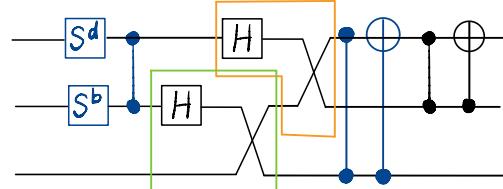
Lem 26 Def 1-5, Def 7, C₂, C₃, C₅, C₇, C₁₅, R₇, R₁₀, R₁₁, R₁₆, R₁₇, R₁₈, R₁₉, R₃₁ & R₅₉ imply

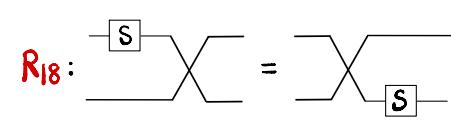
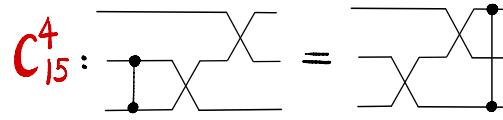
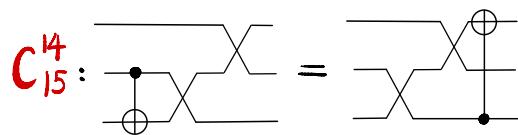
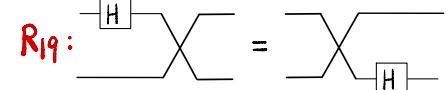
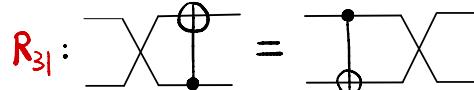
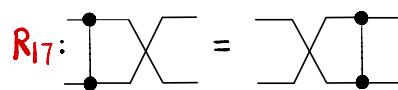
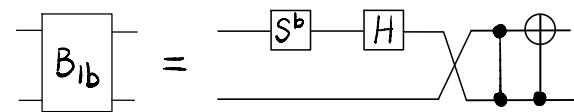
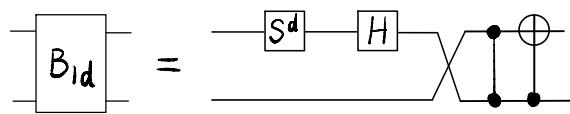


Proof: 7.4-6.(4)-(6). LHS :=



C₇, R₁₇, R₃₁
C₁₅⁴, C₁₅¹⁴





Lem 26

$$7.4-6. (4)-(6) \quad B_{1d} = B_{1,b2} \cdot \underbrace{\dots}_{B_{1,d+2}} \cdot \underbrace{\dots}_{S \cdot S \cdot H \cdot \dots \cdot H \cdot S \cdot S \cdot H \cdot X \cdot X \cdot Z} \cdot w$$

Proof cont:

$$7.4-6.(4)-(6). \text{ RHS} := B_{1,b2} \cdot \underbrace{\dots}_{B_{1,d+2}} \cdot \underbrace{\dots}_{S \cdot S \cdot H \cdot \dots \cdot H \cdot S \cdot S \cdot H \cdot X \cdot X \cdot Z} \cdot w$$

$$\underline{\text{def}} \quad \begin{array}{c} S^d \quad S^2 \quad H \\ \boxed{S^b \quad S^2 \quad H} \quad \text{CNOT} \quad S^2 \quad H \\ S^2 \quad H \end{array} \cdot w$$

$$\underline{R_{17}, R_{31}} \quad \underline{C_{15}^4, C_{15}^{14}} \quad \begin{array}{c} S^d \quad S^2 \quad H \\ \boxed{S^b \quad S^2 \quad H} \quad \text{CNOT} \quad S^2 \quad H \\ S^2 \quad H \end{array} \cdot w$$

$$\underline{R_{18}} \quad \begin{array}{c} S^d \quad S^2 \\ \boxed{S^b \quad S^2} \quad \text{CNOT} \quad H \quad \text{CNOT} \quad S^2 \quad H \\ S^2 \quad H \end{array} \cdot w$$

$$\underline{R_{18}} \quad \begin{array}{c} S^d \\ \boxed{S^b \quad S^2 \quad H} \quad \text{CNOT} \quad S^2 \quad H \\ S^2 \quad H \end{array} \cdot w$$

$$R7: \quad X^2 = H S S H H S H$$

$$R10: \quad Z = S' S' S \quad R11: \quad Z^2 = S' S S$$

$$C_3: S^3 = I \quad R_{16}: \quad \text{Diagram} = \text{_____}$$

Lem 26

$$7.4-6. (4)-(6) \quad B_{1d} B_{1b} = B_{1,d+2} B_{1,b+2} \quad \text{Diagram} \cdot w$$

Proof cont:

$$7.4-6.(4)-(6). \text{ RHS} = \text{Diagram} \cdot w$$

$$X^2 Z^2 \xrightarrow[R_I]{R_{II}} HS^2 H^2 SH S' S^2 \quad X^2 Z \xrightarrow[R_I]{R_{II}} HS^2 H^2 SH S'^2 S$$

$$\frac{R_I}{R_{II}, R_{III}} \quad \text{Diagram} \cdot w$$

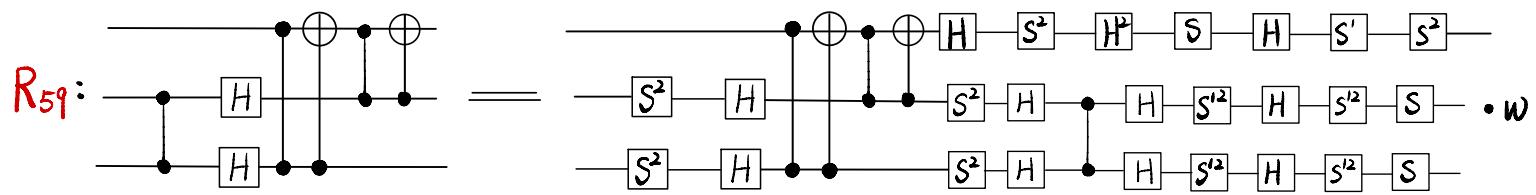
$$\text{Hence, } \text{Diagram} \xrightarrow[WTS]{} \text{Diagram}$$

$$\frac{S^d}{S^b} \quad \text{Diagram} \cdot w$$

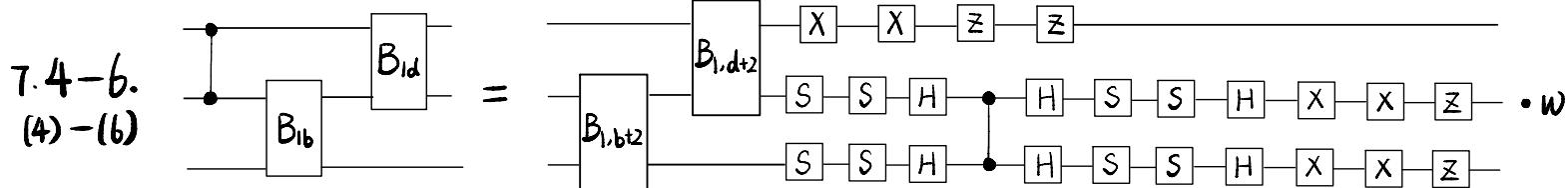
$$C_3 \parallel R_{16}$$

$$\frac{H}{H} \quad \text{WTS} \quad \text{Diagram} \cdot w$$

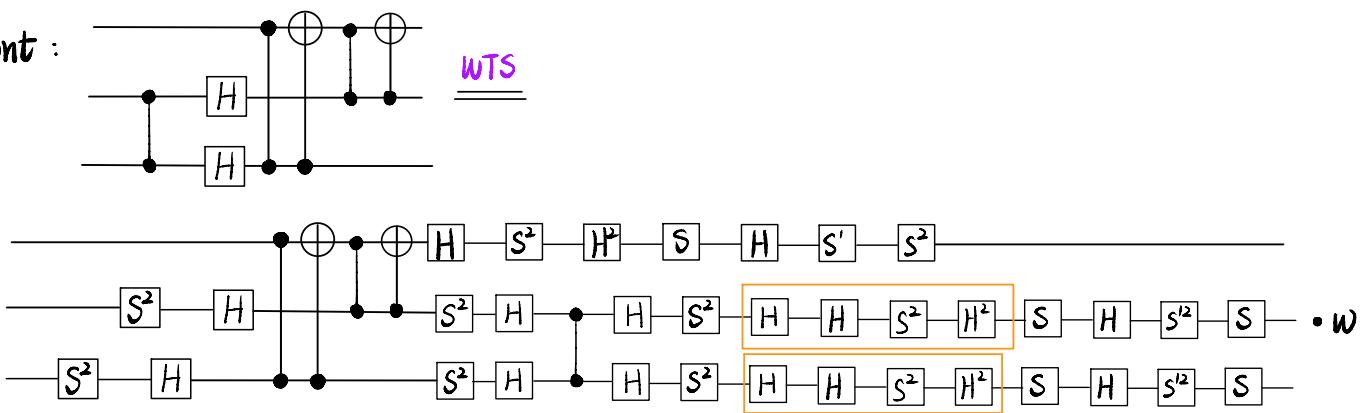
Def 1 : $S' := H H S H H$ $C_3 : S^3 = I$ $C_5 : S'S = SS'$ $C_2 : H^3 = I$



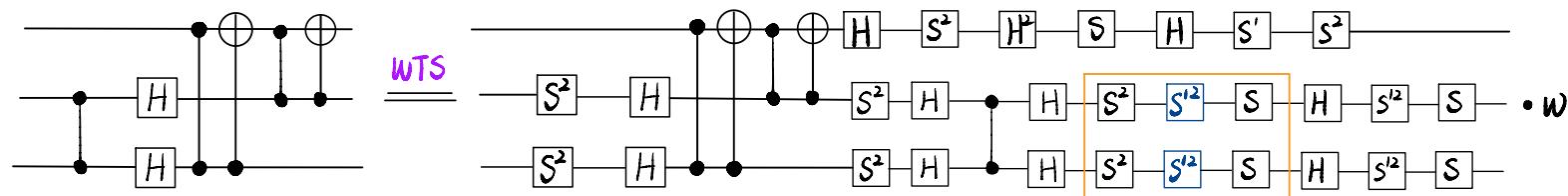
Lem 26



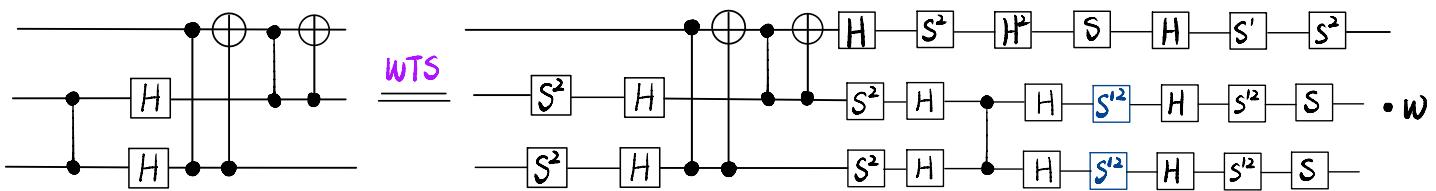
Proof cont:



$C_2 \parallel Def 1$



$C_3 \parallel C_5$



By R_{59} , this completes the proof.

