

Def 3: :=

R₁₇: =

C₃: $S^3 = I$

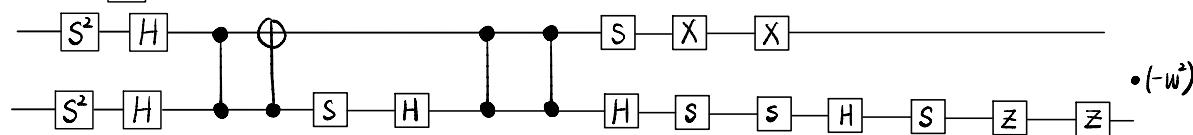
R₁₉: (1) = (2) =

R₁₈: (1) = (2) =

C₇: (1) = (2) =

R₁₈: (1) = (2) =

R₃₄: =



Lem 52 By Def 3, C₃, C₇, C₈, R₁₇, R₁₈, R₁₉ & R₃₄,

4. (g)

$\bullet (-w^2)$

(h)

$\bullet (-w^2)$

Proof: 4. (g). LHS := def

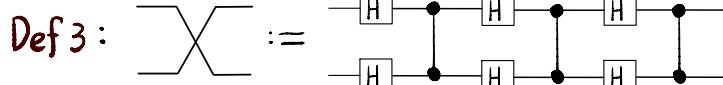
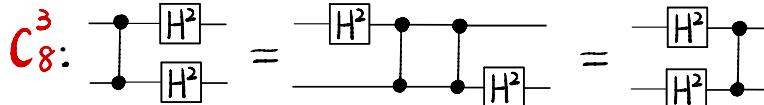
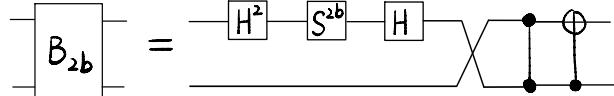
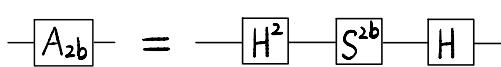
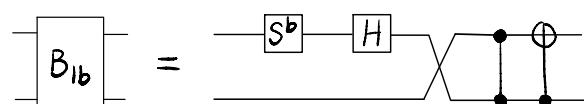
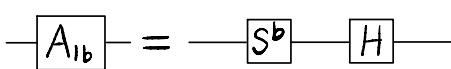
G

R17

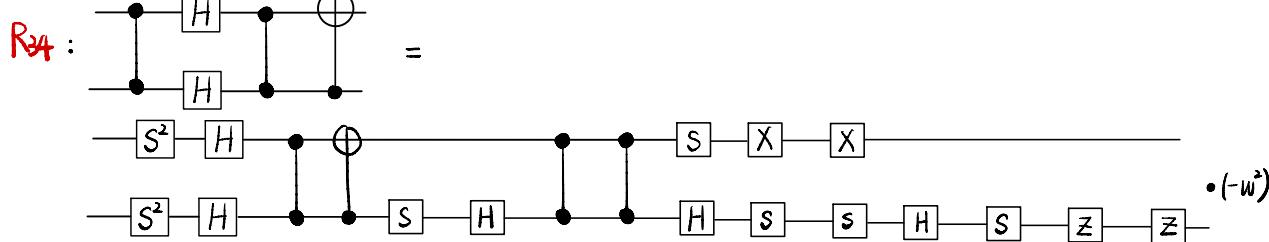
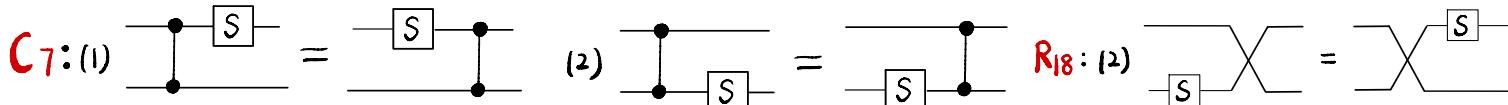
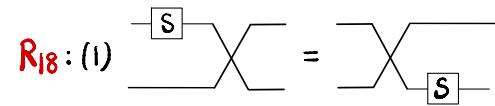
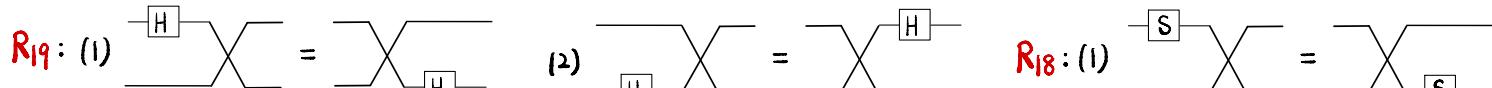
R34

$\bullet (-w^2)$ R19, C₃ R18

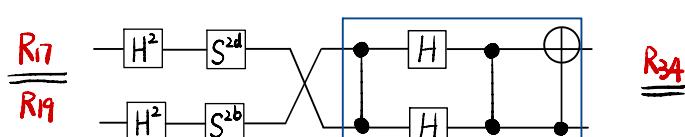
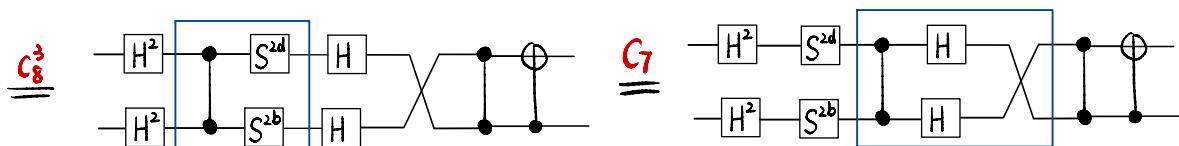
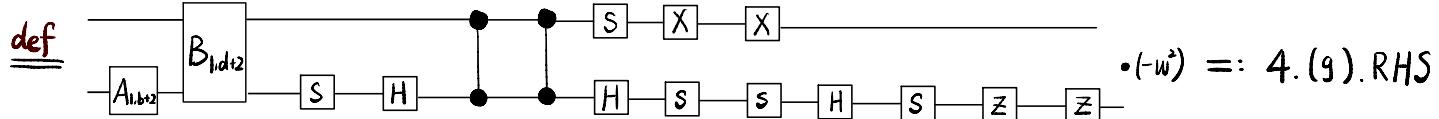
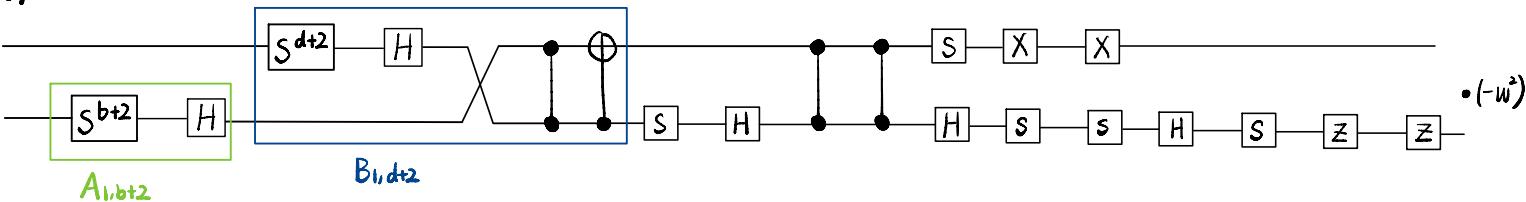
$\bullet (-w^2)$

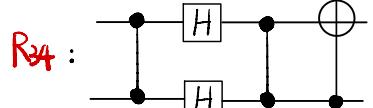
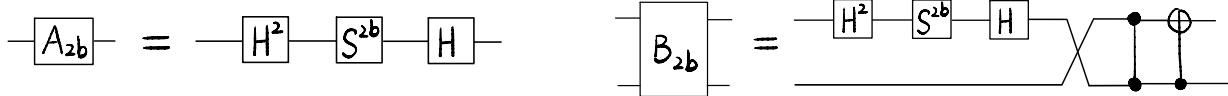


$$C_3: S^3 = I$$

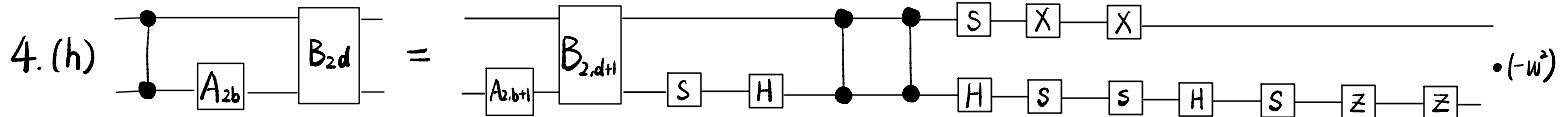
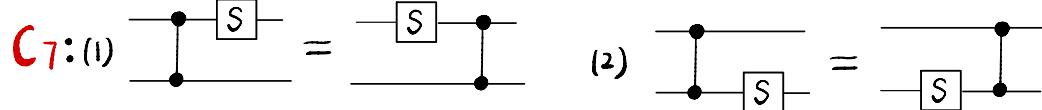
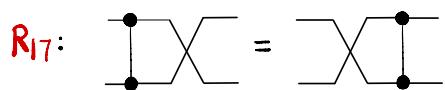
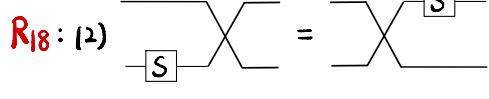
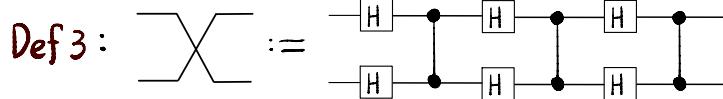
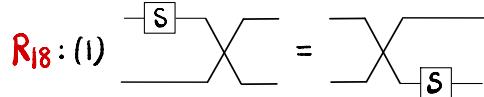
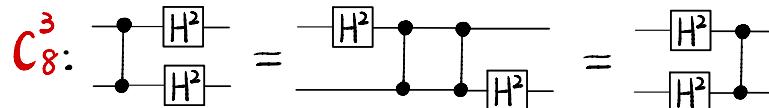
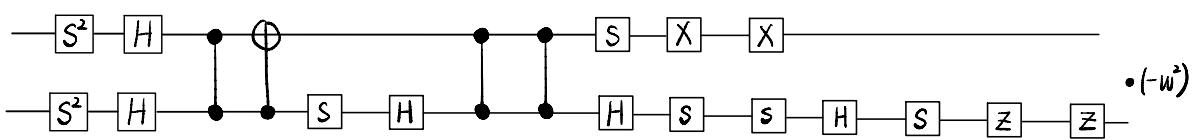


Proof cont.

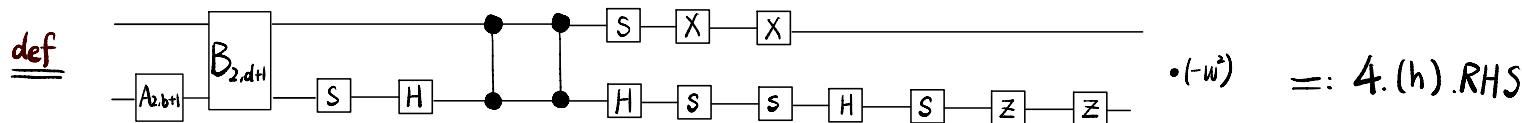
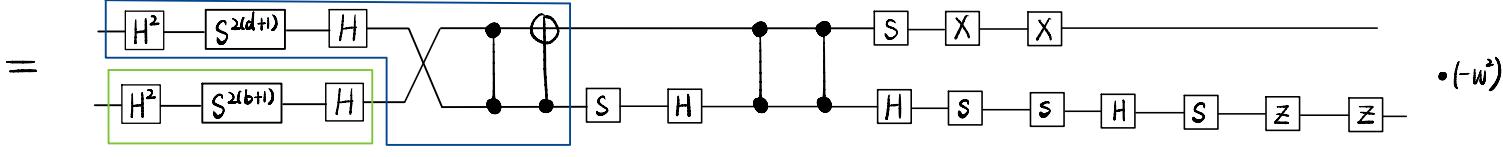
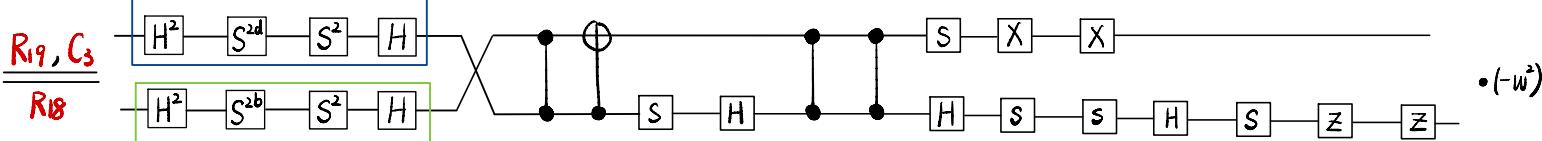
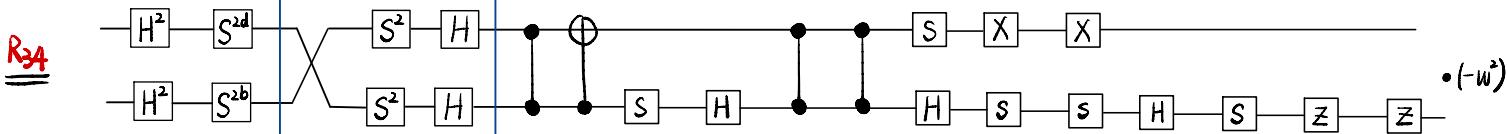
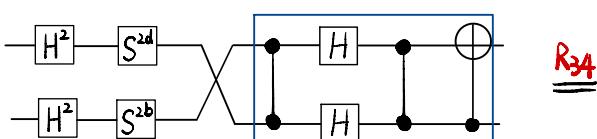




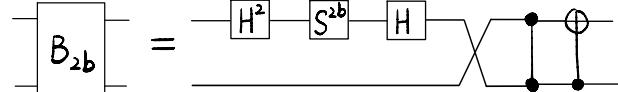
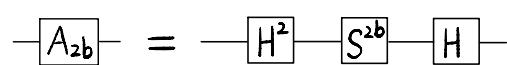
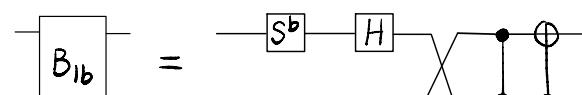
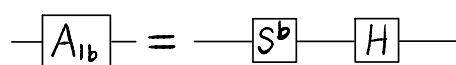
=



Proof cont.



□



$$R_{18} : (1) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

$$(2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \times \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \boxed{S}$$

$$C_3 : S^3 = I$$

$$R_{1q} : (1) \quad \begin{array}{c} \text{---} \\ | \end{array} \boxed{H} \begin{array}{c} \text{---} \\ | \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \end{array} \quad \begin{array}{c} \text{---} \\ | \end{array} \quad \boxed{H}$$

$$(2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \times \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \boxed{H}$$

$$C_8 : (2) \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array}$$

H^2

$$C_7: (1) \quad \text{Diagram} = \text{Diagram} \quad (2) \quad \text{Diagram} = \text{Diagram}$$

$$4.(i) \quad \text{Diagram showing the equivalence of two circuit structures. The left side shows a vertical connection between } A_{1,b} \text{ and } B_{2,d}. \text{ The right side shows a horizontal connection between } A_{1,b+1} \text{ and } B_{2,d+2}, \text{ followed by a sequence of components: } H, H, S, H, S, S, S, H, S, S, Z, Z. \text{ A dot at the end indicates the sequence continues.}$$

Proof cont.

A quantum circuit diagram illustrating a sequence of operations. The circuit starts with H^2 , followed by S^{2d} . A CNOT gate acts on the second qubit. This is followed by a block enclosed in a blue box containing two S gates and one H gate. After this, there is another CNOT gate. The sequence continues with S , H , X , and S gates. Subsequent CNOT gates act on the second and third qubits. The circuit concludes with three S^2 gates and a final measurement z^2 .

R19, C3

R18

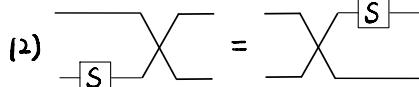
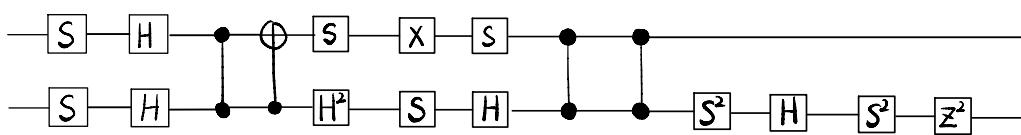
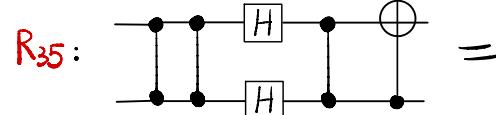
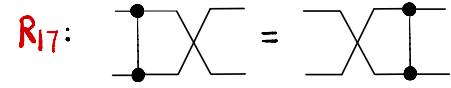
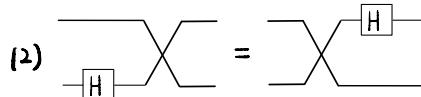
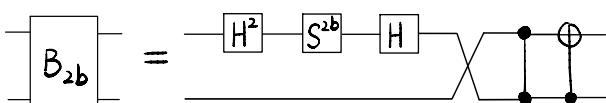
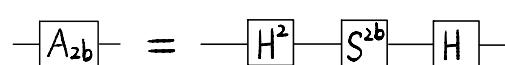
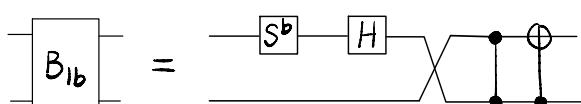
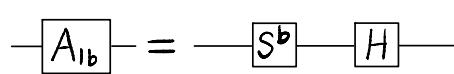
$\bullet w$

$A_{1,b+1}$ $B_{2,d+2}$ $2(d+2) = 2d + 4 = 2d + 1$

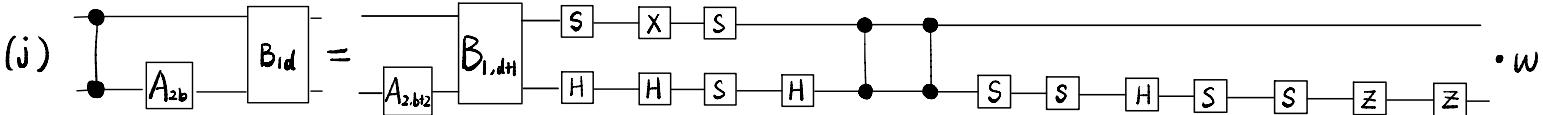
def

$$(j) \quad \begin{array}{c} \bullet \\ \bullet \end{array} \xrightarrow{\quad A_{2b} \quad} \boxed{B_{1d}} = \begin{array}{c} \bullet \\ \bullet \end{array} \xrightarrow{\quad A_{2,b12} \quad} \boxed{B_{1,d11}} \xrightarrow{\quad H \quad} \boxed{H} \xrightarrow{\quad H \quad} \boxed{S} \xrightarrow{\quad H \quad} \boxed{S} \xrightarrow{\quad H \quad} \boxed{S} \xrightarrow{\quad S \quad} \boxed{S} \xrightarrow{\quad S \quad} \boxed{H} \xrightarrow{\quad S \quad} \boxed{S} \xrightarrow{\quad S \quad} \boxed{Z} \xrightarrow{\quad \bar{Z} \quad} \bullet \cdot w$$

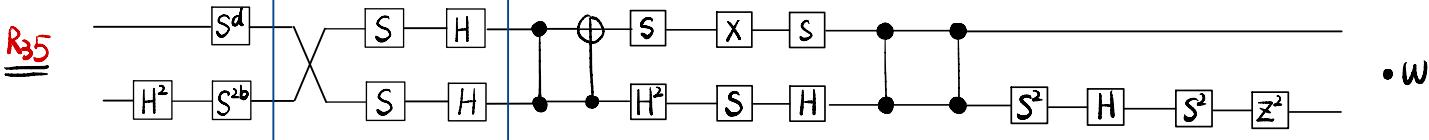
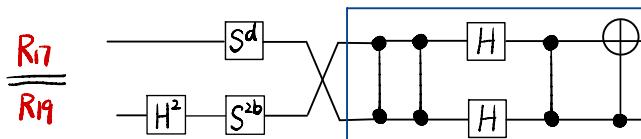
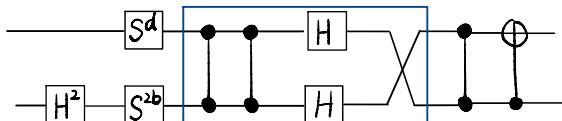
$$4.(j).LHS := \text{Diagram } A_{2b} \xrightarrow{\text{def}} \text{Diagram } H^2 S^{2b} H \oplus \text{Diagram } C_8$$



$$C_3: S^3 = I$$

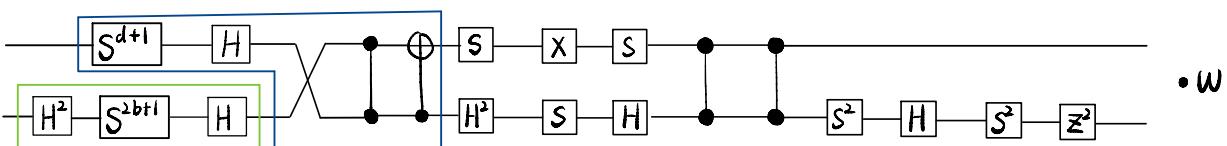


Proof cont.



R19, C3

R18



A_{2,b+2}

$$2(b+2) = 2b+4 = 2b+1$$

B_{1,d+1}

def

