

$$B_{00} = \text{Diagram A} \quad R_{16}: \text{Diagram B} = \text{Diagram C} \quad R_{17}: \text{Diagram D} = \text{Diagram E}$$

$$C_{15}^4: \text{Diagram F} = \text{Diagram G} \quad C_{13}^1: \text{Diagram H} = \text{Diagram I}$$

$$\text{Def 3: } \text{Diagram J} := \text{Diagram K}$$

$$R_{45}: \text{Diagram L} = \text{Diagram M} \quad \text{Def 5: } \text{Diagram N} := \text{Diagram O}$$

Lem 10 Def 3, Def 5, R_{16} , R_{17} , C_{13} , C_{15}^4 & R_{45} imply

$$14.(1) \quad B_{00} = \text{Diagram P} \quad (\text{Diagram P has 7 horizontal lines})$$

$$\text{Proof: } 14.(1). \text{LHS} := \text{Diagram Q} \stackrel{\text{def}}{=} \text{Diagram R} \stackrel{R_{16}}{=} \text{Diagram S}$$

$$\stackrel{R_7}{=} \text{Diagram T} \stackrel{R_{16}/C_{13}^4}{=} \text{Diagram U}$$

$$\stackrel{R_{45}}{=} \text{Diagram V}$$

$$\stackrel{\text{def}}{=} \text{Diagram W}$$

$\therefore 14.(1).\text{RHS}$



$$R_{16}: \text{Diagram} = \text{Diagram}$$

$$R_{17}: \text{Diagram} = \text{Diagram}$$

$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$\text{Def 3: } \text{Diagram} := \text{Diagram}$$

$$C_{13}^1: \text{Diagram} = \text{Diagram}$$

$$C_{15}^4: \text{Diagram} = \text{Diagram}$$

$$\text{Def 5: } \text{Diagram} := \text{Diagram}$$

Lem 11 For $a, b \in \mathbb{Z}_3 \times \mathbb{Z}_3 \setminus \{(0,0)\}$, Def 2-3, Def 5, R_{16} , R_{17} , C_{13} & C_{15} imply

$$\text{Diagram} = \text{Diagram}, \text{ where } \text{Diagram} = \text{Diagram}.$$

Proof: Recall that

$$\text{Diagram} = \text{Diagram}$$

$$\text{Diagram} = \text{Diagram}$$

$$\text{Diagram} = \text{Diagram} \quad a \neq 0$$

Proceed by case distinctions.

Case 1:

$$\text{Diagram} \underset{\text{def}}{=} \text{Diagram}$$

$$\begin{array}{c} R_{16} \\ \equiv \\ \text{Diagram} \end{array}$$

$$\begin{array}{c} R_{17} \\ \equiv \\ C_{15}^4 \\ \text{Diagram} \end{array}$$

$$\begin{array}{c} R_{16} \\ \equiv \\ C_{13}^1 \\ \text{Diagram} \\ = \\ \text{Diagram} \end{array}$$

Case 2:

$$\text{Diagram} \underset{\text{def}}{=} \text{Diagram}$$

$$R_{16}: \text{Diagram} = \text{Diagram}$$

$$R_{17}: \text{Diagram} = \text{Diagram}$$

$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$\text{Def 3: } \text{Diagram} := \text{Diagram}$$

$$C_{13}^1: \text{Diagram} = \text{Diagram}$$

$$C_{15}^4: \text{Diagram} = \text{Diagram}$$

$$\text{Def 5: } \text{Diagram} := \text{Diagram}$$

Lem 11 For $a, b \in \mathbb{Z}_3 \times \mathbb{Z}_3 \setminus \{(0,0)\}$, Def 2-3, Def 5, R_{16} , R_{17} , C_{13} & C_{15} imply

$$\text{Diagram} = \text{Diagram}, \text{ where } \text{Diagram} = \text{Diagram}.$$

Proof cont: $\text{Diagram} = \text{Diagram}$ | $\text{Diagram} = \text{Diagram}$ | $\text{Diagram} = \text{Diagram}$ $a \neq 0$

Case 2:

$$\text{Diagram} \stackrel{\text{def}}{=} \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram}$$

$$\text{Diagram} \stackrel{R_{17}}{=} \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram}$$

Case 3: $a \neq 0$

$$\text{Diagram} \stackrel{\text{def}}{=} \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram}$$

$$\text{Diagram} \stackrel{R_{17}}{=} \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram}$$

$$R_{16} : \text{Diagram} = \text{Diagram}$$

$$R_{17} : \text{Diagram} = \text{Diagram}$$

$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$\text{Def 3: } \text{Diagram} := \text{Diagram}$$

$$C_{13}^1 : \text{Diagram} = \text{Diagram}$$

$$C_{15}^4 : \text{Diagram} = \text{Diagram}$$

$$\text{Def 5: } \text{Diagram} := \text{Diagram}$$

$$R_{46} : \text{Diagram} = \text{Diagram}$$

Lem 12 For $a, b \in \mathbb{Z}_3 \times \mathbb{Z}_3 \setminus \{(0,0)\}$, Def 2-3, Def 5, $R_{16}, R_{17}, C_{13}, C_{15}^4$ & R_{46} imply

$$14.(2)-(9) \quad \text{Diagram} = \text{Diagram}$$

Proof: By Lem 11, for $a, b \in \mathbb{Z}_3 \times \mathbb{Z}_3 \setminus \{(0,0)\}$, $\text{Diagram} = \text{Diagram}$, and

$$14.(2)-(9).LHS := \text{Diagram} \xrightarrow{\substack{R_{16}, R_{17} \\ C_{13}^1, C_{15}^4}} \text{Diagram}$$

$$\xrightarrow{R_{46}} \text{Diagram}$$

$$\xrightarrow{\text{Lem 11}} \text{Diagram} =: 14.(2)-(9).RHS.$$

IV