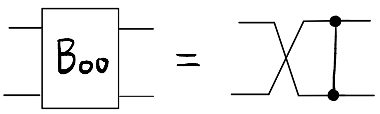
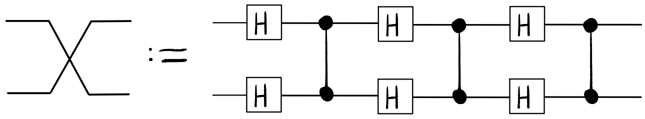
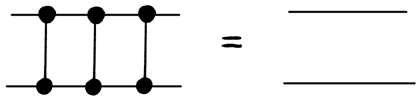
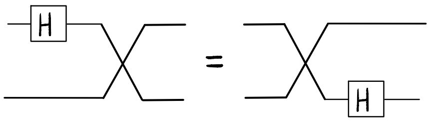
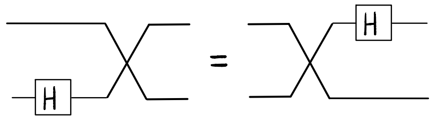


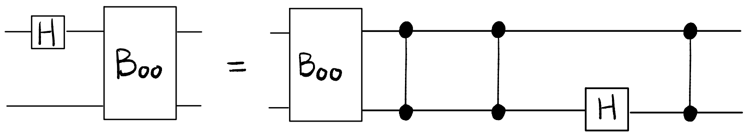
Def 3 : 

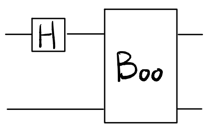
C_6 : 

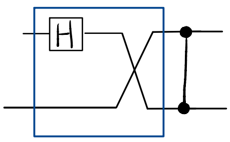
R_{19} : (1) 

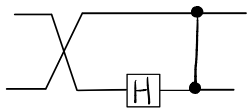
(2) 

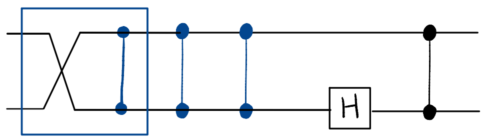
Lem 15 By Def 3 , C_6 & R_{19} ,

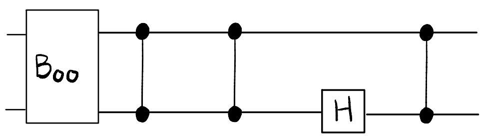
5. (1) 

Proof : 5. (1). LHS := 

$\overset{\text{def}}{=}$ 

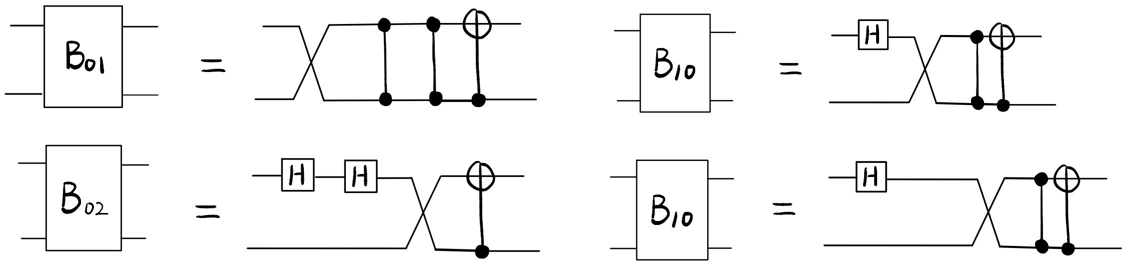
$\overset{R_{19}}{=}$ 

$\overset{C_6}{=}$ 

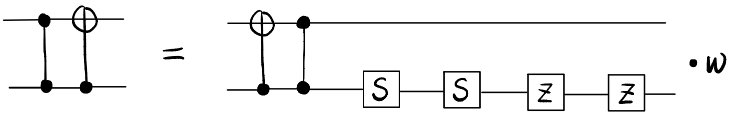
$\overset{\text{def}}{=}$ 

= 5. (1). RHS

∎

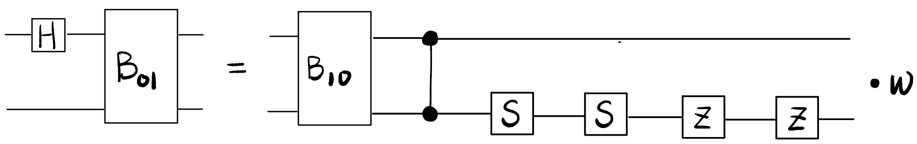


R25:

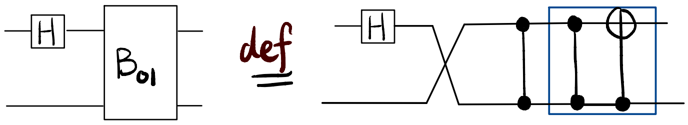


Lem 16 By R25,

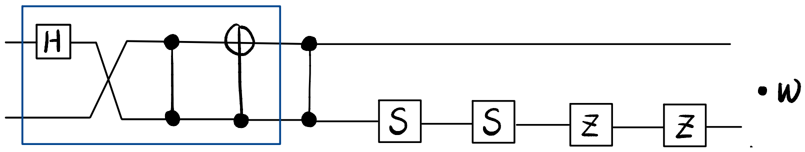
5.(2)



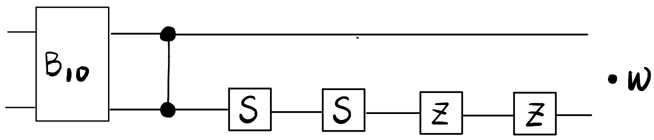
Proof: 5.(2).LHS :=



R25



def

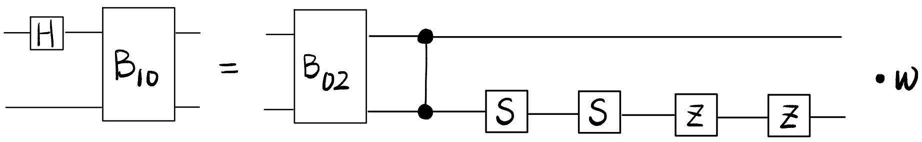


=: 5.(2).RHS

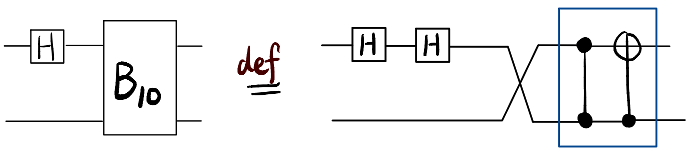
//

Lem 17 By R25,

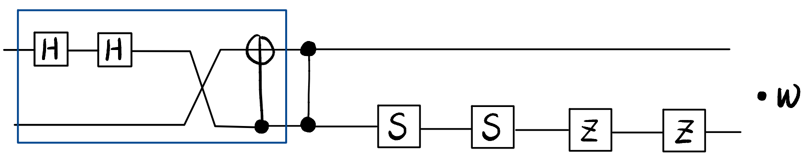
5.(4)



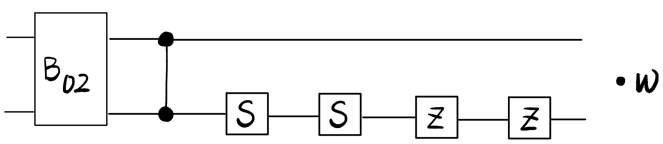
Proof: 5.(4).LHS :=



R25

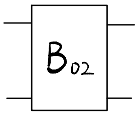


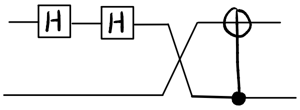
def

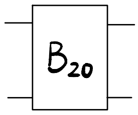


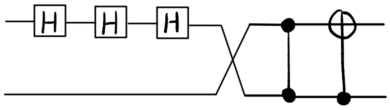
=: 5.(4).RHS

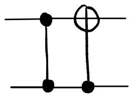
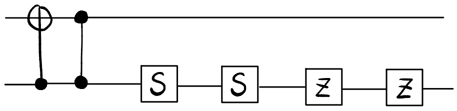
//



=


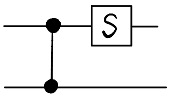


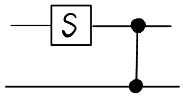
=


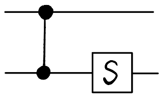
R_{25}:  = 

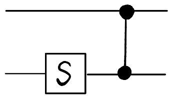
$\cdot w$

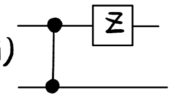
R_3 : $SZ = ZS$ C_1: $w^3 = 1$

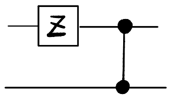
C_7: (1) 

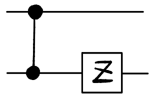
= 

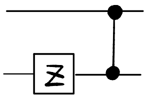
(2) 

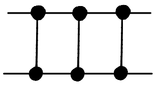
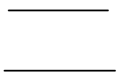
= 

R_{13} : (1) 

= 

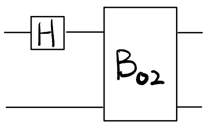
(2) 

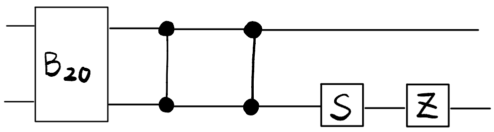
= 

C_6:  = 

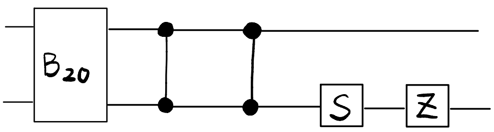
C_3 : $S^3 = I$ R_{12}: $Z^3 = I$

Lem 18 By C_1, C_3, C_6, C_7, R_3, R_{12}, R_{13} & R_{25},

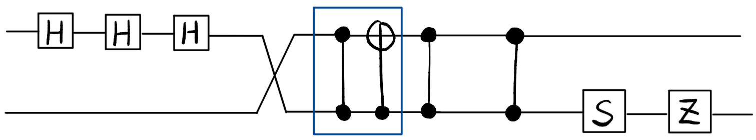
5.(3) 

= 

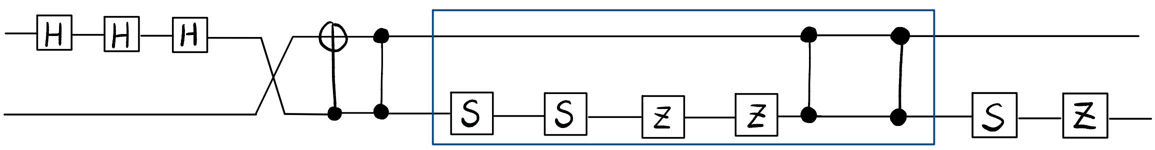
$\cdot w^2$

Proof: 5.(3).RHS := 

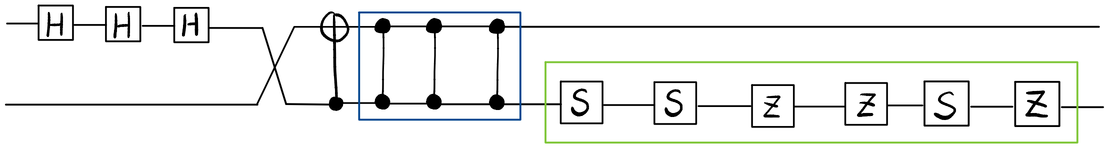
$\cdot w^2$

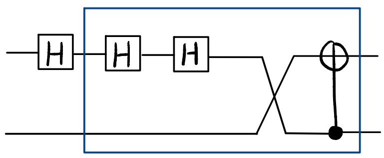
$\underline{\underline{\text{def}}}$ 

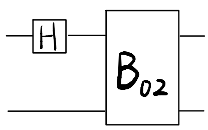
$\cdot w^2$

$\underline{\underline{R_{25}}}$ 

$\boxed{\cdot w^2 \cdot w}$

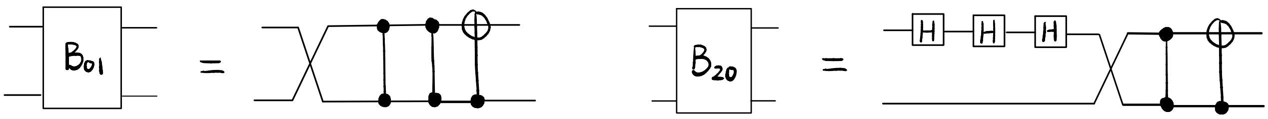
$\underline{\underline{C_7, R_{13}}}$
C_1


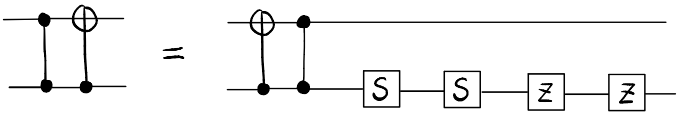
$\underline{\underline{C_6, R_3}}$
C_3, R_{12}


$\underline{\underline{\text{def}}}$ 

=: 5.(3).LHS

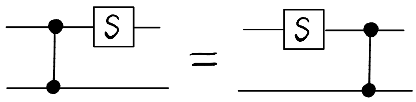
∎

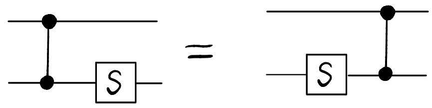


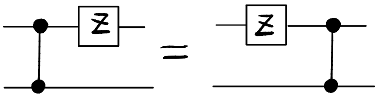
R_{25}: 

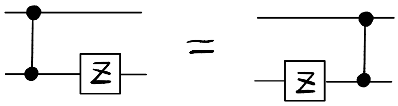
$\cdot w$

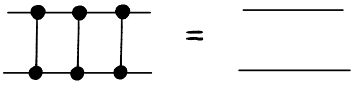
$R3$: $SZ = ZS$ $C1$: $w^3 = 1$

C_7: (1) 

(2) 

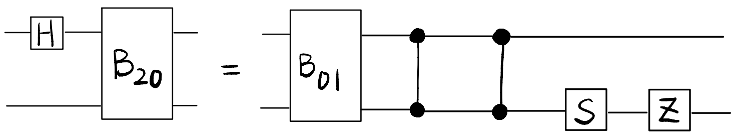
R_{13} : (1) 

(2) 

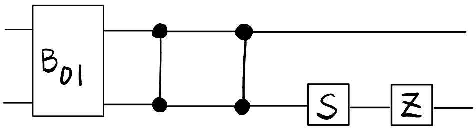
C_6: 

$C3$: $S^3 = I$ $R12$: $Z^3 = I$ $C2$: $H^4 = I$

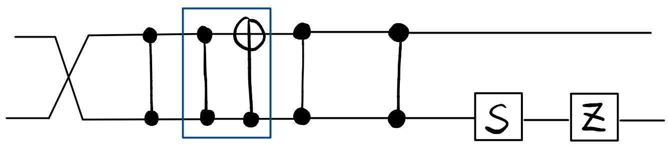
Lem 19 By $C_1, C_2, C_3, C_6, C_7, R_3, R_{12}, R_{13}$ & R_{25},

5.(7) 

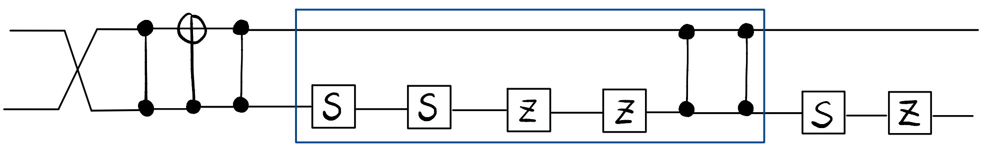
$\cdot w^2$

Proof: 5.(7).RHS := 

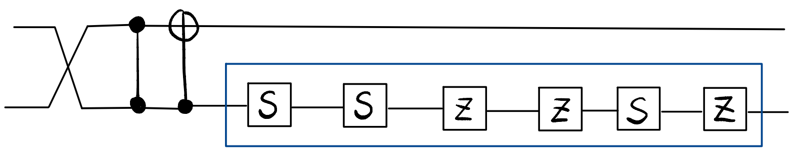
$\cdot w^2$

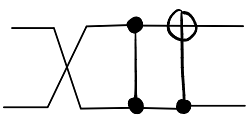
$\underline{\underline{\text{def}}}$ 

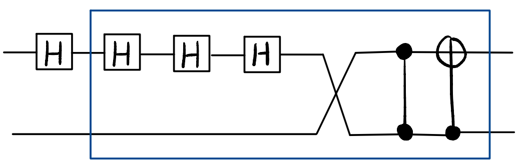
$\cdot w^2$

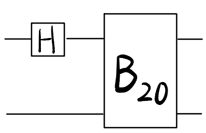
$\underline{\underline{R_{25}}}$ 

$\boxed{\cdot w^2 \cdot w}$

$\underset{C_1, C_6}{\overset{C_7, R_{13}}{=\!=}}$ 

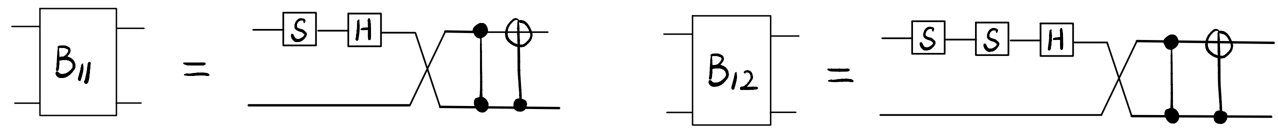
$\underset{C_3, R_{12}}{\overset{R_3}{=\!=}}$ 

$\underline{\underline{C_2}}$ 

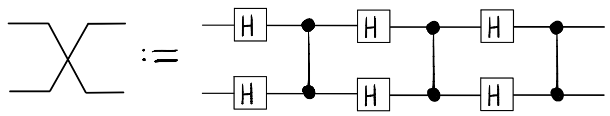
$\underline{\underline{\text{def}}}$ 

=: 5.(7).LHS

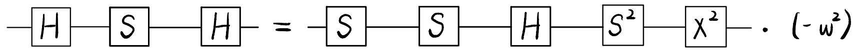
$\boxed{/\!/}$



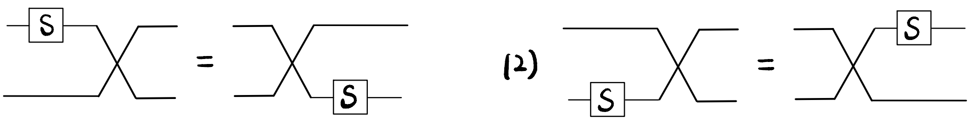
Def 3:



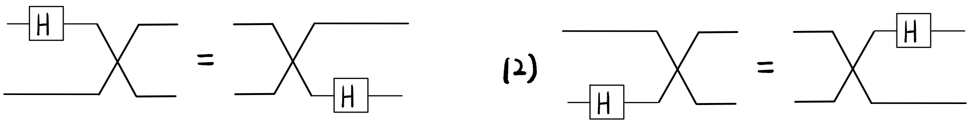
R1:



R18: (1)



R19: (1)

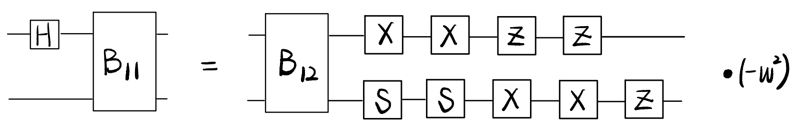


C7: (1)

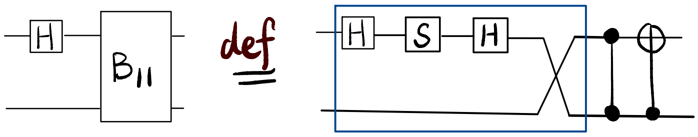
R14: (1)

Lem 20 By Def 3, R1, R12, R14, R18, R19, R24, R26 & C7,

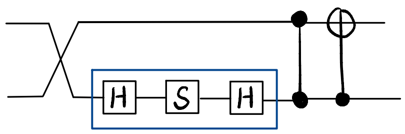
5.(5)



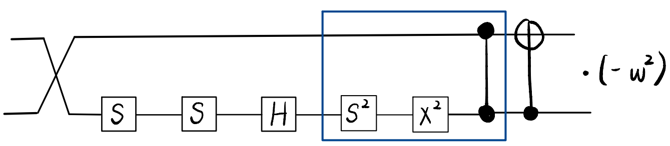
Proof: 5.(5). LHS :=



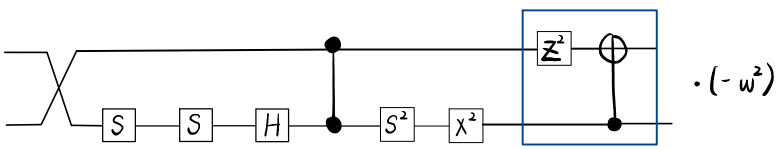
R18 / R19

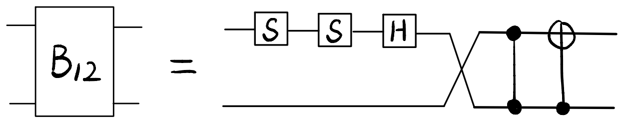


R1

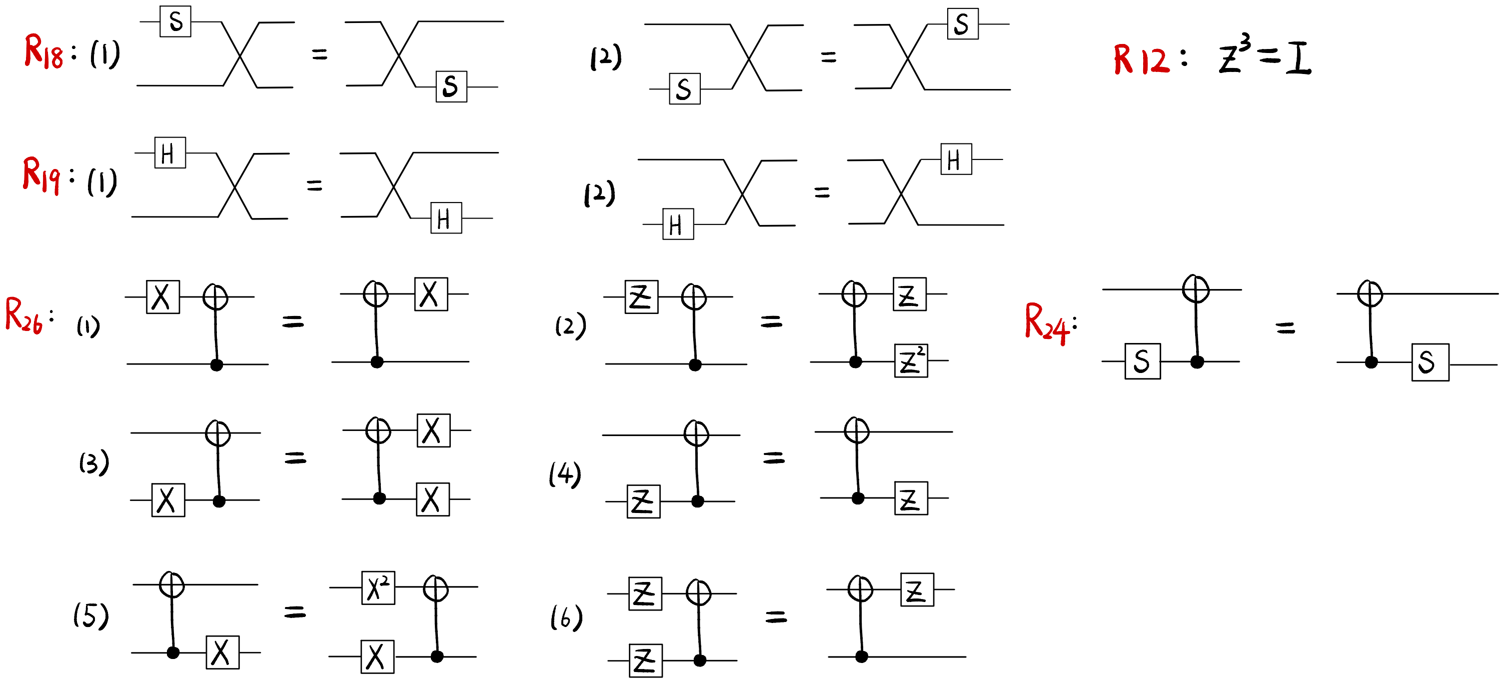


C7 / R14

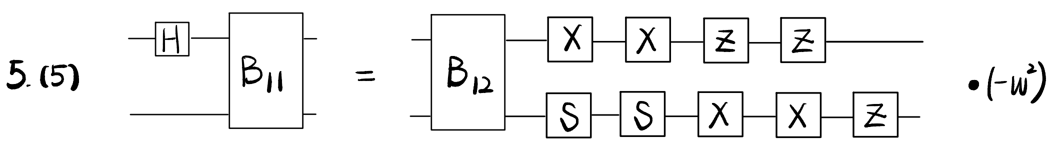




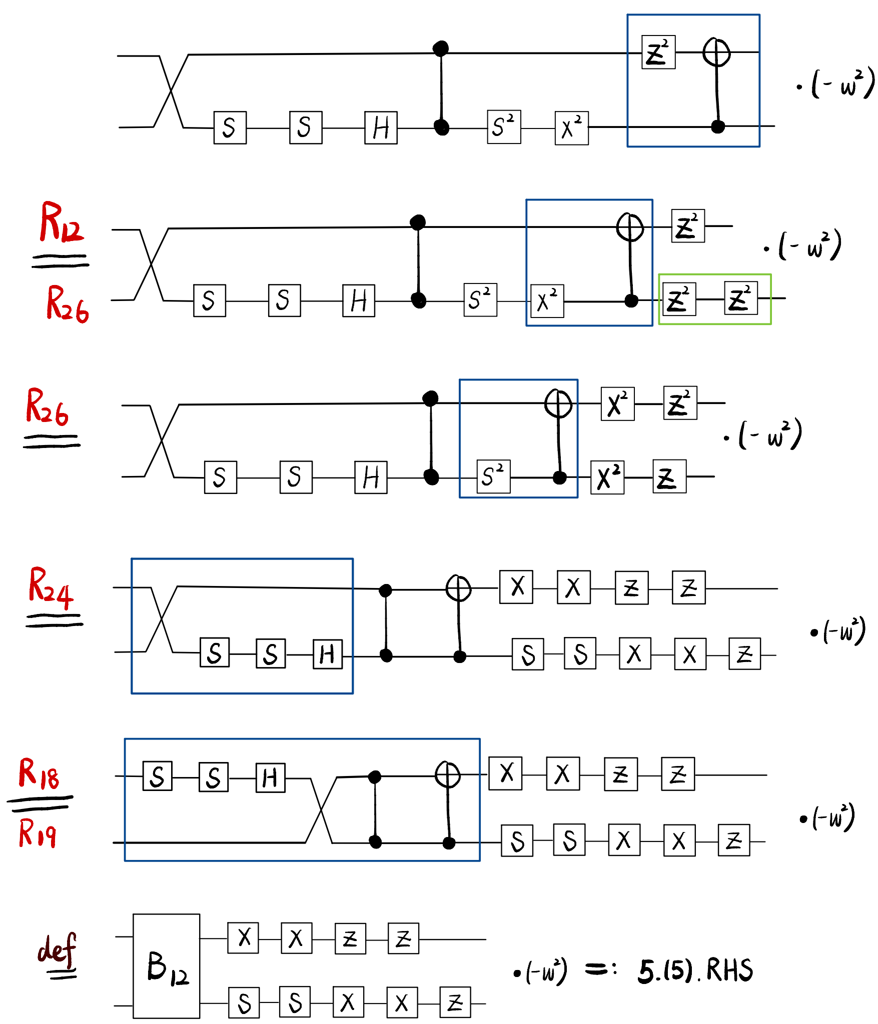
B_{12}
=
S S H



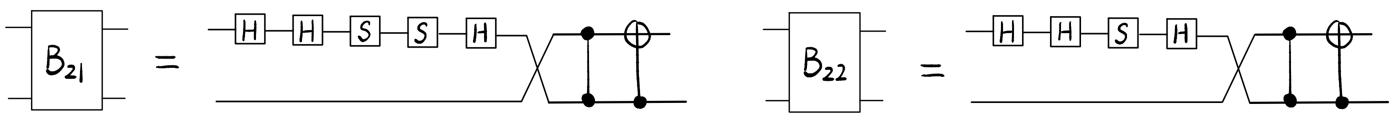
R18: (1)
S
=
S
(2)
S
=
S
R12: $Z^3 = I$
R19: (1)
H
=
H
(2)
H
=
H
R26: (1)
X
=
X
(2)
Z
=
Z
Z^2
R24:
S
=
S
(3)
X
=
X
X
(4)
Z
=
Z
(5)
X
=
X^2
X
(6)
Z
Z
=
Z



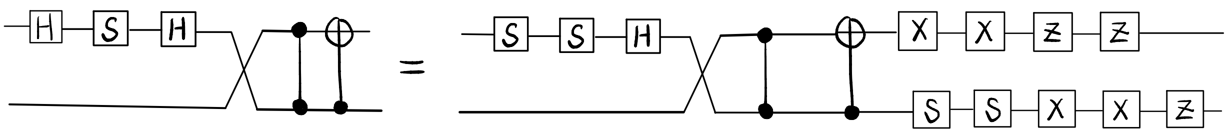
5. (5)
H
B_{11}
=
B_{12}
X X Z Z
S S X X Z
$\cdot(-w^2)$



S S H S^2 X^2
Z^2
$\cdot(-w^2)$
R12
R26
S S H S^2 X^2
Z^2
Z^2 Z^2
$\cdot(-w^2)$
R26
S S H S^2
X^2 Z^2
X^2 Z
$\cdot(-w^2)$
R24
S S H
X X Z Z
S S X X Z
$\cdot(-w^2)$
R18
R19
S S H
X X Z Z
S S X X Z
$\cdot(-w^2)$
def
B_{12}
X X Z Z
S S X X Z
$\cdot(-w^2)$ =: 5.(5).RHS

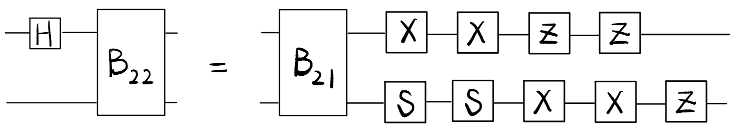


Lem 20



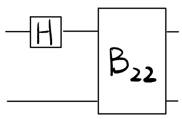
$\bullet(-\omega^2)$

Lem 21 5.(9)

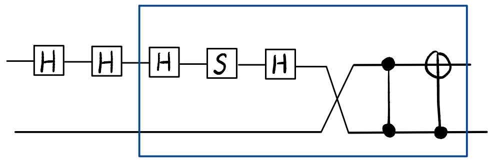


$\bullet(-\omega^2)$

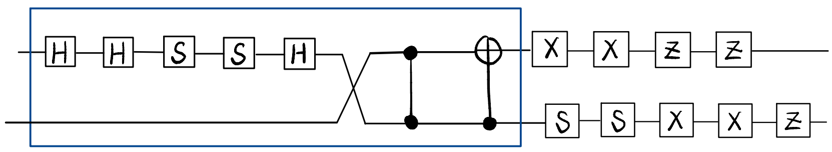
Proof: 5.(9).LHS :=



$\underline{\underline{\text{def}}}$

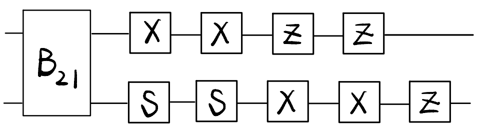


$\underline{\underline{\text{Lem 20}}}$



$\bullet(-\omega^2)$

$\underline{\underline{\text{def}}}$



$\bullet(-\omega^2)$ =: 5.(9).RHS

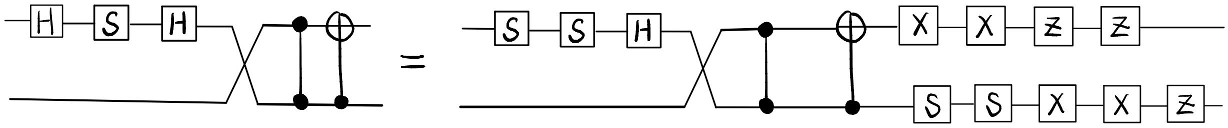
∎

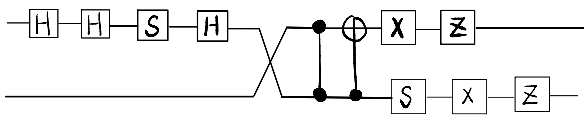
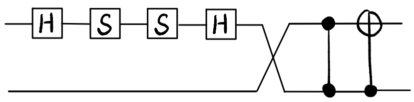
$C_0: (-1)^2 = 1 \qquad C_1: \omega^3 = 1 \qquad C_3: S^3 = I \qquad R_9: X^3 = I \qquad R_{12}: Z^3 = I$

$R_3: SZ = ZS \qquad R_8: \boxed{Z}\boxed{X} = \boxed{X}\boxed{Z}\cdot \omega^2$

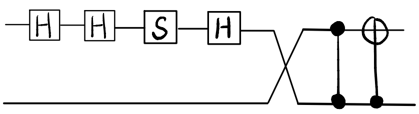
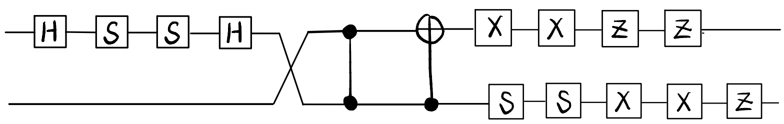
$R_6: \boxed{S}\boxed{X} = \boxed{X}\boxed{S}\boxed{Z}\cdot \omega^2$

$R_6': \boxed{S}\boxed{X}\boxed{Z}\boxed{Z}\cdot \omega = \boxed{X}\boxed{S}$

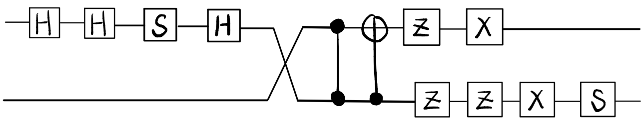
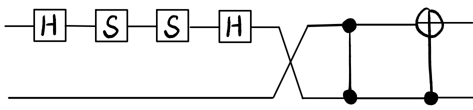
Lem 20  $\cdot(-\omega^2)$ (1)

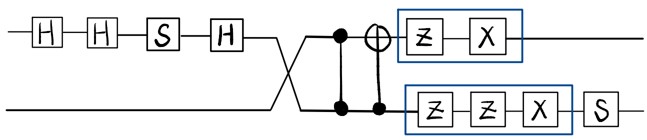
Lem S Lem 20 implies  $\cdot(-\omega^2)$ = 

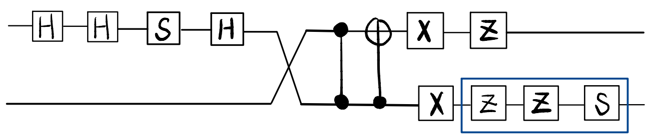
Proof: By C_0 & C_1, multiply both sides of (1) by $(-\omega)$, followed by appending both sides of (1) by H to the left.

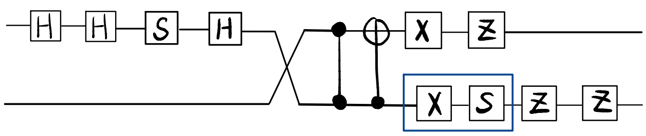
 $\cdot(-\omega)$ =  (2)

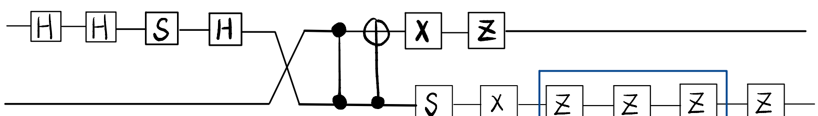
By C_3, R_9 & R_{12}, append both sides of (2) by $ZX \otimes Z^2XS$ to the right. Composition in diagrammatic order.

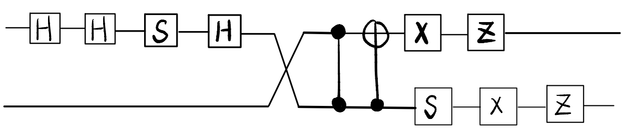
 $\cdot(-\omega)$ =  (3)

(3). LHS =  $\cdot(-\omega)$

$\overset{R_8}{=}$  $\cdot(-\omega)\ \boxed{\cdot\omega^2\cdot\omega^2\cdot\omega^2}$

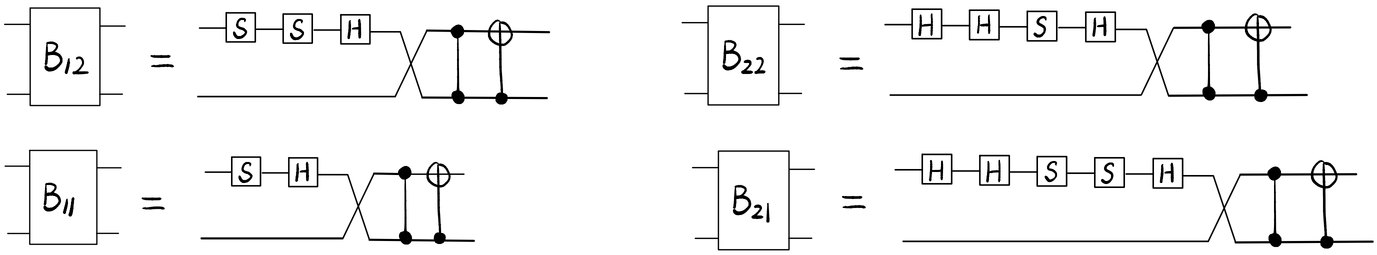
$\overset{R_3}{\underset{C_1}{=}}$  $\cdot(-\omega)$

$\overset{R_6'}{=}$  $\cdot(-\omega)\cdot\omega$

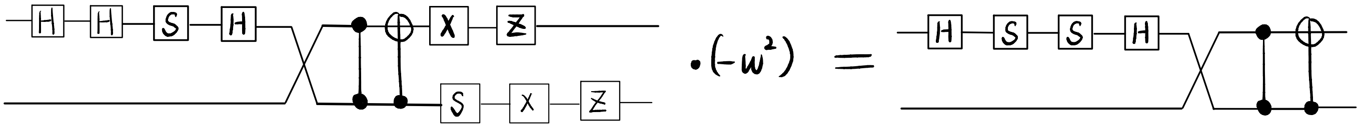
$\overset{R_{12}}{=}$  $\cdot(-\omega^2)$

We can reduce this process to the completeness of single-qutrit Clifford relations.

$\blacksquare$

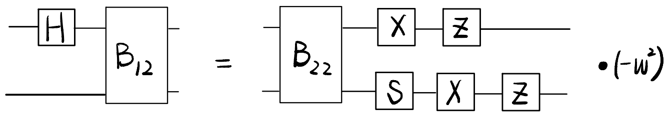


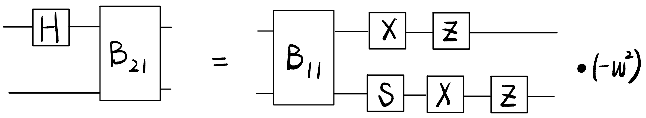
Lem S

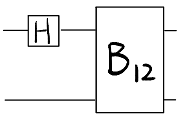


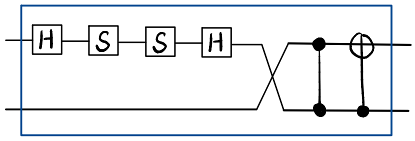
C2 : $H^4 = I$

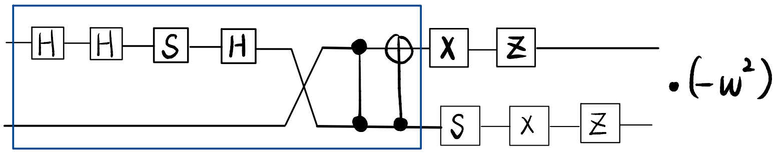
Lem 22 By Lem S & C_2,

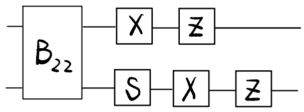
5.(6) 

(8) 

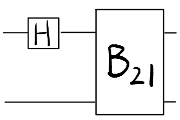
Proof: 5.(6). LHS := 

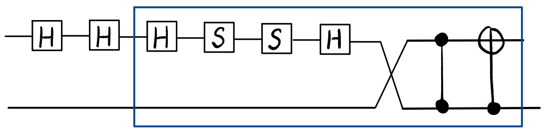
$\underline{\underline{\text{def}}}$ 

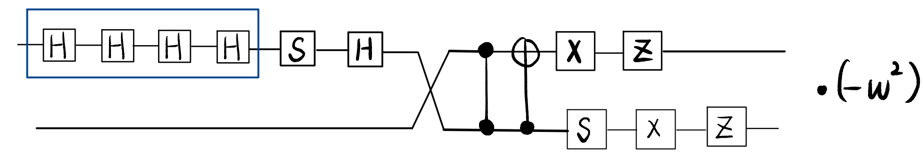
$\underline{\underline{\text{Lem S}}}$ 

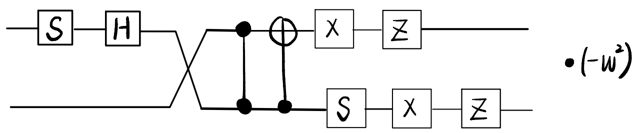
$\underline{\underline{\text{def}}}$ 

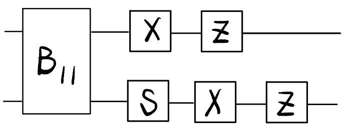
$\cdot(-w^2)$ =: 5.(6). RHS

5.(8). LHS := 

$\underline{\underline{\text{def}}}$ 

$\underline{\underline{\text{Lem S}}}$ 

$\underline{\underline{C_2}}$ 

$\underline{\underline{\text{def}}}$ 

$\cdot(-w^2)$ =: 5.(8). RHS //