

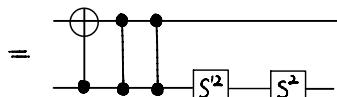
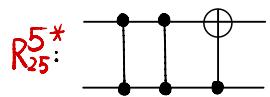
$C_0 : (-1)^2 = 1$

$C_1 : w^3 = 1$

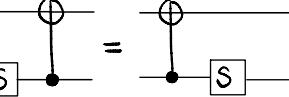
$C_2 : H^4 = I$

$C_3 : S^3 = I$

$C_5 : SS' = S'S$



$\cdot w^2 \quad R_{24}$



Def 2: :=

$C_7^*:$ =

$R_{15}^*:$ =

$C_{16}^{3'}:$

$R_{24}^I:$

$C_8:$

Lem Y

$R_{51}:$ = • $(-w)$

Proof cont.

$R_{51}:$ \equiv • w^2

Def 2, $C_2 \parallel R_{25}^*, C_{16}^{3'}$

$R_{51}:$ \equiv • w^2

$R_{51}:$ \equiv • w^2

$C_1, C_2 \parallel R_{24}, R_{24}^I, C_7^*, R_{15}^*, C_8$

$R_{51}:$ \equiv \equiv \equiv • w^2

Def 1-2 $\parallel C_2, C_3$

$R_{51}:$ \equiv

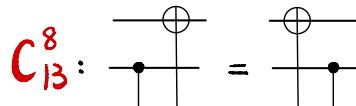
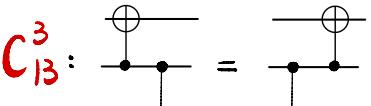
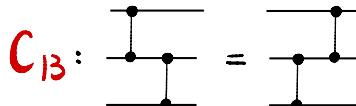
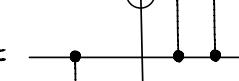
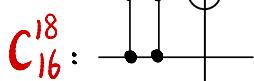
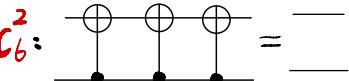
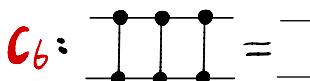
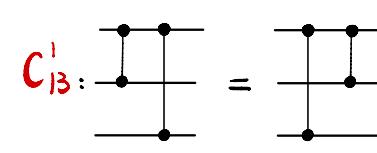
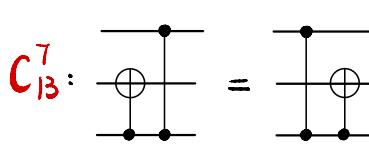
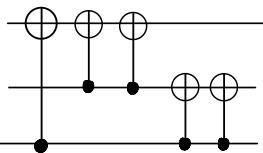
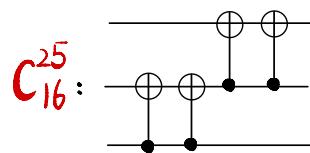
$C_0 : (-1)^2 = 1$

$C_1 : w^3 = 1$

$C_2 : H^4 = I$

$C_3 : S^3 = I$

$C_5 : SS' = S'S$



Lem Y

R_{51} :

Proof cont.

R_{51} :

$C_{13}^7 \parallel C_{16}^{25}$

R_{51} :

$C_{13}^1 \parallel C_6^*, C_6^2$

R_{51} :

Then $R_{51}.LHS :=$

$=: R_{51}.RHS$



$$C_0 : (-1)^2 = 1$$

$$C_1 : w^3 = 1$$

$$C_2 : H^4 = I$$

$$C_3 : S^3 = I$$

$$\text{Def 2: } \begin{array}{c} \oplus \\ \text{---} \\ | \quad | \\ \text{---} \end{array} := \begin{array}{c} \text{H} \quad \bullet \quad \text{H}^3 \\ \text{---} \quad | \quad \text{---} \\ | \quad | \\ \text{---} \end{array}$$

$$\text{Def 1: } \begin{array}{c} \text{S}' \\ \text{---} \end{array} := \begin{array}{c} \text{H} \quad \text{H} \quad \text{S} \quad \text{H} \quad \text{H} \quad \text{H} \end{array} \quad C_5 : SS' = S'S$$

$$R_{23}^1: \begin{array}{c} \text{S} \quad \oplus \\ \text{---} \quad | \\ | \quad | \quad | \\ \text{---} \quad | \quad \text{S}' \end{array} = \begin{array}{c} \oplus \quad \bullet \quad \text{S} \\ \text{---} \quad | \quad \text{---} \\ | \quad | \quad | \\ \text{---} \quad | \quad \text{S}' \end{array} \cdot w^2 \quad C_4^2: \begin{array}{c} \text{S} \quad \text{H}^3 \quad \text{S} \\ \text{---} \quad | \quad \text{---} \end{array} = \begin{array}{c} \text{H} \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \quad \text{---} \end{array} \cdot (-w^2)$$

$$\text{Def 7: } \begin{array}{c} \oplus \\ \text{---} \\ | \quad | \\ \text{---} \end{array} := \begin{array}{c} \text{---} \quad \text{---} \\ | \quad | \\ \text{---} \quad \text{---} \end{array} \quad \text{Def 5: } \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} := \begin{array}{c} \text{---} \quad \text{---} \\ | \quad | \\ \text{---} \quad \text{---} \end{array} \quad \text{Def 4: } \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} := \begin{array}{c} \text{---} \quad \text{---} \\ | \quad | \\ \text{---} \quad \text{---} \end{array}$$

Lem \exists Def 0-2, Def 4-5, Def 7, C_0 - 8 , C_{13} , C_{16} , R_{15} , R_{23} , R_{24} , R_{25} imply

$$R_{52}: \begin{array}{c} \text{---} \quad \text{---} \\ | \quad | \\ \text{---} \quad \text{---} \end{array} = \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \text{H} \quad \text{S}' \quad \text{H} \quad \text{S} \quad \text{H}^2 \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \end{array} \cdot (-w)$$

$$\text{Proof: } R_{52} \cdot \text{RHS} := \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \text{H} \quad \boxed{\text{S} \quad \text{H} \quad \text{S}' \quad \text{H}} \quad \text{S} \quad \text{H}^2 \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \end{array} \cdot (-w)$$

$$\text{SH} \boxed{\text{S}' \text{H}} \xrightarrow{\text{Def 1}} \text{SHH}^2 \text{S} \quad \text{H}^2 \text{H} \xrightarrow{\text{C}_4^2} \text{HS}^2 \boxed{\text{HH}^3} \cdot (-w^2) \xrightarrow{\text{C}_2} \text{HS}^2 \cdot (-w^2)$$

$$\frac{\text{Def 1}}{\text{C}_2, \text{C}_4^2} \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \text{H} \quad \boxed{\text{H} \quad \text{S}^2 \quad \text{S}} \quad \text{H}^2 \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \end{array} \cdot (-w) \cdot (-w^2)$$

$$\frac{\text{C}_3}{\text{C}_0, \text{C}_1} \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \boxed{\text{H} \quad \text{H}^2} \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \quad | \quad | \quad | \quad | \end{array}$$

$$\text{Def 2} \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \boxed{\text{S} \quad \text{S}^2} \quad \text{S}^2 \quad \text{H} \\ \text{---} \quad | \quad | \quad | \quad | \end{array}$$

$$R_{23}^1: \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \boxed{\text{S}' \quad \text{S}^2 \quad \text{H}^2 \quad \text{S}} \\ \text{---} \quad | \quad | \quad | \quad | \end{array} \cdot w^2$$

$$\text{S}' \boxed{\text{S}^2 \text{H}^2 \text{S}} \xrightarrow{\text{C}_5} \text{S}^2 \boxed{\text{S}' \text{H}^2 \text{S}} \xrightarrow{\text{Def 1}} \text{S}^2 \text{H} \text{S} \boxed{\text{H}^2 \text{H}^2 \text{S}} \xrightarrow{\text{C}_2} \text{S}^2 \text{H}^2 \text{S}$$

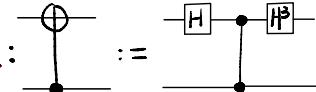
$$\frac{\text{Def 1, C}_2}{\text{C}_3, \text{C}_5} \begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array} \begin{array}{c} \text{H} \quad \text{H}^3 \quad \text{S} \quad \text{H} \\ \text{---} \quad | \quad | \quad | \end{array} \cdot w^2$$

$$C_0 : (-1)^2 = 1$$

$$C_1 : w^3 = 1$$

$$C_2 : H^4 = I$$

$$C_3 : S^3 = I$$

Def 2:  :=

$$C_8^6 : \text{Diagram} = \text{Diagram}$$

$$C_8^{6*} : \text{Diagram} = \text{Diagram}$$

Lem Z

$$R_{52} : \text{Diagram} = \text{Diagram} \cdot (-w)$$

Proof cont.

$$R_{52} \cdot \text{RHS} = \text{Diagram} \cdot w^2$$

$$\frac{\text{Def 2}}{C_2} : \text{Diagram} \cdot w^2$$

$$\frac{\text{Def 2}}{C_2} : \text{Diagram} \cdot w^2$$

$$\frac{C_8^6}{C_2} : \text{Diagram} \cdot w^2$$

$$\frac{C_2}{C_2} : \text{Diagram} \cdot w^2$$

Hence $R_{52} : \text{Diagram} \quad \text{WTS} \quad \text{Diagram} \cdot w^2$

$$C_8^{6*} \parallel C_2$$

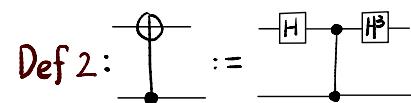
$$R_{52} : \text{Diagram} \quad \text{WTS} \quad \text{Diagram} \cdot w^2$$

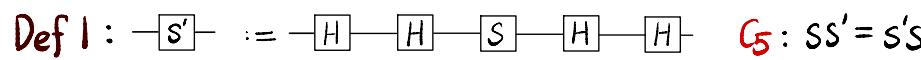
$$C_0 : (-1)^2 = 1$$

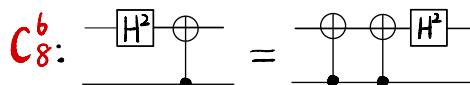
$$C_1 : w^3 = 1$$

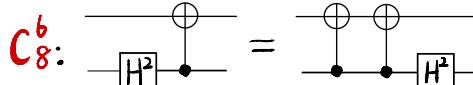
$$C_2 : H^4 = I$$

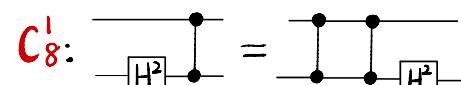
$$C_3 : S^3 = I$$

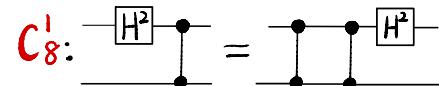
Def 2: 

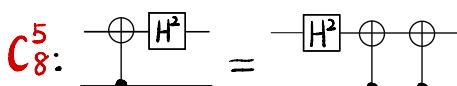
Def 1:  C₅: SS' = s's

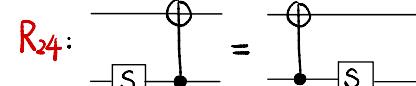
C₈^b: 

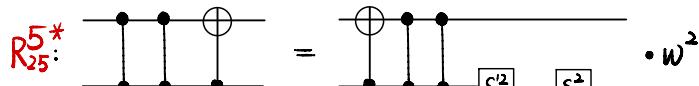
C₈^b: 

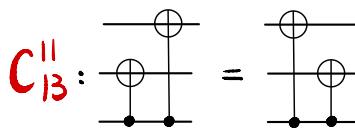
C₈¹: 

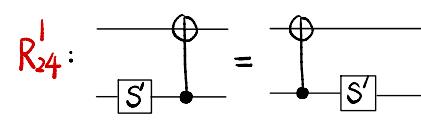
C₈¹: 

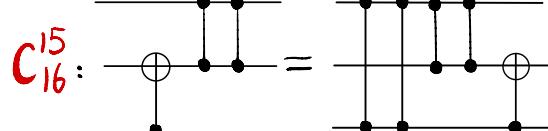
C₈⁵: 

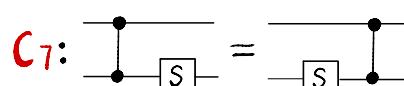
R₂₄: 

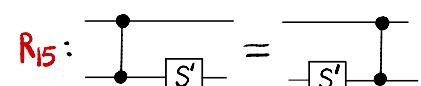
R₂₅^{*}: 

C₁₃^{II}: 

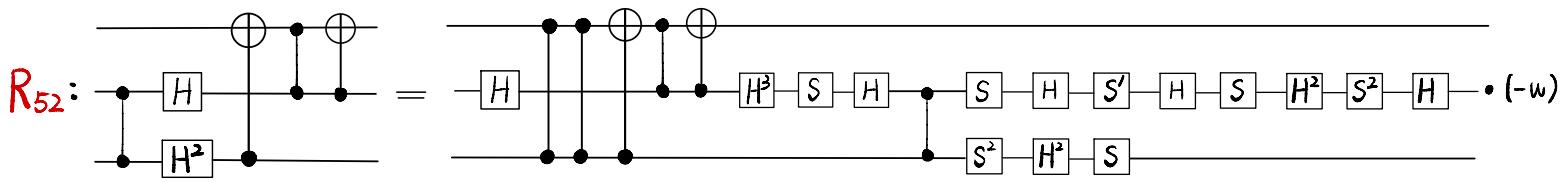
R₂₄^I: 

C₁₆¹⁵: 

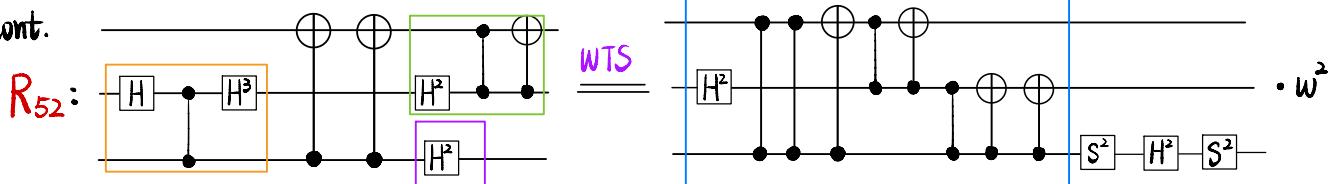
C₇: 

R₁₅: 

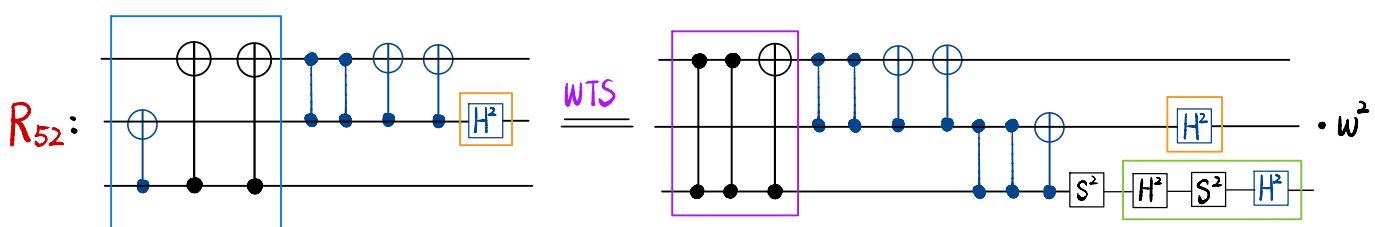
Lem Z

R₅₂: 

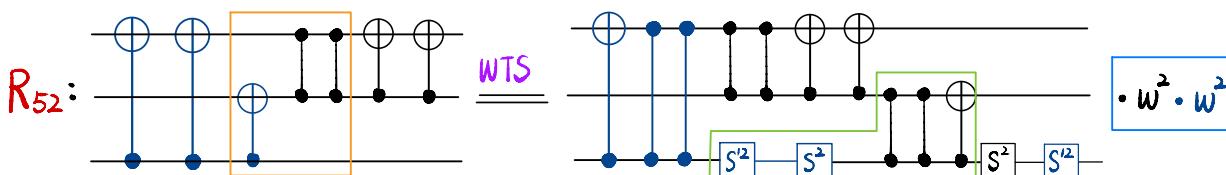
Proof cont.

R₅₂: 

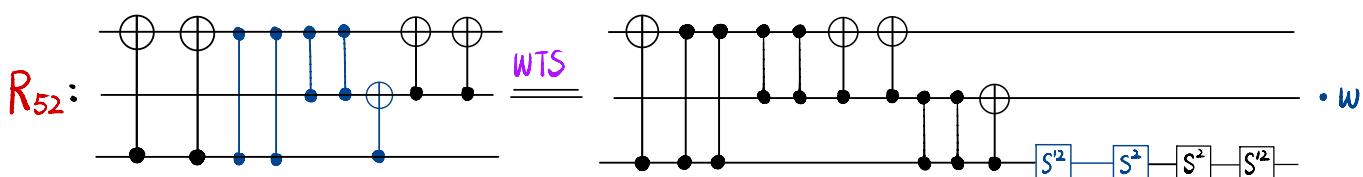
Def 2, C₂ ||| C₈¹, C₈^b, C₈⁵

R₅₂: 

C₂, C₁₃^{II} ||| Def 1, R₂₅^{*}

R₅₂: 

C₁₆¹⁵ ||| C₁, R₂₄, R₂₄^I, C₇, R₁₅

R₅₂: 

$C_0 : (-1)^2 = 1$

$C_1 : w^3 = 1$

$C_2 : H^4 = I$

$C_3 : S^3 = I$

$C_6^* : \text{Diagram} = \text{Diagram}$

$\text{Def 1} : [S'] := [H][H][S][H][H] \quad C_5 : SS' = S'S$

$C_6^* : \text{Diagram} = \text{Diagram}$

$C_{16}^{26} : \text{Diagram} = \text{Diagram}$

$R_{25}^{5*} : \text{Diagram} = \text{Diagram} \cdot w^2$

$R_{24}^* : [S] = [S]$

$R_{24}^{1*} : [S] = [S']$

$C_{16} : \text{Diagram} = \text{Diagram}$

Lem Z

$R_{52} : \text{Diagram} = \text{Diagram} \cdot (-w)$

Proof cont.

$R_{52} : \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram} \cdot w$

$\text{Def 1}, C_6^{26} \parallel C_2, C_3, C_5, C_6^{2*}$

$R_{52} : \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram} \cdot w$

$\text{Def 1}, C_1, C_2, C_3 \parallel R_{25}^{5*}$

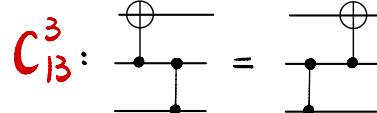
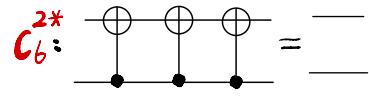
$R_{52} : \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram} \cdot w$

$C_1, C_5, C_6^* \parallel R_{24}^*, R_{24}^{1*}$

$R_{52} : \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$

$\text{Def 1}, C_2, C_3 \parallel C_6^{16}$

$R_{52} : \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$



Lem Z

$R_{52}:$

Proof cont.

$R_{52}:$

Then $R_{52} \cdot \text{LHS} :=$

$\underline{\underline{C_6^{2*}}}$

□