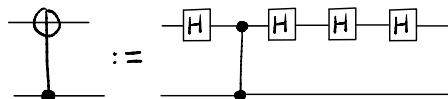
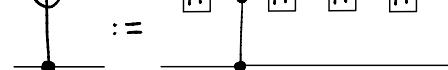
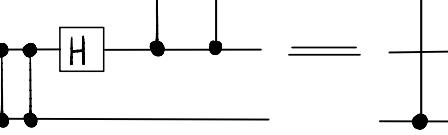
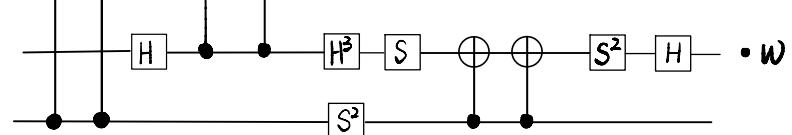
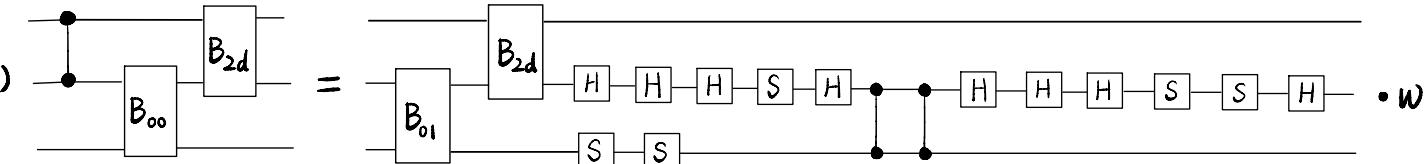
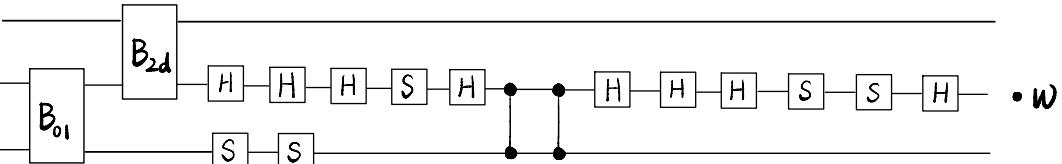


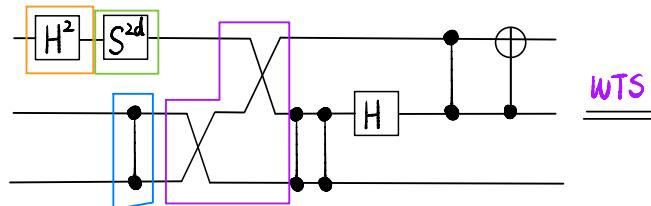
$$C_2 : H^4 = I \quad C_3 : S^3 = I \quad R_{16} : \text{Diagram} = \text{Diagram} \quad C_6 : \text{Diagram} = \text{Diagram}$$

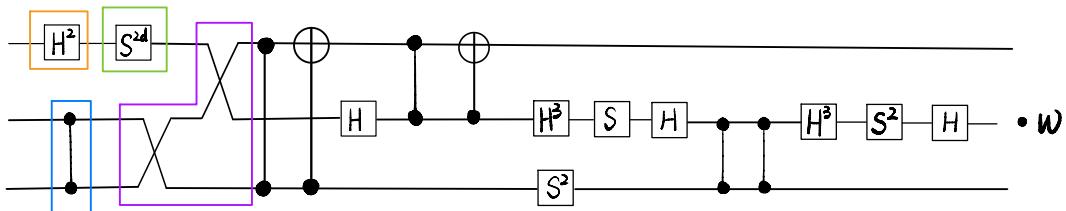
Def 2:  := 

R₄₈:  =  • w

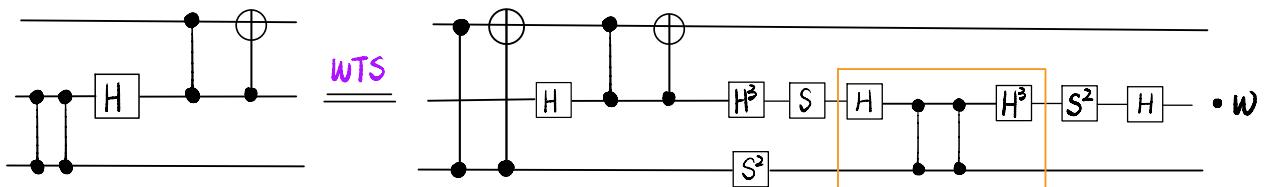
Lem 15

7.1.(7)-(9)  =  • w

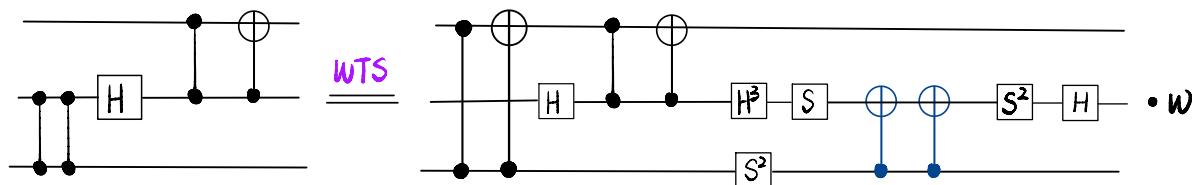
Proof cont.: 



$C_2, C_3 \parallel R_{16}, C_6$

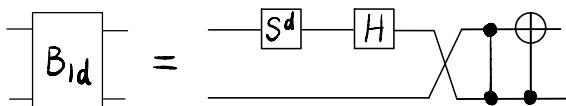
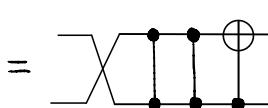
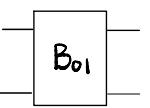
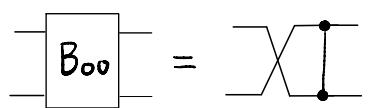


$C_2 \parallel \text{Def 2}$

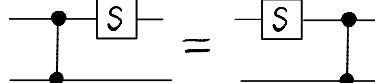


By R₄₈, this completes the proof.

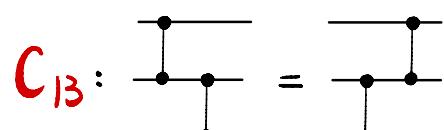
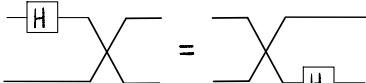




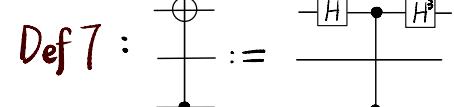
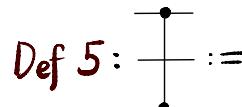
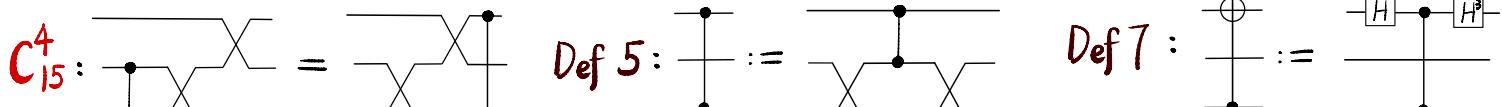
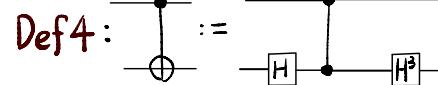
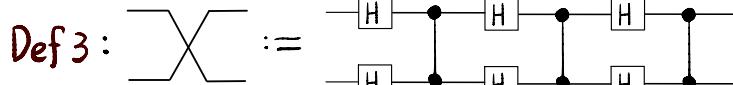
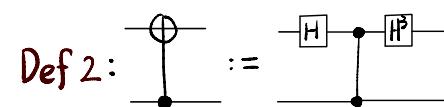
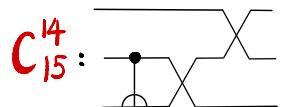
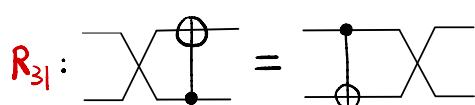
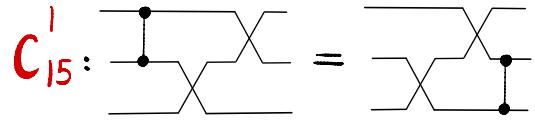
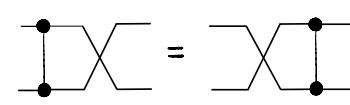
$C_7:$



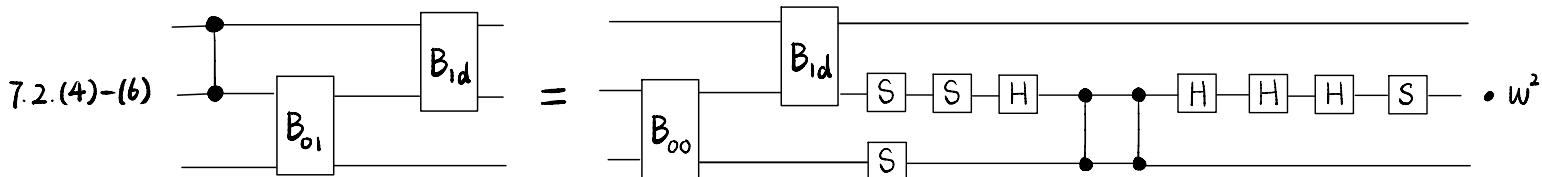
$R_{19}:$



$R_{17}:$

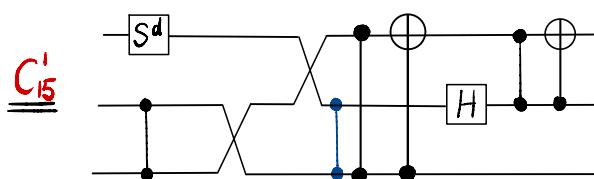
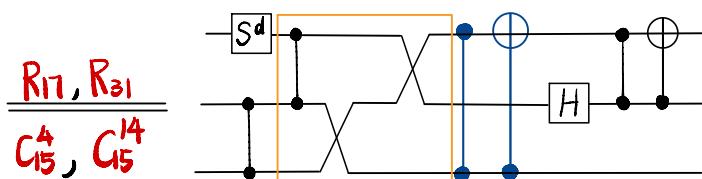
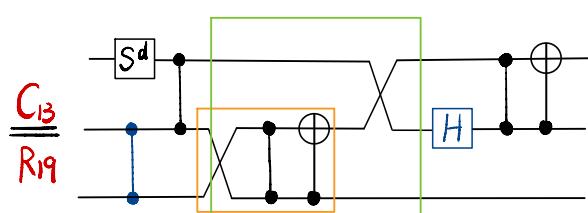
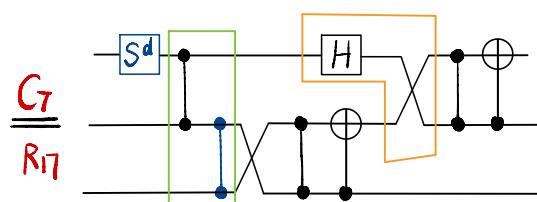
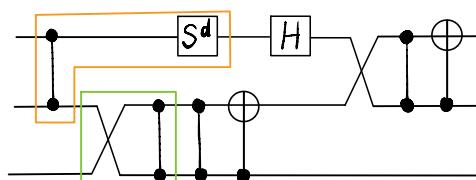


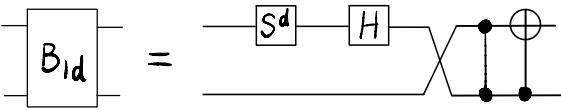
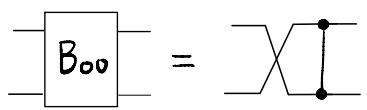
Lem 16 Def 2-5, Def 7, C_3 , C_7 , C_{15} , R_{16} , R_{17} , R_{19} , R_{31} & R_{49} imply



Proof: 7.2.(4)-(6). LHS :=

def





$R_{16}:$

$C_3: S^3 = I \quad C_6:$

$R_{17}:$

$R_{19}:$

$R_{49}:$

Lem 16

7.2.(4)-(b)

Proof cont:

7.2.(4)-(b). LHS =

7.2.(4)-(b). RHS :=

def

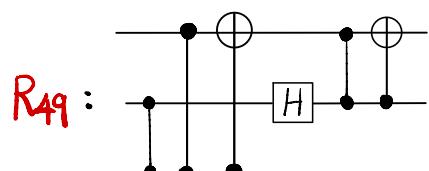
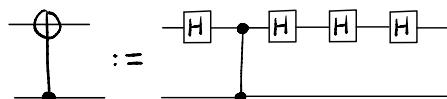
R_{17}

Hence,

$C_3 \parallel R_{16}, C_6$

$$C_2 : H^4 = I$$

Def 2:



Lem 16

$$7.2.(4)-(b) \quad \text{Circuit} = \text{Circuit} \cdot w^2$$

Proof cont:

$C_2 \parallel \text{Def 2}$

By R_{49} , this completes the proof.



$$B_{00} = \text{X gate}, \quad B_{2d} = \text{H}^2 \text{ gate} \otimes S^{2d} \text{ gate} \otimes \text{H} \text{ gate}, \quad C_8: \quad \text{H}^2 \text{ gate} = \text{H}^2 \text{ gate} \otimes \text{H}^2 \text{ gate} \otimes \text{H}^2 \text{ gate}$$

$$B_{02} = \text{H}^2 \quad C_7: \quad R_{19}: \quad$$

$$\text{Def 7: } \begin{array}{c} \textcircled{+} \\ \text{---} \\ \text{---} \end{array} := \begin{array}{c} \text{---} \\ \text{---} \xrightarrow{\text{H}} \bullet \xleftarrow{\text{H}} \text{---} \\ \text{---} \end{array} \quad R_{17}: \begin{array}{c} \bullet \\ \text{---} \\ \text{---} \\ \bullet \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \bullet \\ \bullet \end{array} \quad C_{15}^1: \begin{array}{c} \bullet \\ \text{---} \\ \text{---} \\ \bullet \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \bullet \\ \bullet \end{array}$$

$$R_{3|} : \text{Diagram with } \oplus \text{ at top} = \text{Diagram with } \ominus \text{ at top}$$

$$C_{15}^{14} : \text{Diagram with } \bullet \text{ at top} = \text{Diagram with } \oplus \text{ at top}$$

$$\text{Def 2: } \text{Diagram with } \oplus \text{ at top} := \text{Diagram with } H \text{ and } H^{\dagger} \text{ connected}$$

$$\text{Def 3: } \text{X} := \text{H} \cdot \text{H} \cdot \text{H} \quad \text{Def 5: } \text{+} := \text{X} \cdot \text{X}$$

$$\text{Def 4: } \begin{array}{c} \bullet \\ \parallel \end{array} := \begin{array}{c} \bullet \\ \parallel \\ \text{---} \\ | \quad | \\ \text{H} \quad \text{H}^3 \end{array}$$

Lem 17 Def 2-5, Def 7, C_2 , C_3 , C_7 , C_8 , C_{15} , R_{16} , R_{17} , R_{19} , R_{31} & R_{50} imply

$$7.3.(7)-(9) \quad = \quad \begin{array}{c} \text{Circuit Diagram} \\ \text{Left: } B_{02} \text{ and } B_{2d} \text{ in series.} \\ \text{Right: } B_{00}, B_{2d}, \text{ and a sequence of gates } S, S, H, H, H, H, S. \end{array} \bullet w^2$$

Proof: 7.3.(7)-(9). LHS :=

The circuit diagram illustrates the decomposition of a two-qubit gate B_{2d} (represented by a box) into a sequence of operations. The decomposition is defined as follows:

- The first part of the decomposition consists of a vertical sequence of three operations: a H^2 gate (green box), a S^{2d} gate (orange box), and another H^2 gate (green box).
- The second part of the decomposition consists of a CNOT gate (represented by a box with an 'X') and a measurement (indicated by a circle with a plus sign).
- The entire sequence is enclosed in a green box.

The diagram shows a quantum circuit with four horizontal wires. On the left, there is a red label $C_8 \equiv R_{19}$. The circuit consists of several components:

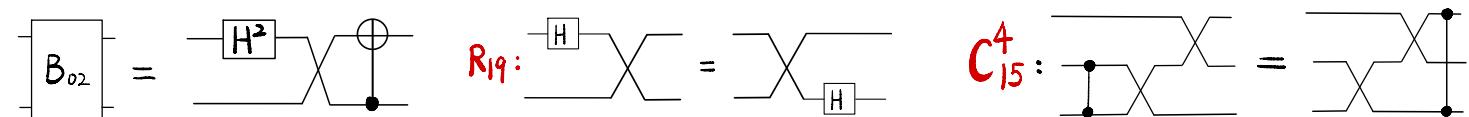
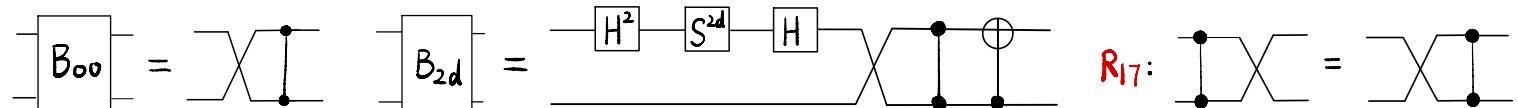
- A blue box labeled H^2 is connected to the top wire.
- An orange box labeled S^{2d} is connected to the top wire.
- A blue box labeled H^2 is connected to the second wire.
- A green box labeled \oplus is connected to the third wire.
- A blue box labeled H is connected to the fourth wire.
- Control points are indicated by small black dots on the wires.
- Measurement symbols (circles with a plus sign) are placed at the end of the wires.

 The circuit is enclosed in a light gray rectangular frame.

Quantum circuit diagram for C_7, R_{31}, G_{14}, G_5 . The circuit consists of the following sequence of operations:

- H^2
- S^{2d}
- Controlled operation (green box):
 - Two H^2 gates (one in series with a control line)
 - Two H gates (one in series with a control line)
 - Two I gates (one in series with a control line)
- Measurement

A blue line connects the output of the green box to the input of the final H gate.



$C_2: H^4 = I$ $C_3: S^3 = I$ R_{16} : = _____

Lem 17

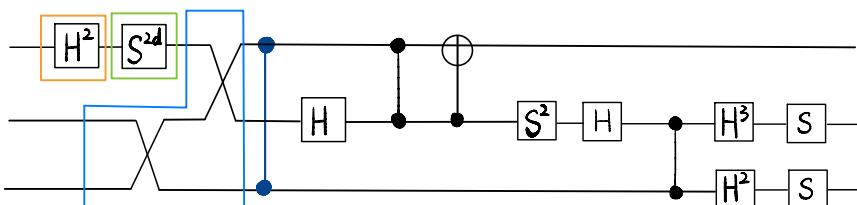
7.3.(7)-(9) • w^2

Proof cont:

7.3.(7)-(9).RHS := • w^2

def • w^2

R19 • w^2

R17
 C_{15}^4  • w^2

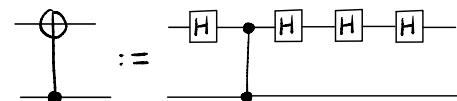
Hence,

WTS

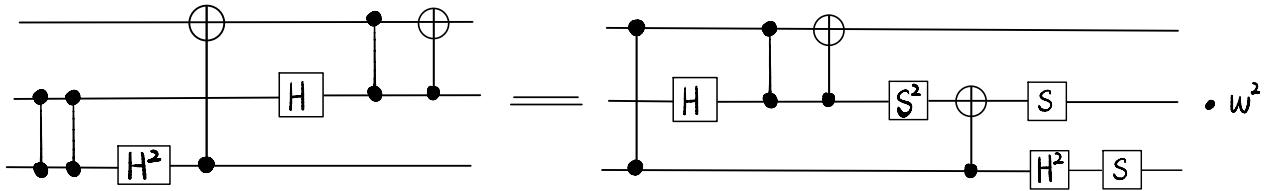
• w^2 $\underline{C_2, G_3}$ $\underline{R16}$

$C_2 : H^4 = I$

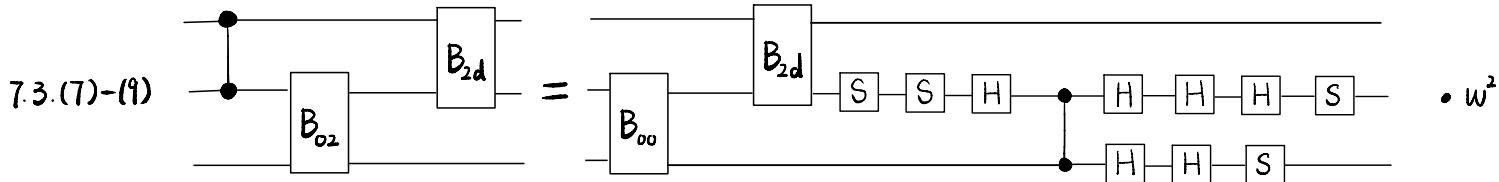
Def 2:



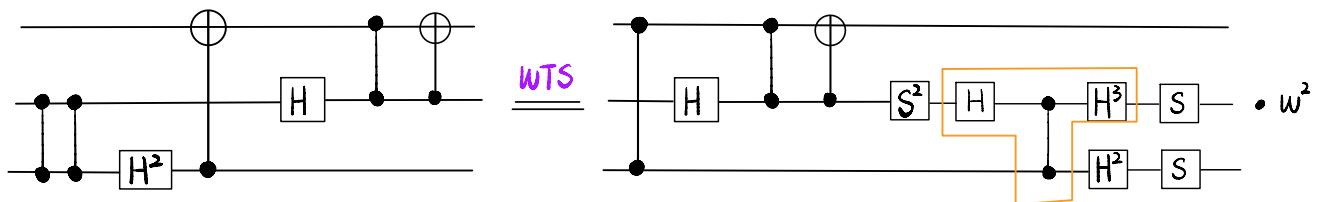
$R_{50} :$



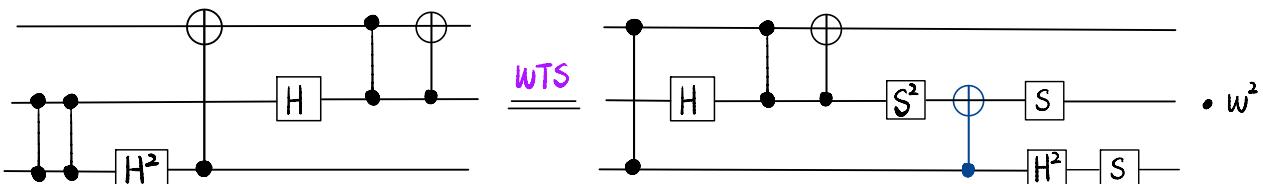
Lem 17



Proof cont:



$C_2 \parallel Def 2$



By R_{50} , this completes the proof.

