

$$R_{17}: \quad \text{CNOT} = \text{CNOT}$$

$$\text{Def 4: } \oplus := \text{H} \text{H}^3$$

$$\text{Def 2: } \oplus := \text{H} \text{H}^3$$

$$\text{Def 7: } \oplus := \text{H} \text{H}^3$$

$$\text{Def 5: } \oplus := \text{CNOT}$$

$$C_{15}^1: \quad \text{CNOT} = \text{CNOT}$$

$$R_{31}: \quad \text{CNOT} = \text{CNOT}$$

$$C_{15}^{14}: \quad \text{CNOT} = \text{CNOT}$$

$$\text{Def 3: } \text{CNOT} := \text{H} \text{H} \text{H} \text{H}$$

$$C_{15}^4: \quad \text{CNOT} = \text{CNOT}$$

$$C_7: \quad \text{S} = \text{S}$$

$$R_{19}: \quad \text{H} = \text{CNOT}$$

$$C_8: \quad \text{H}^2 = \text{H}^2$$

Lem 29 Def 1-5, Def 7, $C_2, C_3, C_5, C_7, C_{15}, R_5, R_{10}, R_{11}, R_{16}, R_{17}, R_{18}, R_{19}, R_{31}$ & R_{60} imply

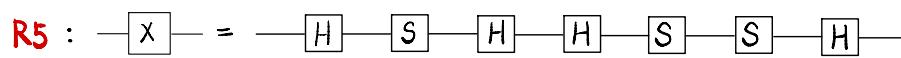
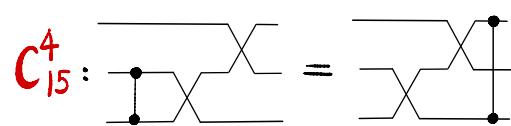
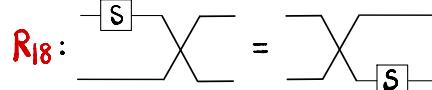
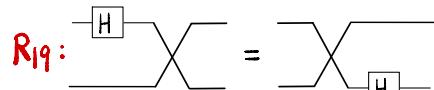
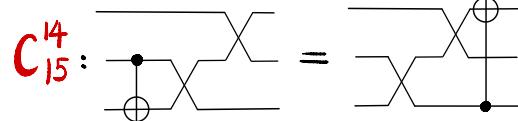
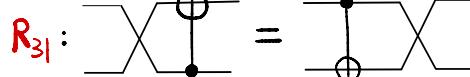
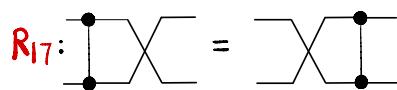
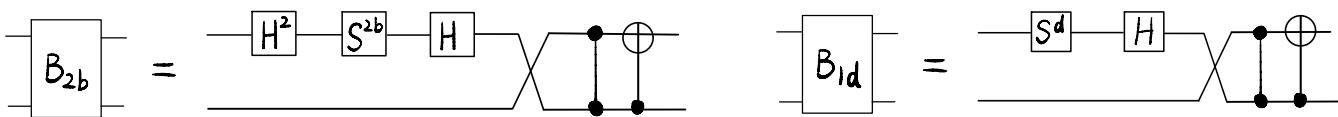
$$\begin{array}{c} 7.7-9. \\ (4)-(6) \end{array} \quad B_{2b} \quad B_{1d} = B_{2,b+2} \quad B_{1,d+1} \quad X \quad Z \\ \text{H} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{Z} \quad \text{Z} \\ \text{H} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{S} \quad \text{H} \quad \text{S} \quad \text{Z} \quad \text{Z} \end{array}$$

$$\text{Proof: } 7.7-9.(4)-(6). \text{ LHS} := B_{2b} B_{1d} \stackrel{\text{def}}{=} \text{H}^2 \text{S}^{2b} \text{H} \quad \text{CNOT} \quad \text{S}^d \text{H}$$

$$\frac{R_{17}, R_{31}}{C_{15}^4, C_5^{14}}$$

$$\frac{C_8}{C_7}$$

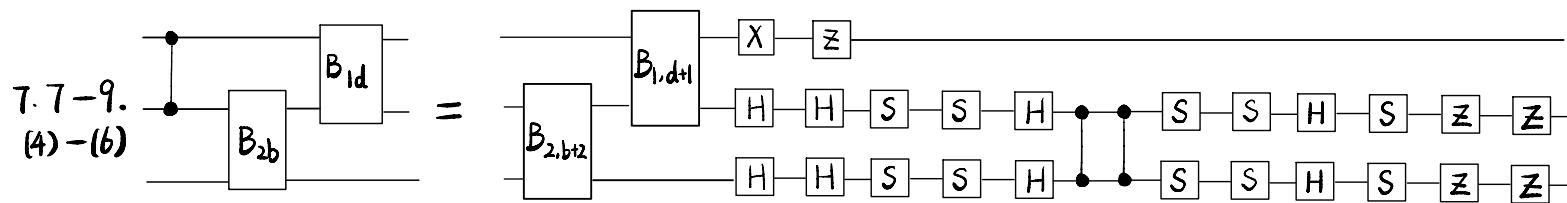
$$\frac{R_{19}}{C_5^1}$$



$$C_3: \quad S^3 = I$$

$$R_{10}: \quad \boxed{\bar{z}} = \boxed{S'} \boxed{S'} \boxed{S}$$

Lem 29



Proof cont:

$$\begin{aligned} & 7.7-9.(4)-(6).RHS := \\ & \quad B_{1,d+1} \quad X \quad z \\ & \quad B_{2,b+2} \quad H \quad H \quad S \quad S \quad H \quad S \quad S \quad H \quad S \quad z \quad z \\ & \quad H \quad H \quad S \quad S \quad H \quad S \quad S \quad H \quad S \quad z \quad z \end{aligned}$$

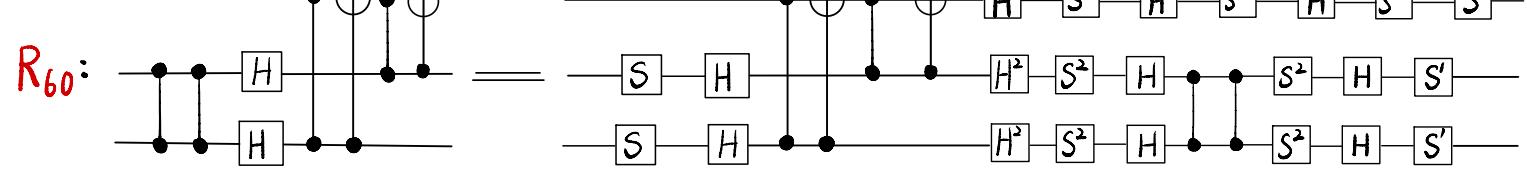
$$2(b+2) = 2b+1$$

$$\begin{aligned} & \text{def} \\ & \boxed{H^2 S^{2b} S H} \quad \boxed{S^d S H} \quad \boxed{H^2 S^2 H} \quad \boxed{X} \quad \boxed{z} \\ & \quad \boxed{H^2 S^{2b} S H} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{z^2} \end{aligned}$$

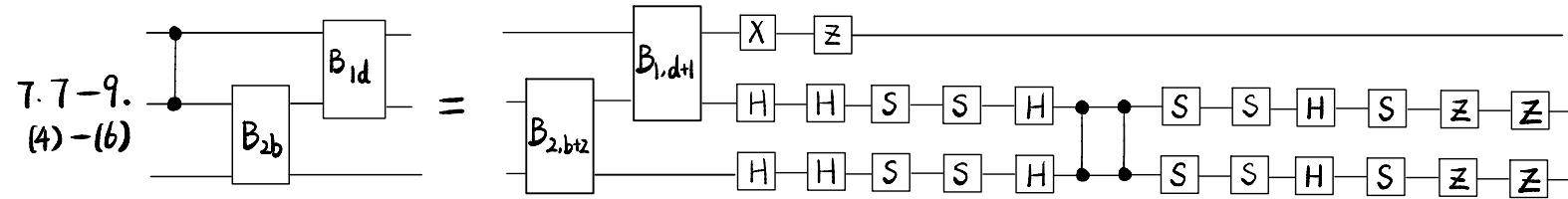
$$\begin{aligned} & R_{17}, R_{31} \\ & C_{15}^4, C_{15}^{14} \\ & \boxed{S^d S H} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{X} \quad \boxed{z} \\ & \quad \boxed{H^2 S^{2b} S H} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{z^2} \\ & \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{z^2} \end{aligned}$$

$$\begin{aligned} & R_{18} \\ & R_{19} \\ & \boxed{S^d} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{X} \quad \boxed{z} \\ & \quad \boxed{H^2 S^{2b} S H} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S} \quad \boxed{z^2} \\ & \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{S} \quad \boxed{z^2} \end{aligned}$$

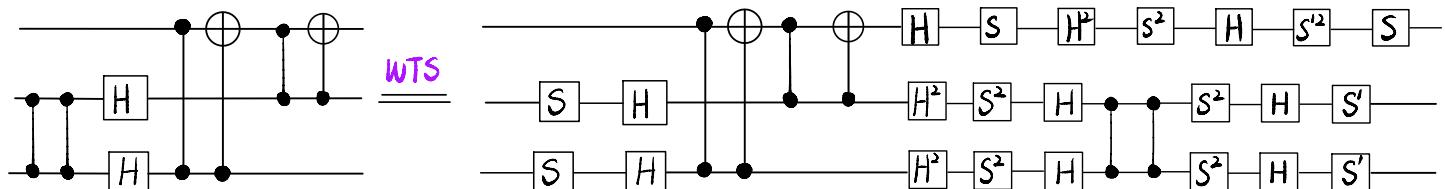
$$\begin{aligned} & R_5 \\ & R_{10}, R_{11} \\ & \boxed{S^d} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{S} \\ & \quad \boxed{H^2 S^{2b} S H} \quad \boxed{H^2 S^2 H} \quad \boxed{S^2 H} \quad \boxed{S^2 H} \quad \boxed{S} \end{aligned}$$



Lem 29



Proof cont : Hence



By R_{60} , this completes the proof.

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