

$$\begin{array}{ll}
R_{23}: \quad \text{Diagram} = \text{Diagram} \cdot w^2 & R_{25}: \quad \text{Diagram} = \text{Diagram} \cdot w \\
C_7: \quad \text{Diagram} = \text{Diagram} & C_1: w^3 = 1 \quad C_3: S^3 = I \\
R_{24}^I: \quad \text{Diagram} = \text{Diagram} & C_5: SS' = S'S \\
R_{15}: \quad \text{Diagram} = \text{Diagram} & C_6: \quad \text{Diagram} = \text{Diagram}
\end{array}$$

Lem B1 R_{23}^8 :

$$\begin{array}{l}
\text{Diagram} = \text{Diagram} \cdot w \\
\text{Proof: } R_{23}^8 \cdot \text{LHS} := \text{Diagram} \stackrel{R_{23}}{=} \text{Diagram} \cdot w^2 \stackrel{R_{23}}{=} \text{Diagram} \cdot w^2 \cdot w^2 \\
\text{C}_1, \text{C}_7 \quad \text{Diagram} \cdot w =: R_{23}^8 \cdot \text{RHS.} \quad \boxed{\checkmark}
\end{array}$$

Lem B2 R_{25}^5 :

$$\begin{array}{l}
\text{Diagram} = \text{Diagram} \cdot w^2 \\
\text{Proof: } R_{25}^5 \cdot \text{LHS} := \text{Diagram} \stackrel{R_{25}}{=} \text{Diagram} \cdot w \stackrel{R_{25}}{=} \text{Diagram} \cdot w \cdot w \\
\text{C}_7 \quad \text{Diagram} \cdot w^2 =: R_{25}^5 \cdot \text{RHS.} \quad \boxed{\checkmark}
\end{array}$$

Lem B3 R_{23-25} :

$$\begin{array}{l}
\text{Diagram} = \text{Diagram} \cdot w \\
\text{Proof: } R_{23-25} \cdot \text{LHS} := \text{Diagram} \stackrel{R_{23}}{=} \text{Diagram} \cdot w \stackrel{R_{23}^8}{=} \text{Diagram} \cdot w \stackrel{R_{24}^I}{=} \text{Diagram} \cdot w \cdot w \\
\text{R}_{25}^5 \quad \text{Diagram} \cdot w^2 \cdot w^2 =: \text{Diagram} \cdot w \quad \text{C}_7 \quad \text{Diagram} \cdot w =: \text{Diagram} \cdot w \cdot w \\
\text{C}_3, \text{C}_5 \quad \text{Diagram} \cdot w =: R_{23-25} \cdot \text{RHS.} \quad \boxed{\checkmark}
\end{array}$$

$$\begin{array}{l}
R_{25}: \quad \text{Diagram} = \text{Diagram} \cdot w \quad R_{24}^1: \quad \text{Diagram} = \text{Diagram} \\
R_{24}: \quad \text{Diagram} = \text{Diagram} \quad C_5: SS' = S'S \quad R_{23}: \quad \text{Diagram} = \text{Diagram} \cdot w^2 \quad C_1: w^3 = 1 \\
C_7: \quad \text{Diagram} = \text{Diagram} \quad R_{15}: \quad \text{Diagram} = \text{Diagram} \quad C_6: \quad \text{Diagram} = \text{Diagram} \\
C_6^{2*}: \quad \text{Diagram} = \text{Diagram}
\end{array}$$

$$\text{Lem B4} \quad R_{25}^6: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

$$\text{Proof: } R_{25}^6. \text{LHS} := \text{Diagram} \stackrel{R_{25}}{=} \text{Diagram} \cdot w \stackrel{R_{24}}{=} \text{Diagram} \cdot w$$

$$\underline{\underline{R_{25}}} \quad \text{Diagram} \cdot w \cdot w \stackrel{C_3}{=} \text{Diagram} \cdot w^2 =: R_{25}^6. \text{RHS.} \quad \text{VI}$$

$$\text{Lem B5} \quad R_{23}^9: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

$$\text{Proof: } R_{23}^9. \text{LHS} := \text{Diagram} \stackrel{R_{23}}{=} \text{Diagram} \cdot w^2 \stackrel{R_{23}}{=} \text{Diagram} \cdot w^2 \cdot w^2$$

$$\underline{\underline{R_{25}}} \quad C_1 \quad \text{Diagram} \cdot w \cdot w \stackrel{C_3, C_5}{=} \text{Diagram} \cdot w^2 =: R_{23}^9. \text{RHS.} \quad \text{VI}$$

$$\text{Lem B7} \quad R_{25}^7: \quad \text{Diagram} = \text{Diagram} \cdot w$$

$$\text{Proof: } R_{25}^7. \text{LHS} := \text{Diagram} \stackrel{R_{25}^6}{=} \text{Diagram} \cdot w^2$$

$$\underline{\underline{R_{25}^6}} \quad \text{Diagram} \cdot w^2 \cdot w^2 \stackrel{C_1}{=} \text{Diagram} \cdot w^2$$

$$\underline{\underline{C_3}} \quad \text{Diagram} \cdot w =: R_{25}^7. \text{RHS.}$$

VI

2

$$R_{16}: \quad \text{Diagram} = \text{Diagram}$$

$$R_{17}: \quad \text{Diagram} = \text{Diagram}$$

$$R_{22}: \quad \text{Diagram} = \text{Diagram}$$

$$R_{18}: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Diagram} = \text{Diagram}$$

$$R_{31}: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Lem B17} \quad R_{23-25}: \quad \text{Diagram} = \text{Diagram} \cdot w \quad \text{implies}$$

$$R_{23-25}^1: \quad \text{Diagram} = \text{Diagram} \cdot w$$

$$\text{Proof: } R_{23-25}^1. \text{LHS} := \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram} \stackrel{\text{R18}}{=} \text{Diagram} \stackrel{R_{31}}{=} \text{Diagram}$$

$$R_{23-25} \stackrel{\text{R17, R18}}{=} \text{Diagram} \stackrel{R_{31}}{=} \text{Diagram} \cdot w$$

$$R_{16} \stackrel{\text{R16}}{=} \text{Diagram} \cdot w =: R_{23-25}^1. \text{RHS} \quad \square$$

$$R_{25}^5: \quad \text{Diagram} = \text{Diagram} \cdot w^2 \quad \text{implies}$$

$$R_{25}'^5: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

$$\text{Proof: } R_{25}'^5. \text{LHS} := \text{Diagram} \stackrel{R_{16}}{=} \text{Diagram} \stackrel{\text{R16}}{=} \text{Diagram}$$

$$R_{25}^5 \stackrel{\text{R18, R22}}{=} \text{Diagram} \stackrel{\text{R17, R31}}{=} \text{Diagram} \cdot w^2$$

$$R_{16} \stackrel{\text{R16}}{=} \text{Diagram} \cdot w^2 =: R_{25}'^5. \text{RHS.} \quad \square$$

$R_{16}:$ =
 $R_{18}:$ =
 $R_{17}:$ =
 $C_6^{2*}:$ =
 $C_8^8:$ =
 $C_8^7:$ =
 $C_8:$ =
 $R_{23}^{q'}:$ =

Def 1: $[S'] := [H] [H] [S] [H] [H]$

Lem B 25 $R_{23}^{q'}:$ = $\cdot w^2$ implies

$R_{23}^{q'}:$ = $\cdot w^2$

Proof: $R_{23}^{q'}$. LHS := $\stackrel{R_{16}}{=} \boxed{\text{orange box}}$ $\stackrel{R_{18}}{=} \boxed{\text{orange box}}$ $\cdot w^2$

$\stackrel{R_{23}}{=} \boxed{\text{orange box}}$ $\cdot w^2$ $\stackrel{R_{17}, R_{18}}{=} \boxed{\text{orange box}}$ $\cdot w^2$

$\stackrel{R_{16}}{=} \boxed{\text{orange box}}$ $\cdot w^2 =: R_{23}^{q'}$. RHS

IV

Lem B 28

$R_{23}^{10}:$ = $\cdot w^2$

Proof: R_{23}^{10} . LHS := $\stackrel{\text{Def 1}}{=} \boxed{\text{orange box}}$ $\cdot w^2$

$\stackrel{R_{23}^{q'}}{=} \boxed{\text{orange box}}$ $\cdot w^2$ $\stackrel{C_8^7}{=} \boxed{\text{orange box}}$ $\cdot w^2$ $\stackrel{\text{Def 1}}{=} \boxed{\text{orange box}}$ $\cdot w^2 =: R_{23}^{10}$. LHS

IV

$$\text{Def 4: } \oplus := \begin{array}{c} \bullet \\ \oplus \\ \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \text{H} \quad \text{H}^3 \end{array}$$

$$\text{Def 2: } \oplus := \begin{array}{c} \oplus \\ \bullet \\ \bullet \end{array} = \begin{array}{c} \text{H} \quad \bullet \\ \bullet \quad \text{H}^3 \end{array}$$

$$C_8^1: \begin{array}{c} \text{H}^2 \\ \bullet \\ \bullet \end{array} = \begin{array}{c} \bullet \\ \bullet \quad \text{H}^2 \end{array}$$

$$C_7: \begin{array}{c} \bullet \\ \bullet \quad S \\ \bullet \end{array} = \begin{array}{c} S \\ \bullet \quad \bullet \end{array}$$

$$C_2: H^4 = I \quad C_3: S^3 = I$$

Lem B29 $R_{23-25}^1:$ $\cdot w \text{ implies}$

$$R_{23-25}^2:$$
 $\cdot w$

Proof: $R_{23-25}^2. \text{LHS} :=$ $\stackrel{\text{Def 2}}{=} \begin{array}{c} \text{S}^2 \quad \text{H} \quad \oplus \\ \text{S}^2 \quad \text{H} \end{array} \stackrel{\text{C}_8^1}{=} \begin{array}{c} \text{S}^2 \quad \text{H}^2 \quad \text{H}^3 \\ \text{S}^2 \quad \text{H} \end{array}$

$$\stackrel{\text{C}_7}{=} \begin{array}{c} \text{S}^2 \quad \text{H} \quad \text{S}^2 \quad \text{H} \\ \text{S}^2 \quad \text{H} \quad \text{H}^3 \quad \text{H} \end{array} \stackrel{\text{Def 4}}{=} \begin{array}{c} \text{S}^2 \quad \text{H} \\ \text{S}^2 \quad \text{H} \end{array}$$

$$\stackrel{\text{R}_{23-25}^1}{=} \begin{array}{c} \text{S}^2 \quad \text{S}^2 \quad \text{H} \\ \text{S}^2 \quad \text{H} \end{array} \cdot w \stackrel{\text{C}_3}{=} \begin{array}{c} \text{S} \quad \text{H} \\ \text{S}^2 \quad \text{H} \end{array} \cdot w =: R_{23-25}^2. \text{RHS}$$

IV

Def 2: :=

Def 4: :=

R_{23}^8 : =

R_{16} : =

R_{18} : =

R_{22} : =

R_{31} : =

R_{17} : =

Lem B30 R_{25}^6 : = $\cdot w^2$ implies

$R_{25}^{6'}$: = $\cdot w^2$

Proof: $R_{25}^{6'}$. LHS := $\stackrel{R_{16}}{=} \text{orange box}$ $\stackrel{R_{17}}{=} \text{orange box}$ $\stackrel{R_{31}}{=}$

$\stackrel{R_{25}^6}{=} \text{orange box}$ $\cdot w^2 \stackrel{R_{18}, R_{22}}{=} \text{orange box}$ $\stackrel{R_{17}, R_{31}}{=} \text{orange box}$ $\cdot w^2$

$\stackrel{R_{16}}{=} \text{orange box}$ $\cdot w^2 =: R_{25}^{6'}$. RHS

IV

Lem B31 R_{23}^8 : =

$R_{23}^{8'}$: =

$\cdot w$ implies $R_{23}^{8'}$: =

Proof: Reasoning analogously as before.

V

$$C_7: \text{Diagram} = \text{Diagram} \quad C_1: w^3 = 1 \quad C_6: \text{Diagram} = \text{Diagram} \quad R_{15}: \text{Diagram} = \text{Diagram}$$

$$C_1: w^3 = 1 \quad C_6^*: \text{Diagram} = \text{Diagram} \quad C_2: H^4 = I \quad C_3: S^3 = I$$

$$\text{Def 1: } [S'] := [H] [H] [S] [H] [H] \quad C_6^1: \text{Diagram} = \text{Diagram}$$

Lem B35

$$R_{23}^{10}: \text{Diagram} = \text{Diagram} \cdot w^2 \text{ implies } R_{23}^{11}: \text{Diagram} = \text{Diagram} \cdot w,$$

$$R_{23}^{12}: \text{Diagram} = \text{Diagram} \cdot w^2 \quad \& \quad R_{23}^{13}: \text{Diagram} = \text{Diagram} \cdot w,$$

$$R_{23}^{14}: \text{Diagram} = \text{Diagram} \cdot w$$

Proof: $R_{23}^{11} \cdot \text{LHS} :=$

$$\frac{C_1, G}{R_{15}} \text{Diagram} \cdot w =: R_{23}^{11} \cdot \text{RHS}$$

R_{23}^{12} & R_{23}^{13} can be proved analogously as before.

$$R_{23}^{14} \cdot \text{LHS} :=$$

□

Lem B36

$$R_{23}^{8'}: \text{Diagram} = \text{Diagram} \cdot w \text{ implies}$$

$$R_{23}^{15}: \text{Diagram} = \text{Diagram} \cdot w^2 = \text{Diagram} \cdot w^2$$

Proof: $R_{23}^{15} \cdot \text{RHS} :=$

$$\frac{\text{Def 1, } C_1}{C_2, C_3, C_6} \text{Diagram} =: R_{23}^{15} \cdot \text{LHS}$$

□

7

$$\begin{array}{c}
 C_6^*: \quad \text{Diagram} = \text{Diagram} \\
 R_{24}: \quad \text{Diagram} = \text{Diagram} \\
 C_7: \quad \text{Diagram} = \text{Diagram} \\
 R_{15}: \quad \text{Diagram} = \text{Diagram} \\
 R_5^{25}: \quad \text{Diagram} = \text{Diagram} \\
 C_1: w^3 = I \\
 C_3: S^3 = I
 \end{array}$$

Lem B40 $R_{25}^7:$ $\text{Diagram} = \text{Diagram} \cdot w \text{ implies}$

$R_{25}^{7'}:$ $\text{Diagram} = \text{Diagram} \cdot w$

Proof: Reasoning analogously as before. IV

Lem B42 $R_{23}^{13}:$ $\text{Diagram} = \text{Diagram} \cdot w \text{ implies } R_{23}^{16}:$ $\text{Diagram} = \text{Diagram} \cdot w$

Proof: $R_{23}^{16} \cdot \text{LHS} :=$ $\text{Diagram} \underset{\text{R}_{23}^{13}}{=} \text{Diagram} \cdot w \underset{\text{R}_{24}^{13}}{=} \text{Diagram} \cdot w \cdot w$

$\underline{\underline{R_{25}^5}}$ $\text{Diagram} \cdot w^2 \cdot w^2$

$\underline{\underline{C_1, C_3, C_6}}$ $\text{Diagram} \cdot w =: R_{23}^{16} \cdot \text{RHS}$ VII

$$C_6^2: \quad \text{Diagram} = \text{Diagram} \quad C_6^1: \quad \text{Diagram} = \text{Diagram} \quad C_8^6: \quad \text{Diagram} = \text{Diagram}$$

$$C_8^8: \quad \text{Diagram} = \text{Diagram} \quad C_2: H^4 = I$$

Lem B45 $C_{12} = R_{36}$: $\text{Diagram} = \text{Diagram}$ iff $C'_{12}: \quad \text{Diagram} = \text{Diagram}$

Proof: $C_{12} = R_{36}$: $\text{Diagram} = \text{Diagram}$
 $C_6^1 \equiv C_6^2$

$$C'_{12}: \quad \text{Diagram} = \text{Diagram} \quad \equiv C_2$$

$$C'_{12}: \quad \text{Diagram} = \text{Diagram}$$

$$C_8^6 \equiv C_8^8$$

$$C'_{12}: \quad \text{Diagram} = \text{Diagram}$$

$$\equiv C_2$$

$$C'_{12}: \quad \text{Diagram} = \text{Diagram}$$

□