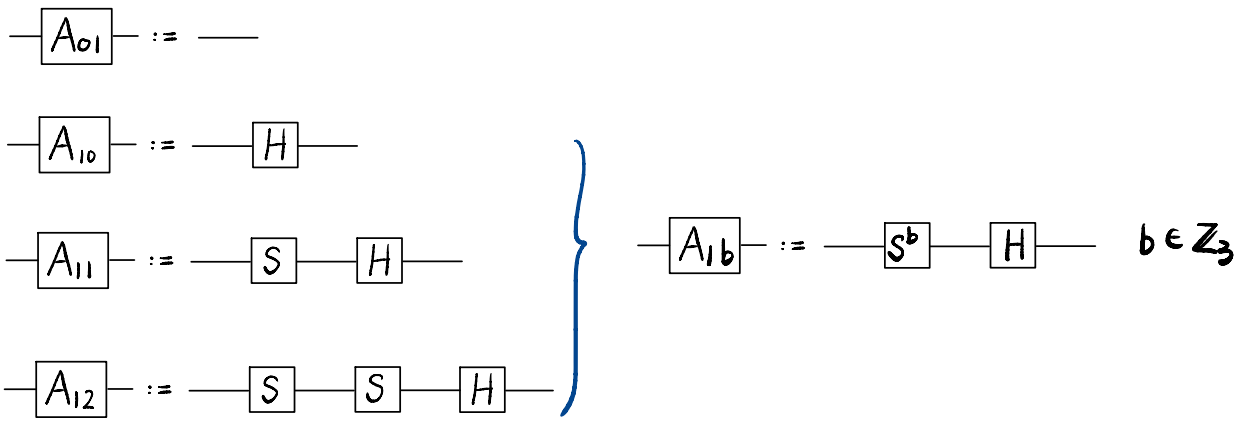


C_3 : $S^3 = I$

Lem 3 By definition and C_3, we have

2.(1) — S — A_{01} — = — A_{01} — S —

(3) — S — A_{10} — = — A_{11} —

(4) — S — A_{11} — = — A_{12} —

(5) — S — A_{12} — = — A_{10} —

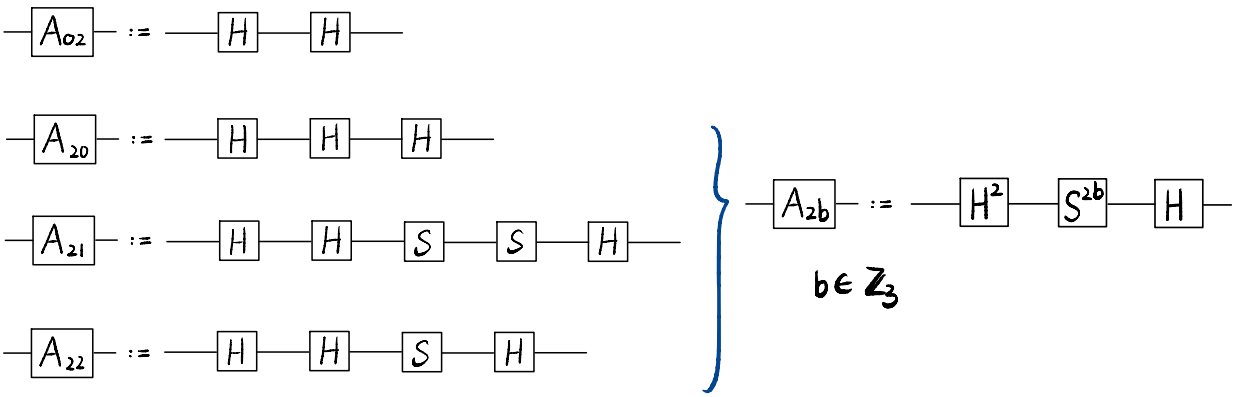
Proof: 2.(1). LHS := — S — =: — A_{01} — S — = 2.(1). RHS

2.(3)/(4)/(5). LHS = — S — A_{1b} — := — S — S^b — H —

= — S^{b+1} — H —

=: — $A_{1,b+1}$ — = 2.(3)/(4)/(5). RHS

Note that 2.(5). LHS := — S — S — S — H — $\overset{C_3}{=}$ — H — =: — A_{10} — = 2.(5). RHS.



C3 : $S^3 = I$

R2 : $S\,H\,H = H\,H\,S\,Z^2$

R3 : $SZ = ZS$

R4 : $HZ^2H^\dagger = X$ $\quad Z^2\,H = H\,X$

Lem 4 By definition, R2, R3 & R4, we have 2.(2) $S\,A_{02} = A_{02}\,S\,Z^2$

(6) $S\,A_{20} = A_{22}\,X$

(7) $S\,A_{21} = A_{20}\,X$

(8) $S\,A_{22} = A_{21}\,X$

Proof: 2.(2). LHS := $S\,H\,H \overset{R2}{=} H\,H\,S\,Z^2$

$=: A_{02}\,S\,Z^2 =$ 2.(2). RHS.

2.(6)/(7)/(8). LHS = $S\,A_{2b} := S\,H^2\,S^{2b}\,H \overset{R2}{=} H\,H\,S\,Z^2\,S^{2b}\,H$

$\overset{R3}{=} H\,H\,S\,S^{2b}\,Z^2\,H = H\,H\,S^{2b+1}\,Z^2\,H$

$= H\,H\,S^{2(b+2)}\,Z^2\,H \overset{R4}{=} H\,H\,S^{2(b+2)}\,H\,X$ ($A_{2,b+2}$)

$=: A_{2,b+2}\,X =$ 2.(6)/(7)/(8). RHS

Note that 2.(7). LHS := $S\,H^2\,S^2\,H \overset{R2}{=} H\,H\,S\,Z^2\,S^2\,H$

$\overset{R3}{=} H\,H\,S\,S^2\,Z^2\,H \overset{C3}{=} H\,H\,Z^2\,H$

$\overset{R4}{=} H\,H\,H\,X =: A_{20}\,X =$ 2.(7). RHS.