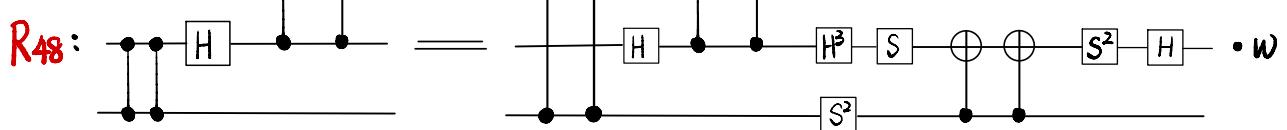
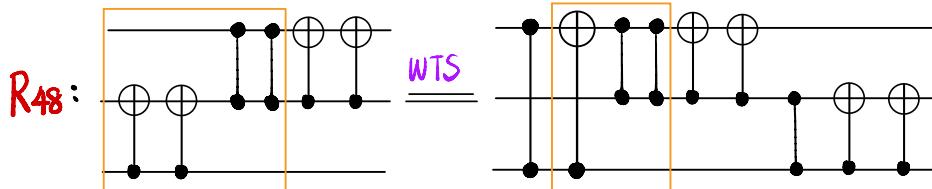


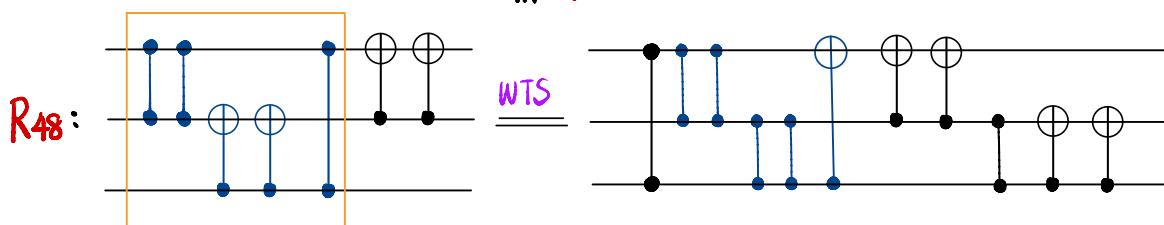
Lem P



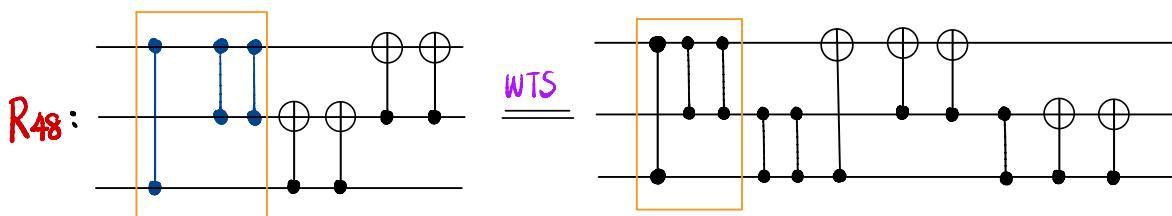
Proof cont.



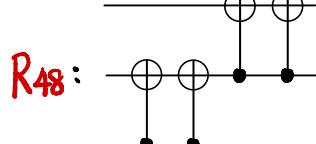
$C_{16}^{16} \equiv C_{16}^{3'}$



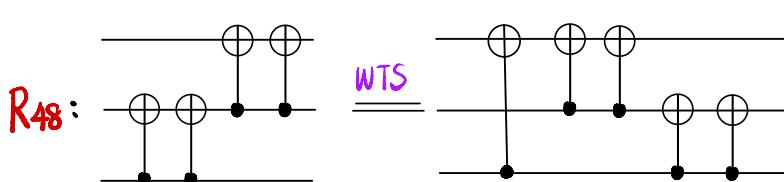
$C_{13}^1 \equiv C_{13}^7$

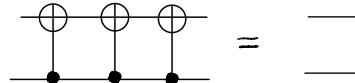
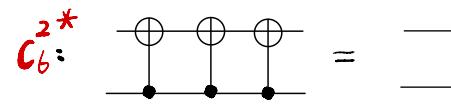
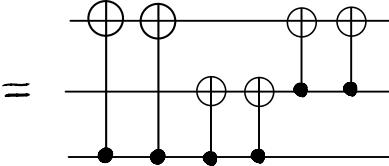
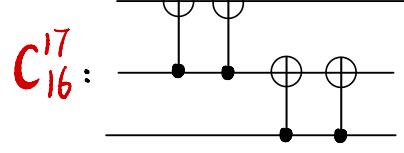


$\equiv C_6^*$

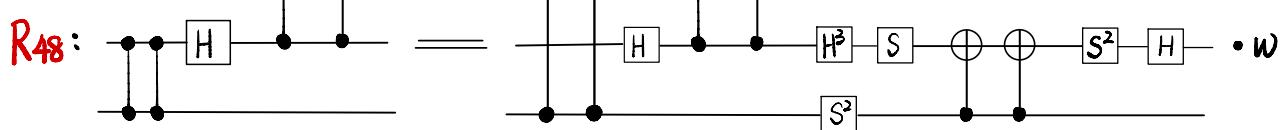


$C_{13}^3 \equiv C_{13}^8, C_6$

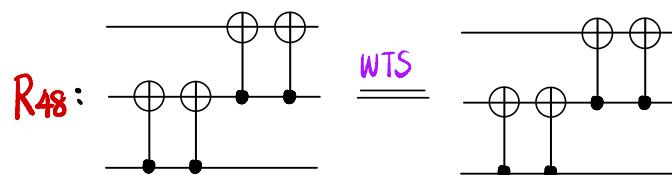
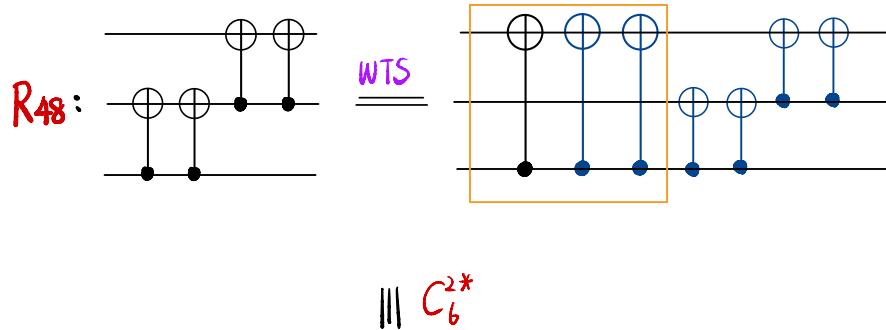
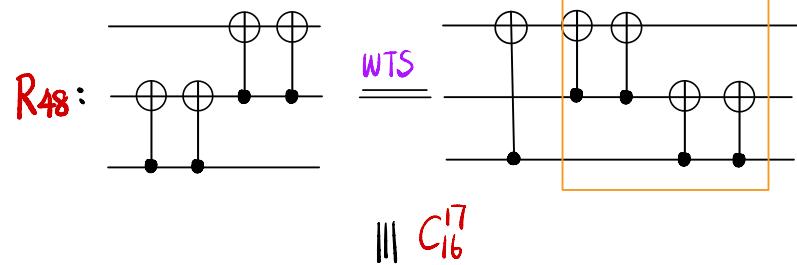




Lem P

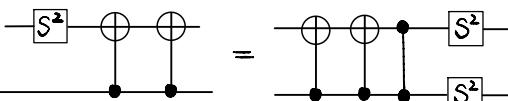
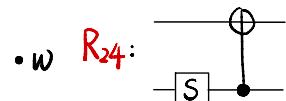
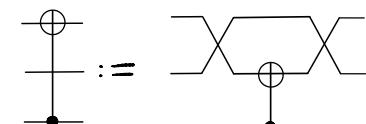
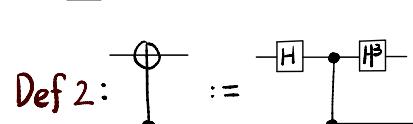
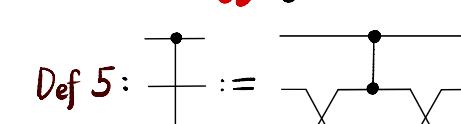
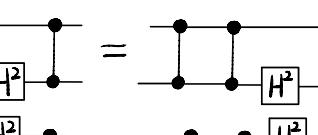
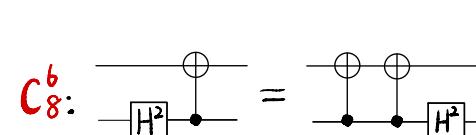
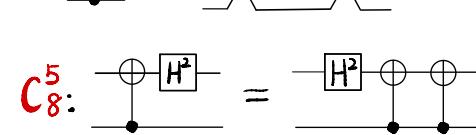


Proof cont.

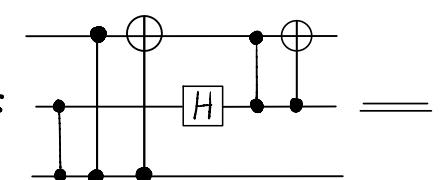
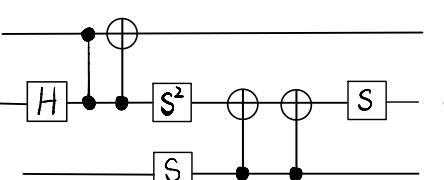


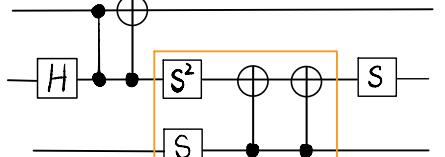
This completes the proof.

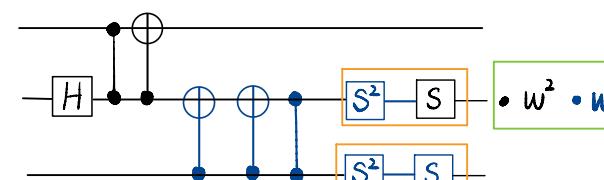
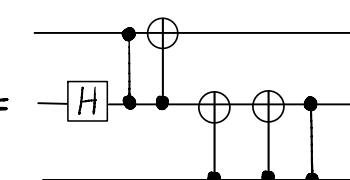
□

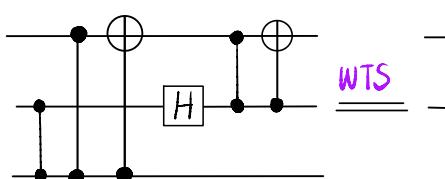
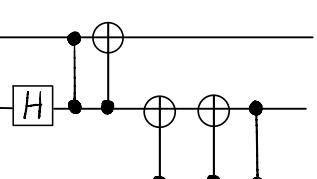
$R_{23-25}:$    $\cdot w$   $R_{24}:$    $C_1: w^3 = I$   $C_2: H^4 = I$   
 $C_3: S^3 = I$   
 $Def 7:$    
 $Def 2:$    
 $Def 5:$    
 $C_8^1:$    
 $C_8^6:$    
 $C_8^5:$  

Lem Q Def 2, Def 5, Def 7,  $C_1, C_2, C_3, C_5, G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}, G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16}$ ,  $R_{23-25}, R_{24}, R_{25}^*$  imply

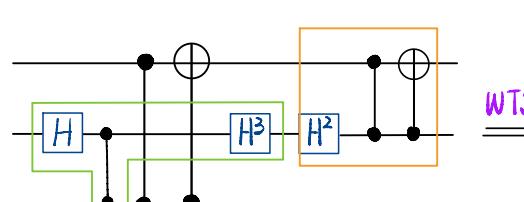
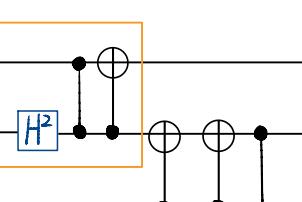
$R_{49}:$    $=$    $\cdot w^2$

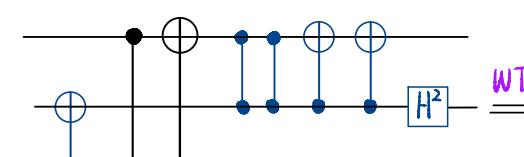
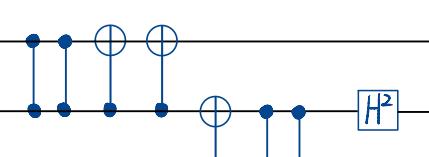
Proof:  $R_{49} \cdot \text{RHS} :=$    $\cdot w^2$

$\frac{R_{23-25}}{R_{24}}$    $\cdot w^2 \cdot w$   $\equiv_{C_1, C_3}$  

Hence  $R_{49}:$    $\equiv_{WTS}$  

$\equiv_{C_2}$

$R_{49}:$    $\equiv_{WTS}$    
 $\text{Def 2} \equiv_{C_8^1, C_8^5, C_8^6}$

$R_{49}:$    $\equiv_{WTS}$  

$$\begin{array}{c}
 C_{16}^{14}: \quad \text{Diagram} = \text{Diagram} \\
 C_{16}^{18}: \quad \text{Diagram} = \text{Diagram} \\
 C_{16}^3: \quad \text{Diagram} = \text{Diagram} \\
 C_{16}^3: \quad \text{Diagram} = \text{Diagram} \\
 C_6^2: \quad \text{Diagram} = \text{Diagram} \\
 C_3: \quad \text{Diagram} = \text{Diagram}
 \end{array}$$

Lem Q

$$R_{49}: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

Proof cont.

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$C_2 \parallel C_{16}^{14}$

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$\parallel C_3$

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$C_{16}^{18} \parallel C_6^2$

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$C_3 \parallel C_{16}^3$

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$\begin{array}{l}
 C_6: \quad \text{Diagram} = \text{Diagram} \\
 R_{25}: \quad \text{Diagram} = \text{Diagram} \cdot w^2 \quad C_1: w^3 = 1 \quad C_2: H^4 = I \\
 R_{25}^*: \quad \text{Diagram} = \text{Diagram} \cdot w \quad C_3: S^3 = I \quad C_5: S'S = SS' \\
 C_{13}^8: \quad \text{Diagram} = \text{Diagram} \quad C_{13}^7: \quad \text{Diagram} = \text{Diagram} \\
 C_{13}^{11}: \quad \text{Diagram} = \text{Diagram}
 \end{array}$$

Lem Q

$$R_{49}: \quad \text{Diagram} = \text{Diagram} \cdot w^2$$

Proof cont.

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

||| C<sub>6</sub>

$$R_{49}: \quad \text{Diagram} \xrightarrow{\text{WTS}} \text{Diagram}$$

$$R_{49}. \text{RHS} := \text{Diagram} \xrightarrow{\substack{R_{25}^5 \\ C_5}} \text{Diagram} \cdot w$$

$$\xrightarrow{\substack{C_{13}^2 \\ C_{13}^8}} \text{Diagram} \cdot w \quad \xrightarrow{C_6} \text{Diagram} \cdot w$$

$$\xrightarrow{\substack{C_{13}^7 \\ C_{13}^{11}}} \text{Diagram} \cdot w \quad \xrightarrow{R_{25}^*} \text{Diagram} =: R_{49}. \text{RHS.} \quad \square$$

$$R_{23}^8: \begin{array}{c} S^2 \\ \square \end{array} \oplus = \begin{array}{c} \oplus \\ \square \end{array} \cdot w$$

$$C_1: w^3=1 \quad C_2: H^4=I \quad C_3: S^3=I$$

$$\text{Def 1: } \begin{array}{c} S' \\ \square \end{array} := \begin{array}{c} H \\ \square \end{array} \begin{array}{c} H \\ \square \end{array} \begin{array}{c} S \\ \square \end{array} \begin{array}{c} H \\ \square \end{array} \begin{array}{c} H \\ \square \end{array}$$

$$\text{Def 7: } \begin{array}{c} \oplus \\ \square \end{array} := \begin{array}{c} X \\ \square \end{array} \begin{array}{c} X \\ \square \end{array}$$

$$\text{Def 2: } \begin{array}{c} \oplus \\ \square \end{array} := \begin{array}{c} H \\ \square \end{array} \begin{array}{c} \bullet \\ \square \end{array} \begin{array}{c} H \\ \square \end{array}$$

$$\text{Def 5: } \begin{array}{c} \bullet \\ \square \end{array} := \begin{array}{c} X \\ \square \end{array} \begin{array}{c} X \\ \square \end{array}$$

$$C_8^{6*}: \begin{array}{c} \oplus \\ H^2 \end{array} = \begin{array}{c} \oplus \\ H^2 \end{array} \cdot H^2$$

Lem R Def 1-2, Def 5, Def 7,  $C_1, C_2, C_3, C_5, C_7, C_8, C_{13}, C_{16}, R_{23}, R_{24}, R_{25}^*$  imply

$$R_{50}: \begin{array}{c} \oplus \\ \square \end{array} \cdot H = \begin{array}{c} H \\ \square \end{array} \cdot \begin{array}{c} S^2 \\ \square \end{array} \oplus \begin{array}{c} S \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} S \\ \square \end{array} \cdot w^2$$

$$\text{Proof: } R_{50} \cdot \text{RHS} := \begin{array}{c} \oplus \\ H \end{array} \cdot \begin{array}{c} S^2 \\ \square \end{array} \oplus \begin{array}{c} S \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} S \\ \square \end{array} \cdot w^2$$

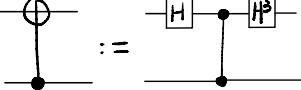
$$\underline{\underline{R_{23}^8}}: \begin{array}{c} \oplus \\ H \end{array} \cdot \begin{array}{c} S^2 \\ \square \end{array} \oplus \begin{array}{c} S \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} S \\ \square \end{array} \cdot w^2 \cdot w$$

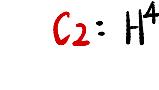
$$\underline{\underline{C_1, C_3}} \quad \underline{\underline{\text{Def 1}}} \quad \begin{array}{c} \oplus \\ H \end{array} \cdot \begin{array}{c} S^2 \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} S^2 \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \cdot \begin{array}{c} S \\ \square \end{array} \quad \underline{\underline{C_2}} \quad \underline{\underline{C_3}}$$

$$\text{Hence } R_{50}: \begin{array}{c} \oplus \\ H^2 \end{array} \cdot \begin{array}{c} H \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \quad \text{WTS} \quad \begin{array}{c} \oplus \\ H^2 \end{array} \quad \text{WTS} \quad \underline{\underline{C_8^{6*}}}$$

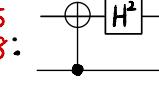
$$R_{50}: \begin{array}{c} \oplus \\ H^2 \end{array} \cdot \begin{array}{c} H \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \quad \text{WTS} \quad \begin{array}{c} \oplus \\ H^2 \end{array} \quad \text{WTS} \quad \underline{\underline{C_2}}$$

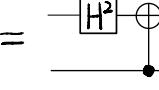
$$R_{50}: \begin{array}{c} \oplus \\ H^2 \end{array} \cdot \begin{array}{c} H^3 \\ \square \end{array} \cdot \begin{array}{c} H^2 \\ \square \end{array} \quad \text{WTS} \quad \begin{array}{c} \oplus \\ H^2 \end{array} \cdot \begin{array}{c} \oplus \\ H^2 \end{array}$$

Def 2:   $\vdash$  

$C_1: w^3 = I$      $C_2: H^4 = I$      $C_8^5: \quad \vdash$  

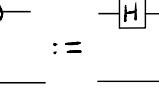
$C_8^1: \quad \vdash$  

$C_8^6: \quad \vdash$  

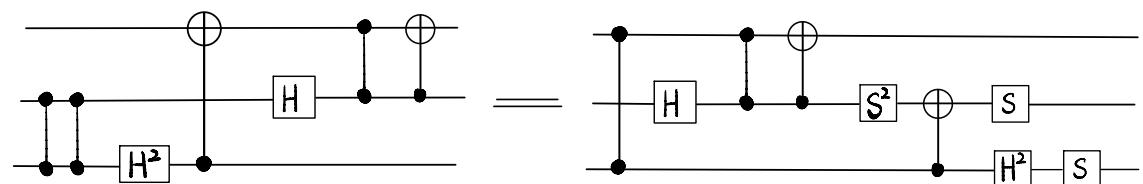
$C_{13}^3: \quad \vdash$  

$C_{16}^{17}: \quad \vdash$  

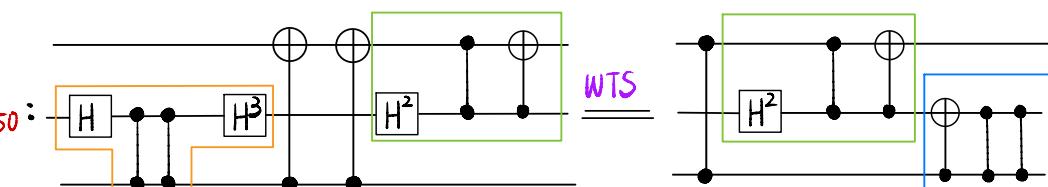
$C_3: S^3 = I$

$C_6^2: \quad \vdash$  

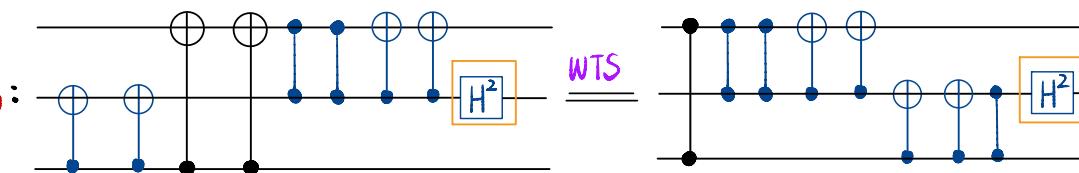
Lem R

$R_{50}: \quad \vdash$  

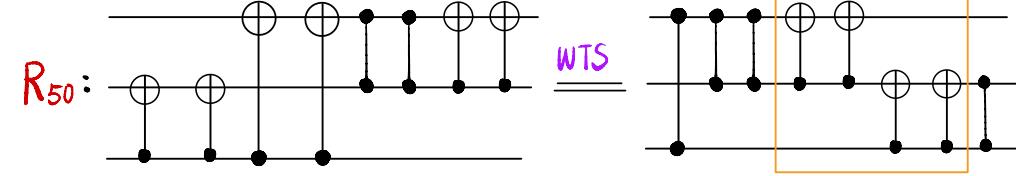
Proof cont.

$R_{50}: \quad \vdash$  

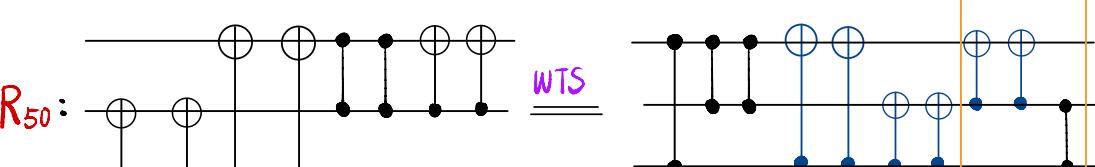
$\text{Def 2 } \parallel C_8^1, C_8^5, C_8^6$

$R_{50}: \quad \vdash$  

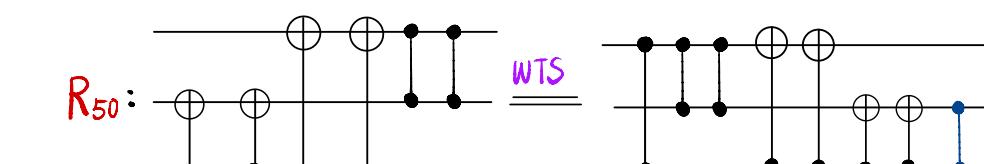
$\parallel C_2$

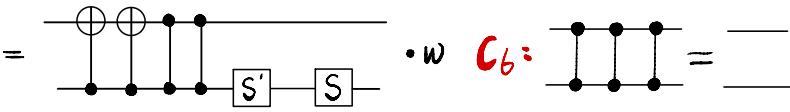
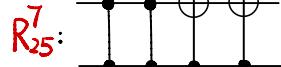
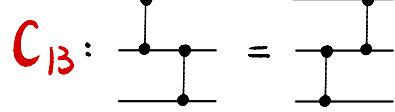
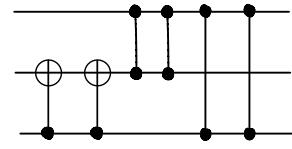
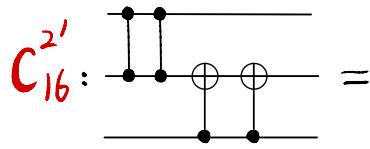
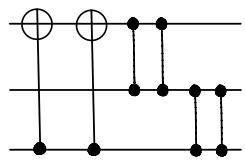
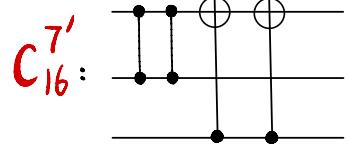
$R_{50}: \quad \vdash$  

$\parallel C_{16}^{17}$

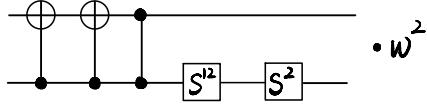
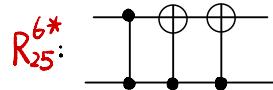
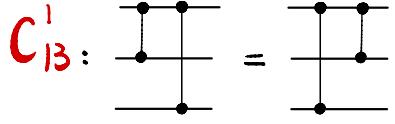
$R_{50}: \quad \vdash$  

$C_6^2 \parallel C_{13}^3$

$R_{50}: \quad \vdash$  

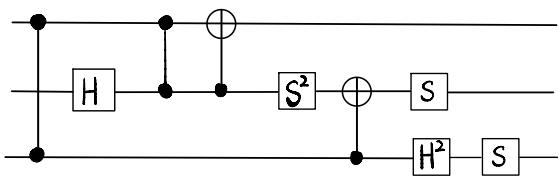
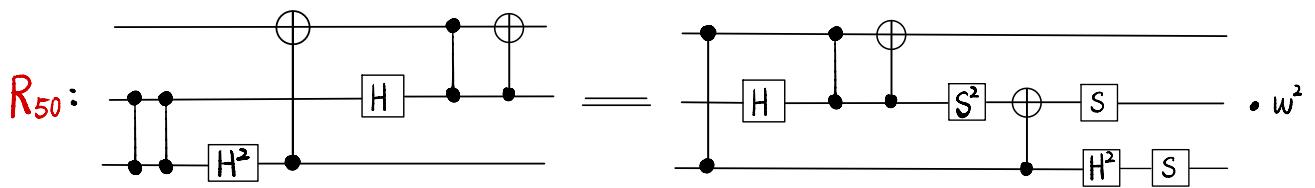


$\cdot w \quad C_6:$

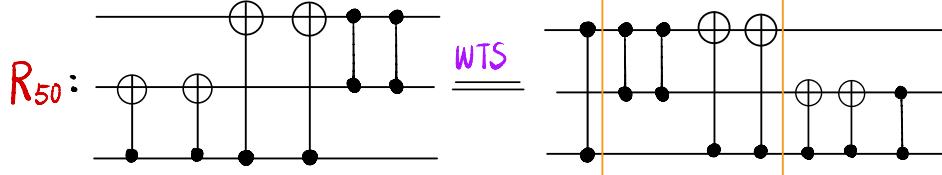


$\cdot w^2$

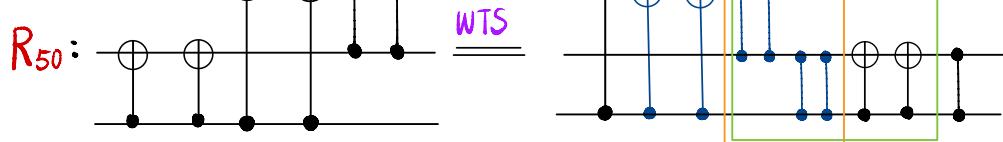
Lem R



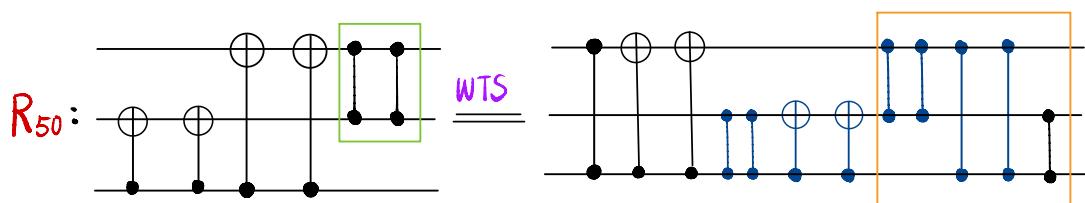
Proof cont.



|||  $C_{16}^{7'}$



$C_B \parallel C_{16}^{2'}$



$C_B \parallel C_6, C_B^1$

