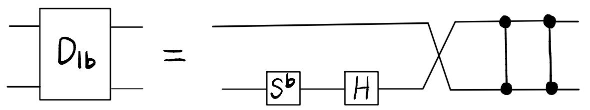
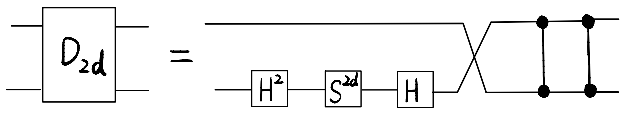
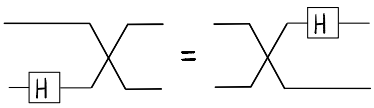
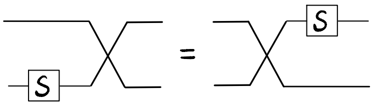
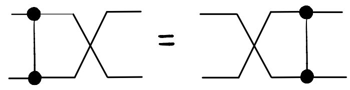
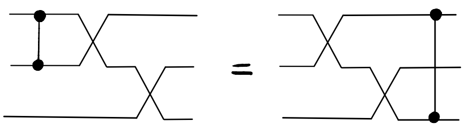


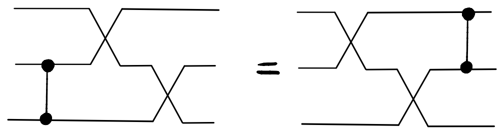


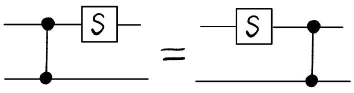
R19 : 

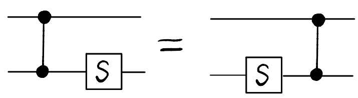
R18 : 

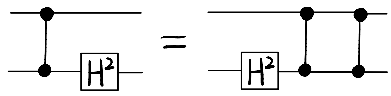
R17 : 

C_{15}^2 : 

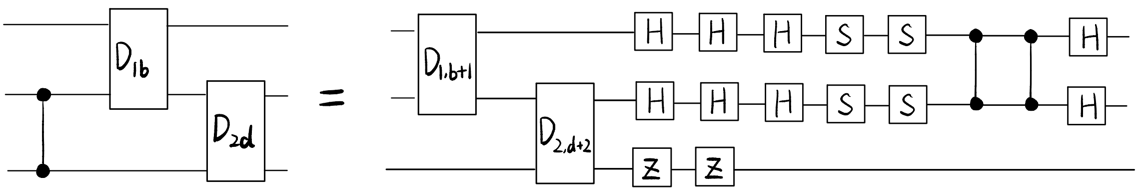
C_{15} : 

C_7 : (1) 

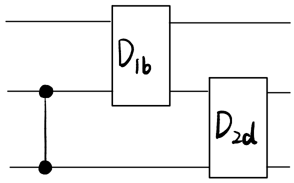
(2) 

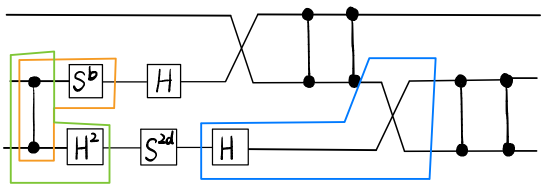
C_8 : 

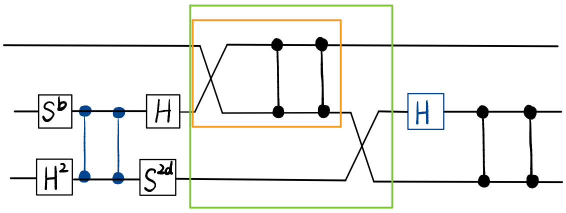
Lem 8 Def 1–2, Def 5, Def 7, C2–3, C6–8, R_{11}, R_{16}, R_{17}, R_{18}, R_{19}, R_{43}, C_{13} & C_{15} imply

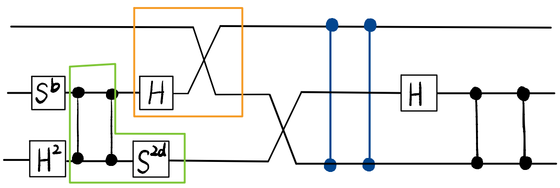
10.(h) 

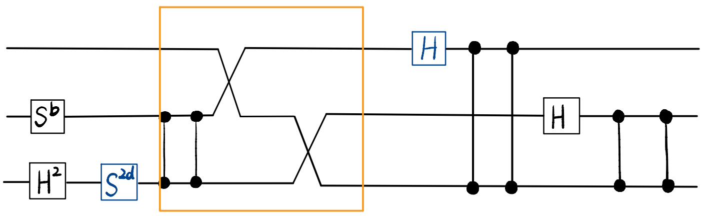
, $b, d \in \mathbb{Z}_3$.

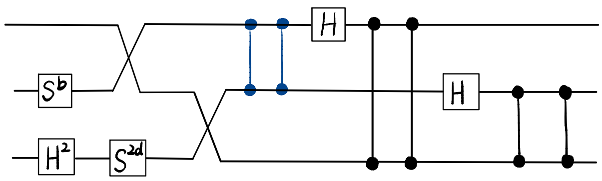
Proof: 10.(h). LHS := 

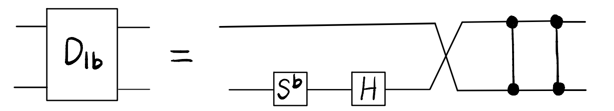
$\stackrel{def}{=}$ 

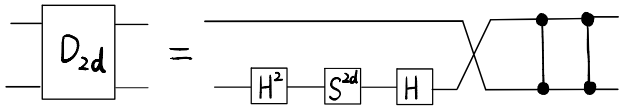
$\overline{\overline{C_7, C_8^3}}$ R_{19} 

$\overline{\overline{C_{15}^2}}$ R_{17} 

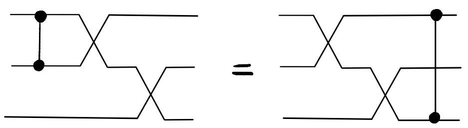
$\overline{\overline{R_{19}}}$ C_7 

$\overline{\overline{C_{15}}}$ 





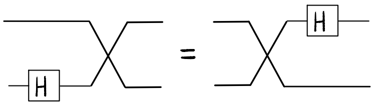
C^2_{15} :



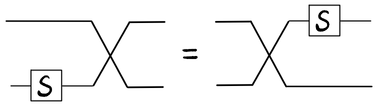
$C2: H^4 = I$

$C3: S^3 = I$

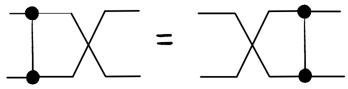
R_{19} :



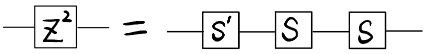
R_{18} :



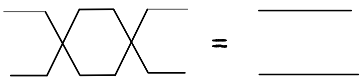
R_{17} :



R_{11} :

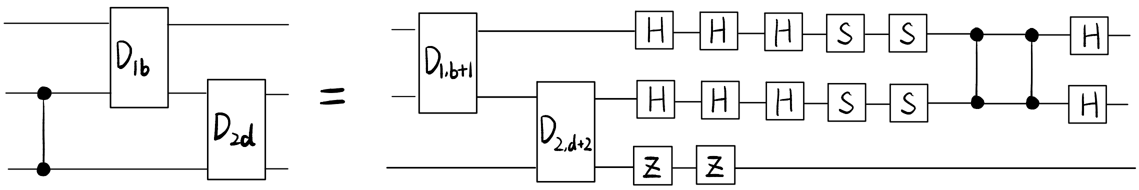


R_{16} :



Lem 8

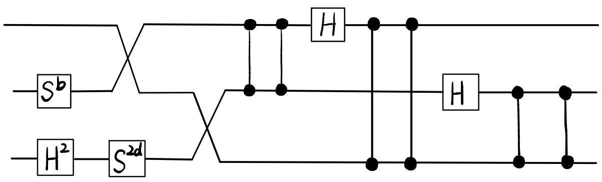
10.(h)



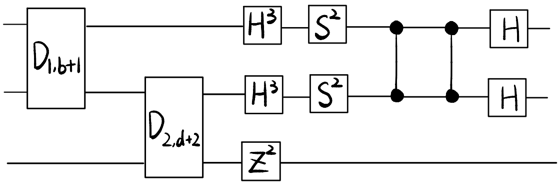
, $b, d \in \mathbb{Z}_3$.

Proof cont :

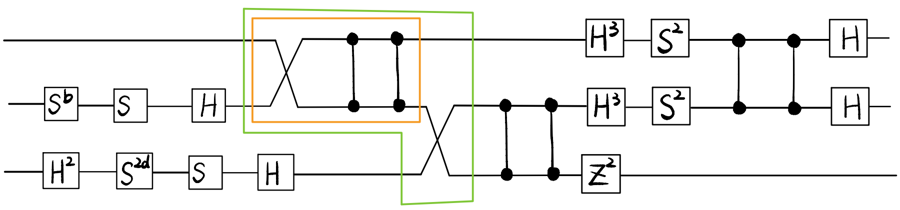
10.(h).LHS =



10.(h).RHS :=

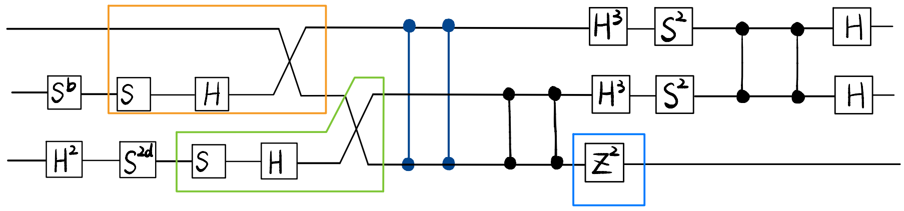


$\underline{\underline{\text{def}}}$

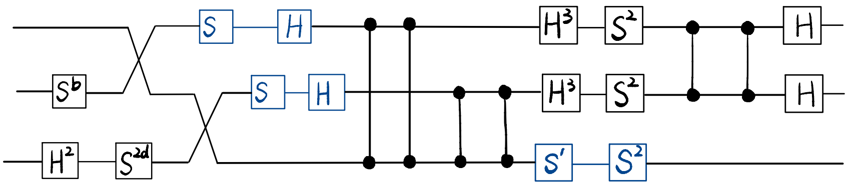


$2(d+2) = 2d+1$

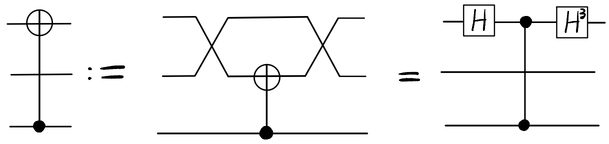
$\dfrac{C^2_{15}}{R_{17}}$



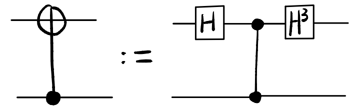
$\dfrac{R_{18}, R_{19}}{R_{11}, \text{Def }1}$



Def 7 :

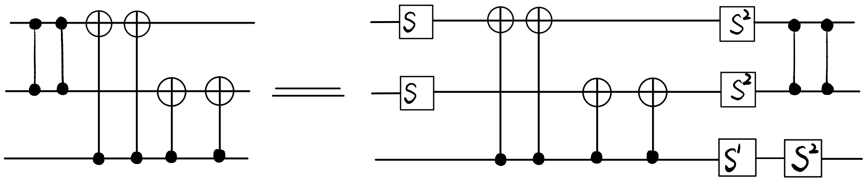


Def 2 :



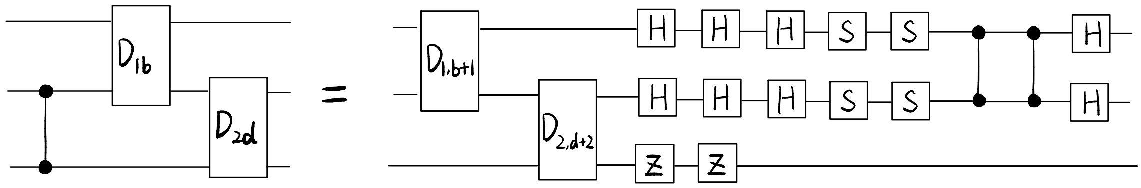
C2 : $H^4 = I$

R_{44} :



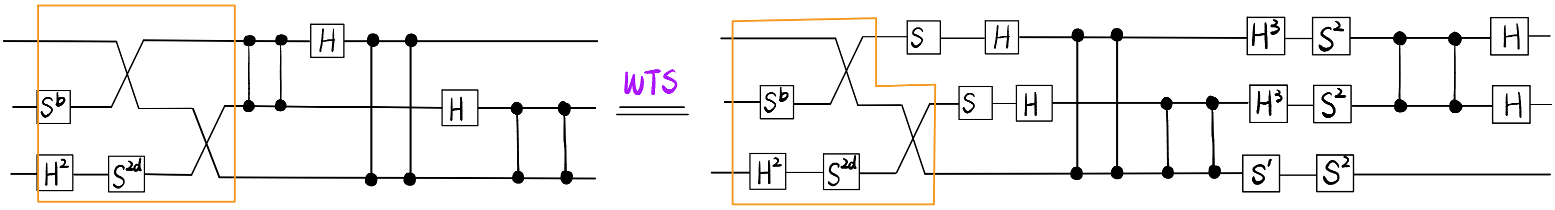
Lem 8

10.(h)

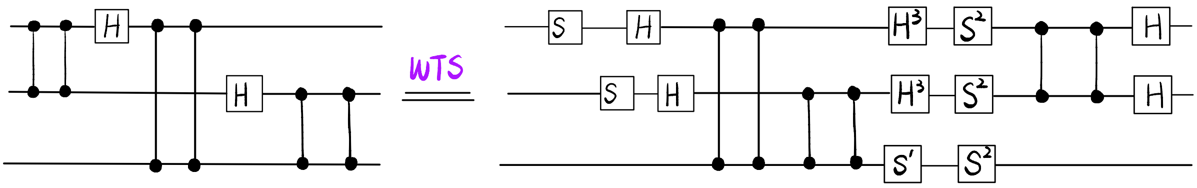


, $b, d \in \mathbb{Z}_3$.

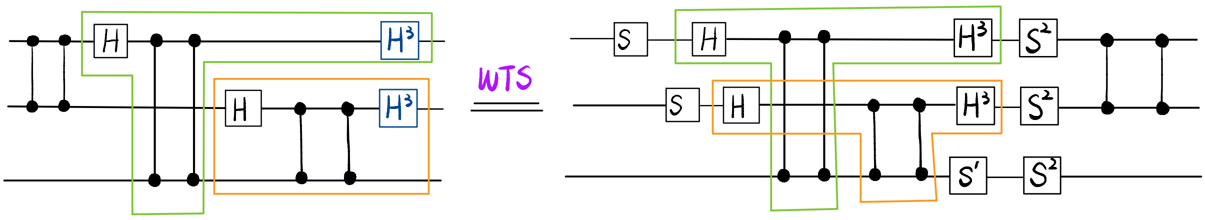
Proof cont : Hence



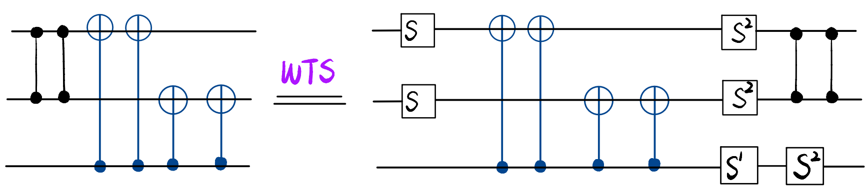
$C_2, C_3 \equiv R_{16}$



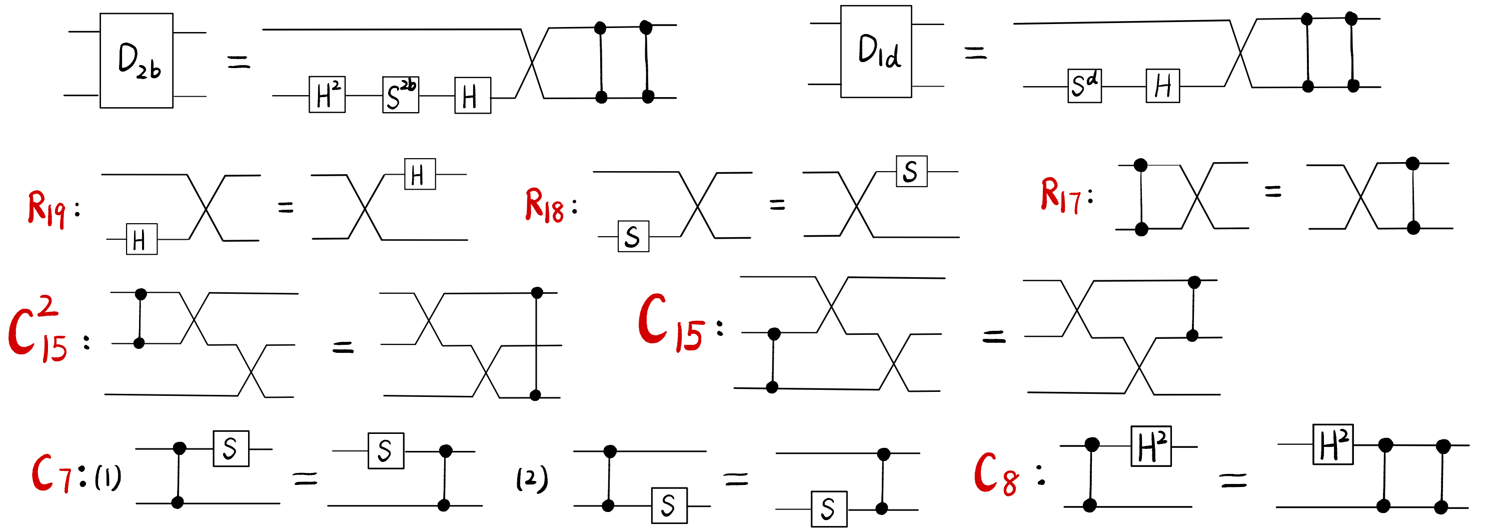
$C_2 \equiv$



$C_2 \equiv$ Def 2, Def 7

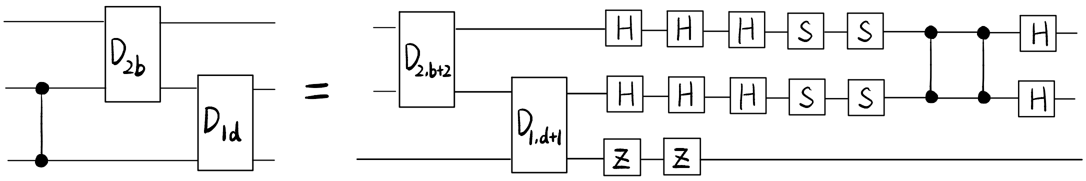


By R_{44}, this completes the proof. $\square$



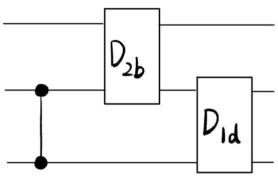
Lem 9 Def 1–2, Def 5, Def 7, C_{2-3}, C_{6-8}, R_{11}, R_{16}, R_{17}, R_{18}, R_{19}, R_{43}, C_{13} & C_{15} imply

10. (i)

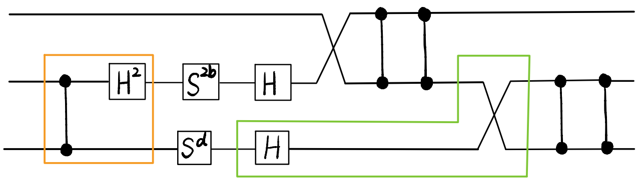


, $b, d \in \mathbb{Z}_3$.

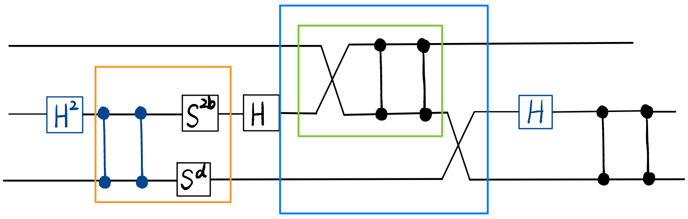
Proof: 10.(i).LHS :=



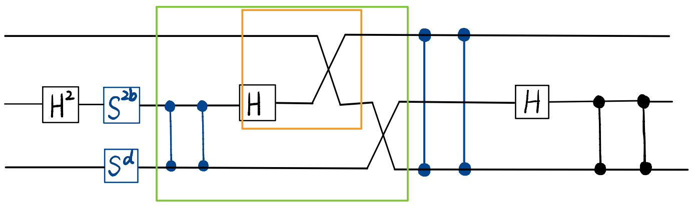
$\overset{def}{=}$



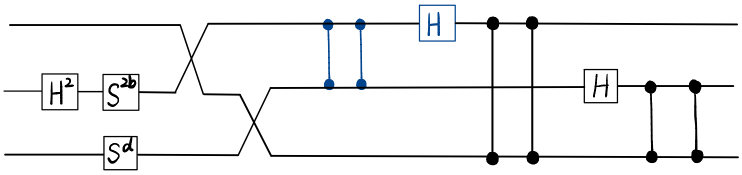
$\frac{C_8}{R_{19}}$



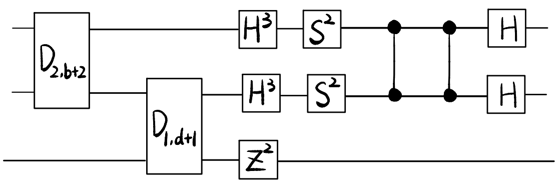
$\frac{C_7, R_{17}}{C_{15}^2}$



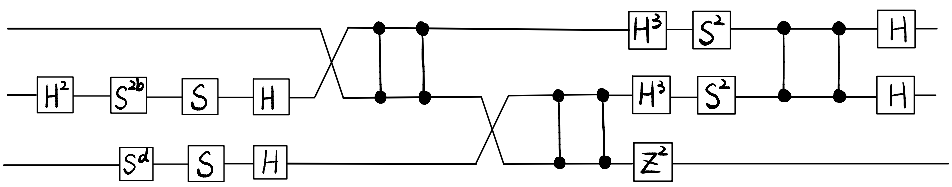
$\frac{C_{15}}{R_{19}}$



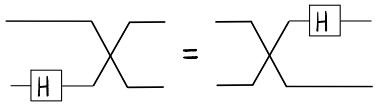
10.(i).RHS :=

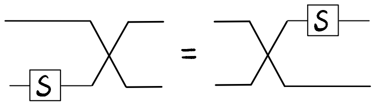


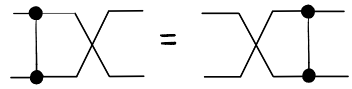
$\overset{def}{=}$

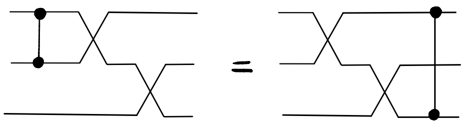


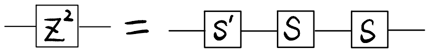
$2(b+2) = 2b+1$

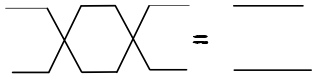
R_{19}:


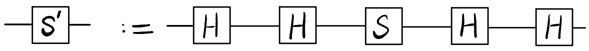
R_{18}:


R_{17}:


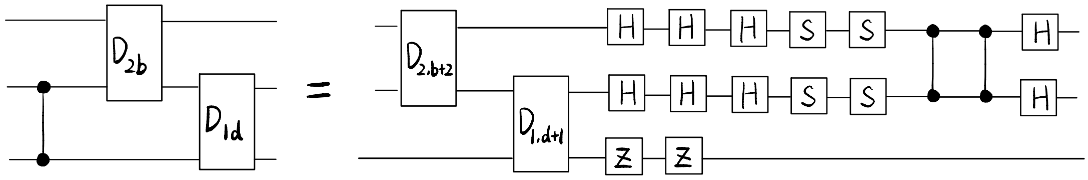
C_{15}^{2}:


$R11$:


R_{16}:


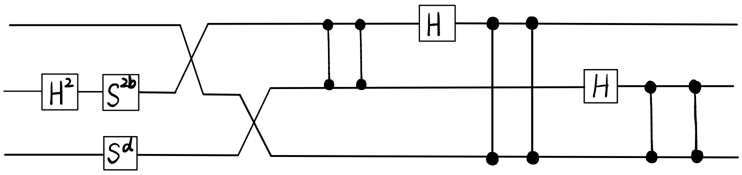
Def 1:


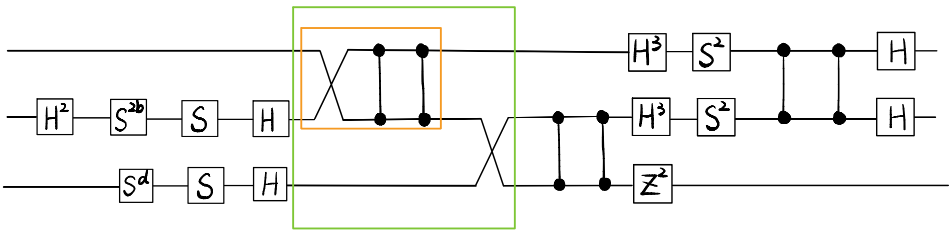
Lem 9

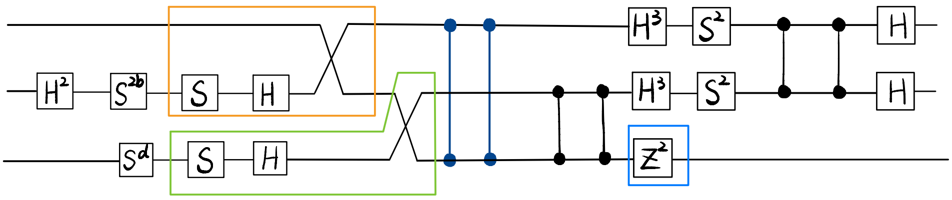
10. (i)


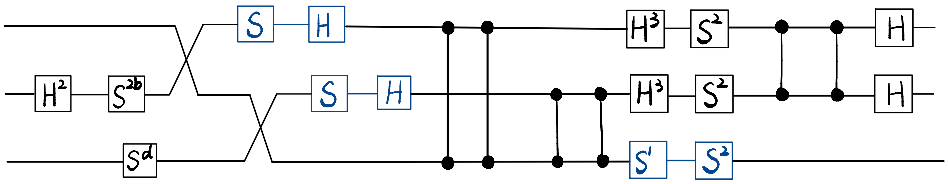
, $b, d \in \mathbb{Z}_3$.

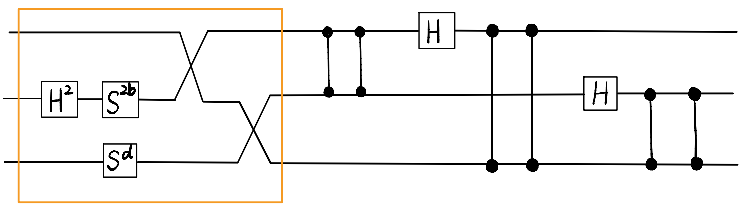
Proof cont :

10.(i).LHS =


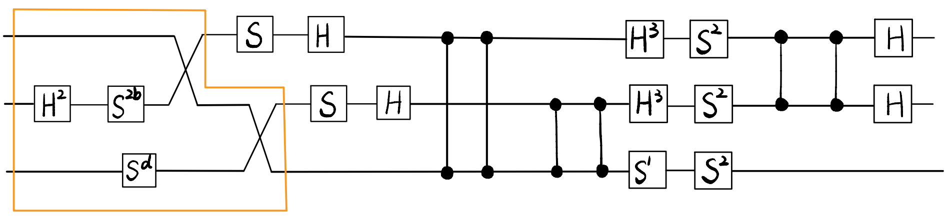
10.(i).RHS =


$\overset{R_{17}}{\underset{C_{15}^{2}}{=}}$


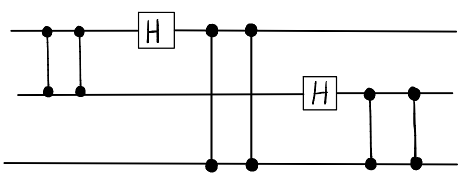
$\overset{R_{18}, R_{19}}{\underset{R_{11}, \text{Def} 1}{=}}$


Hence


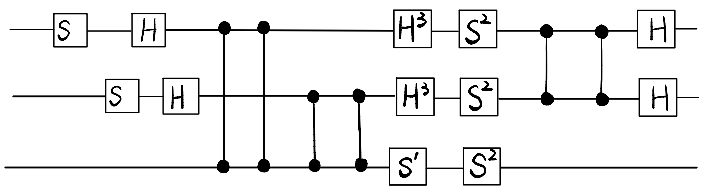
$\overset{\text{WTS}}{=}$

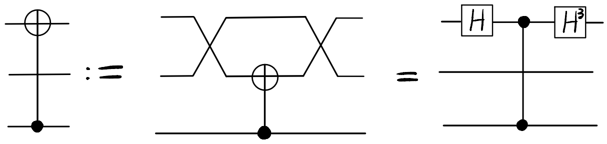
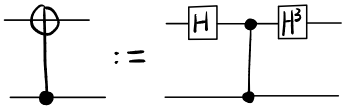


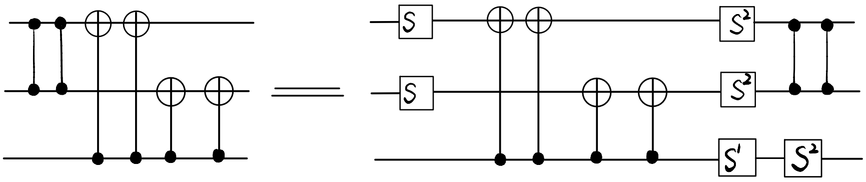
$\overset{C_2, C_3}{\underset{R_{16}}{\equiv}}$



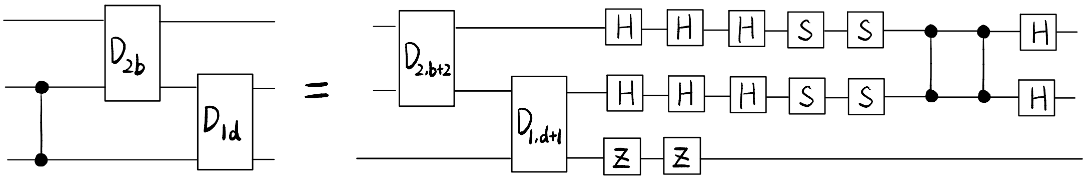
$\overset{\text{WTS}}{=}$

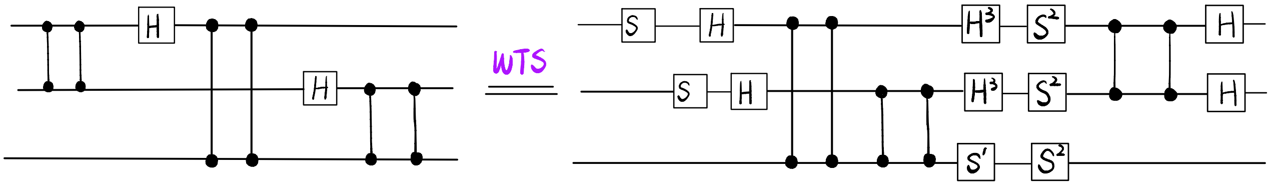


Def 7 :  Def 2:  C2 : $H^4 = I$

R44: 

Lem 9

10. (i)  , $b, d \in \mathbb{Z}_3$.

Proof cont : 

Reason analogously as in Lem 8 , by R44, we complete the proof. ◻