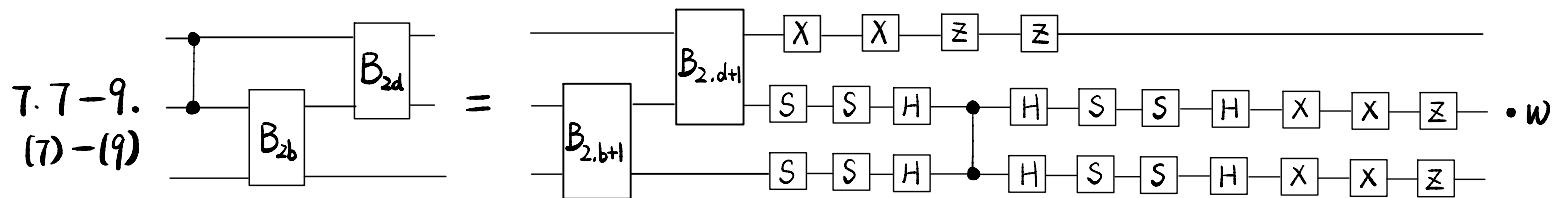
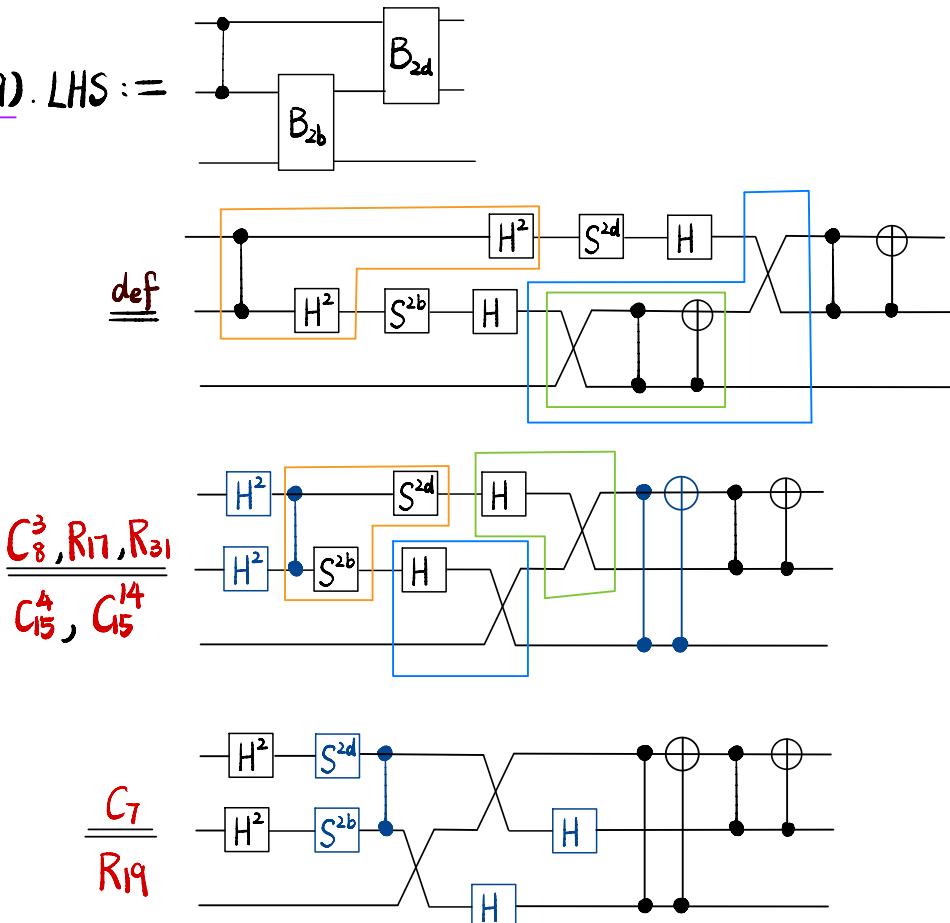
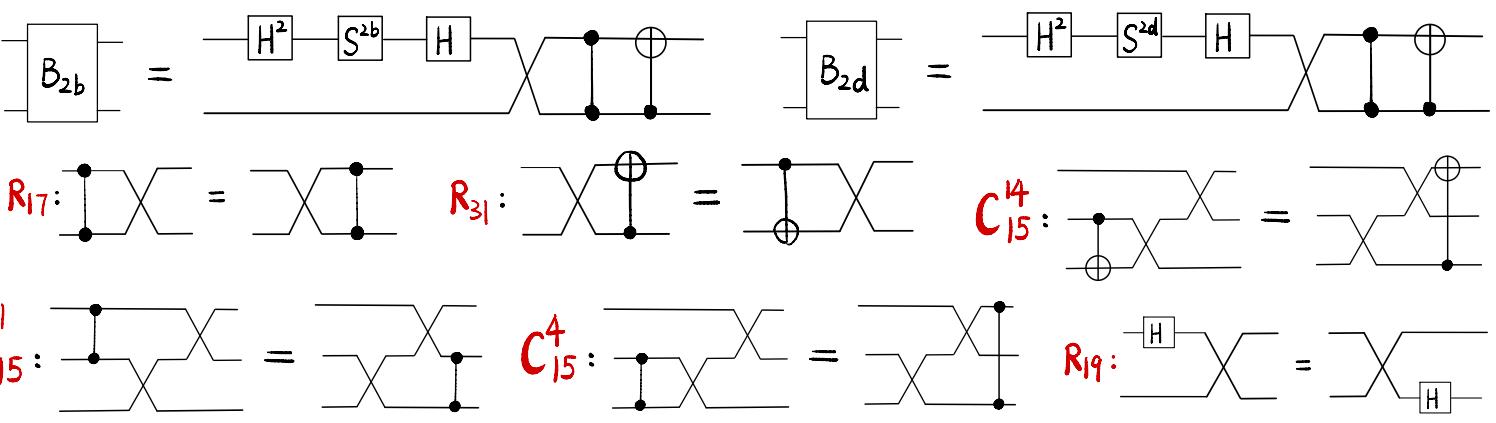


Lem 27 Def 1-5, Def 7, C_2 , C_3 , C_7 , C_{15} , R_7 , R_{10} , R_{11} , R_{16} , R_{17} , R_{18} , R_{19} , R_{31} & R_{59} imply



Proof: 7.7-9.(7)-(9). LHS :=





Lem 27

$$7 \cdot 7 - 9 \cdot (7) - (9) \cdot B_{2b} = B_{2,d+1} \cdot \begin{array}{ccccccccc} X & X & \bar{z} & \bar{z} \\ S & S & H & & H & S & S & H & X & X & \bar{z} \\ S & S & H & & H & S & S & H & X & X & \bar{z} \end{array} \cdot w$$

Proof cont:

$$7 \cdot 7 - 9 \cdot (7) - (9) \cdot LHS = \begin{array}{c} \text{Quantum circuit diagram for } LHS \text{ with orange boxes highlighting specific regions.} \end{array}$$

$$\underline{C_{15}^1} \begin{array}{c} \text{Quantum circuit diagram for } C_{15}^1 \text{ with blue lines connecting specific points.} \end{array}$$

$$7 \cdot 7 - 9 \cdot (7) - (9) \cdot RHS := B_{2,b+1} \cdot \begin{array}{ccccccccc} X & X & \bar{z} & \bar{z} \\ S & S & H & & H & S & S & H & X & X & \bar{z} \\ S & S & H & & H & S & S & H & X & X & \bar{z} \end{array} \cdot w$$

$$2(b+1) = 2b + 2 \quad 2(d+1) = 2d + 2$$

$$\stackrel{\text{def}}{=} \begin{array}{c} \text{Quantum circuit diagram for RHS with green and orange boxes highlighting specific regions.} \end{array}$$

R_{17}, R_{31}

C_{15}^4, C_{15}^{14}

$$\begin{array}{c} \text{Quantum circuit diagram for } C_{15}^4 \text{ and } C_{15}^{14} \text{ with blue lines connecting specific points.} \end{array}$$

$$R_{18}: \text{Diagram} = \text{Diagram}$$

$$R_{19}: \text{Diagram} = \text{Diagram}$$

$$R_{16}: \text{Diagram} = \text{Diagram}$$

$$R_7: \text{Diagram} = \text{Diagram} \quad C_3: S^3 = I$$

$$R_{10}: \text{Diagram} = \text{Diagram} \quad R_{11}: \text{Diagram} = \text{Diagram}$$

Lem 27

$$7.7-9. (7)-(9) \cdot w: \text{Diagram} = \text{Diagram}$$

Proof cont: 7.7-9.(7)-(9).RHS =

$$\begin{array}{c} \text{Diagram} \\ \text{Diagram} \\ \text{Diagram} \end{array} \cdot w$$

$$\frac{R_7}{R_{10}, R_{11}} X^2 Z^2 \stackrel{R_7}{=} HS^2 H^2 SHS' S^2 \quad X^2 Z \stackrel{R_7}{=} HS^2 H^2 SH S'^2 S$$

$$\begin{array}{c} \text{Diagram} \\ \text{Diagram} \end{array} \cdot w$$

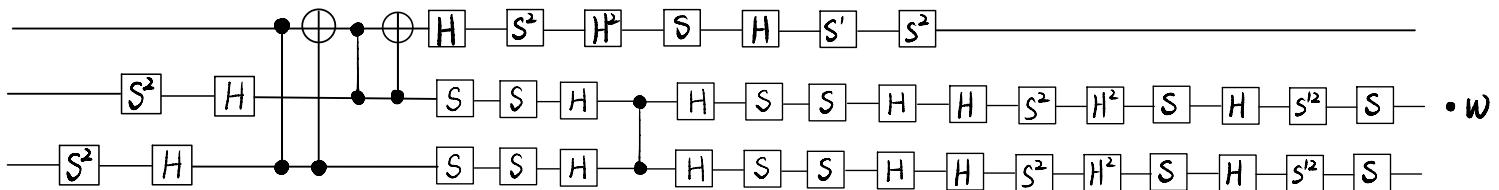
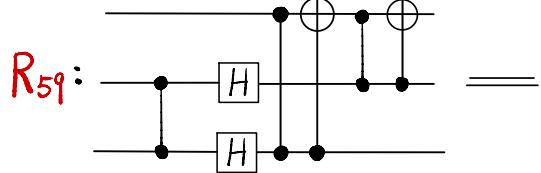
R₉
R₈

$$\begin{array}{c} \text{Diagram} \\ \text{Diagram} \\ \text{Diagram} \end{array} \cdot w$$

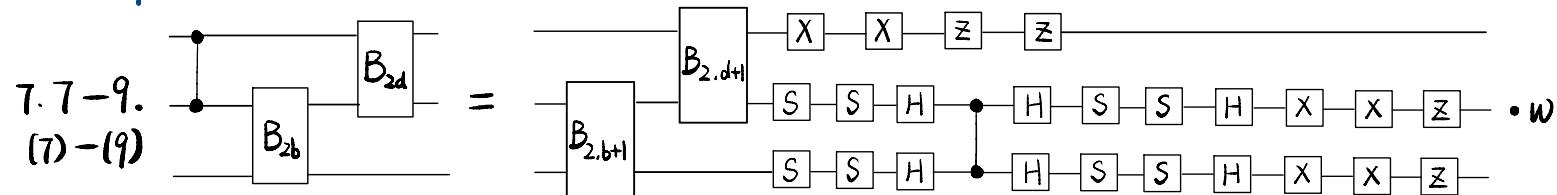
Hence,

$$\begin{array}{c} \text{Diagram} \\ \text{Diagram} \end{array} \text{WTS}$$

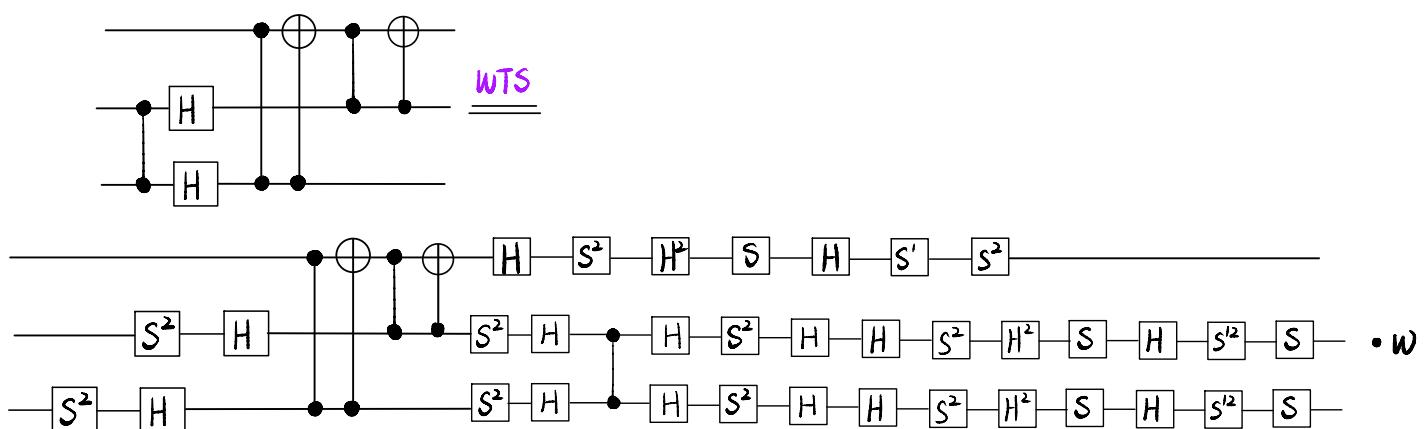
$$\begin{array}{c} \text{Diagram} \\ \text{Diagram} \\ \text{Diagram} \end{array} \cdot w$$



Lem 27

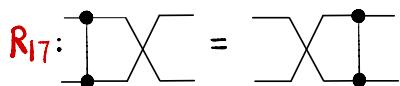
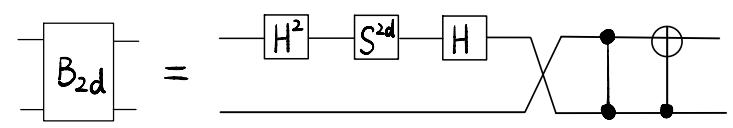
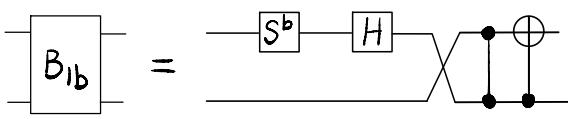


Proof cont. :



By R_{59} , this completes the proof.





Def 4: $\vdash :=$

$$\text{Def 2: } \text{ := } \begin{array}{c} \text{---} \\ | \quad | \\ \oplus \end{array} \quad \begin{array}{c} \text{---} \\ | \quad | \\ \boxed{\mathbf{H}} \quad \bullet \quad \boxed{\mathbf{H}^3} \end{array}$$

$$\text{Def 7 : } \begin{array}{c} \oplus \\ \hline \end{array} := \begin{array}{c} -[H] \quad \bullet \quad [H^3] \\ \hline \end{array}$$

Def 5: $\vdash :=$  

$$C_{15}^1 : \quad \text{Diagram} = \quad \text{Diagram}$$

$$R_{31} : \quad \begin{array}{c} \text{Diagram of } R_{31} \\ \text{with two nodes} \end{array} = \quad \begin{array}{c} \text{Diagram of } R_{31} \\ \text{with one node} \end{array}$$

$$C_{15}^{14} : \quad \text{Diagram} = \quad \text{Diagram}$$

$$\text{Def 3: } \text{X} := \begin{array}{c} \text{H} \\ \text{H} \end{array} \quad \begin{array}{c} \text{H} \\ \text{H} \end{array} \quad \begin{array}{c} \text{H} \\ \text{H} \end{array}$$

$$C_{15}^4 : \quad \text{Diagram} = \quad \text{Diagram}$$

$$C_7: \quad \begin{array}{c} \text{Diagram of } C_7 \text{ showing two nodes connected by a vertical line, with one node connected to a horizontal line labeled } S \text{ and the other to a dashed line.} \\ \hline \end{array} = \quad \begin{array}{c} \text{Diagram of } C_7 \text{ showing two nodes connected by a vertical line, with both nodes connected to horizontal lines labeled } S \text{ and a dashed line.} \\ \hline \end{array}$$

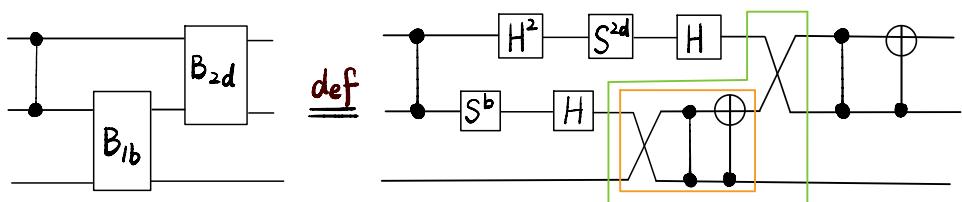
$$R_{1g} : \quad \begin{array}{c} \text{---} \\ | \\ \text{H} \\ | \\ \text{---} \end{array} \quad \times \quad = \quad \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{H} \\ | \\ \text{---} \end{array}$$

$$C_8: \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \text{---} \end{array} \quad \boxed{H^2} \quad = \quad \begin{array}{c} \text{---} \\ | \\ \bullet \\ | \\ \bullet \\ | \\ \bullet \\ | \\ \text{---} \end{array}$$

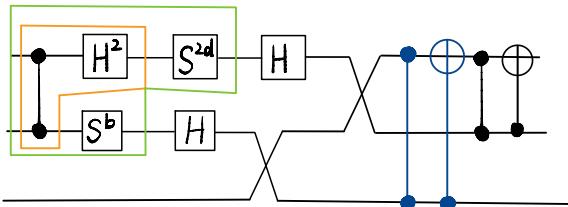
Lem 28 Def 1-5, Def 7, C₂, C₃, C₅, C₇, C₁₅, R₅, R₁₀, R₁₁, R₁₆, R₁₇, R₁₈, R₁₉, R₃₁ & R₆₀ imply

$$7.4-6. = \begin{array}{c} B_{1b} \\ \parallel \\ B_{2d} \end{array} = \begin{array}{c} B_{1,b+1} \\ \parallel \\ B_{2,d+2} \end{array} X \otimes \begin{array}{c} z \\ \parallel \\ z \end{array}$$

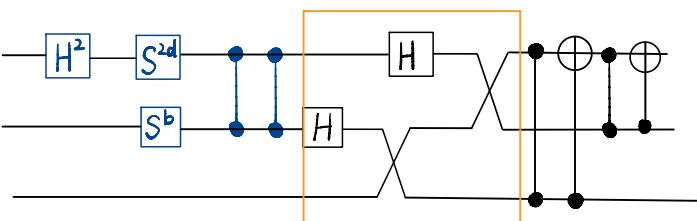
Proof: 7.4 - 6.(7) - (9). LHS :=



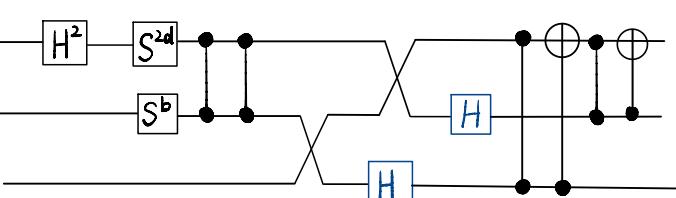
$$\underline{R_{17}, R_{31}}$$

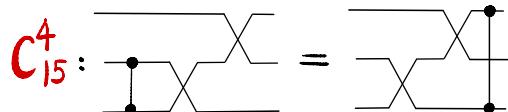
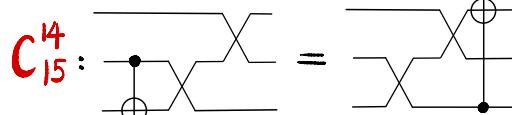
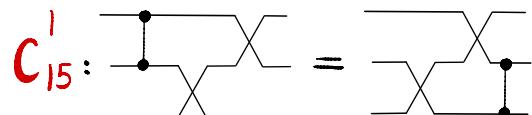
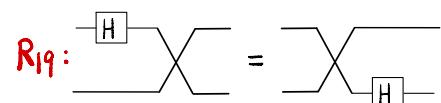
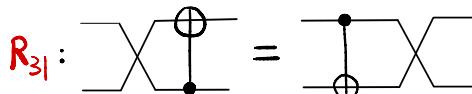
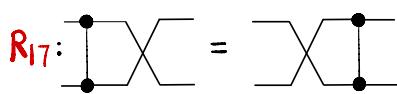
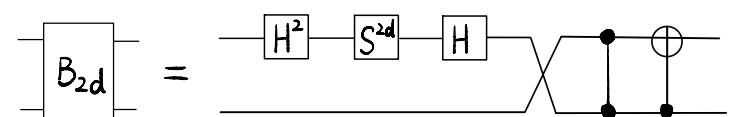
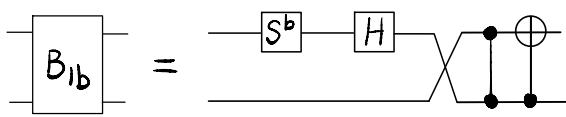


C₈
C₇

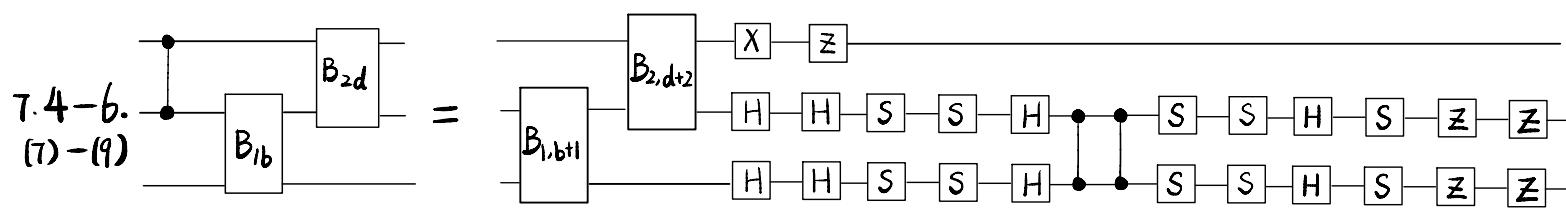


R19

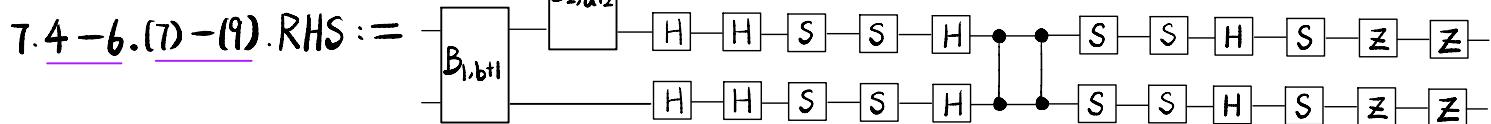
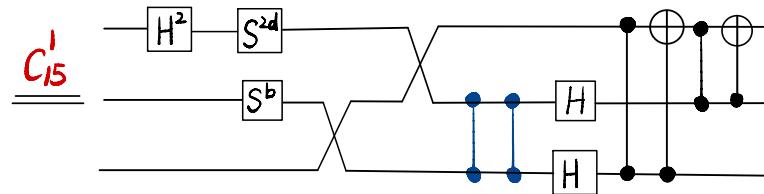
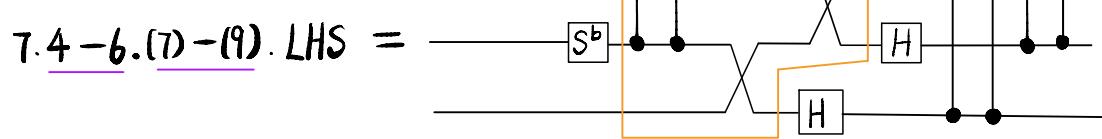




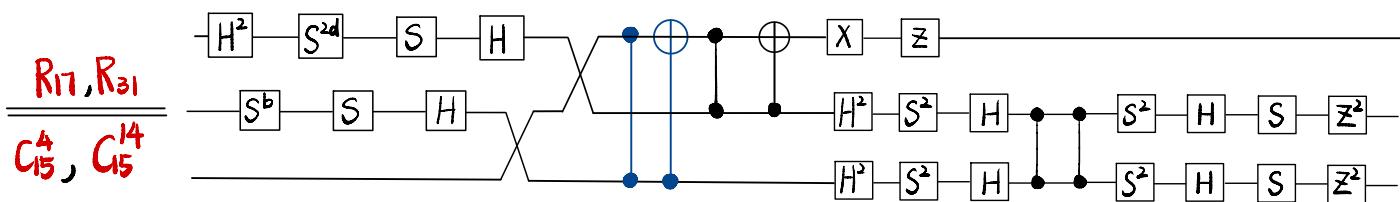
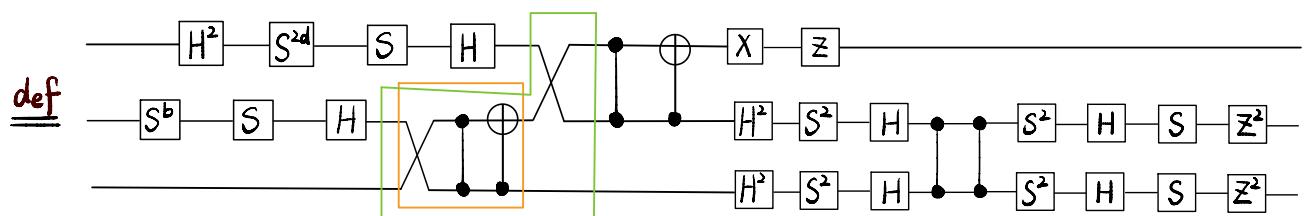
Lem 28



Proof cont:



$$2(d+2) = 2d+1$$



$$R_{18}: \text{Diagram showing } S \text{ and } S \text{ canceling each other out.}$$

$$R_{19}: \text{Diagram showing } H \text{ and } H \text{ canceling each other out.}$$

$$C_3: S^3 = I$$

$$R_5: X = H S H H S S H$$

$$R_{10}: Z = S' S' S$$

$$R_{11}: Z^2 = S' S S$$

$$R_{60}: \text{Diagram showing a complex circuit with multiple layers of gates (H, S, S^2, H, S^2, H, S) and control lines.}$$

Lem 28

$$7.4-6. (7)-(9). \text{LHS} = \text{Diagram showing the left-hand side of the equation.}$$

Proof cont : 7.4-6.(7)-(9).RHS =

$$\text{Diagram showing the right-hand side of the equation. It includes two parts: one with an orange box around the first four qubits and another with an orange box around the first three qubits.}$$

$$\frac{R_{18}}{R_{19}} \text{Diagram showing the simplified circuit after applying R18 and R19. It highlights the first three qubits with a green box and the last two with a blue box.}$$

$$XZ \xrightarrow[R_{10}]{R_5} HSH^2S^2HS'^2S \quad SZ^2 = SS'S^2 \xrightarrow[C_5]{C_3} S'$$

$$\frac{R_5}{R_{10}, R_{11}} \text{Diagram showing the simplified circuit after applying R5 and R10/R11. It highlights the last two qubits with a blue box.}$$

Hence,

$$\text{Diagram showing the final simplified circuit, labeled 'WTS' (Without Technical Specification). It shows the circuit from the previous step with the last two qubits highlighted in blue.}$$

By R_{60} , this completes the proof.