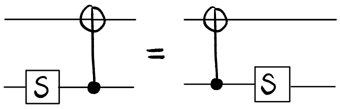
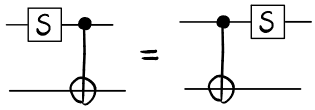
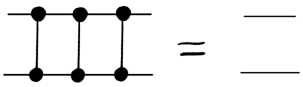
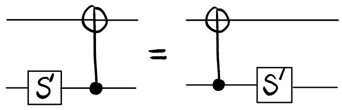
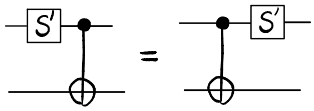
$C1: \omega^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$

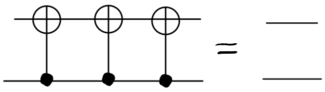
R_{24}: 

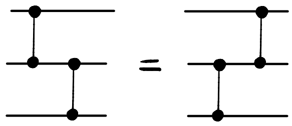
R_{24}^2: 

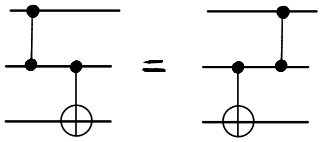
C_6: 

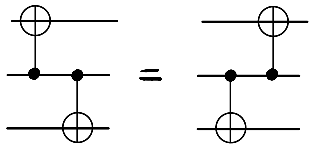
R_{24}^1: 

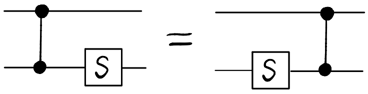
R_{24}^3: 

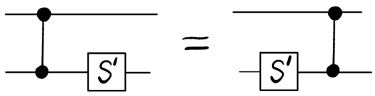
C_6^2: 

C_{13}: 

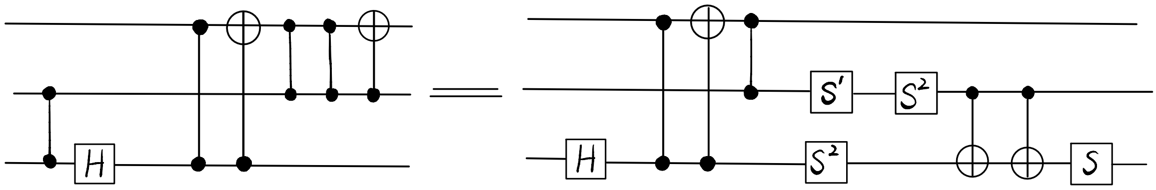
C_{13}^4: 

C_{13}^9: 

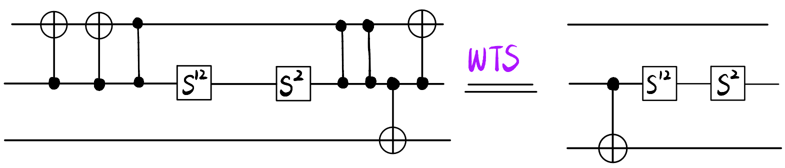
C_7: 

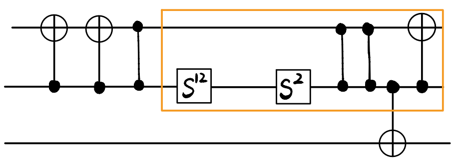
R_{15}: 

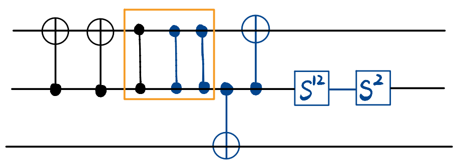
Lem T

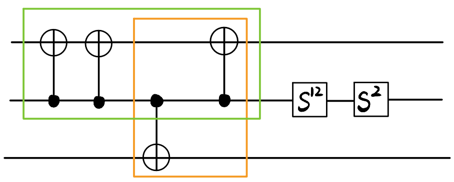
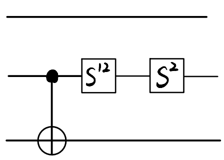
R_{54}: 

Proof cont.

R_{54}: 

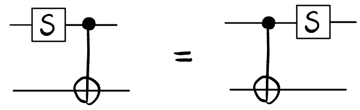
Then R_{54}.LHS:= 

$\underset{R_{24}^1, R_{24}^2, R_{24}^3}{\overset{C_7, R_{15}, R_{24}}{=\!=}}$ 

$\overset{C_6}{=\!=}$  $\underset{C_6^2}{\overset{C_{13}^9}{=\!=}}$  $=:$ R_{54}.RHS

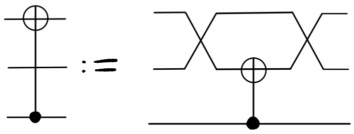
$C1: \omega^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$ $C5: SS' = S'S$

R^2_{24}:

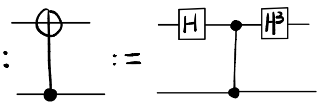


Def 1: $S' := H\,H\,S\,H\,H$

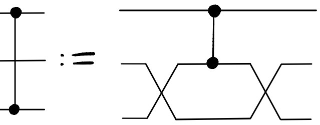
Def 7:



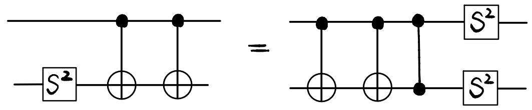
Def 2:



Def 5:

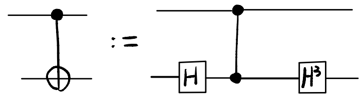


$R^1_{23\text{-}25}$:



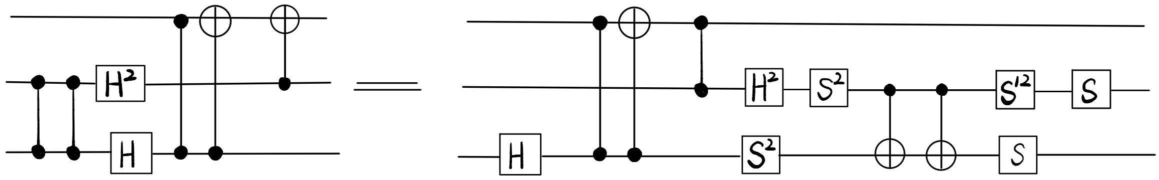
$\cdot\, \omega$

Def 4:



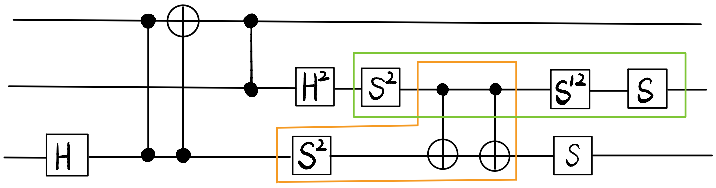
Lem U Def 1-2, Def 4-5, Def 7, $C_1, C_2, C_3, C_5, C_6, C_7, C_8, C_{13}, C_{16}, R_{15}, R_{23}, R_{24}, R_{25}$ imply

R_{58}:



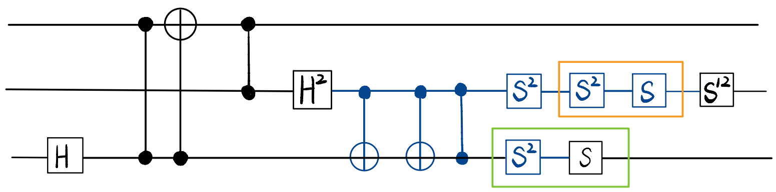
$\cdot\, \omega$

Proof: R_{58}.RHS :=



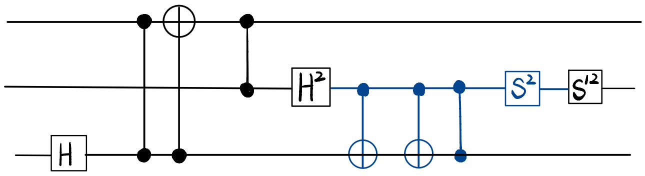
$\cdot\, \omega$

$\overset{R^2_{24}, C_5}{\underset{R^1_{23\text{-}25}}{=}}$



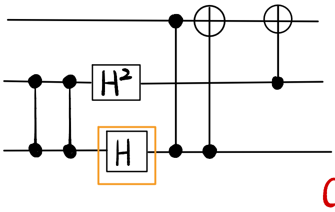
$\cdot\, \omega \cdot \omega$

$\overset{C_3}{=}$

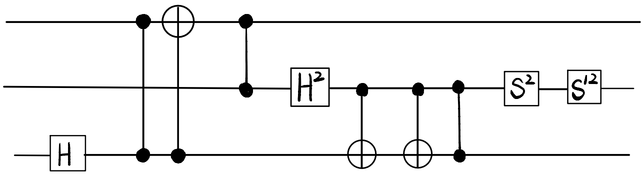


$\cdot\, \omega^2$

Hence R_{58}:



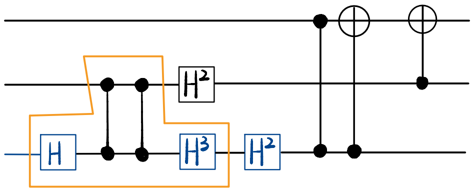
$\overset{\text{WTS}}{=}$



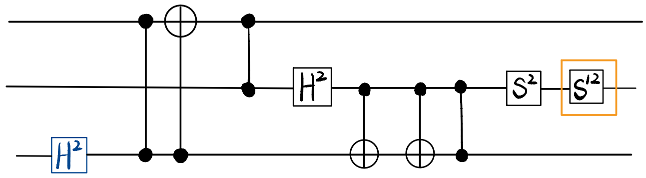
$\cdot\, \omega^2$

C_2 |||

R_{58}:



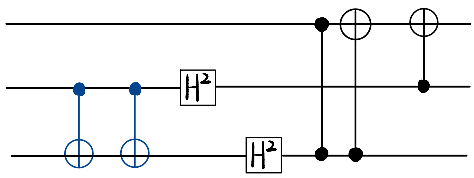
$\overset{\text{WTS}}{=}$



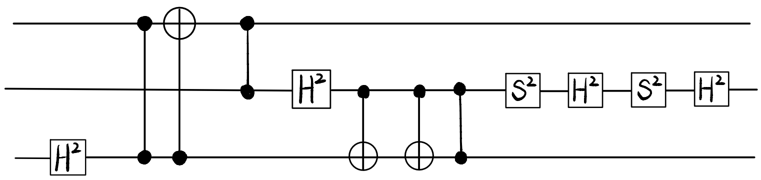
$\cdot\, \omega^2$

C_2, Def 1 ||| Def 4

R_{58}:

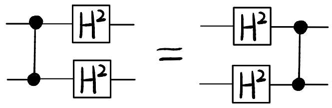


$\overset{\text{WTS}}{=}$

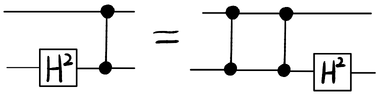


$\cdot\, \omega^2$

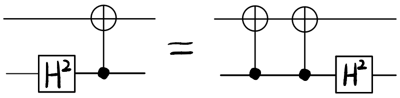
$C1: w^3 = 1$ $C2: H^4 = I$ $C3: S^3 = I$ $C5: SS' = S'S$ $C_8^3:$



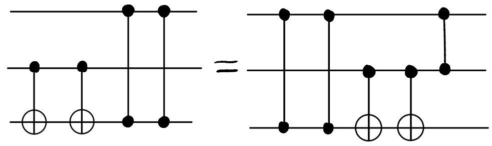
$C_8^{1*}:$



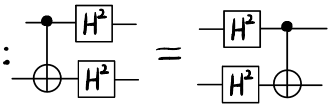
$C_8^{6*}:$



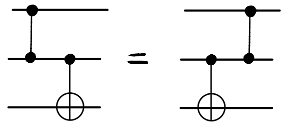
$C_{16}^{22}:$



$C_8^{14}:$

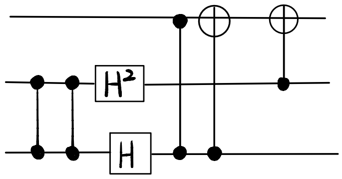


$C_{13}^4:$

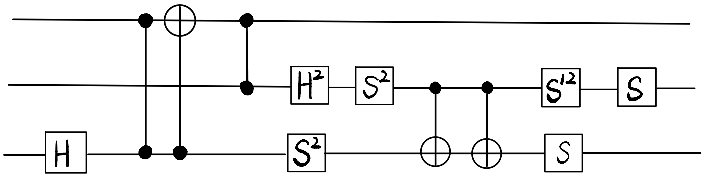


Lem U

$R_{58}:$



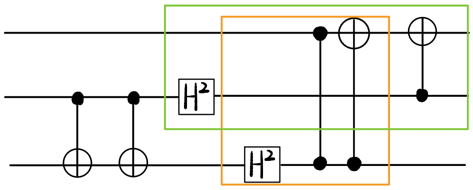
$=$



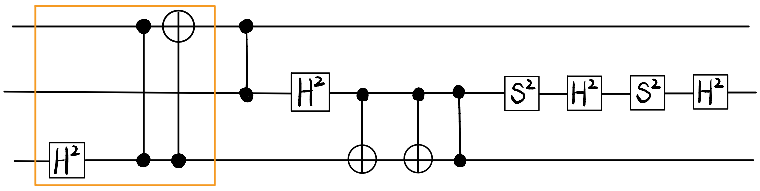
$\cdot w$

Proof cont.

$R_{58}:$



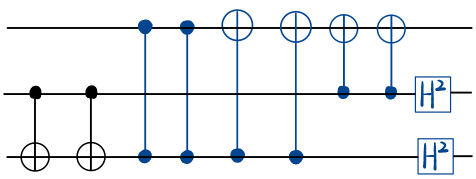
WTS $\equiv$



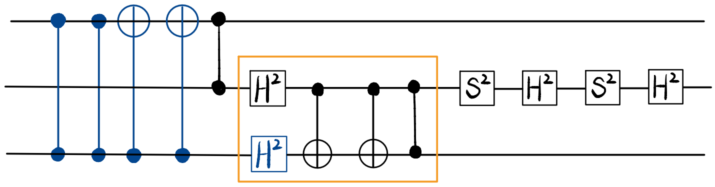
$\cdot w^2$

$\equiv C_8^{1*}, C_8^{6*}$

$R_{58}:$



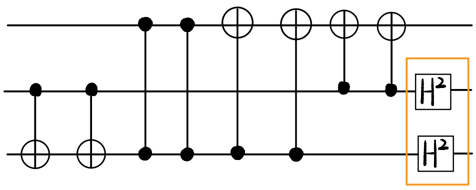
WTS $\equiv$



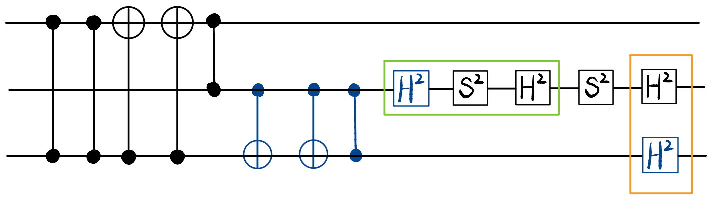
$\cdot w^2$

$C_8^3 \equiv C_8^{14}$

$R_{58}:$



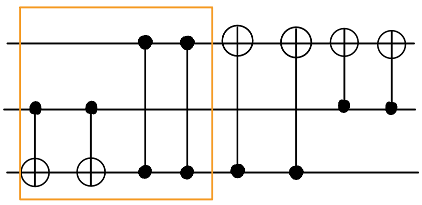
WTS $\equiv$



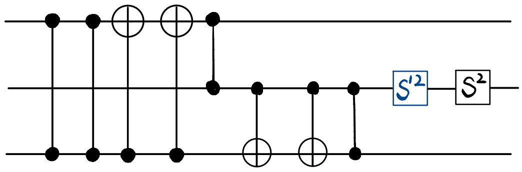
$\cdot w^2$

Def1 $\equiv$ C_2

$R_{58}:$



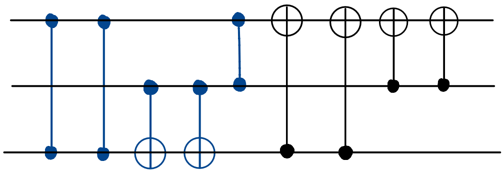
WTS $\equiv$



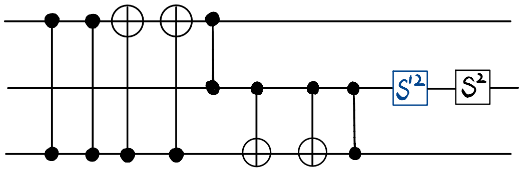
$\cdot w^2$

$\equiv C_{16}^{22}$

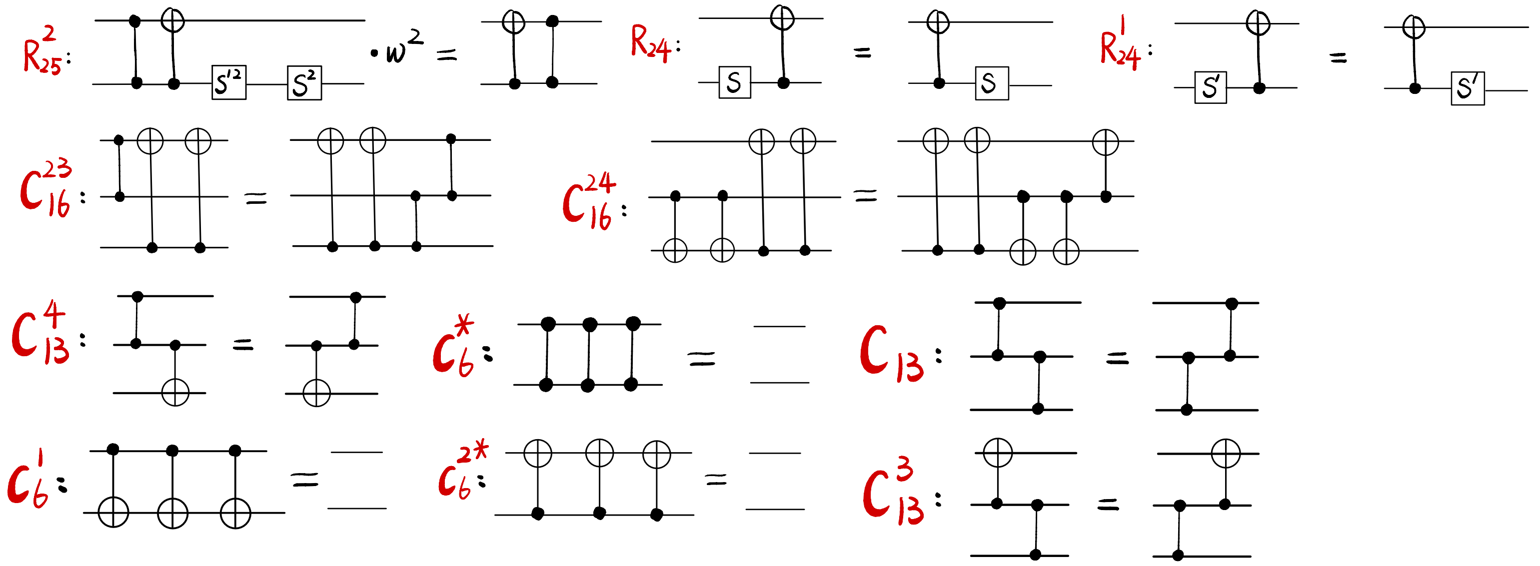
$R_{58}:$



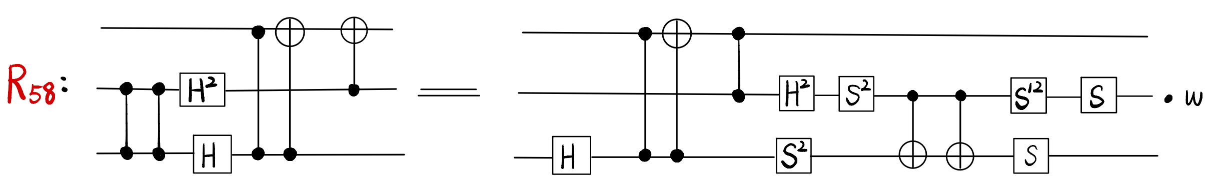
WTS $\equiv$



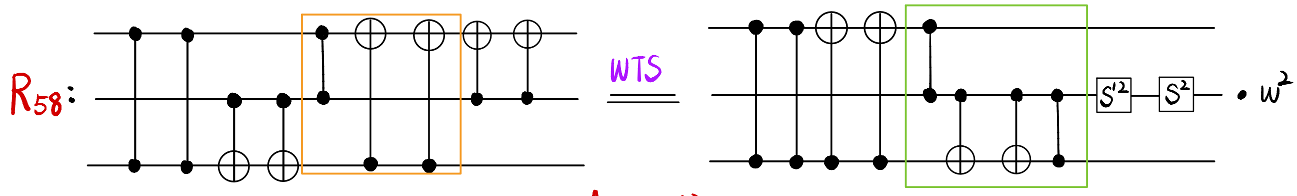
$\cdot w^2$



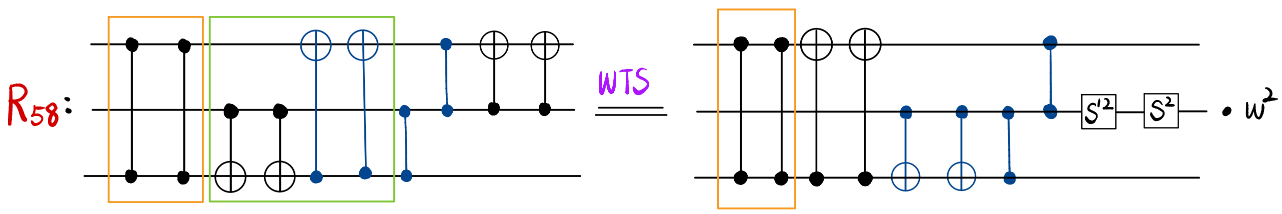
Lem U



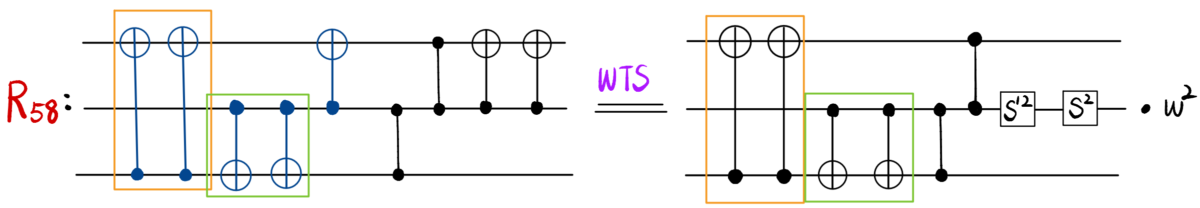
Proof cont.



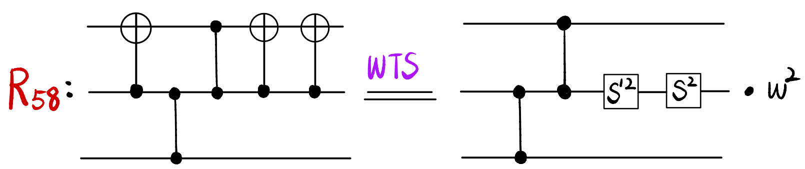
C^4_{13} ||| C^{23}_{16}, C_{13}



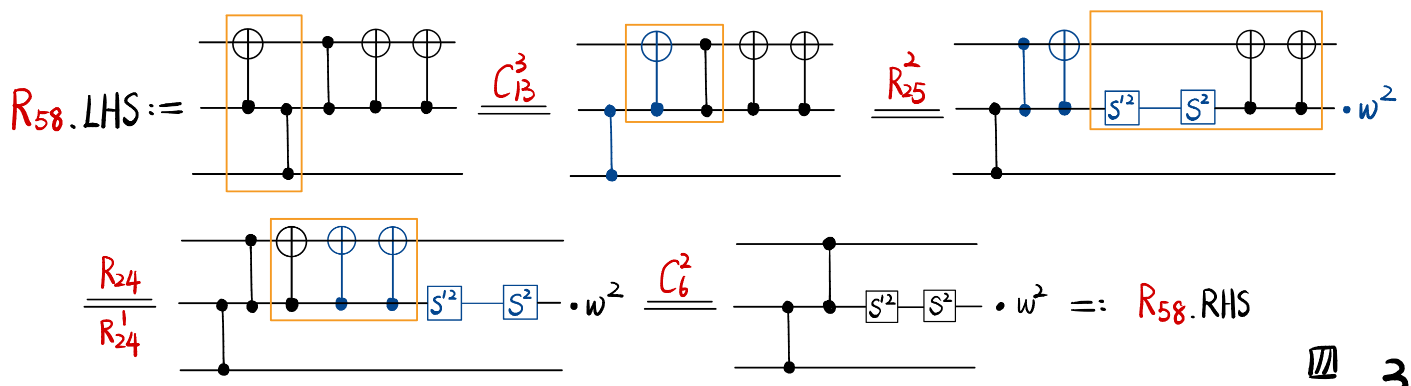
C^{24}_{16} ||| C^*_6



C^1_6 ||| C^{2*}_6

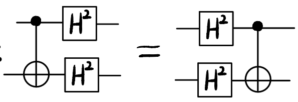


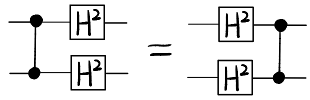
Then

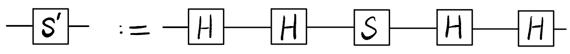


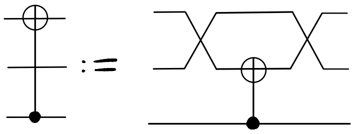
∎

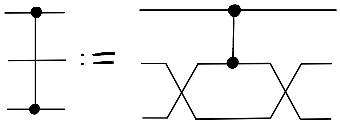
$C1: \omega^3 = 1$ $\quad C2: H^4 = I$ $\quad C3: S^3 = I$

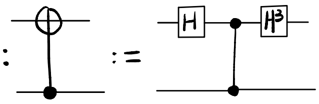
C_8^{14} :


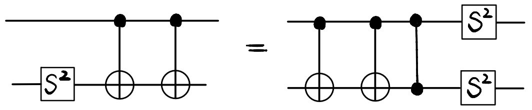
C_8^{3} :


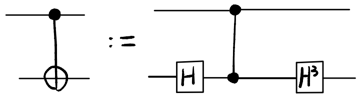
Def 1 :


Def 7 :


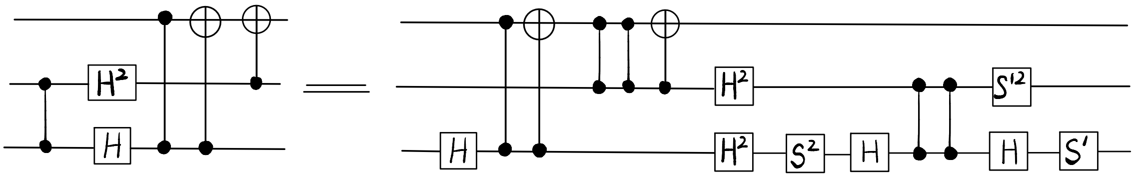
Def 5 :


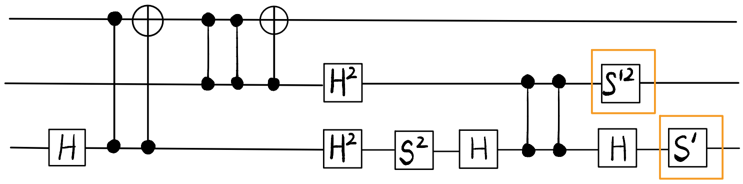
Def 2 :


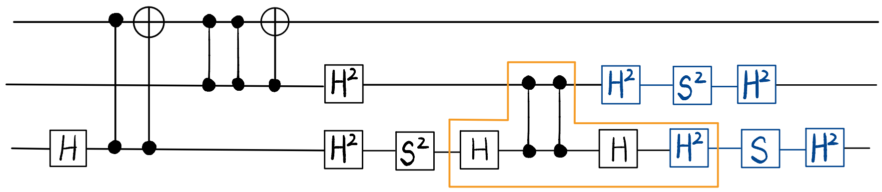
$R'_{23\text{-}25}$:

$\cdot\, \omega$

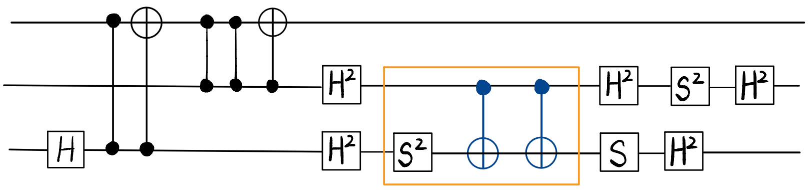
Def 4 :


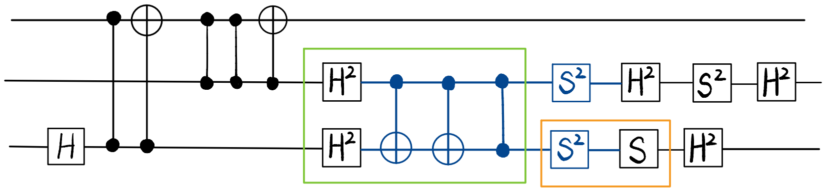
Lem V Def 1-2, Def 4-5, Def 7, $C_1, C_2, C_3, C_5, C_6, C_7, C_8, C_{13}, C_{16}, R_{15}, R_{23}, R_{25}$ imply

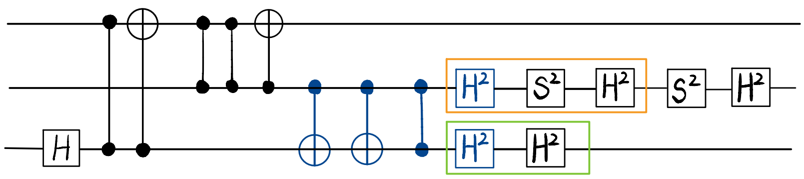
R_{55} :

$\cdot\, \omega$

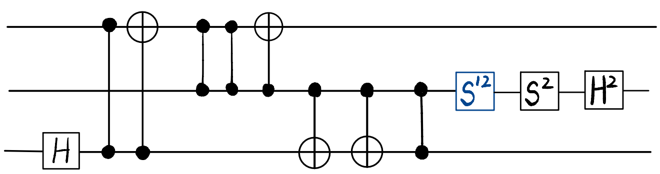
Proof: R_{55}.RHS :=

$\cdot\, \omega$

$\overline{\overline{\text{Def 1}}}$ C_2

$\cdot\, \omega$

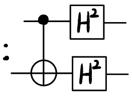
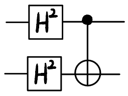
$\overline{\overline{\text{Def 4}}}$ C_2

$\cdot\, \omega$

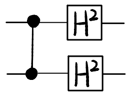
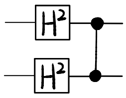
$\overline{\overline{R'_{23\text{-}25}}}$

$\cdot\, \omega \cdot \omega$

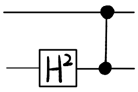
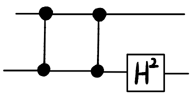
$\overline{\overline{C_8^3, C_8^{14}}}$ C_3

$\cdot\, \omega^2$

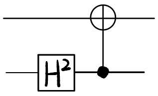
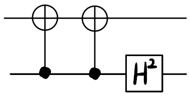
$\overline{\overline{\text{Def 1}}}$ C_2

$\cdot\, \omega^2$

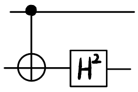
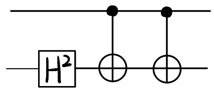
$C_1: \omega^3 = 1$ $\quad C_2: H^4 = I$ $\quad C_3: S^3 = I$

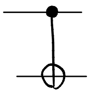
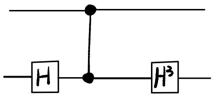
C_8^{14}:  = 

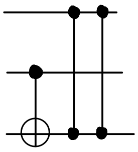
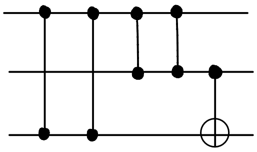
C_8^{3}:  = 

C_8^{1*}:  = 

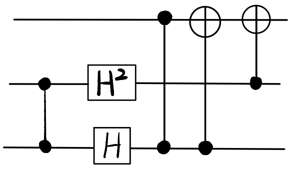
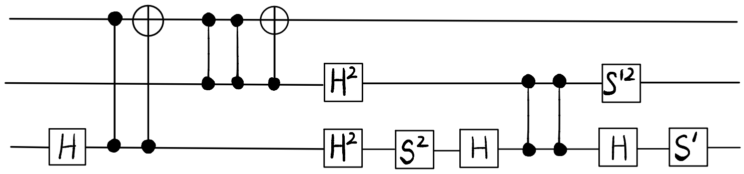
C_8^{6*}:  = 

C_8^{7}:  = 

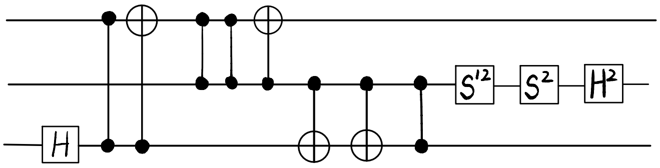
Def4:  := 

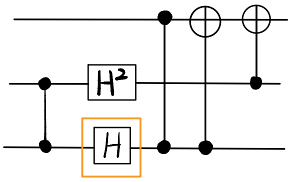
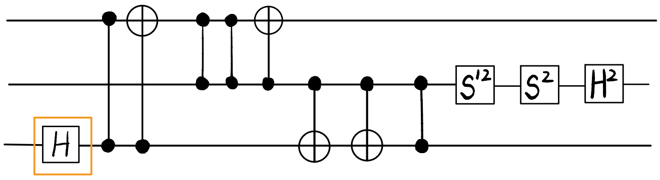
C_{16}^{19}:  = 

Lem V

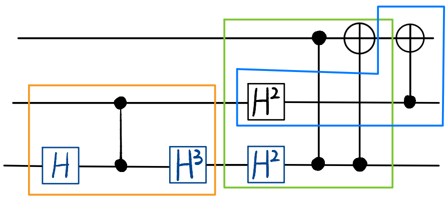
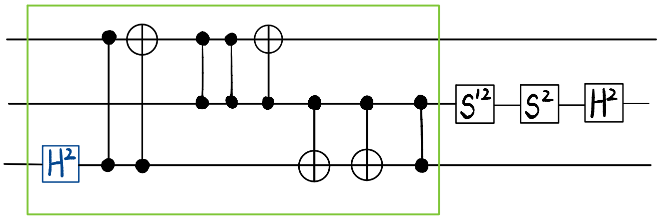
R_{55}:  =  $\cdot\, \omega$

Proof cont.

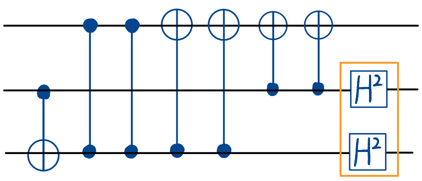
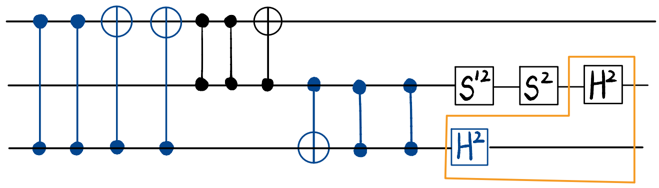
R_{55}.RHS =  $\cdot\, \omega^2$

Hence R_{55}:  $\overset{\text{WTS}}{=}$  $\cdot\, \omega^2$

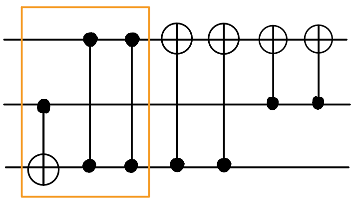
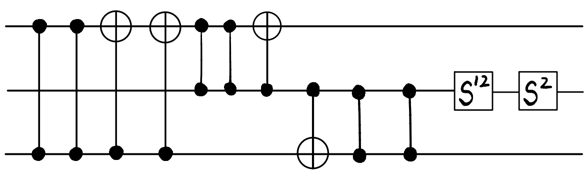
C_2 $\equiv$

R_{55}:  $\overset{\text{WTS}}{=}$  $\cdot\, \omega^2$

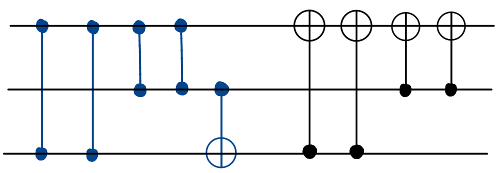
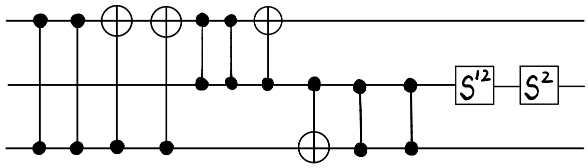
Def 4 $\equiv$ $C_8^{1*}, C_8^{6*}, C_8^{7}$

R_{55}:  $\overset{\text{WTS}}{=}$  $\cdot\, \omega^2$

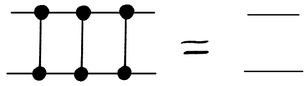
C_2 $\equiv$

R_{55}:  $\overset{\text{WTS}}{=}$  $\cdot\, \omega^2$

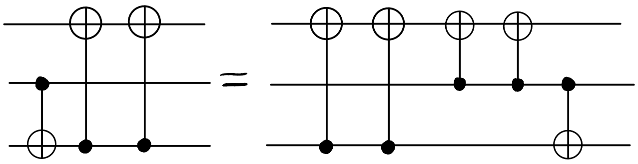
C_{16}^{19} $\equiv$

R_{55}:  $\overset{\text{WTS}}{=}$  $\cdot\, \omega^2$

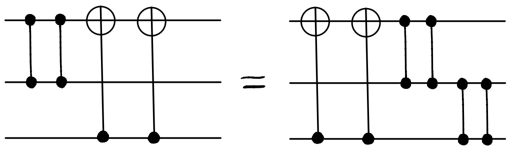
$C_1: \omega^3 = 1$ $C_2: H^4 = I$ $C_3: S^3 = I$ C_6^*:



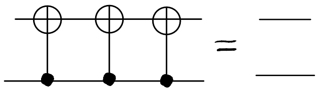
C_{16}^{21}:



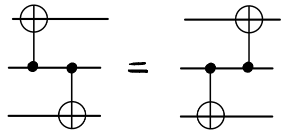
$C_{16}^{7'}$:



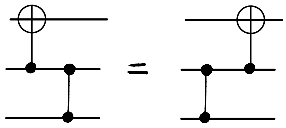
C_6^{2*}:



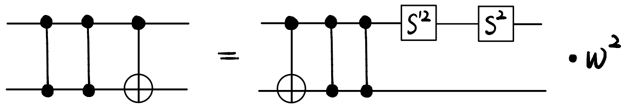
C_{13}^{9}:



C_{13}^{3}:

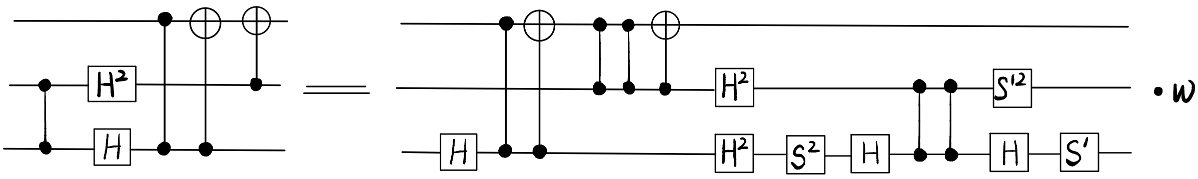


$R_{25}^{5'}$:



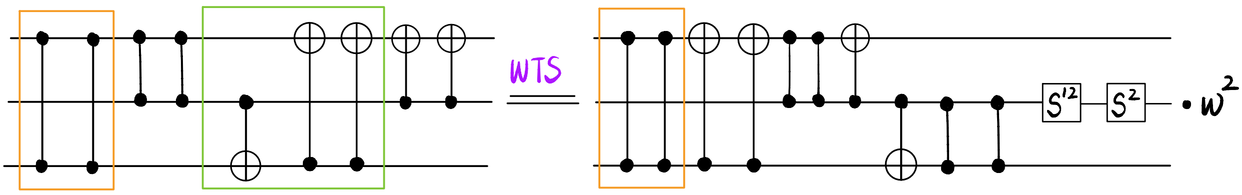
Lem V

R_{55}:



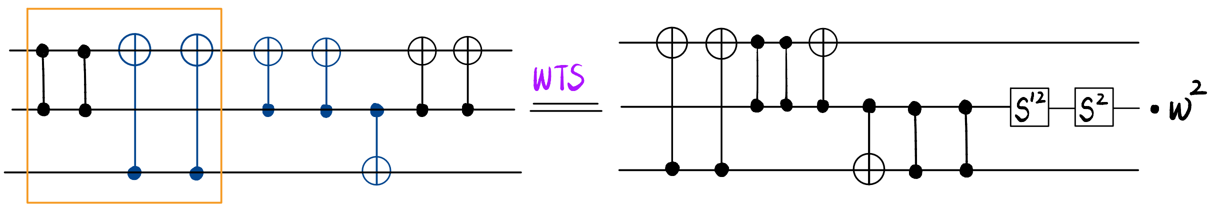
Proof cont.

R_{55}:



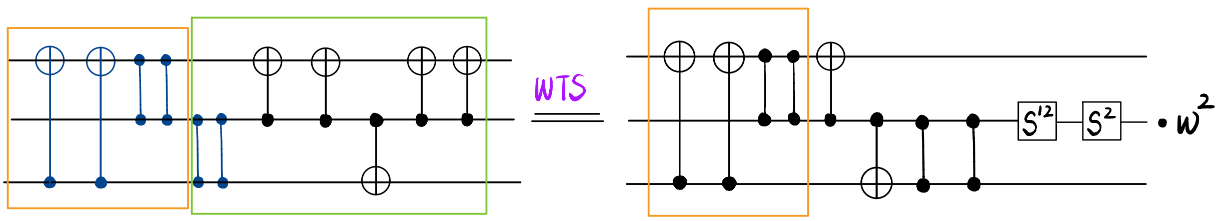
$C_6^* \;|||\; C_{16}^{21}$

R_{55}:



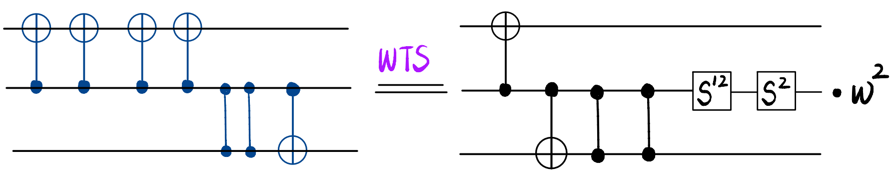
$C_{16}^{7'} \;|||$

R_{55}:

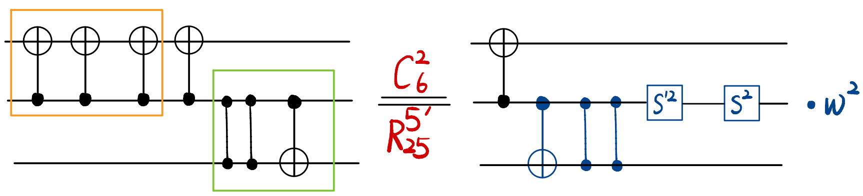


$C_6, C_6^{2*} \;|||\; C_{13}^{9}, C_{13}^{3}$

R_{55}:



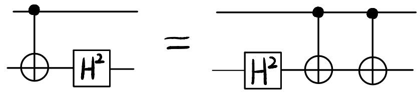
Then $R_{55}.\mathrm{LHS} :=$

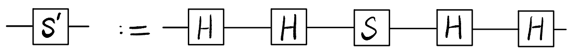


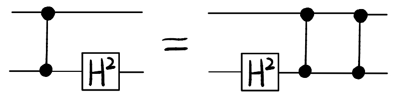
$=: R_{55}.\mathrm{RHS}$

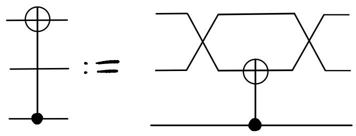
▯

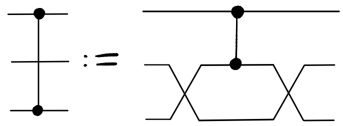
$C_1: \omega^3 = 1$ $C_2: H^4 = I$ $C_3: S^3 = I$ $C_5: SS' = S'S$

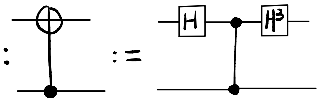
C_8^7: 

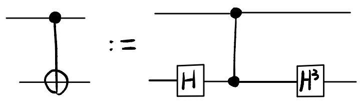
Def 1 : 

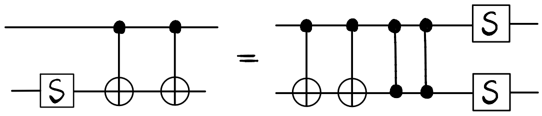
C_8: 

Def 7 : 

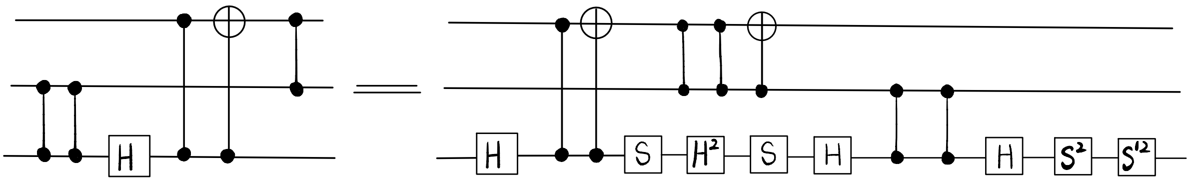
Def 5 : 

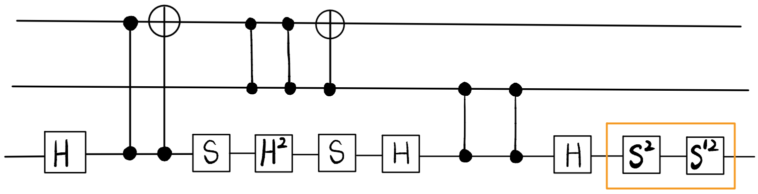
Def 2 : 

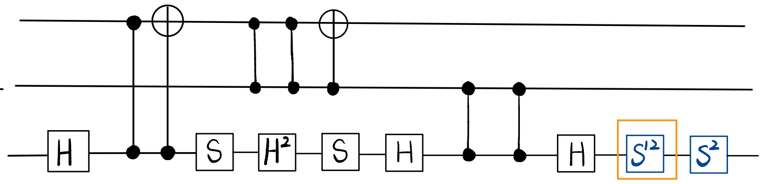
Def4 : 

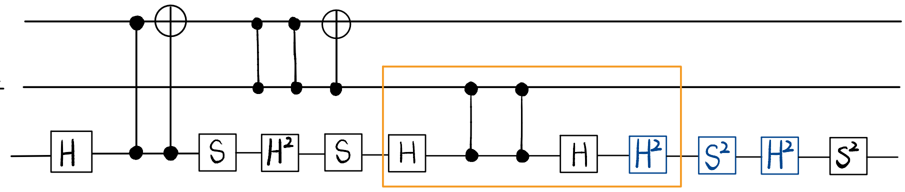
$R_{23}^{9'}$:  $\cdot\, \omega^2$

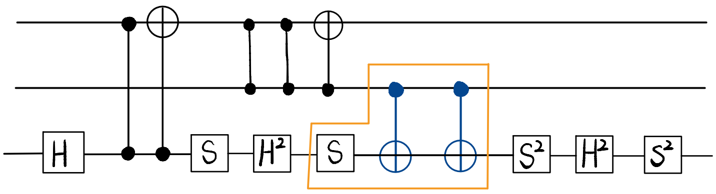
Lem W Def1-2, Def4-5, Def7, $C_1, C_2, C_3, C_5, C_6, C_7, C_8, C_{13}, C_{16}, R_{15}, R_{23}, R_{25}$ imply

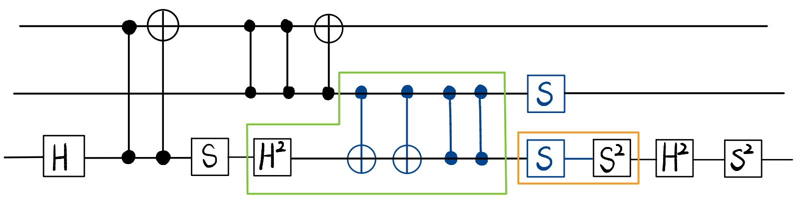
R_{56}: 

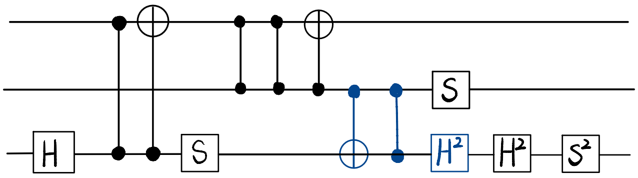
Proof: R_{56}.RHS := 

$\underline{\underline{C_5}}$ 

$\underline{\underline{\text{Def 1}}}$ 

$\underline{\underline{\text{Def 4}}}$ C_2 

$\underline{\underline{R_{23}^{9'}}}$  $\cdot\, \omega^2$

$\underline{\underline{C_8, C_8^7}}$ C_3  $\cdot\, \omega^2$