

$$\boxed{A_{01}} := \text{---}$$

$$\boxed{A_{02}} := \text{---} \boxed{H} \text{---} \boxed{H} \text{---}$$

$$\boxed{A_{10}} := \text{---} \boxed{H} \text{---}$$

$$\boxed{A_{20}} := \text{---} \boxed{H} \text{---} \boxed{H} \text{---} \boxed{H} \text{---}$$

C2: $H^4 = I$

Lem 1 By definition and **C2**, we have 1. (1) $\text{---} \boxed{H} \text{---} \boxed{A_{01}} \text{---} = \text{---} \boxed{A_{10}} \text{---}$

$$(2) \text{---} \boxed{H} \text{---} \boxed{A_{02}} \text{---} = \text{---} \boxed{A_{20}} \text{---}$$

$$(3) \text{---} \boxed{H} \text{---} \boxed{A_{10}} \text{---} = \text{---} \boxed{A_{02}} \text{---}$$

$$(6) \text{---} \boxed{H} \text{---} \boxed{A_{20}} \text{---} = \text{---} \boxed{A_{01}} \text{---}$$

Proof 1. (1). LHS := $\text{---} \boxed{H} \text{---} =: \text{---} \boxed{A_{10}} \text{---} = 1. (1). \text{RHS}$

$$1. (2). \text{LHS} := \text{---} \boxed{H} \text{---} \boxed{H} \text{---} \boxed{H} \text{---} =: \text{---} \boxed{A_{20}} \text{---} = 1. (2). \text{RHS}$$

$$1. (3). \text{LHS} := \text{---} \boxed{H} \text{---} \boxed{H} \text{---} =: \text{---} \boxed{A_{02}} \text{---} = 1. (3). \text{RHS}$$

$$1. (6). \text{LHS} := \text{---} \boxed{H} \text{---} \boxed{H} \text{---} \boxed{H} \text{---} \boxed{H} \text{---} \stackrel{\text{C2}}{=} \text{---} =: \text{---} \boxed{A_{01}} \text{---} = 1. (6). \text{RHS.}$$



$$-A_{11}- := -S-H-$$

$$-A_{21}- := -H-H-S-S-H-$$

$$-A_{12}- := -S-S-H-$$

$$-A_{22}- := -H-H-S-H-$$

$$C_2: H^4 = I$$

$$R_1: -H-S-H- = -S-S-H-S^2-X^2- \cdot (-w^2)$$

$$R'_1: -H-S-S-H- = -H-H-S-H-X-S- \cdot (-w)$$

Lem 2 By definition, C_2 & R_1 , we have 1. (4) $-H-A_{11}- = -A_{12}-S^2-X^2- \cdot (-w^2)$

$$(5) -H-A_{12}- = -A_{22}-X-S- \cdot (-w)$$

$$(7) -H-A_{21}- = -A_{11}-X-S- \cdot (-w)$$

$$(8) -H-A_{22}- = -A_{21}-S^2-X^2- \cdot (-w^2)$$

$$\text{Proof: 1. (4). LHS} := -H-S-H- \stackrel{R_1}{=} -\boxed{S-S-H}-S^2-X^2- \cdot (-w^2)$$

$$=:-A_{12}-S^2-X^2- \cdot (-w^2) = 1. (4). \text{RHS}$$

$$1. (5). \text{LHS} := -H-S-S-H- \stackrel{R'_1}{=} -\boxed{H-H-S-H}-X-S- \cdot (-w)$$

$$=:-A_{22}-X-S- \cdot (-w) = 1. (5). \text{RHS}$$

$$1. (7). \text{LHS} := -H-H-\boxed{H-S-S-H}-$$

$$\stackrel{R'_1}{=} -\boxed{H-H-H-H}-S-H-X-S- \cdot (-w)$$

$$\stackrel{C_2}{=} -\boxed{S-H}-X-S- \cdot (-w)$$

$$=:-A_{11}-X-S- \cdot (-w) = 1. (7). \text{RHS}$$

$$1. (8). \text{LHS} := -H-H-\boxed{H-S-H}-$$

$$\stackrel{R_1}{=} -\boxed{H-H-S-S-H}-S^2-X^2- \cdot (-w^2)$$

$$=:-A_{21}-S^2-X^2- \cdot (-w^2) = 1. (8). \text{RHS}$$

