

$$C_{15}^2: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^3: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^4: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^5: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Def 2: } \text{Diagram} := \text{Diagram}$$

$$R_{19}: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Diagram} = \text{Diagram}$$

$$\text{Def 4: } \text{Diagram} := \text{Diagram}$$

Lem E Def 2, Def 4, G5 & R19 imply

$$C_{15}^{10}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{11}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{12}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{13}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{14}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{15}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{16}: \quad \text{Diagram} = \text{Diagram}$$

$$C_{15}^{17}: \quad \text{Diagram} = \text{Diagram}$$

$$\text{Proof: } C_{15}^{10}. \text{LHS} := \text{Diagram} \quad \underline{\underline{\text{Def 4}}} \quad \text{Diagram}$$

$$\text{Diagram} \quad \underline{\underline{\text{R19}}} \quad \text{Diagram}$$

$$\underline{\underline{C_5^2}} \quad \text{Diagram} \quad \underline{\underline{\text{R19}}} \quad \text{Diagram}$$

$$\text{Diagram} \quad \underline{\underline{\text{Lem D}}} \quad \text{Diagram} =: C_{15}^{10}. \text{RHS.}$$

$$C_{15}^{11}. \text{LHS} := \text{Diagram} \quad \underline{\underline{\text{Def 2}}} \quad \text{Diagram}$$

$$\text{Diagram} \quad \underline{\underline{\text{R19}}} \quad \text{Diagram}$$

$$\underline{\underline{C_5^2}} \quad \text{Diagram} \quad \underline{\underline{\text{R19}}} \quad \text{Diagram}$$

$$\text{Diagram} \quad \underline{\underline{\text{Lem D}}} \quad \text{Diagram} =: C_{15}^{11}. \text{RHS.}$$

Reasoning analogously, we can prove $C_{15}^{12} - C_{15}^{17}$ are the consequence of Def 2, Def 4, G5 & R19.

$$C_{13}: \quad \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{16}: \quad \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} = \begin{array}{c} \text{Diagram 7} \end{array}$$

$$R_{17}: \quad \begin{array}{c} \text{Diagram 8} \\ \text{Diagram 9} \end{array} = \begin{array}{c} \text{Diagram 10} \end{array}$$

$$C_{15}: \quad \begin{array}{c} \text{Diagram 11} \\ \text{Diagram 12} \end{array} = \begin{array}{c} \text{Diagram 13} \\ \text{Diagram 14} \end{array}$$

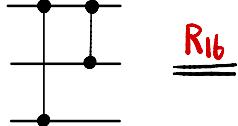
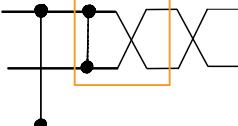
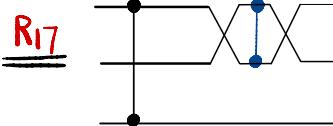
$$C_{15}^3: \quad \begin{array}{c} \text{Diagram 15} \\ \text{Diagram 16} \end{array} = \begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \end{array}$$

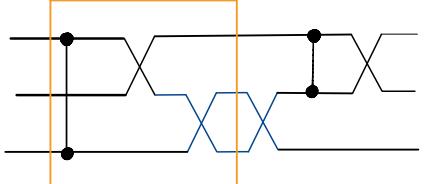
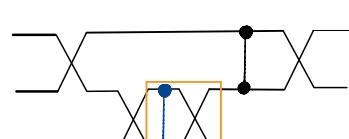
$$C_{15}^4: \quad \begin{array}{c} \text{Diagram 19} \\ \text{Diagram 20} \end{array} = \begin{array}{c} \text{Diagram 21} \\ \text{Diagram 22} \end{array}$$

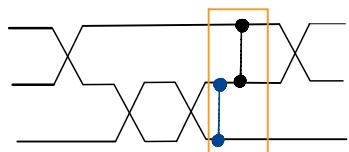
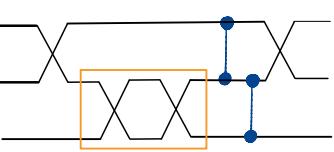
Lem F Def 5, R_{16} , R_{17} , G_3 & G_5 imply

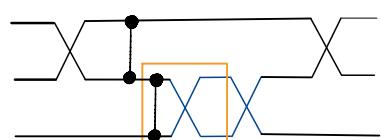
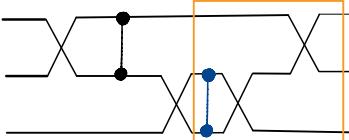
$$C_{13}^1: \quad \begin{array}{c} \text{Diagram 23} \\ \text{Diagram 24} \end{array} = \begin{array}{c} \text{Diagram 25} \\ \text{Diagram 26} \end{array}$$

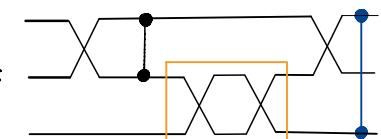
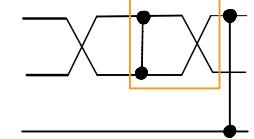
$$C_{13}^2: \quad \begin{array}{c} \text{Diagram 27} \\ \text{Diagram 28} \end{array} = \begin{array}{c} \text{Diagram 29} \\ \text{Diagram 30} \end{array}$$

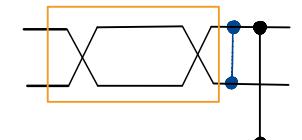
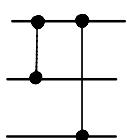
Proof: $C_{13}^1 \cdot \text{RHS} :=$  $\stackrel{R_{16}}{=} \quad$  $\stackrel{R_{17}}{=} \quad$ 

$$\stackrel{R_{16}}{=} \quad$$
  $\stackrel{C_{15}^3}{=} \quad$ 

$$\stackrel{R_{17}}{=} \quad$$
  $\stackrel{C_{13}}{=} \quad$ 

$$\stackrel{R_{16}}{=} \quad$$
  $\stackrel{R_{17}}{=} \quad$ 

$$\stackrel{C_{15}^4}{=} \quad$$
  $\stackrel{R_{16}}{=} \quad$ 

$$\stackrel{R_{17}}{=} \quad$$
  $\stackrel{R_{16}}{=} \quad$  $=: C_{13}^1 \cdot \text{LHS}$

$$C_{13}: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{16}: \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} = \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array}$$

$$R_{17}: \begin{array}{c} \text{Diagram 9} \\ \text{Diagram 10} \end{array} = \begin{array}{c} \text{Diagram 11} \\ \text{Diagram 12} \end{array}$$

$$C_{15}^2: \begin{array}{c} \text{Diagram 13} \\ \text{Diagram 14} \end{array} = \begin{array}{c} \text{Diagram 15} \\ \text{Diagram 16} \end{array}$$

$$C_{15}^3: \begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \end{array} = \begin{array}{c} \text{Diagram 19} \\ \text{Diagram 20} \end{array}$$

$$C_{15}^4: \begin{array}{c} \text{Diagram 21} \\ \text{Diagram 22} \end{array} = \begin{array}{c} \text{Diagram 23} \\ \text{Diagram 24} \end{array}$$

$$C_{15}^5: \begin{array}{c} \text{Diagram 25} \\ \text{Diagram 26} \end{array} = \begin{array}{c} \text{Diagram 27} \\ \text{Diagram 28} \end{array}$$

Lem F Def 5, R_{16} , R_{17} , G_3 & G_5 imply

$$C_{13}^1: \begin{array}{c} \text{Diagram 29} \\ \text{Diagram 30} \end{array} = \begin{array}{c} \text{Diagram 31} \\ \text{Diagram 32} \end{array}$$

$$C_{13}^2: \begin{array}{c} \text{Diagram 33} \\ \text{Diagram 34} \end{array} = \begin{array}{c} \text{Diagram 35} \\ \text{Diagram 36} \end{array}$$

Method 1

$$\text{Proof: } C_{13}^2 \cdot \text{RHS} := \begin{array}{c} \text{Diagram 37} \\ \text{Diagram 38} \end{array} \stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 39} \\ \text{Diagram 40} \end{array} \stackrel{R_{17}}{=} \begin{array}{c} \text{Diagram 41} \\ \text{Diagram 42} \end{array}$$

$$\stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 43} \\ \text{Diagram 44} \end{array} \stackrel{C_5}{=} \begin{array}{c} \text{Diagram 45} \\ \text{Diagram 46} \end{array}$$

$$\stackrel{R_{17}}{=} \begin{array}{c} \text{Diagram 47} \\ \text{Diagram 48} \end{array} \stackrel{C_{13}}{=} \begin{array}{c} \text{Diagram 49} \\ \text{Diagram 50} \end{array}$$

$$\stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 51} \\ \text{Diagram 52} \end{array} \stackrel{R_{17}}{=} \begin{array}{c} \text{Diagram 53} \\ \text{Diagram 54} \end{array}$$

$$\stackrel{C_4}{=} \begin{array}{c} \text{Diagram 55} \\ \text{Diagram 56} \end{array} \stackrel{C_2}{=} \begin{array}{c} \text{Diagram 57} \\ \text{Diagram 58} \end{array} \stackrel{C_3}{=} \begin{array}{c} \text{Diagram 59} \\ \text{Diagram 60} \end{array}$$

$$\stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 61} \\ \text{Diagram 62} \end{array} \stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 63} \\ \text{Diagram 64} \end{array} =: C_{13}^2 \cdot \text{LHS}$$

□

$$C_{13}: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{16}: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$R_{17}: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{15}^2: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{15}^3: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{15}^4: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{15}^5: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Lem F Def 5, R_{16} , R_{17} , G_3 & G_5 imply

$$C_{13}^1: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^2: \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Method 2

Proof: C_{13}^2 . RHS :=

$$\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \stackrel{R_{16}}{=} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\begin{array}{c} C_{15}^4 \\ \equiv \\ C_{15}^5 \end{array} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\begin{array}{c} C_{13}^1 \\ \equiv \\ R_{16} \end{array} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\begin{array}{c} C_{15}^2 \\ \equiv \\ C_{15}^3 \end{array} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}$$

$$\begin{array}{c} R_{16} \\ \equiv \\ \text{Diagram 1} \end{array}$$

$$\begin{array}{c} R_{16} \\ \equiv \\ \text{Diagram 1} \end{array} =: C_{13}^2 \cdot \text{LHS}$$

□

$$C_{13} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^1 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^2 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Def 2: } \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} := \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Def 4: } \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} := \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

Lem F' Def 2, Def 4 & C_{13} imply

$$C_{13}^3 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^4 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^5 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^6 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^7 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^8 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Proof: } C_{13}^3. \text{LHS} := \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{C_{13}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} =: C_{13}^3. \text{RHS.}$$

$$C_{13}^4. \text{LHS} := \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{C_{13}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} =: C_{13}^4. \text{RHS.}$$

$$C_{13}^5. \text{LHS} := \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} = \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \xrightarrow{C_{13}^1} \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array}$$

$$= \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} =: C_{13}^5. \text{RHS.}$$

Reasoning analogously, we can prove C_{13}^6 is the consequence of Def 4 & C_{13} .

$$C_{13}^7. \text{LHS} := \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} = \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \xrightarrow{C_{13}^2} \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array}$$

$$= \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} =: C_{13}^7. \text{RHS.}$$

Reasoning analogously, we can prove C_{13}^8 is the consequence of Def 2 & C_{13} .

$$C_{13}^3 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^6 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^8 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Def 2: } \begin{array}{c} \text{Diagram 1} \end{array} := \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Def 4: } \begin{array}{c} \text{Diagram 1} \end{array} := \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \quad C_2 : H^4 = I$$

Lem F" Def 2, Def 4 & G_3 imply

$$C_{13}^9 : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^{10} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^{11} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^{12} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^{13} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$C_{13}^{14} : \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array}$$

$$\text{Proof: } C_{13}^9 \cdot \text{LHS} := \begin{array}{c} \text{Diagram 1} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} =: C_{13}^9 \cdot \text{RHS.}$$

$$C_{13}^{10} \cdot \text{LHS} := \begin{array}{c} \text{Diagram 1} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} =: C_{13}^{10} \cdot \text{RHS.}$$

$$C_{13}^{11} \cdot \text{LHS} := \begin{array}{c} \text{Diagram 1} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} =: C_{13}^{11} \cdot \text{RHS.}$$

$$C_{13}^{12} \cdot \text{LHS} := \begin{array}{c} \text{Diagram 1} \end{array} \xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} =: C_{13}^{12} \cdot \text{RHS.}$$

$$\xrightarrow{\text{Def 2}} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \xrightarrow{\text{Def 4}} \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} =: C_{13}^{12} \cdot \text{RHS.}$$

Reasoning analogously, we can prove C_{13}^{13} & C_{13}^{14} .