

Lem 20

$$7.4-6.(1) \quad B_{1b} \quad = \quad \text{[Circuit Diagram]} \cdot w^2$$

Proof cont:

$$7.4-6.(1). LHS = \text{[Circuit Diagram]}$$

$$7.4-6.(1). RHS := \text{[Circuit Diagram]} \cdot w^2$$

$$\text{def} \quad \text{[Circuit Diagram]} \cdot w^2$$

$$\frac{R_{19}}{\text{Def 4}} \quad \text{[Circuit Diagram]} \cdot w^2$$

$$\frac{R_{17}, R_{31}}{C_{15}^4, C_{15}^{14}} \quad \text{[Circuit Diagram]} \cdot w^2$$

$$\underline{\underline{R_{19}}} \quad \text{[Circuit Diagram]} \cdot w^2$$

$$C_3 : S^3 = I \quad R_{16} : \text{Diagram} = \text{Diagram}$$

$$C_8^b: \quad \begin{array}{c} \text{---} \\ \oplus \\ \text{---} \\ | \quad | \\ H^2 \quad \bullet \end{array} = \quad \begin{array}{c} \text{---} \\ \oplus \\ \text{---} \\ | \quad | \\ \bullet \quad \bullet \quad H^2 \end{array}$$

$$C_2: H^4 = I$$

$$R_{53} = \begin{array}{c} \text{Circuit Diagram} \\ \text{with } H \text{ and CNOT gates} \end{array} = \begin{array}{c} \text{Circuit Diagram} \\ \text{with } S, S^2, \text{ and CNOT gates} \end{array} \cdot w^2$$

Lem 20

$$7.4-6.(1) \quad \text{Diagram showing the equivalence of two circuit configurations. The left side shows a series connection of resistors } B_{1b}, B_{00}, \text{ and } B_{02}. \text{ The right side shows a more complex network with resistors } B_{1b}, B_{02}, \text{ and various switches labeled H (horizontal), S (series), and } w^2. \text{ The two configurations are connected by an equals sign.}$$

Proof cont : Hence

A quantum circuit diagram illustrating the WTS (Weighted Toom-Cook) algorithm. The circuit consists of two horizontal wires representing qubits. On the left wire, there is a box labeled S^b . A sequence of three controlled-NOT gates (CNOTs) acts on both wires. The first CNOT has its control on the S^b wire and target on the right wire. The second CNOT has its control on the right wire and target on the S^b wire. The third CNOT has its control on the right wire and target on the left wire. Following these, there is a single-qubit gate H on the right wire. Finally, a sequence of three CNOT gates returns the wires to their initial state: the first CNOT has control on the right wire and target on the left wire; the second CNOT has control on the left wire and target on the right wire; and the third CNOT has control on the right wire and target on the left wire. A circled plus sign (+) is placed above the right wire, indicating the final measurement point.

$$C_3 \equiv R_{16}$$

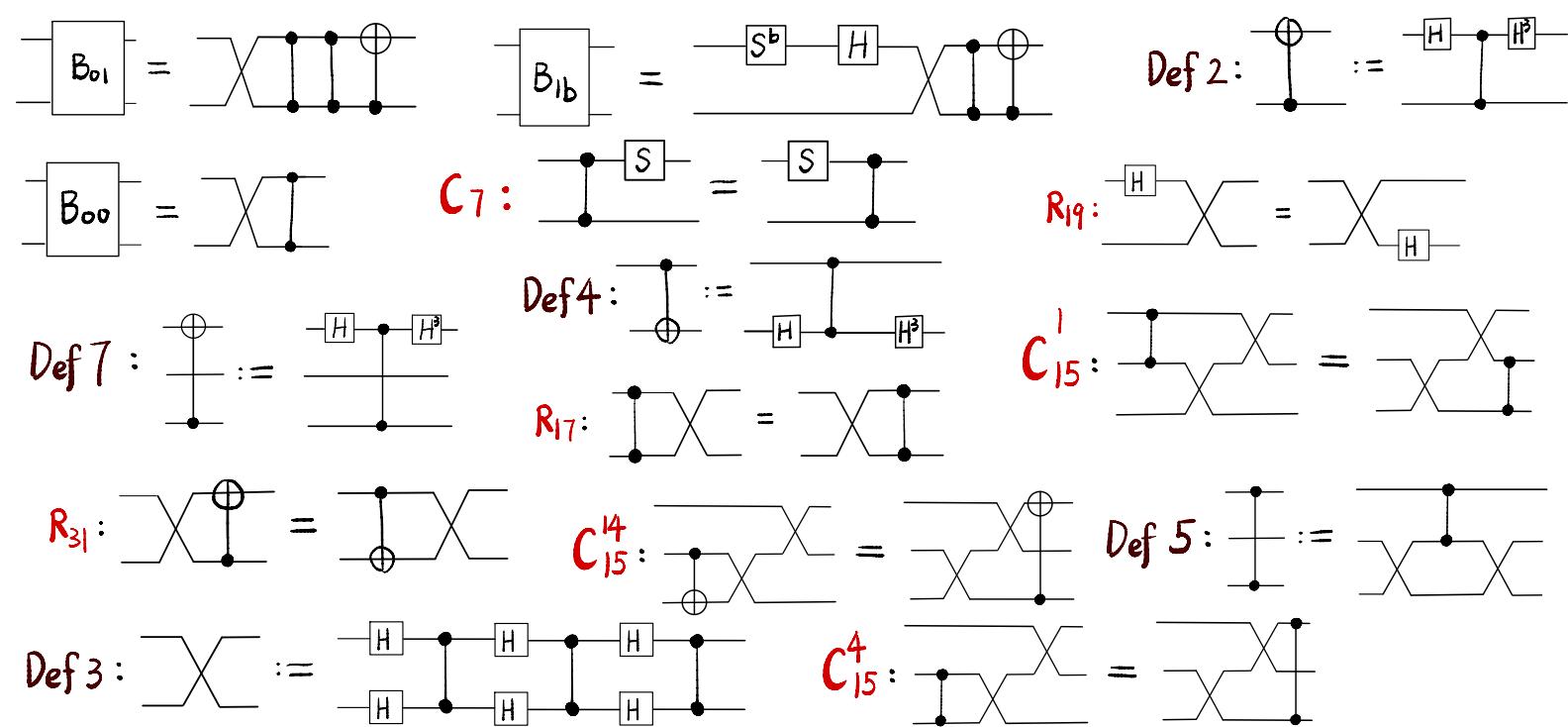
WTS

$$C_2 \equiv C_8^6$$

The diagram shows a quantum circuit with four horizontal wires. The top wire has three control circles at positions 1, 2, and 3. The second wire from the top has two blue dots at positions 2 and 3. The third wire from the top has a single black dot at position 1. The bottom wire has three black dots at positions 1, 2, and 3. There are several rectangular boxes representing gates: an H gate on the first wire at position 1, an S gate on the second wire at position 3, an S^2 gate on the third wire at position 2, and another S gate on the fourth wire at position 3. The labels WTS , \bullet , and w^2 are placed near the top and right sides of the circuit.

By R₅₃, this completes the proof.

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Lem 21 Def 2-5, Def 7, $C_3, C_7, C_5, R_{1b}, R_{17}, R_{19}, R_{31}$ & R_{54} imply

7.4-6.(2) =

Proof: 7.4-6.(2). LHS := $\stackrel{\text{def}}{=}$
 $\boxed{S^b} \quad \boxed{H} \quad \boxed{\text{CNOT}} \quad \boxed{Z} \quad \boxed{Z} \quad \boxed{H} \quad \boxed{H} \quad \boxed{H} \quad \boxed{S}$
 $\boxed{C_7, R_{17}, R_{31}} \quad \boxed{C_{15}^4, C_{15}^{14}}$
 $\frac{R_{19}}{C_{15}^1}$

7.4-6.(2). RHS := $\stackrel{\text{def}}{=}$

$$R_{19}: \text{Diagram} = \text{Diagram}$$

$$R_{31}: \text{Diagram} = \text{Diagram}$$

$$R_{17}: \text{Diagram} = \text{Diagram}$$

$$C_{15}^1: \text{Diagram} = \text{Diagram}$$

$$C_{15}^{14}: \text{Diagram} = \text{Diagram}$$

$$C_2: H^4 = I$$

$$C_{15}^4: \text{Diagram} = \text{Diagram}$$

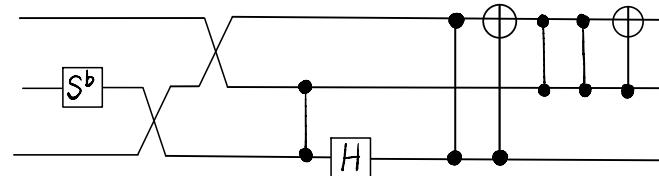
$$\text{Def 4: } \text{Diagram} := \text{Diagram}$$

$$R_{11}: \text{Diagram} = \text{Diagram}$$

Lem 2

$$7.4-6.(2) \text{ LHS} = \text{Diagram} = \text{Diagram}$$

Proof cont : 7.4-6.(2). LHS =



$$7.4-6.(2). \text{ RHS} = \text{Diagram}$$

$$\underline{C_2, R_{17}, R_{31}} \\ \underline{C_{15}^4, C_{15}^{14}, \text{Def 4}}$$

$$\text{Diagram}$$

Hence

$$\text{Diagram} \quad \underline{\text{WTS}}$$

$$\text{Diagram}$$

$$C_3 : S^3 = I \quad R_{16} : \text{Diagram} = \text{Diagram}$$

$R_{54}:$

Lem 2 |

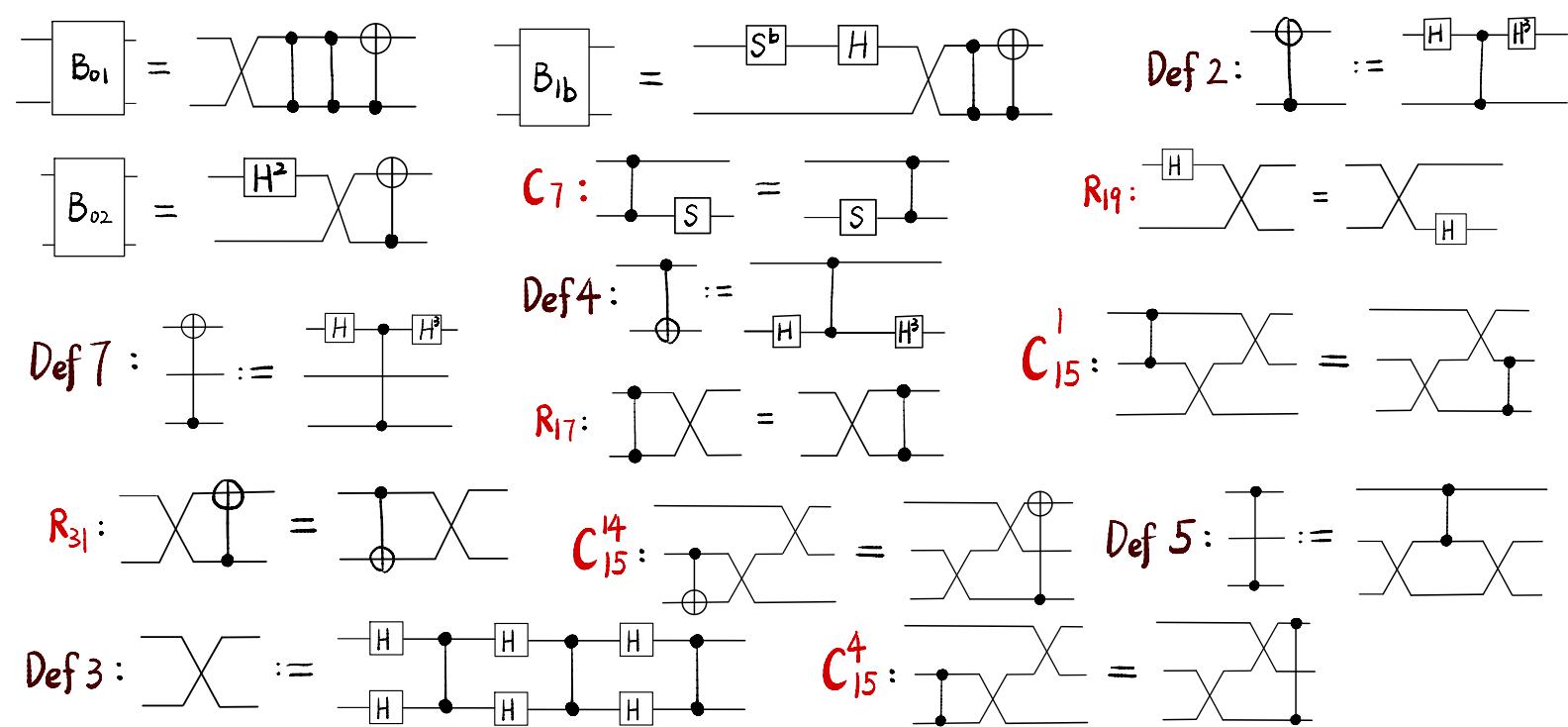
7.4-6.(2)

Proof cont :

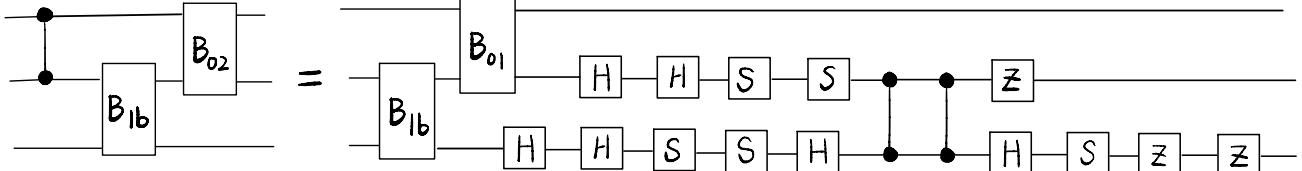
$C_3 \parallel R_{16}$

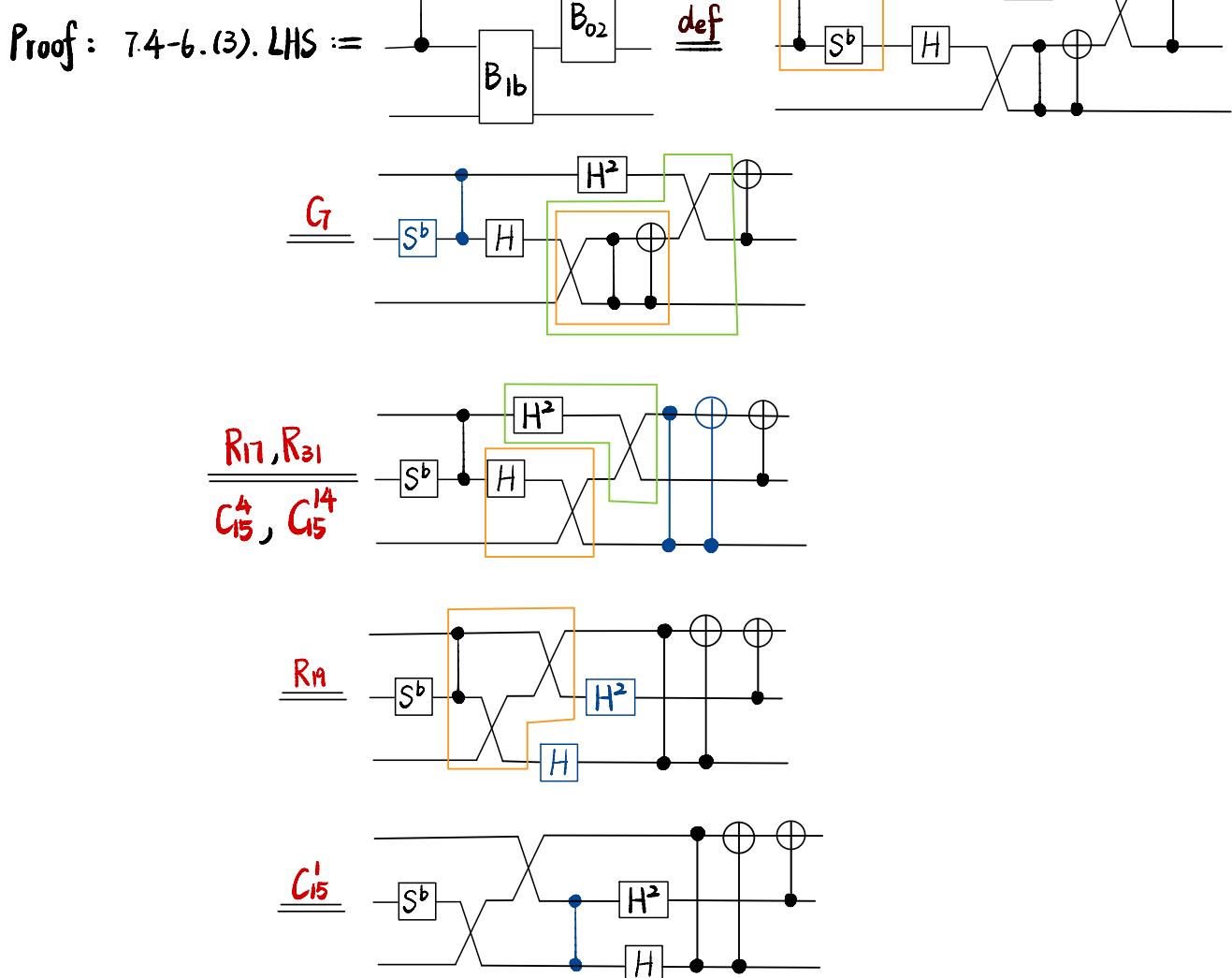
By R_{54} , this completes the proof.

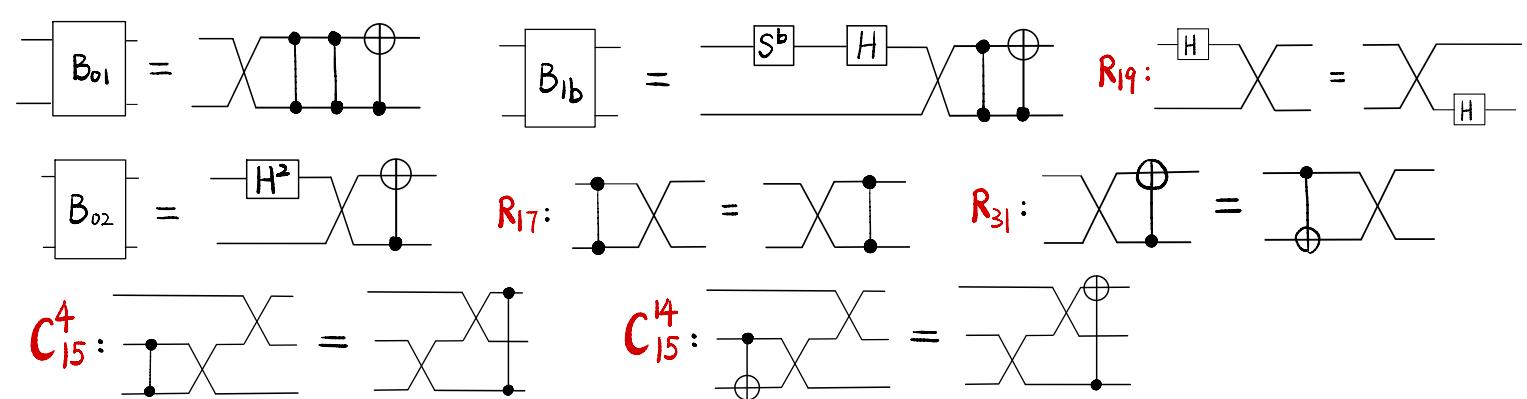




Lem 22 Def 1 - 5, Def 7, $C_3, C_5, C_7, C_{15}, R_{10}, R_{11}, R_{16}, R_{17}, R_{19}, R_{31}$ & R_{55} imply

7.4-6.(3)  • w





Lem 22

7.4-6.(3)

• w

Proof cont:

7.4-6.(3). RHS :=

• w

def

• w

R17, R31
C15, G5

• w

Rn

• w

Hence

WTS

• w