

# STAT 535: Assignment

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## 1 Pseudo-code for the Test

### 1.1 Discrete MC Test

The DiscreteMCTest file is used to test the invariance and the irreducibility of a discrete state space Markov chain. Here is the psuedo-code for this algorithm:

1. Give an index for each possible state in the state space. The mapping between the index and the state space is bidirectional.
2. Build the target distribution for the model provided by enumerating all possible configurations via ExhaustiveDebugRandom.
  - (a) Initialize the probabilities to be a Counter.
  - (b) while(state i is not listed by the exhaustive method){
 

probabilities[i] +=  $p_i$

 }
3. target distribution=probabilities/sum(probabilities)
4. Build the transition matrix for each kernel. Each element in the transition matrix is calculated empirically.
5. Invariance test of each transition kernel. Matrix multiplication of the transition matrix and the target distribution to see if this is equal to the target distribution.
6. Check the irreducibility of the mixture of kernels.
  - (a) Suppose we have  $m$  candidate Kernels  $K_1, K_2, \dots, K_m$ , then  $\frac{1}{m+1}[\sum_{i=1}^m K_i + I]$  where  $I$  is the identity matrix is the mixture kernel.
  - (b) Start at an arbitrary state, check if we are able to reach all other states in the space in  $n$  steps if  $n$  is the state space size.

## 1.2 ExhaustiveDebugRandom

The four main functions in this class are `nextInt`, `nextBernoulli`, `nextCategorical`, `hasNext`.

The `hasNext` function calls the `init()` function and initialize all the variables values. The next categorical is addressed below

1. translated is a list of index which represent the element of probabilities is greater than 0
2. initialize old Decision. Compare current Depth and old deepest incrementable branch, there are three cases based on their value.
3. add current decision to newDecisions list.
4. if current decision is less than length of translated minus 1, `hasNext` equals tree, assign current Depth to new deep incrementable branch.
5. current depth +=1
6. sample a category
7. `pr *= probabilities[sampled category]`

## 2 Test for Invariance and Irreducibility

1. Invariance: We have the property that if each kernel is invariance, then the mixture of the kernels is also invariant. We start from the test of invariance for each kernel since it is easier for us to find which one does not pass the invariance test.
2. Irreducibility: The test procedure for invariance does not hold for irreducibility since the property above does not hold. Even when each individual kernel is not irreducible, the mixture of the kernels can still be irreducible. Therefore, we test the irreducibility of the mixture kernel.

## 3 Posterior Distribution of Permutation

