EEE4118F – Process Control & Instrumentation

Lab Report – Part B: Robust Control System Design

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Executive Summary

This report serves to describe the process of designing a controller to be implemented in a servo motor system. Servo motors are devices used to drive and rotate parts of machinery with high efficiency and accuracy. A controller is needed in closed loop to ensure that the output is accurate to the input and that the settling time is correct. The controller should be robust, so as to ensure the system behaves as expected when plant conditions change.

With the controller implemented, the system should have a response time of 1.2 s, have an overshoot of less than 12.5%, have zero steady state error, and have no oscillation at the output. These response specifications should be the same, even when the plant operating condition is changed, as well as when the plant attenuation changes. The settling time should also be maintained in the presence of an output disturbance.

A P/PI cascade controller is chosen to fulfil the design specifications. The inner loop of the cascade controller is the velocity loop, with the controller transfer function of: $G_{vel}(s) = \frac{1.72(s+6.558)}{s}$. The outer loop is the position loop, with the controller transfer function of: $G_{pos}(s) = 1.5$.

Using a sample time of 50 ms, the discrete transfer function for the controller is found to be: $G_{vel}(z) = \frac{2.002z-1.438}{z-1}$. This controller is implemented digitally via a computer and a DAC which interfaces with the servo motor system. This is done via C# code, using difference equations of the following forms: $Velocity_{setpoint} = 1.5\left(Position_{setpoint} - Position\right)$ and $Output = 2.002 * Velocity_{error} - 1.438 * Velocity_{error,previous} + Output_{previous}$. Where $Velocity_{error} = Velocity_{setpoint} - Velocity$. The output equation is what gets sent to the DAC and allows for the control of the system.

The system is simulated, and these results are then compared with the actual lab system. Both are injected with the same steps, and output disturbance. The simulation results and lab results are found to be almost identical. For low inertia, and low inertia with a magnetic break, the system is found to have zero steady state error, no overshoot, robust behaviour under different plant conditions and good rejection of output disturbance. The main issue found at this stage is that the settling time is 2 s, rather than 1.2 s.

It is found, via simulation, that the controller configuration is not suitable for the high inertia case. This has not been confirmed using the physical system due to time constraints. The system is unstable when the high inertia transfer function is used. This indicates that the servo motor system with the final controller design implemented is not suitable for driving heavy loads. This means that the application of the system would be limited to lighter loads.

In order to fix this error, the gain could be changed, however this still results in oscillations. It becomes evident that the PI controller would need to be redesigned in order for the controller to be truly robust. However, despite the controller not fully meeting the technical specifications, it still is robust when applied to lighter loads, and could find application for driving lighter loads, and in systems that cannot tolerate any overshoot in the output.

Thus, it is recommended that the controller configuration described in this report be limited to applications described above. Otherwise, should the system be needed to drive heavier loads, or to give faster responses, a new velocity controller would need to be designed.

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Introduction

This report outlines the process of designing a robust controller for a DC Servo Motor System.

Design Problem

Servo motors are electrical devices which are widely used in industry. They are used to rotate parts of machinery with high efficiency and accuracy. For the sake of this project, a servo motor system has been set up in the laboratory. The plant system can be represented using the block diagram in the figure below:

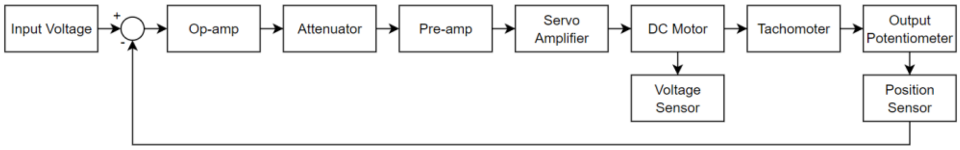

Figure 1: Servo motor block diagram

The generalised transfer function of the system can be given as: $P_{pos}(s) = \frac{k}{s(s/a+1)}$, which approximately describes the output position, and $P_{vel}(s) = \frac{k}{s/a+1}$, which is the transfer function for the motor velocity.

For the system set up in the lab, there are three operating conditions, namely low inertia, low inertia with a magnetic break, and high inertia. These three operating conditions results in three different transfer functions which can be used to describe the system. Open-loop step tests have been performed to obtained these transfer functions, and the final results are summarised in the report.

Servo motor systems are only able to operate with high accuracy due to the feedback and control embedded into the system. In open loop the system is unstable, as the output position oscillated, and the motor continues to rotate. When the loop is closed, without any form of controller in place, the output oscillates before settling, and there is a high steady state error.

It is evident that a controller is important to ensure accuracy in the system output, as well as to provide faster settling times for the system. The theory behind the specific controller type to be implemented is outlined in this report.

The system may also experience some output disturbance, as well as changes to the plant. A controller is needed to ensure that these changes/disturbances do not affect the output of the system.

Technical Specifications

The controller design outlined in this report is for a robust controller, that should theoretically, and practically, be able to handle the different operating conditions, as well changes to the plant, and output disturbances.

Given the overall complexity of the system, a cascade controller is required to fully control the system. The cascade controller should consist of two loops; the inner loop being velocity control, and the outer loop being position control.

The specifications for the response of the system once the controller is implemented is as follows:

- Settling/response time of 1.2 seconds
- Overshoot in velocity loop less than 25%
- Overshoot in position loop less than 12.5%
- Perfect tracking; zero steady state error

- Have no oscillations at the output
- Minimal controller action (no high amplitude oscillations of >0.4V)

The controller should ensure the above response specifications under the following plant conditions:

- Change of plant; change in plant attenuation
- The three operating conditions, namely low inertia, low inertia with a magnetic break, and high inertia.
- Maintain the same settling time in the presence of output disturbance (overshoot limitations do not apply in this case)

The controller should be implemented digitally, and the design should allow a 10° margin for digital design.

Theory

A brief overview of the PID controller is given in this section, and an overview of cascade control is supplied.

A PID controller, or proportional integral derivative controller, is a controller which is widely used in industry. A PID controller works by continuously calculating the error value of the loop, which is the difference between the setpoint and the measured output or process variable.

The PID consists of three terms, each serving a different purpose. The proportional term is used to find the error between the set point and actual output. The integral is used to calculate the sum of the past terms, and when the error is eliminated from the system, the integral term will stop increasing. The derivative term is used to predict the expected error values in the future, based on the present value. [1]

The general form of the PID controller is shown below:

$$u(t) = k_p(e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{de(t)}{dt}$$

The PID controller has various configurations – not all terms need to be implemented. For the purpose of the servo motor system in the laboratory, a PI controller is used in the inner/velocity loop and a P controller is used in the outer/position loop.

The purpose of the proportional controller in the position loop is to output a setpoint for the velocity loop, which is proportional to the current error of the position loop (position setpoint – output position).

The purpose of the PI controller is to ensure that there is zero steady state error in the output. To increase the speed of the response, the integral gain can be decreased. The proportional term here serves a similar purpose to the proportional controller in the position loop. The reason for omitting the derivative term of the normal PID controller is because the controller becomes sensitive to measurement noise.

Cascade control is the cascading of two unique control loops. Cascade control is useful in systems where a measurable secondary variable (i.e., velocity) directly influences the main controlled variable (i.e., position). [2]

The advantages of cascade control are that they allow improved speed of the response, especially when the inner loop responds faster than the outer loop. It also allows for isolation of load disturbances [2], which is beneficial in the servo system.

Preliminary Design Steps

System Identification Summary

In the previous lab report, the operating zones and system identification methods are outlined in depth. The three transfer functions for the system, for the three operating states are shown in the table below, as well as their respective impact on the system:

Table 1: System Identification Results

Operating State	Transfer Function (velocity)	Impact on system
Low inertia	$T_{LI} = \frac{4.756}{1.231s + 1}$	Taken as nominal case
Low inertia + magnetic break	$T_{LI+B} = \frac{2.533}{0.3968s + 1}$	Decreased maximum velocity Increased time constant
High inertia (heavy disk)	$T_{HI} = \frac{4.759}{6.8823s + 1}$	Increased maximum velocity Decreased time constant

The step responses of the above transfer functions were compared to the data used to model the function, and both responses were found to be very close to each other.

Preliminary Closed Loop System Testing

An early step in the controller design process included the testing of different types of controller configurations. This was done to become familiar with the system, and what effects certain controllers would have on the response of the system. The results of these initial tests are as follows:

Simple gain controllers, implemented in just the position loop, and then the velocity loop, were able to provide faster settling time, and reduce oscillations, but increased overshoot and did not fix any tracking errors. Using just a lead controller in the position loop yielded the best response for these tests. This gave the best response in terms of settling time and overshoot, however the there was still steady state error and the system still responded inconsistently to different plant conditions.

The above results made the benefit of controlling both the position and velocity loops evident, as well as indicating that a slightly more complex controller configuration would be needed.

Final Control Design

Design Results

For the final controller, a P/PI cascade controller was chosen. The inner loop of the system is velocity, which is controlled using a PI controller. The outer loop of the system is the position loop, which is controller using a P controller. The block diagram for the system, with the controllers implemented can be found below:

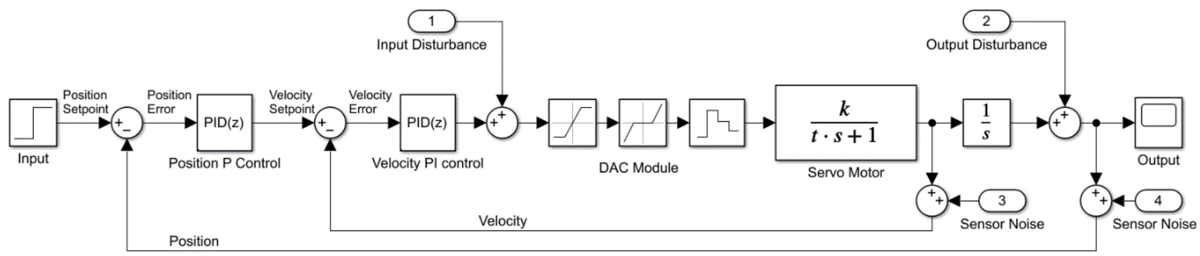

Figure 2: Block diagram of final control loop configuration

Using the technical specifications outlined previously, the controller was designed using the QFT toolbox (Inverse Nichols Chart), and the Root Locus on MATLAB. The detailed steps are provided as appendix A to this report. The final design results are as follows:

The velocity controller is found to be:

$$G_{vel}(s) = \frac{1.72(s + 6.558)}{s}$$

The position controller is given by:

$$G_{pos}(s) = 1.5$$

Controller Digital Implementation and Analysis

Upon implementation of the final P/PI cascade controller designed, the equivalent digital controller can be found, and an initial analysis of the controller can be conducted.

The velocity controller. $G_{vel}(s)$ is converted to a discrete controller used the Tustin method, with a sampling time of 50 ms. This gives the following equation:

$$G_{vel}(z) = \frac{2.002z - 1.438}{z - 1}$$

Since the controller is to be tested in the laboratory on the physical system, where saturation, a dead-zone and sampling effects are present, this digital controller is used in simulation to get accurate results. The step response, for a step from 0 to 5, is shown for all three operating states is found. The step response for just the velocity loop, and the position and velocity controls, are shown in the figures below:

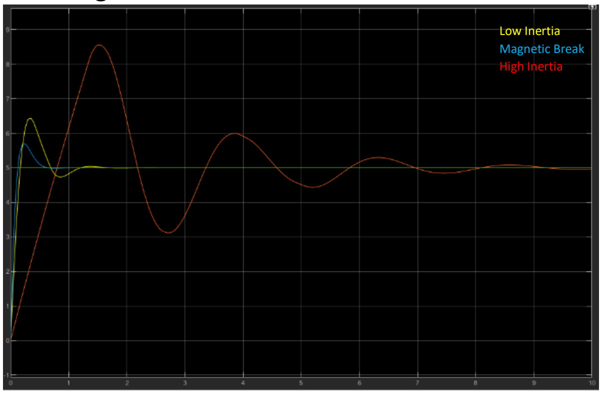

Figure 4: Step test for velocity loop with controller implemented

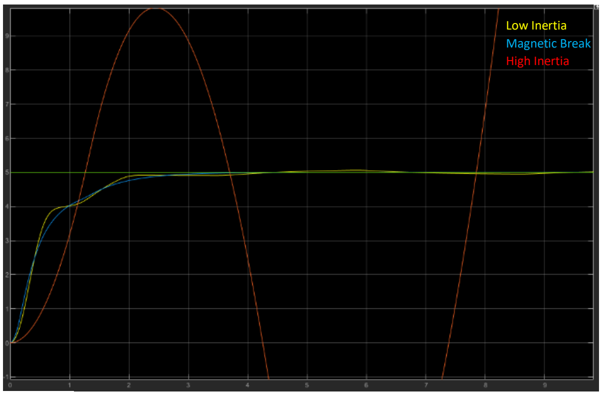

Figure 3: Step test for overall system with controller implemented

From the previous figures, it is evident that the controller works as expected for two of the operating conditions, namely the low inertia, and low inertia with a magnetic break. For the high inertia case, there are oscillations present in the velocity loop, however it does eventually settle (after about 9 seconds). However, when the gain in the position loop is added, the system becomes unstable for the high inertia case.

These oscillations are because the loop gain is too high. To solve this, the position loop gain can be lowered to one third of its initial value, giving $G_{pos}(s) = 0.5$. However, by lowering the position gain to this value, the settling time for the other two conditions increases by a factor of 5. Thus, it becomes evident that the velocity controller values should be re-evaluated, to ensure there are no oscillations on the output of the high inertia plant.

Due to time constraints, a new controller could not be designed, and for the purpose of laboratory testing (where high inertia is not actually tested) the original controller design is implemented. A summary of the results of this controller implementation, is shown in the following table.
From the table it can be seen that the overshoot goals have been achieved, however the controller settles in 2 seconds rather than 1.2 seconds.

Table 2: Summary of results in simulation based on design regulations

Parameter	Symbol	Nominal Value	Group's Value
Controller response time	τ_v	1.2s	2s
Overshoot in velocity loop	M_{pv}	25%	10%
Margin for digital design	$\emptyset_d$	10°	10°
Unsaturated step size	y_U	10°	5V
Overshoot in position loop	M_{pp}	12.5%	No overshoot

Evaluation of Final Design

Digital Implementation

The chosen controller will be digitally implemented, using a computer that is interfaced with the system via a DAC. In order to implement the controller digitally, the control transfer function needs to be written as a difference equation. The difference equations for the two controllers chosen are as follows:

$$Velocity_{setpoint} = 1.5\left(Position_{setpoint} - Position\right)$$

$$Output = 2.002 * Velocity_{error} - 1.438 * Velocity_{error,previous} + Output_{previous}$$

Where $Velocity_{error} = Velocity_{setpoint} - Velocity$. The output equation is what gets sent to the DAC and allows for the control of the system.

Laboratory Test Plan

Once the controller is properly implemented, the test plan to ensure that it function as simulated is as follows:

1. A positive input step from 0V to 5V, which is inputted from the internal input function.
2. Output disturbance rejection test, which is to be achieved by rotating the disk attached to the motor shaft. The input is then set to external, in able to perform the following tests.
3. Applying the magnet to the disk, and performing multiple steps, to ensure that the system is still stable and that the output still tracks the setpoint.
4. Remove the magnetic break. Change the plant attenuation from a value of 3, to a value of 7. Then apply a step test. Output disturbance rejection is tested as in 2. Multiple steps are performed, and then the system is allowed to settle.

Due to time constraints, the high inertia case is not tested in the laboratory.

Laboratory Test Results

The output of the system, with the controller implemented, is captured in the following figure. All of the tests described previously are present in this figure, and labels are provided to indicate the location of specific tests.

The time at which each test occurs is:

- 5V step at t=5s.
- Output disturbance present from t=12.5s to t=15s
- Magnetic break applied between t=20s and t=30s. External input step tests performed from t=33s and t=40s.
- Magnetic break removed and plant attenuation changed at t=44s. External input step tests performed from t=46s and t=59s. Output disturbance applied at t=49s.

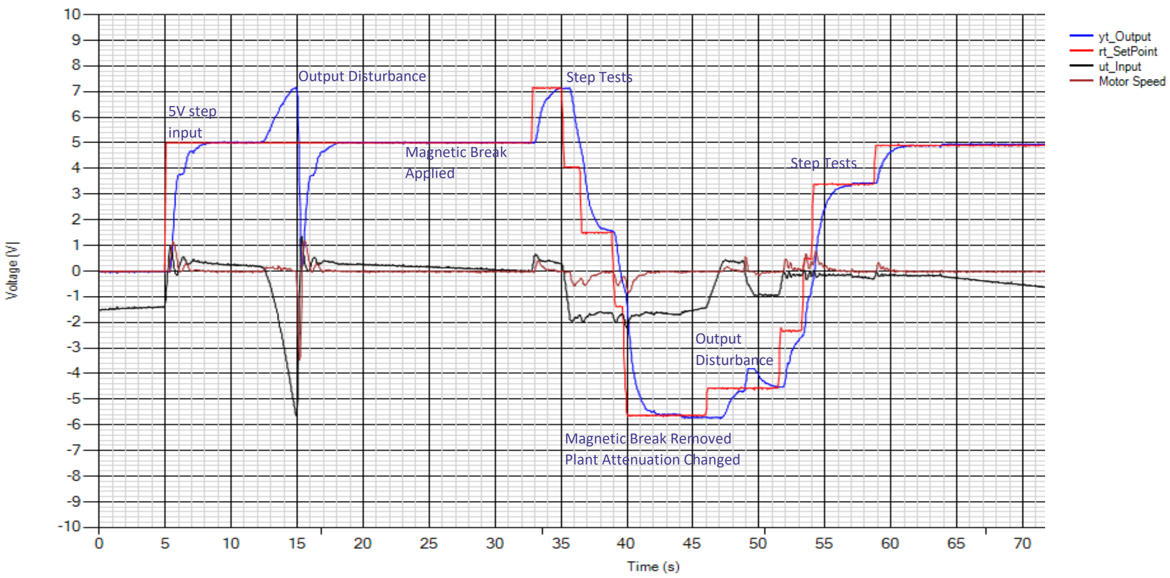

Figure 5: Lab results from testing

From the above figure, it is evident that the controller is very robust. There is zero steady state error, for the three cases tested. The settling time to 5% is consistently 2 seconds. The output disturbance results in overshoot; however, the system returns to the previous values within 2 seconds.

Simulation Results

The simulation model that is used in order to test the controller is shown below:

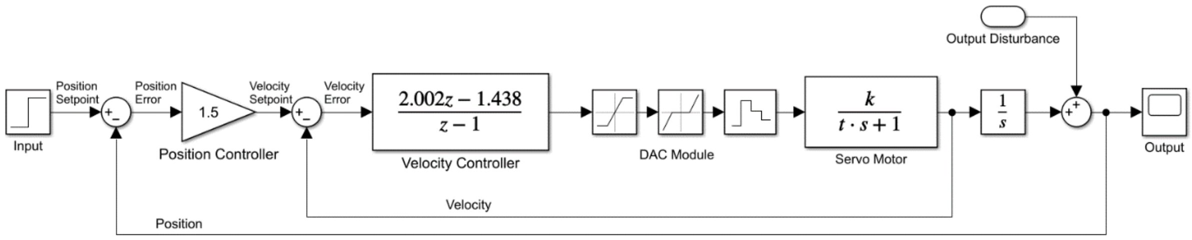

Figure 6: Simulation model used for testing final controller design

The general formula for the servo motor transfer function is used in the above figure. This is chosen as it accounts for the various components within the plant (those listed in the introduction). The DAC module consists of three blocks, namely a saturation block, to account for the limits of -10V and +10V that the DAC can handle in the lab. A dead-zone is added, which accounts for the range of voltages for which the system does not react. Finally, a zero-order hold is implemented, which converts the signal from digital to analogue.

The above simulation model is used, with the various transfer function tested, and the different output disturbances present, and various step inputs. The change in attenuation is modelled as a gain of 0.8, since increasing the attenuation decreases the output gain of the plant by some factor (the exact factor is unknown). This gives the results shown in the following figure.

The inputs were set to approximately match that which was done in the lab. The output disturbances were approximated using ramps and steps, the ensure they only effected the system when the output disturbance was applied. The times at which these steps occur are the same as those listed in the previous section.

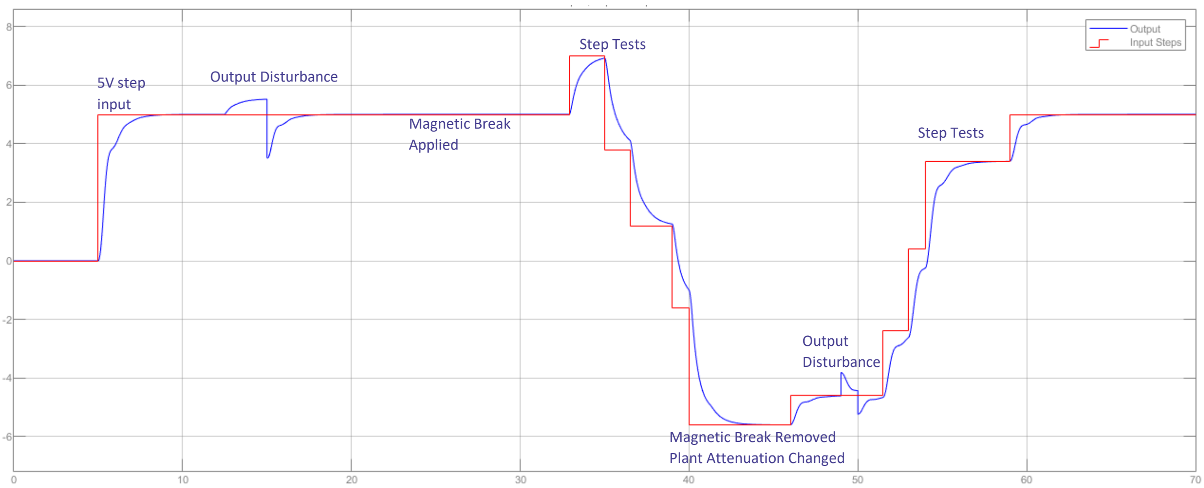

Figure 7: Simulation results from tests

Comparison of Simulation Results and Lab Results

From figure 5, and figure 7, it can be seen that the simulation results closely represent the lab results obtained. The shape of the response for the various plant cases is almost identical. The general characteristics are the same, such as settling time, zero steady state error, overshoot, and ability to reject output disturbances. The main discrepancies can be seen when the output disturbances are applied (at t=12.5s, and t=49s). For the output disturbance, which is applied to the nominal plant, there is more overshoot in the actual lab results than in simulation. However, since overshoot is not an important characteristic in terms of output disturbance rejection (at least in the case of this lab test), the differences between the two is not important to the final results.

The second output disturbance was harder to simulate, since it was just a step input, and since the model for the change in attenuation does not fully represent the actual change to the plant, the simulation is not going to accurately represent the actual output. A more precise model for how a change in attenuation effects the system would need to be found, otherwise the transfer functions and characteristics at the different attenuation would need to be found using a step test. This would be important to do if the system was not easily accessible for testing during the design phase.

Conclusion and Recommendations

This report has outlined the design and implementation of a cascade P/PI controller which was used on a physical servo motor system in the laboratory.

The main problems were outlined, and found to be inaccuracies in the output of the system, as well as oscillations in the output. These are unfavourable characteristics for the system as servo motor systems are used in high precision and efficiency applications. The technical specifications were also outlined, and these were used as guidelines for the controller design. A summary of the system identification is provided, as well as results from some preliminary tests performed with different controller configurations.

The final controller is found to be $G_{vel}(s) = \frac{1.72(s+6.558)}{s}$ and $G_{pos}(s) = 1.5$. These controllers were converted to the z-domain, so that they could be implemented digitally. This gave: $G_{vel}(z) = \frac{2.002z-1.438}{z-1}$ and $G_{pos}(z) = 1.5$. These controllers were then applied in simulation, and the step test results are provided.

From the results of these step tests, it is evident that the controller configuration chosen was theoretically effective on the low inertia and magnetic break cases, however the high inertia step test produced large oscillation on the output, and was unstable for the overall system. The reason for these oscillations and instability is because the gain in the position loop was too high, however lowering the gain decreased the response time of the other two plant cases.

This led the group to believe that the velocity controller would likely need to be redesigned, however due to time constraints this was not possible. This choice is unfortunate, since it means that the servo motor would not be able to drive heavier loads. This limits the application of the servo motor system, with the specified controller configuration, to only be able to drive lighter loads. This in turn means that the controller is not as robust as it should be.

Another issue with the design, is that despite the efforts in designing the controller, it still resulted in a slower settling time than expected. This could be due to the fact that the design was mostly done using the QFT, and only verified using the root locus afterwards. The response time was found to be 2 seconds, rather than 1.2 seconds.

The exact steps taken to test the controller are outlined in the report. In order to apply the controller, the z-domain transfer functions were written in terms of difference equations, which were then written in a C# program which sent the output to the DAC. The results of testing in the lab closely match the simulated results.

From the laboratory tests, it was found that the system reacts well to changes in plant attenuation, it is not affected by the magnetic break, effectively rejects output disturbance, and in all tested cases had zero steady state error. The resultant output graph is almost identical t the response produced using simulation.

Thus, despite the design not meeting all of the technical specification outlined in the report, the design produced might be usable in some applications, such as accurately driving smaller/lighter loads, as well in systems where overshoot cannot be tolerated at all, but a slightly slower response is acceptable.

But, for the case of larger loads, or if a faster response time is needed, simply changing the gain would not be a suitable solution. The velocity loop controller would likely need to be tuned more precisely, or completely redesigned, in order to better fit the technical specifications.

References

[1] T. R. Kuphaldt, “Chapter 32: PID Control Terminology,” in *Lessons In Industrial Instrumentation*, Samurai Media Limited, 2017.

[2] S. Ranjith, “Cascade control loop working, application, avantages,” 22 November 2018. [Online]. Available: https://automationforum.co/cascade-control-loop-working-application-avantages/. [Accessed 30 April 2022].

Appendices

Appendix A: Detailed Design Steps

Velocity Controller:

First, the templates need to be produced. These templates account for the uncertainties in the plant, and are generally drawn for the identified critical frequencies. A nominal plant is chosen (in this case, the low inertia plant was chosen), and the point on the template that represents the nominal plant is indicated. The templates plotted below are for the phase crossover frequencies for each plant case. The solid dot indicates the position of the nominal plant:

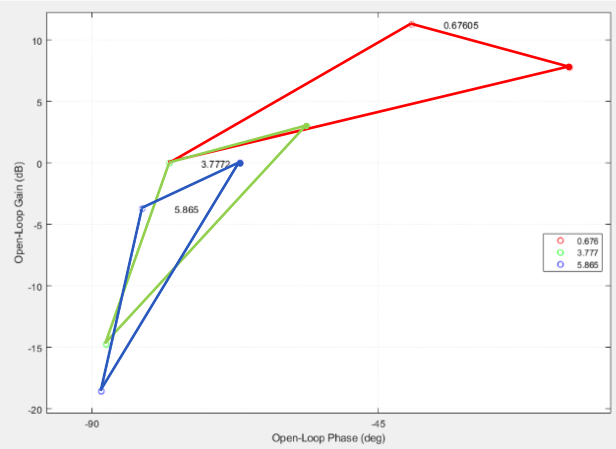

Figure 8: Templates for velocity transfer function, at critical frequencies

These critical frequencies are found by finding the phase crossover frequency of all three responses. These frequencies are found to be: 3.7772 rad/s for the nominal case, 5.8650 rad/s for the low inertia + magnetic break case, and 0.6760 rad/s for the high inertia case.

These templates are used to draw the -3dB nominal bounds on the Inverse Nichols Chart, and these bounds are then used to perform the controller design. The design of a PI controller can be achieved by the combination of an integration, gain, and a real zero in the controller formula. First, the integrator term is added, which ensures zero steady state error. Then, some gain is added, and finally a zero is used to provide a phase shift (and some additional gain) to the right:

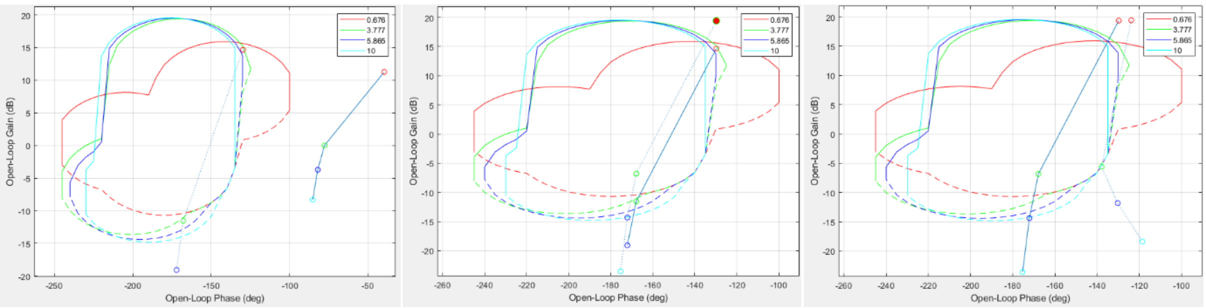

Figure 9: From left to right: Integrator term added, gain of 1.72 added, zero of 6.558 added

The final Inverse Nichols Plot is shown below in the figure below, as well as the corresponding root locus plot, with the additional controller terms:

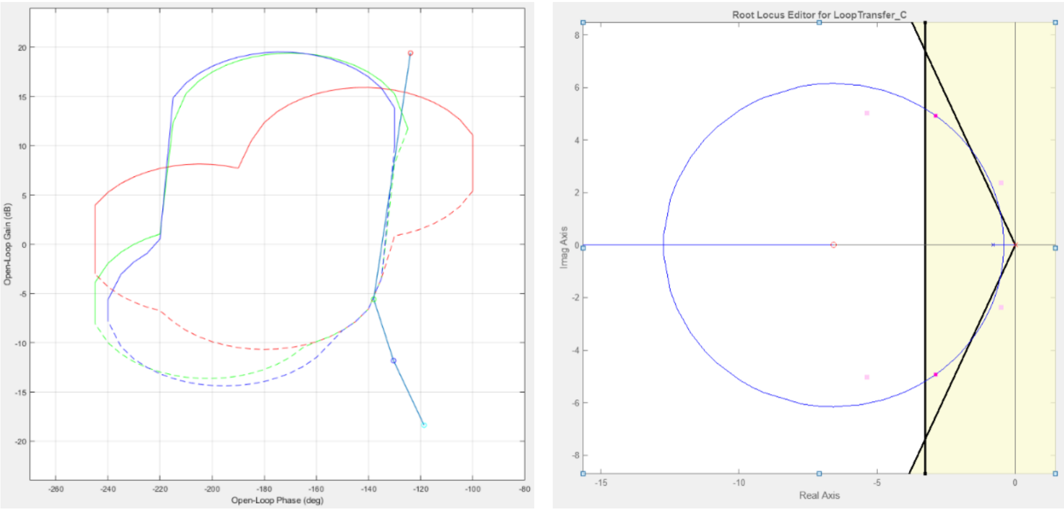

Figure 10: Final Inverse Nichols plot and root locus plot

From the root locus plot above, it can be seen that for the gain value chosen, the velocity loop does not settle within 1.2s.

Once the velocity controller is obtained, a position controller is found. A value for the P controller is first found by simply experimenting with various values within the position control loop, with the velocity controller implemented, in the Simulink simulation files, for the nominal plant. It is found that for a gain higher than around 3, the system starts to oscillate and becomes unstable. For a gain value of 1, the output settles too slowly. The final value for gain is chosen to be 1.5, as this ensures no oscillations in the output, but speeds up the response time.

This gain value can be verified by plotting the root locus. In order to plot these, instead of using T_{LI} as the plant, the plant is taken as the whole inner feedback loop, which can be written as:

$$T_{pos}(s) = \frac{G_{vel} * T_{LI}}{1 + G_{vel} * T_{LI}}$$

The root locus is shown in the figure below:

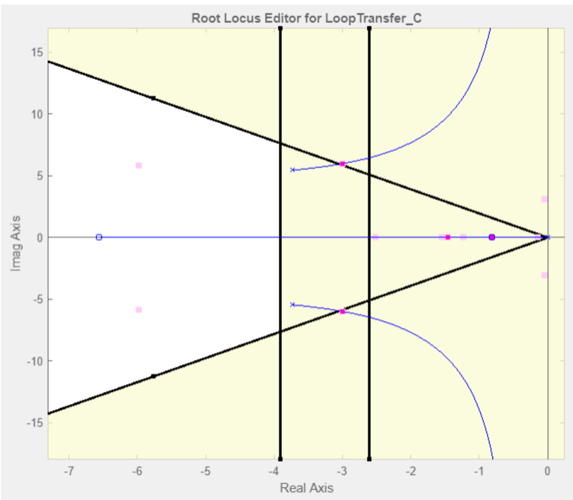

Figure 11: Root locus plot for overall system

From the above it can be seen that the settling time is still too low (the left line indicates the bound for 1 second settling time, and the right line indicates the bound for 2 seconds). The gain value does ensure that the overshoot is not too high.