Non-Linear Least Squares Curve Fitting

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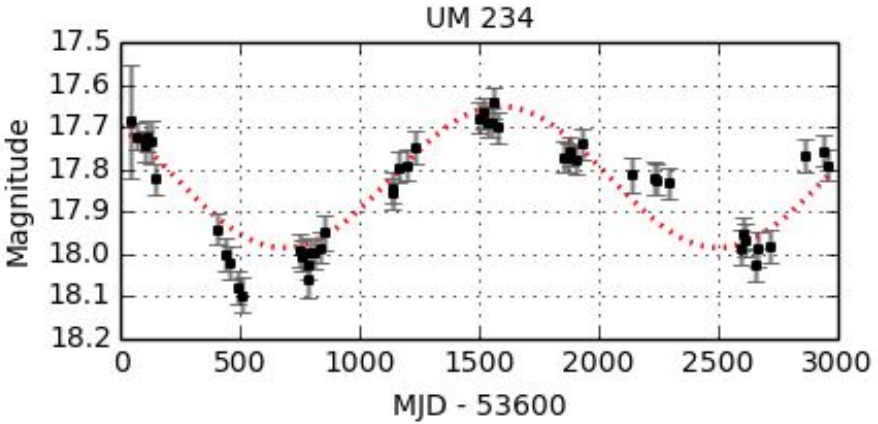
Background



- Graham, looking for strong periodic signals
- 111 Candidates from CRTS
- Fitting a periodic model to the light curve

Image from:
Graham, M. J., Djorgovski, S. G., Stern, et. al (2015). A systematic search for close supermassive black hole binaries in the Catalina Real-time Transient Survey. Monthly Notices of the Royal Astronomical Society, 453(2), 1562-1576. doi:10.1093/mnras/stv1726

Model

Parameters:

- Amplitude
- Frequency
- Phase
- Mean

$$A * \sin(\omega * t + \phi) + \mu$$

Least Squares

Levenberg–Marquardt Algorithm (LM)

- Damped Least-Squares
- Developed in the early 1960's
  - First published by Kenneth Levenberg(1944)
  - Rediscovered later by Donald Marquardt(1963)
- Combines Gradient Descent method and Gauss-Newton method
- Iterative

$$\chi^2(\boldsymbol{\theta}) \approx \chi^2(\boldsymbol{\theta}_c) + \nabla_{\boldsymbol{\theta}}\chi^2(\boldsymbol{\theta}_c)\delta\boldsymbol{\theta} + \delta\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\kappa}\delta\boldsymbol{\theta}$$

$$\kappa_{jk} = \frac{1}{2}\frac{\partial^2\chi^2(\boldsymbol{\theta}_c)}{\partial\theta_j\partial\theta_k}.$$

$$\nabla_{\delta\boldsymbol{\theta}}\chi^2(\boldsymbol{\theta}) \approx \nabla_{\boldsymbol{\theta}}\chi^2(\boldsymbol{\theta}_c) + \boldsymbol{\kappa}\delta\boldsymbol{\theta}$$

Gavin, H. P. (n.d.). The Levenberg-Marquardt algorithm for nonlinear least squares curve-fitting problems. Retrieved 2020, from http://people.duke.edu/~hpgavin/ce281/lm.pdf
Lam, M. Statistical Modeling: Likelihood Maximization and Model Comparison, Rochester Institute of Technology (2020)

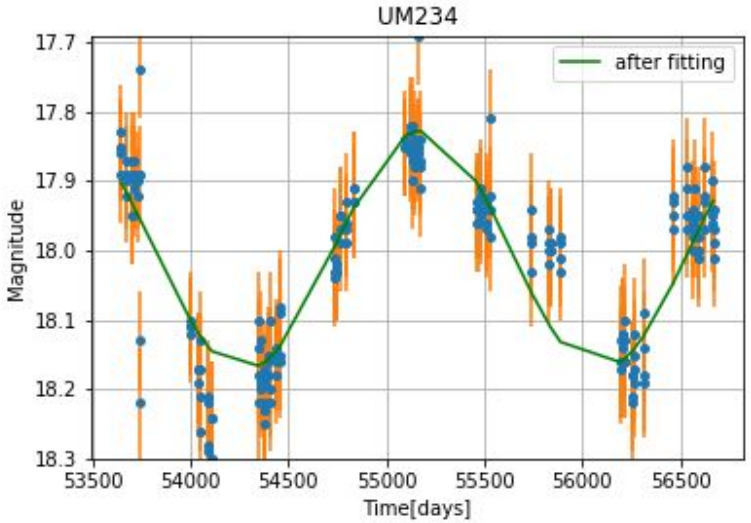
Python Implementation

- Iterations
- Solve for $\delta\boldsymbol{\theta}$

$$\delta\boldsymbol{\theta} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_c$$

- Can be compared to Scipy's optimize.leastsq

$$\nabla_{\delta\boldsymbol{\theta}}\chi^2(\boldsymbol{\theta}) \approx \nabla_{\boldsymbol{\theta}}\chi^2(\boldsymbol{\theta}_c) + \boldsymbol{\kappa}\delta\boldsymbol{\theta}$$



Link to code:
https://github.com/SarahV4775/Comp.-Meth.-for-Astrophysics-HW/blob/master/Final%20Project/Comp_ASTP720_Final.py

Questions?