MTH3121 Algebra Problem Sheet 2 : Saral 30 618 428

Question 3 a

Lets call the group represented by R^+ under multiplication,

M and the group represented by R^+ under the operation $x * y = \frac{1}{3} xy$,

O. So for M and O to be isomorphic there must be a bijection such that :
$f : M \longrightarrow O$ such that :

$f(a * b) = f(a) * f(b)$, for all $a, b \in M$

From looking at the $\frac{1}{3}$ it seems like a bijection could be $f(x) = 3x$. Lets see if this is it.

$f(a * b) = f(ab)$ (Just applying the multiplication binary operation)

$f(ab) = 3ab$ (Applying $f(x) = 3x$)

$3ab = \frac{1}{3}(3a)(3b)$ (Separating out some terms)

$\frac{1}{3}(3a)(3b) = \frac{1}{3} f(a) f(b)$ (Applying $f(x) = 3x$)

$\frac{1}{3} f(a) f(b) = f(a) * f(b)$ (Applying the binary operation $x * y = \frac{1}{3} xy$)

Thus we have shown that $f(a * b) = f(a) * f(b)$

But wait is the supposed bijection , $f(x) = 3x$, even a bijection?

First we have to show that it is injective (one - to - one) :

For any $a, b \in R$

$f(a) = 3a$

$f(b) = 3b$

if $f(a) = f(b)$ then $3a = 3b$

$3a = 3b \implies a = b$

Therefore $f(x)$ is injective

Now is it surjective?

For this to be the case we would need that for anthing
$y \in$ (some codomain) there exists an $x \in$ (some domain) such that :

$f(x) = y$

This would imply that :

$y = 3x$

and thus $x = \frac{y}{3}$, we have found an x for every y in the codomain.

This means that f (x) is surjective.

Since it is both injective and surjective, $f(x) = 3x$, is a bijection.

Question 4

**a) Max order of an element of S_6 is 6,
whereas for D_{720} it is 720. Therefore S_6 and D_{720} are not isomorphic.**

**b) Max order of an element of D_{18} is 18,
where as for Z_{36} it is 36. Therefore D_{18} and Z_{36} are not isomorphic.**

**c) Max order of an element of D_{60} is 60,
whereas for S_5 it is 6. Therefore D_{60} and S_5 are not isomorphic.**

**d) The only element of finite order in R under addition is 0. This
has order 1. For R^* under multiplication -1 has order 2 and 1 has order 1.**

**Thus R^+ and R^* have different numbers of elements
of finite order and therefore they are not isomorphic.**

Question 5