

1 Introduction

2 Problem Formulation

2.1 Control System

For our simulations, we are modelling the dynamics of the car with a simple non-linear bicycle model, with the following state (x) and control (u)

$$x = [p_x \ p_y \ \theta \ \delta \ v]^\top \quad u = [a \ \dot{\delta}]^\top$$

where p_x, p_y is the position, θ is the orientation, δ is the steering angle, and v is the velocity. The controls for the bike are acceleration a , and steering angle rate $\dot{\delta}$.

In order to make the system control-affine for easy optimization, we linearize our approximate dynamics model about X_{ref} and U_{ref} to get the following Jacobians for each step $k \in [1, N]$ in the trajectory:

$$A_k = \left. \frac{\partial f}{\partial x} \right|_{x_{ref,k}, u_{ref,k}} \quad B_k = \left. \frac{\partial f}{\partial u} \right|_{x_{ref,k}, u_{ref,k}}$$

where $f(x, u)$ is our approximate discrete dynamics model, X_{ref} is the optimal trajectory computed offline with approximate dynamics model, and U_{ref} is the optimal controls computed offline with approximate dynamics model.

With this, the system becomes control-affine, where the states are given by

$$x_{k+1} = A_k x_k + B_k u_k$$

and the proportional-derivative (PD) controller gives the control

$$u_k = -[P \cdot (x_k - x_{ref,k}) + D(x_k - x_{k-1})]$$

where P, D are the proportional and derivative control matrices to be found by our algorithm.

2.2 Optimizing for PD matrices

In order to automatically tune the PD matrices, we optimize for best PD matrices that minimize the quadratic cost of the trajectory generated by the PID controller, as constrained by control-affine system dynamics. Mathematically, we attempt to solve the following problem:

$$\begin{aligned}
& \min_{P,D} \quad (X - X_{ref})^\top Q (X - X_{ref}) \\
& \text{s.t.} \quad x_1 = x_{ref,1} \\
& \quad \quad x_{k+1} = A_k x_k + B_k u_k \\
& \quad \quad u_k = - [P \cdot (x_k - x_{ref,k}) + D(x_k - x_{k-1})]
\end{aligned}$$

where X, X_{ref} are the generated and reference trajectories respectively (a direct function of the states x_k), $Q \succeq 0$ is a quadratic cost matrix, and A_k, B_k are the Jacobians representing the linearized system dynamics as mentioned in Section 2.1 above.

We note that the cost function is quadratic and the constraints are affine, thus we have a quadratic programming problem.