

Scientific Machine Learning

MAM5 - INUM, Polytech Nice Sophia
DeepXDE

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UNIVERSITÉ **CÔTE D'AZUR**



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Outline

1 Practical work with DeepXDE: PINNs

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Time-domain Maxwell equations in 1D

$$\begin{cases} \varepsilon_r \frac{\partial E}{\partial t} - \frac{\partial H}{\partial x} = 0, \\ \mu_r \frac{\partial H}{\partial t} - \frac{\partial E}{\partial x} = 0. \end{cases}$$

- $E = E(x, t)$: electric field
- $H = H(x, t)$: magnetic field
- $\varepsilon_r = \varepsilon_r(x)$: (relative) electric permittivity
- $\mu_r = \mu_r(x)$: (relative) magnetic permeability
- Initial and boundary conditions defined on a problem basis
- t is the scaled time in m (meter), and the physical time is $\tilde{t} = t/c_0$ in s (second)
- c_0 is the speed of light and is equal to 3×10^8 m/s

Problem 1: standing mode in cavity

Time-domain Maxwell equations in 1D

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- $\Omega = [0,1]$
- $\varepsilon_r(x) = \mu_r(x) = 1, \forall x \in \Omega$
- $E(x=0, t) = E(x=1, t) = 0$ (perfectly metallic walls)
- $E(x, 0) = f(x)$ and $H(x, 0) = g(x)$

Analytical solution: $E_a(x, t)$ and $H_a(x, t)$

$$\begin{cases} E_a(x, t) &= \sin(m\pi x)\cos(\omega t), \\ H_a(x, t) &= \cos(m\pi x)\sin(\omega t). \end{cases}$$

- m is the mode index $\in \mathbb{N}^+$
- ω is the scaled pulsation in m^{-1} and is computed as $\frac{\tilde{\omega}}{c_0}$
- $\omega = m\pi$
- The physical period is $T = \frac{2\pi}{\tilde{\omega}}$ in s

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- m is the mode index $\in \mathbb{N}^+$
- Set $f(x) = E_a(x, 0)$ and $g(x) = H_a(x, 0)$
- Start by considering $m = 1$ and soft imposition of boundary conditions

Time-domain Maxwell equations in 1D

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- $\Omega = [0, 6]$
- $\varepsilon_r(x) = \mu_r(x) = 1, \forall x \in \Omega$
- $E(x = 0, t) = E(x = 6, t)$ and $H(x = 0, t) = H(x = 6, t)$ (periodic boundaries)
- $E(x, 0) = f(x)$ and $H(x, 0) = g(x)$

$$\begin{cases} f(x) = e^{-\alpha(x-x_g)^2}, \\ g(x) = -e^{-\alpha(x-x_g)^2}. \end{cases}$$

- x_g is the position of the pulse at initialization
- α is a coefficient to be chosen, e.g., $\alpha = 10$
- Simulate for $t \in [0, T_f]$ for a given T_f