Deep Learning

Basics in Deep Learning

Lionel Fillatre

2025-2026

Who am I?

- Full Professor, Université Côte d'Azur, Polytech Nice Sophia
- Member of I3S (computer science labs), team "Signals, Images and Systems"
- Research interests: data science, machine learning, deep learning, statistics, signal and image processing, biological data processing
- Head of the Data Science engineer's track ("MAM5 mineure SD")

Outline of Lecture 1

- Logistic and examination
- Introduction
- Some sucessful applications
- Basics in learning theory
- Basics in deep neural networks
- IT tools
- Conclusion

Logistic and Examination

Class website

- Class website
https://lms.univ-cotedazur.fr/2025/course/view.php?id=4225

- Password for the course registration: **kh95@PR63**

- **You must register yourself!**

Goal of the course

- Overview
  - "How" Deep Leaning works
  - Main advantages of DL
  - Main drawbacks of DL
  - Successful applications of DL
  - Use a neural network right away, get results and then understand it!

- Warning: Fast evolving field with a lot of ressources available online

Timeline of this course

1. Basics in Deep Learning *(17/09/2025)*
2. Deep Neural Networks *(24/09/2025)*
3. Convolutional Neural Network *(01/10/2025)*
4. Power of depth + Graded lab *(08/10/2025)*
5. Object Recognition *(15/10/2025)*
6. Transformer *(22/10/2025)*
7. GPT (Generative Pre-trained Transformer) *(29/10/2025)*
8. LLM (Large Language Model) + Final exam on all the lectures *(05/11/2025)*

Examination

- Graded lab (grade G1):
  - Do the lab (Python notebook) during the lab hours
  - Prepare a video to present your lab
  - Teamwork (2 students, maximum 3 students to avoid single student)

- Final examination (1 hour - grade G2)
  - written exam
  - Exercises focused on all the lecture topics
  - Questions on the lectures may be included
  - No questions on Python/Pytorch

- Final grade : $0.5 \times G1 + 0.5 \times G2$

Graded lab

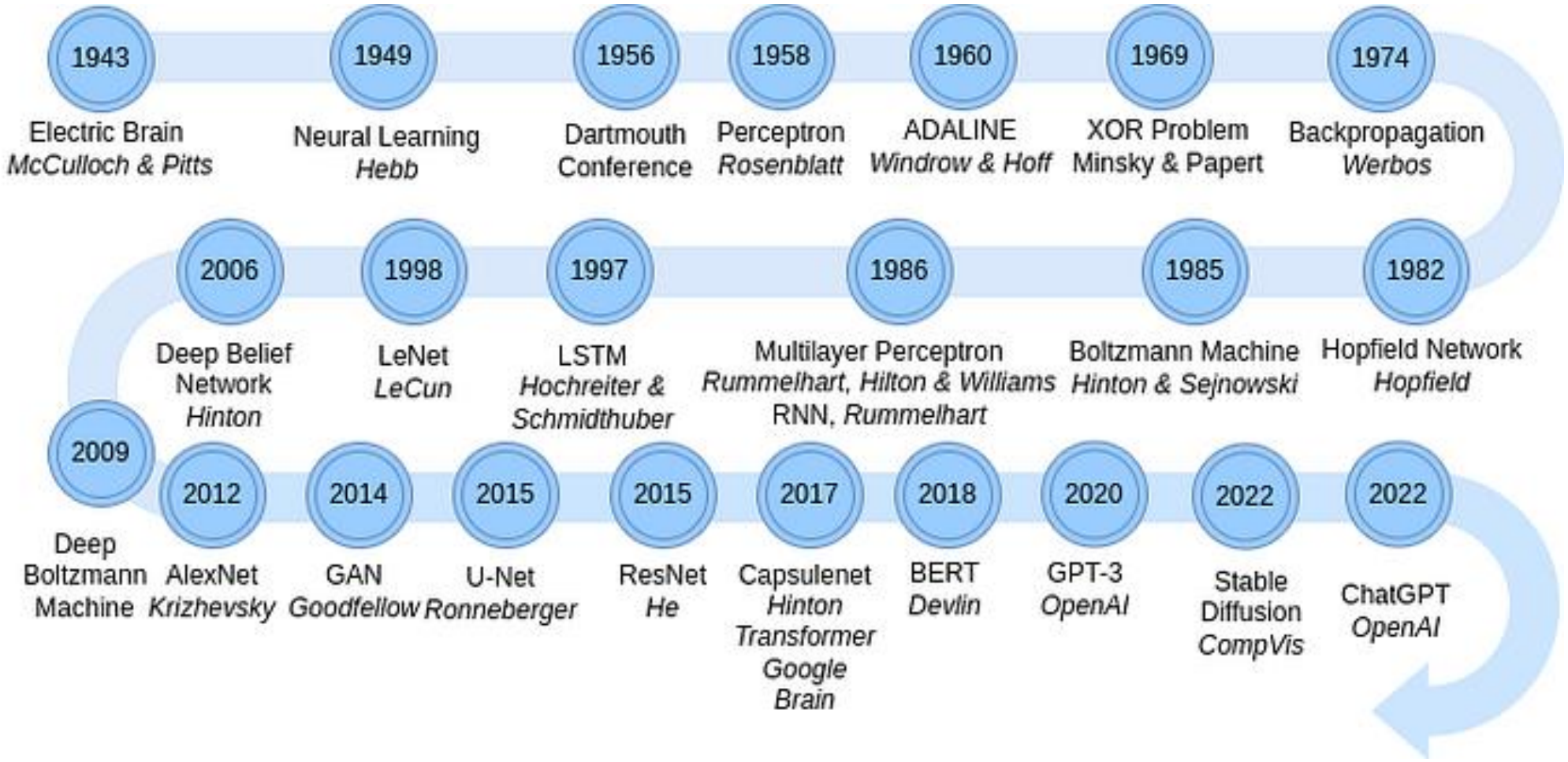
- **For each graded lab, you must post two deliveries**
  - Recorded oral presentation - 10 minutes in video (80% of the score)
  - Notebook in Pytorch (20% of the score, essentially on the legibility of the notebook)

- **Content of the video (slides are strongly recommended)**
  - Short introduction to the lab topic (about 10%)
  - Theoretical analysis of the lab (about 35%)
  - Presentation of your program (about 35%)
  - Discussion about the results (about 20%)
  - **Warning:** The organization of the presentation is free! You can mix everything or separate theory, solution and results.

- For each video, all team members must talk (english or french). You must use a slideshow.
- For each video, record the video and post it somewhere (YouTube, etc.). The URL of the video together with a possible password must be **posted on the course website**.
- The notebook must **run on Google Colab** without any runtime error.

Introduction

Brief Timeline of Neural Networks



https://pub.towardsai.net/a-brief-history-of-neural-nets-472107bc2c9c

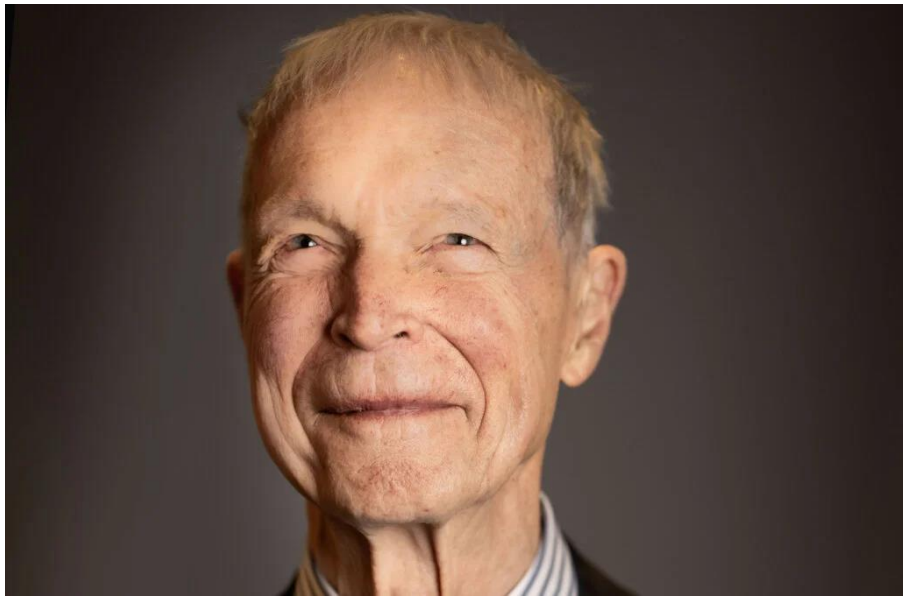
Turing Award for Deep Learning

- The Turing Award, which was introduced in 1966, is often called the Nobel Prize of computing
- Turing Award 2018 given for: “The conceptual and engineering breakthroughs that have made deep neural networks a critical component of computing.”

The Nobel Prize in Physics 2024

John J. Hopfield

“for foundational discoveries and inventions that enable machine learning with artificial neural networks”



Geoffrey Hinton

“for foundational discoveries and inventions that enable machine learning with artificial neural networks”

Why do we need Deep Learning?

- Computer vision
- Natural speech recognition
- Self-driving car
- And many other applications

ImageNet Challenge

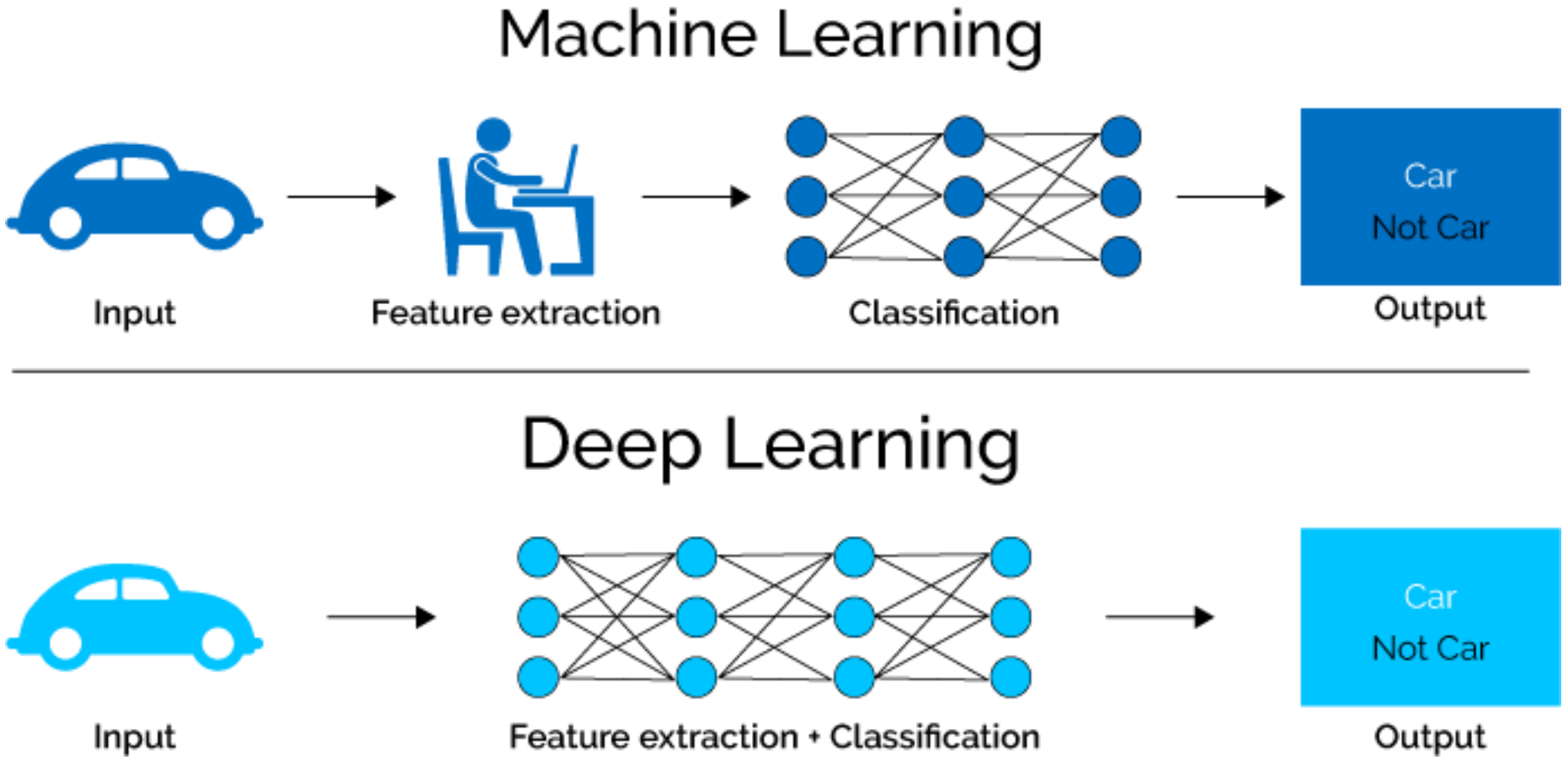
IMAGENET

- 1,000 object classes (categories).
- Images:
  - 1.2 M train
  - 100k test.

Some Cold Water

- Problem: why ? How can we trust a blackbox?

What is Deep Learning



https://www.xenonstack.com/blog/log-analytics-deep-machine-learning/

Why Deep Learning Now?

- Five decades of research in machine learning
- CPUs/GPUs/storage developed for other purposes
- Lots of data from “the internet”
- Tools and culture of collaborative and reproducible science
- Resources and efforts from large corporations

Keras

Caffe

TensorFlow

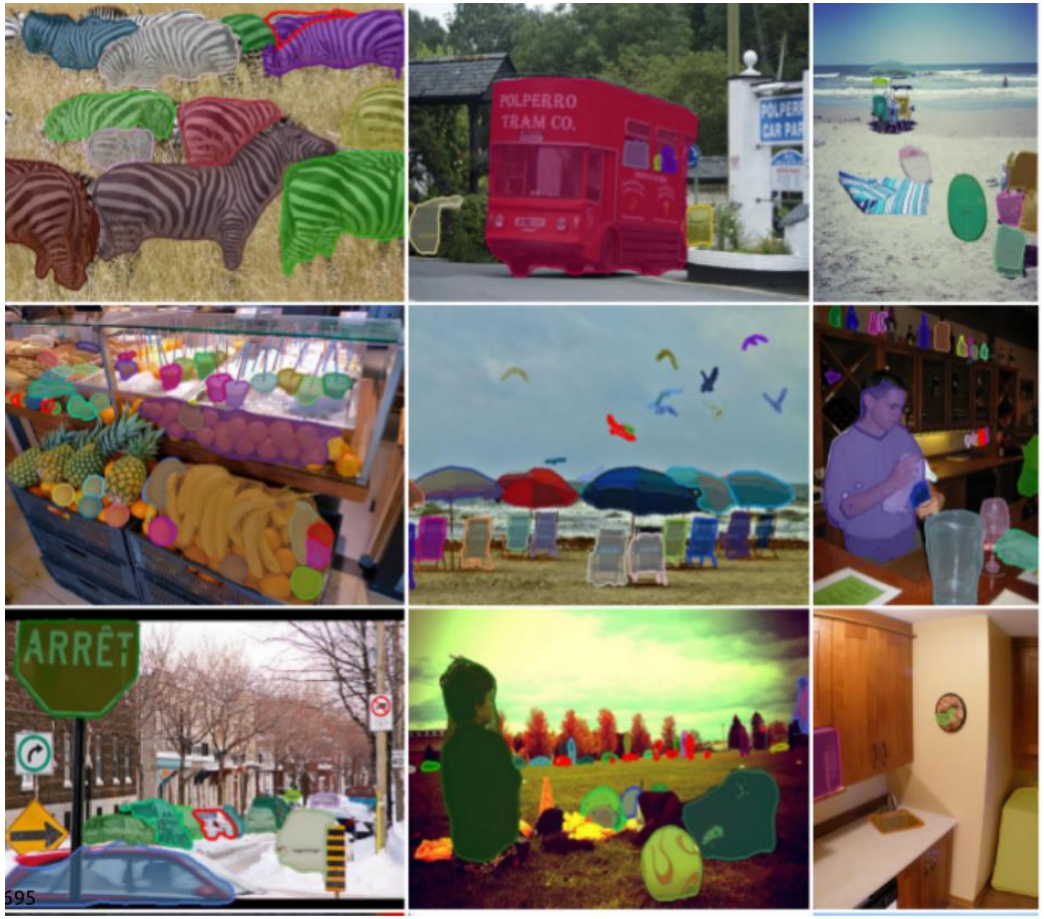
PyTorch

IMAGENET

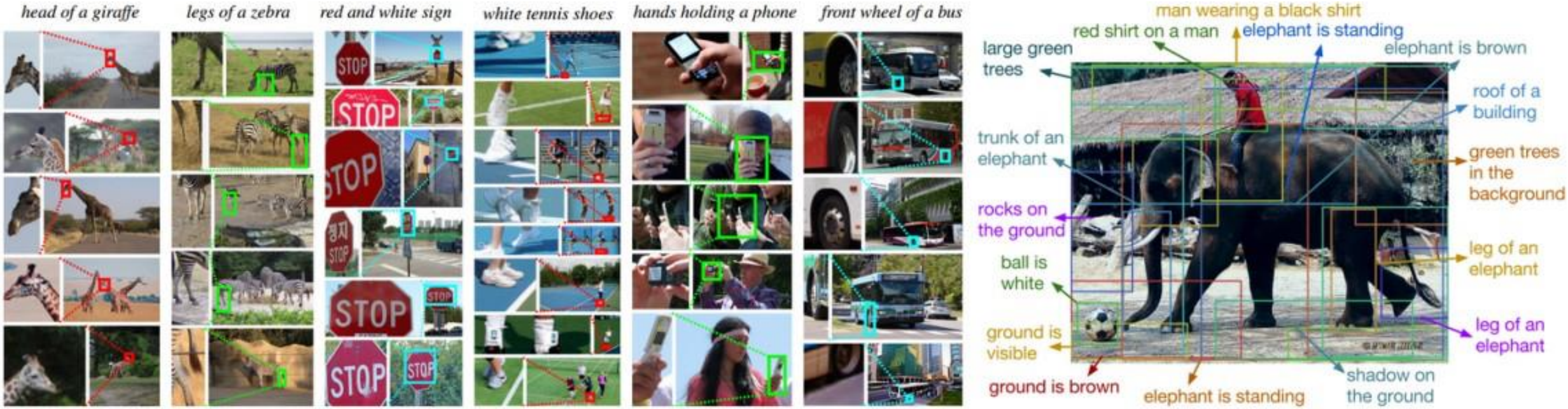
Some succesful applications

Vision

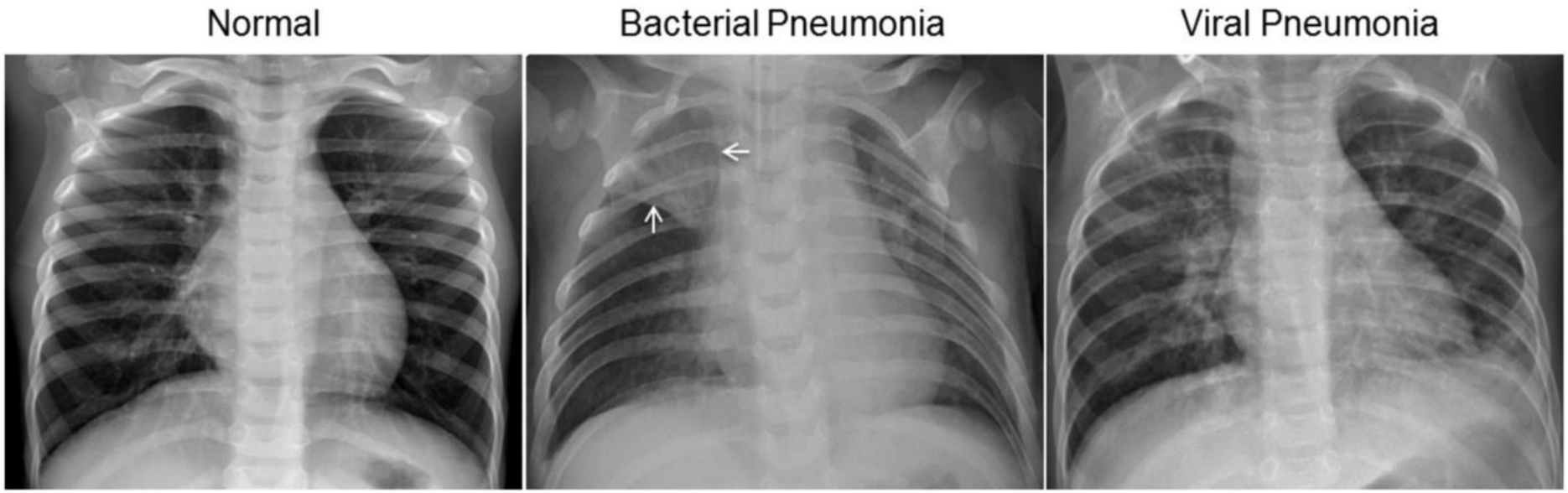
- Object detection and segmentation



Describing photos



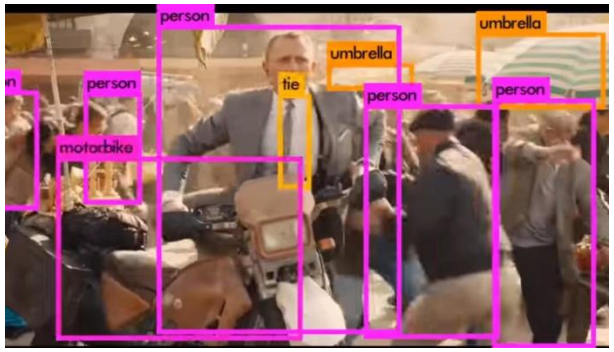
Detecting Pneumonia with Deep Learning



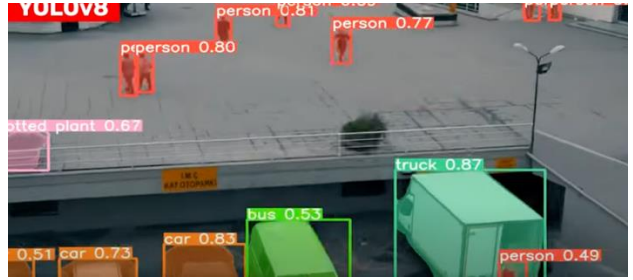
https://www.kaggle.com/paultimothymooney/chest-xray-pneumonia

Video Analysis in Real Time

- https://www.youtube.com/watch?v=VOC3huqHrss



- https://www.youtube.com/watch?v=QgF5PHDCwHw



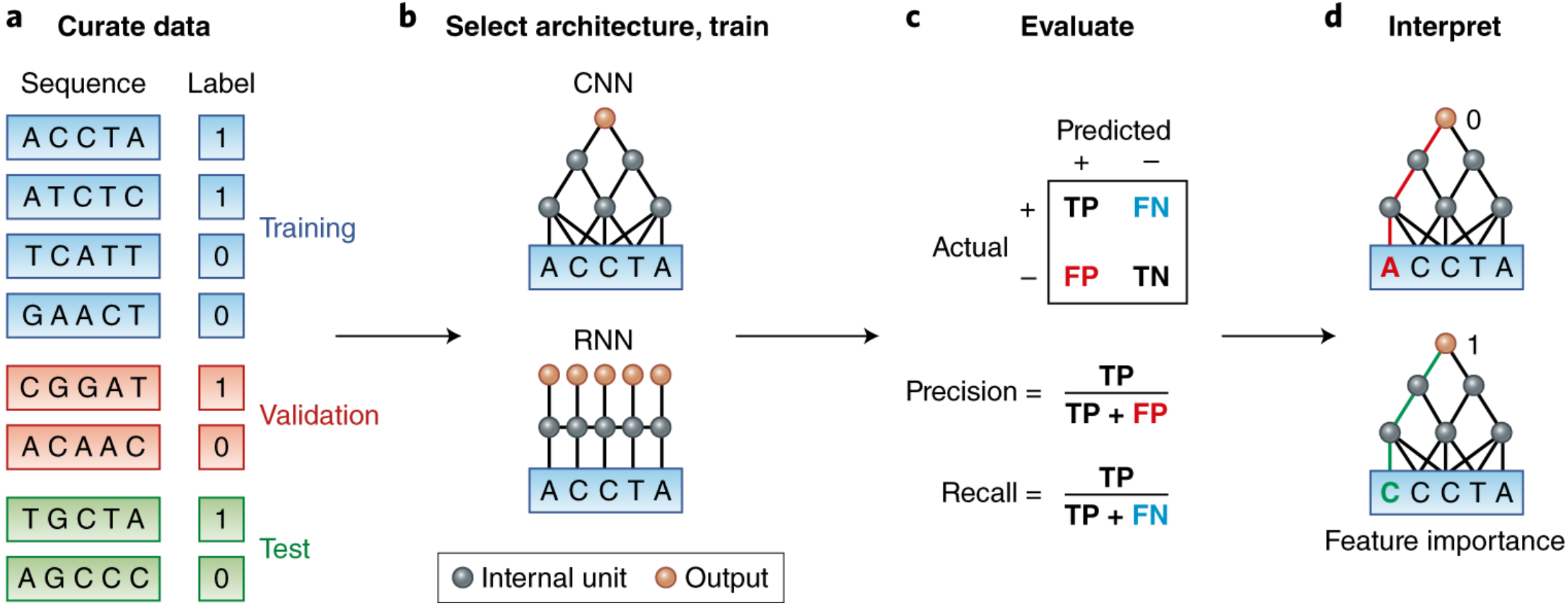
- https://www.youtube.com/watch?v=PYbT_TlZpAI

NLP, question answering

[Google Inbox Smart Reply]

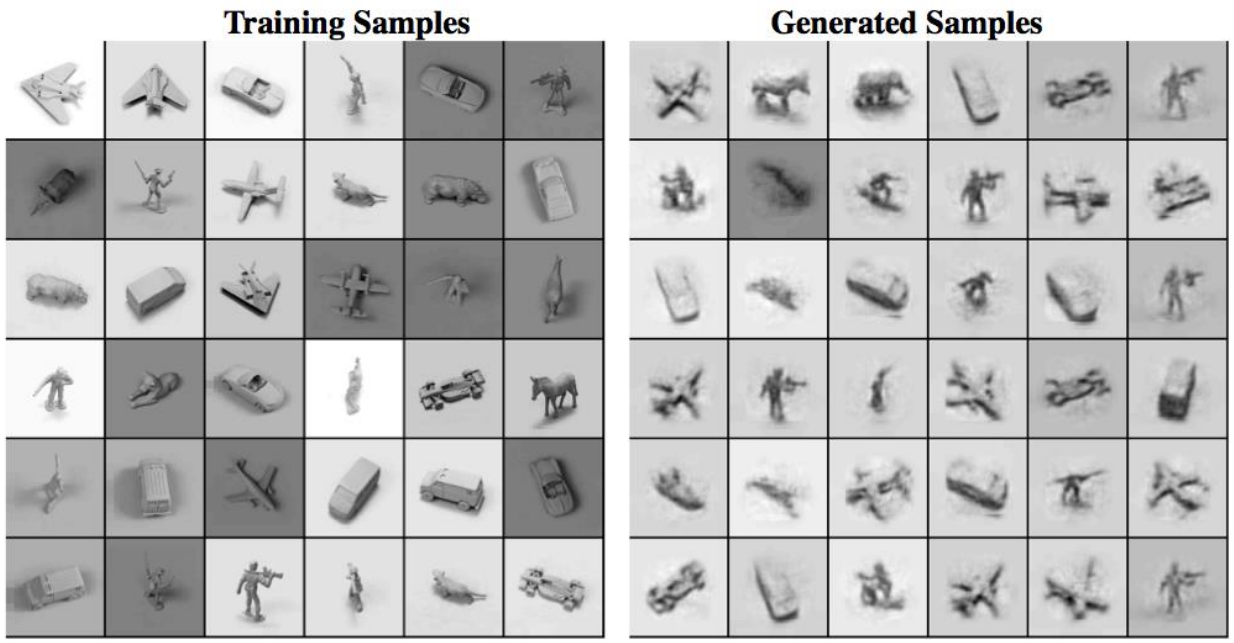
[Amazon Echo / Alexa]

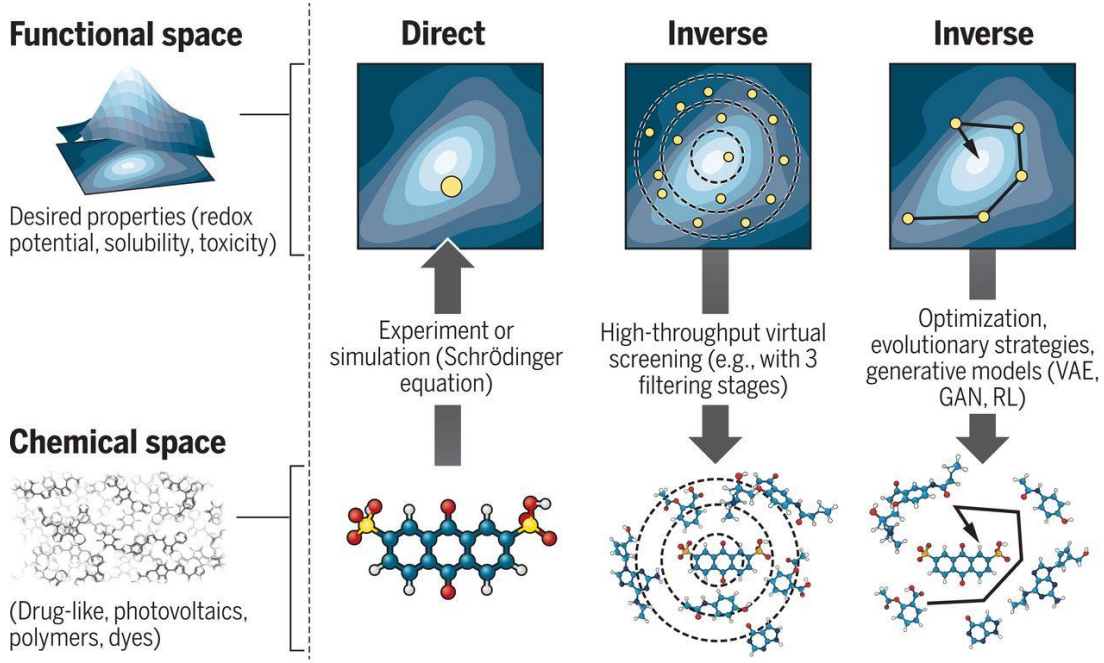
A primer on deep learning in genomics



Zou, J., Huss, M., Abid, A. *et al.* A primer on deep learning in genomics. *Nat Genet* **51,** 12–18 (2019).

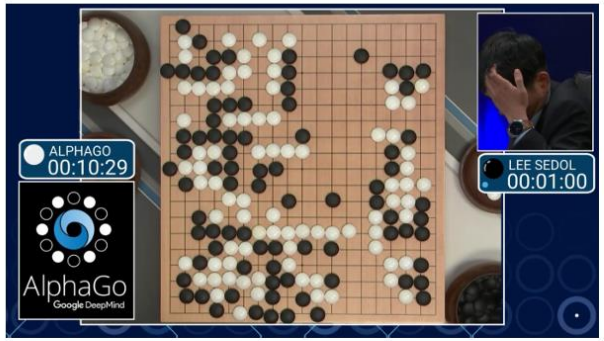
Generative models

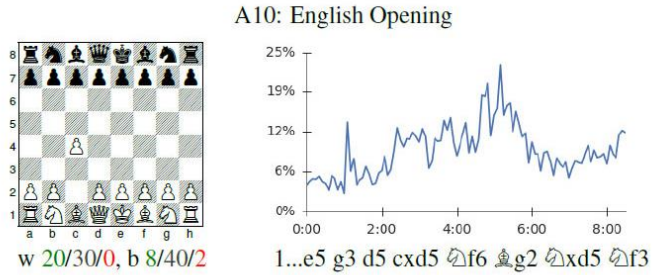




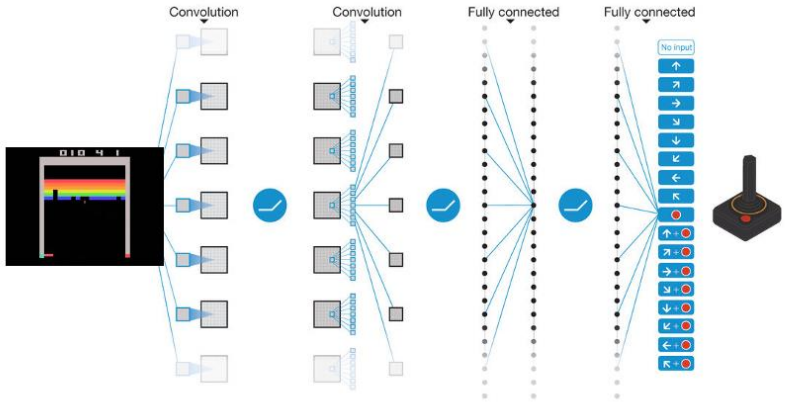
Sanchez-Lengeling, Benjamin and Aspuru-Guzik, Alán.
Inverse molecular design using machine learning: Generative models for matter engineering, Science, 2018

Reinforcement Learning in Games

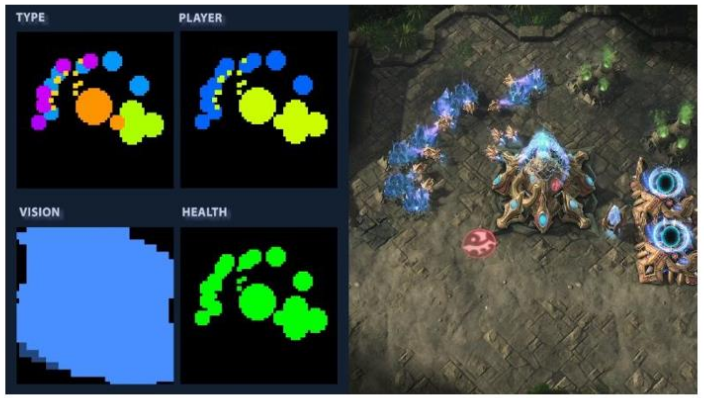




[Deepmind AlphaGo / Zero 2017]

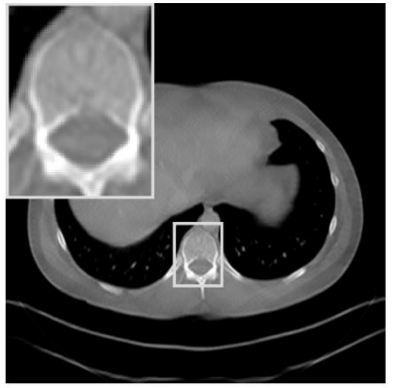


[Atari Games - DeepMind 2016]



[Starcraft 2 for AI research]

Impact of Deep Learning on Mathematics



- **Inverse Problems**
  - Image denoising (Burger, Schuler, Harmeling; 2012)
  - Superresolution (Klatzer, Soukup, Kobler, Hammernik, Pock; 2017)
  - Limited-angle tomography (Bubba, Kutyniok, Lassas, März, Samek, Siltanen, Srinivasan; 2018)
  - Edge detection (Andrade-Loarca, Kutyniok, Öktem, Petersen; 2019)

- **Numerical Analysis of Partial Differential Equations**
  - Schrödinger equation (Rupp, Tkatchenko, Müller, von Lilienfeld; 2012)
  - Black-Scholes PDEs (Grohs, Hornung, Jentzen,von Wurstemberger; 2018)
  - Parametric PDEs (Schwab, Zech; 2018)

$$\frac{d\Psi(x)}{dx} = f(x, \Psi)$$

- **Modelling**
  - Learning equations from data (Sahoo, Lampert, Martius; 2018)

$$f(x_1, x_2) = \frac{\sin(\pi x_1)}{(x_2^2 + 1)}$$

Basics in learning theory

Classification and Regression

- There exists a couple of random variables (X, Y)
  - We know $\mathcal{X}$ and $\mathcal{Y}$ such that $\mathbb{P}(X \in \mathcal{X}) = \mathbb{P}(Y \in \mathcal{Y}) = 1$
  - We observe a realization (x, y) of (X, Y)
  - We want to construct a function $f: \mathcal{X} \to \mathcal{Y}$ to predict y from x

- We choose a loss function $\ell: \mathcal{Y} \times \mathcal{Y} \to [0, +\infty]$ (see next slides)

- We want to find f^* such that

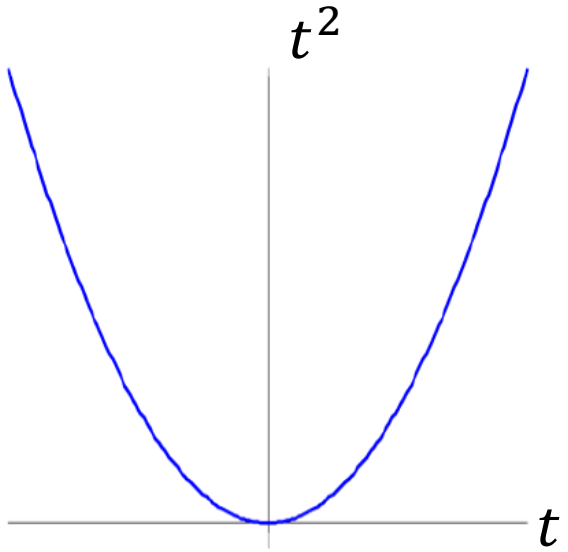
$$f^* \in \operatorname*{argmin}_f \mathbb{E}\big(\ell(f(X), Y)\big)$$

- $\mathbb{E}\big(\ell(f(X), Y)\big)$ is called the risk of f

- Remark: $\mathcal{X}$ is typically $\mathbb{R}^d$, $\mathcal{Y}$ is described in the next slides

Regression

- The set $\mathcal{Y}$ is not-finite
  - Univariate case: $\mathcal{Y} = \mathbb{R}$
  - Multivariate case: $\mathcal{Y} = \mathbb{R}^K$



- In deep-learning, we construct K functions

$$f_k : \mathcal{X} \to \mathbb{R}, \qquad k = 1, \cdots, K$$

such that the output is the vector

$$\hat{y} = f(x) = \left(f_1(x), \cdots, f_K(x)\right)$$

- There are many loss functions, including the famous Euclidean loss function

$$\ell(\hat{y}, y) = \|\hat{y} - y\|^2$$

Classification

- The set $\mathcal{Y}$ is finite
  - Binary case: $\mathcal{Y} = \{-1, +1\}$
  - Multiclass: $\mathcal{Y} = \{1, \cdots, K\}$, $K > 2$

- In deep-learning, for $K \geq 2$, we construct K functions

$$f_k : \mathcal{X} \to [0,1], \qquad k = 1, \cdots, K$$

such that $f(x) = \big(f_1(x), \cdots, f_K(x)\big)$ and $\sum_{k=1}^{K} f_k(x) = 1$

- Output:
  - The « raw » output is $\hat{y} \in \underset{k=1,\cdots,K}{\operatorname{argmin}} f_k(x)$
  - The « soft » output is given by $\hat{y} = \text{softmax}(f(x))$

- There are many loss functions, including the famous 0-1 loss function

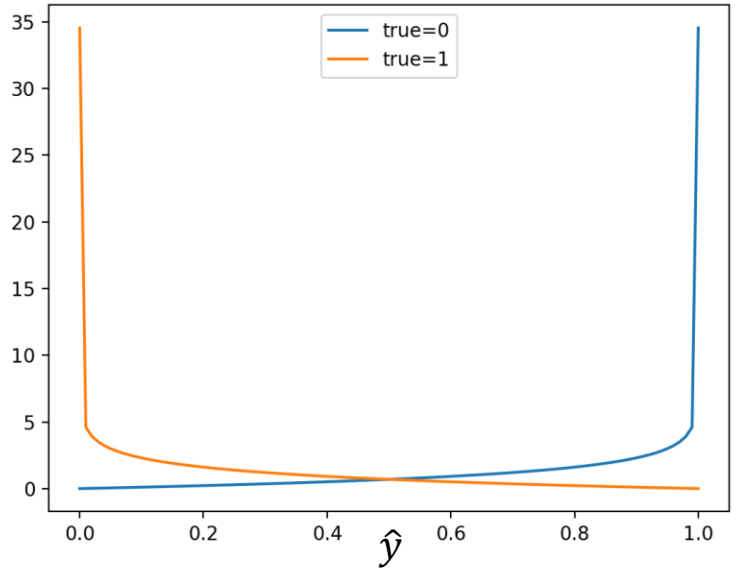
$$\ell(\hat{y}, y) = 1_{\{\hat{y} \neq y\}} = \begin{cases} 1 \text{ if } & \hat{y} \neq y \\ 0 \text{ otherwise} \end{cases}$$

Usual Loss Function for Classification

- Cross-entropy for binary outputs $\hat{y}$ and y
  - To measure a gap between two **binary** values $\hat{y}, y \in \{0,1\}$

$$\ell(\hat{y}, y) = -y \ln \hat{y} - (1 - y) \ln(1 - \hat{y})$$

  - Interpretation:
    - If $\hat{y} = y \in \{0,1\}$, then $\ell(\hat{y}, y) = 0$
    - If $\hat{y} \neq y \in \{0,1\}$, then $\ell(\hat{y}, y) = +\infty$ (in practice, it is a large value)



Binary cross-entropy
(continuous function of $\hat{y}$)

- Cross-entropy for discrete outputs
  - To measure a gap between two **discrete** values $\hat{y}, y \in \{1, \dots, K\}$
  - One-hot encoding: use a binary word of size K such that all bits are '0' except one '1'
  - Example: $\{1,2,3,4\} \Rightarrow \{1000,0100,0010,0001\}$
  - Abuse of notation: the encoded vector of y is the vector $(y[1], y[2], \dots, y[K])$

$$\ell(\hat{y}, y) = -\sum_{j=1}^{K} (\hat{y}[j] \ln y[j] + (1 - \hat{y}[j]) \ln(1 - y[j]))$$

Bayes Risk and Bayes Decision Rule

Theorem: Under certain weak assumptions on the distribution of (X, Y), the function

$$f^b(x) = \underset{y \in \mathcal{Y}}{\operatorname{argmin}}\, \mathbb{E}(\ell(y, Y)|X = x)$$

minimizes the risk:

$$\forall f: \mathcal{X} \to \mathcal{Y}, \qquad \mathbb{E}\left(\ell\left(f^b(X), Y\right)\right) \leq \mathbb{E}\left(\ell(f(X), Y)\right)$$

The function f^b is called the Bayesian decision rule.

- Comments:
  - We may have $\mathbb{E}\left(\ell\left(f^b(X), Y\right)\right) > 0$ (not all the decisions are perfect, i.e., $\mathbb{P}(Y = a|X = x) < 1$ for all a and some x)
  - In practice, we can not calculate the Bayesian decision rule
    - We do not know the distribution of (X, Y)
    - We must represent f^b with a computer

Example of Bayes decision rule

- Classification: binary case $\mathcal{Y} = \{-1, +1\}$ with 0-1 loss function

- Given the observation x, for any value y, we get

$$\mathbb{E}(\ell(y, Y)|X = x) = \ell(y, -1)\mathbb{P}(Y = -1|X = x) + \ell(y, 1)\mathbb{P}(Y = 1|X = x)$$
$$= \begin{cases} \mathbb{P}(Y = -1|X = x) & \text{if } y = 1 \\ \mathbb{P}(Y = 1|X = x) & \text{if } y = -1 \end{cases}$$

- Hence, $f^b(x) = \underset{y \in \mathcal{Y}}{\operatorname{argmin}}\, \mathbb{E}(\ell(y, Y)|X = x)$ is rewritten as

$$f^b(x) = \begin{cases} -1 & \text{if } \mathbb{P}(Y = -1|X = x) \geq \mathbb{P}(Y = 1|X = x) \\ 1 & \text{if } \mathbb{P}(Y = -1|X = x) < \mathbb{P}(Y = 1|X = x) \end{cases}$$

- This is the famous MAP (Maximum A posteriori) rule

Learning in Classification and Regression

- We observe some samples $\mathcal{D} = (x_i, y_i)_{i=1,\cdots,N}$ following the distribution of (X, Y)
  - We are assuming that the samples are independent (i.i.d. assumption)

- We are considering a family $\mathcal{F}$ of functions f_θ parameterized by θ
  - The parameter θ generally belongs to an Euclidean space Θ (e.g., $\theta \in \mathbb{R}^n$)

- We are expecting to solve

$$\theta^* \in \underset{\theta \in \Theta}{\operatorname{argmin}} \, \mathbb{E}\big(\ell(f_\theta(X), Y)\big)$$

but, in practice, a naïve approach consists in minimizing the empirical risk

$$\hat{\theta} = \hat{\theta}(\mathcal{D}) \in \underset{\theta \in \Theta}{\operatorname{argmin}} \frac{1}{N} \sum_{i=1}^{N} \ell(f_\theta(x_i), y_i)$$

Main issues

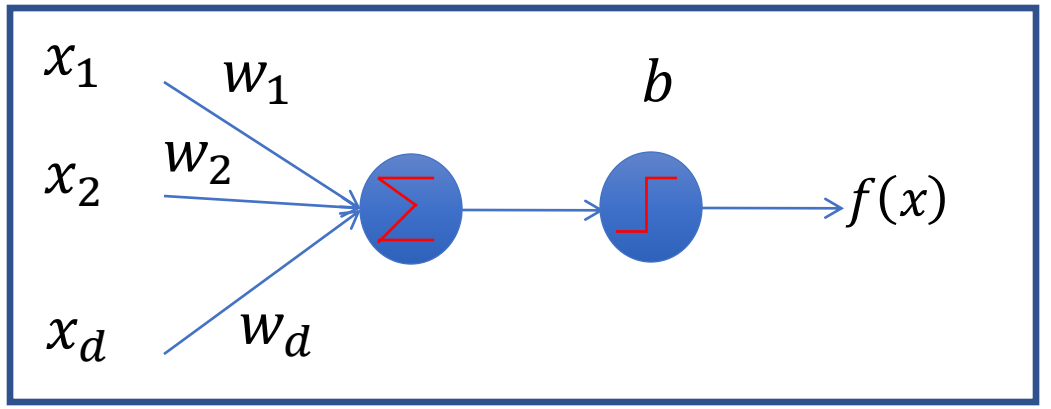
- Optimization
  - The optimization problem can be hard to solve (strong nonlinearity for example)
  - The number of samples is huge (N is huge)
  - The dimension of the parameters is huge (θ has many components)

- Generalization error: is $\hat{\theta} = \hat{\theta}(\mathcal{D})$ close to θ^*?
  - Possible meaning: $\mathbb{E}\left(\ell\left(f_{\hat{\theta}}(X), Y\right)\right) \approx \mathbb{E}\left(\ell(f_{\theta^*}(X), Y)\right)$
  - An other possible meaning: $\sup_{\theta \in \Theta} \left|\mathbb{E}\left(\ell(f_\theta(X), Y)\right) - \frac{1}{N}\sum_{i=1}^{N} \ell(f_\theta(x_i), y_i)\right|$ is bounded?

- Approximation error: is f_{θ^*} close to f^b?
  - Possible meaning: $\mathbb{E}\left(\ell(f_{\theta^*}(X), Y)\right) \approx \mathbb{E}\left(\ell\left(f^b(X), Y\right)\right)$

Basics in deep neural networks

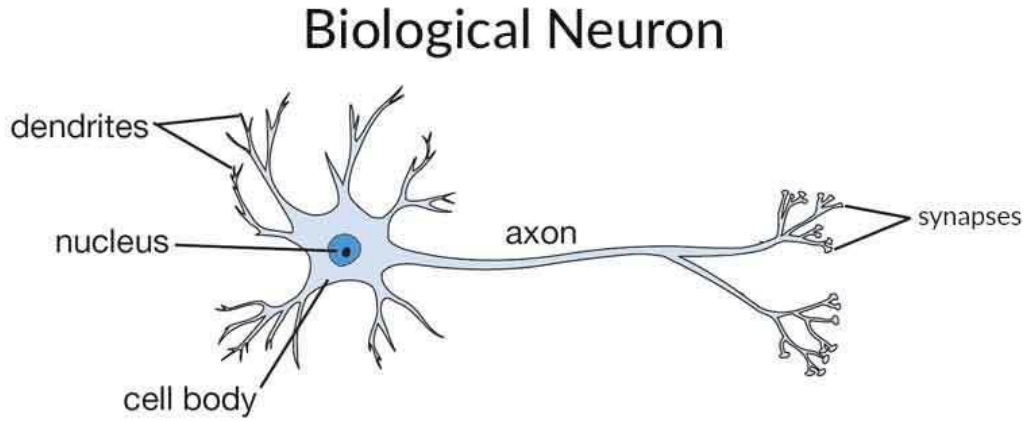
Perceptron (Neuron Metaphor)



- The perceptron (Rosenblatt, 1957) is:

$$f(x) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{n} w_i x_i + b = w^T x + b \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- However inputs are real values and the weights can be different
- This model was originally motivated by biology, with w_i being synaptic weights, x_i the input firing rates and $f(x)$ the output firing rate.
- This is a cartoonish biological model.



F. Rosenblatt, The Perceptron: A Probabilistic Model for Information Storage and Organization in the Brain; 1957

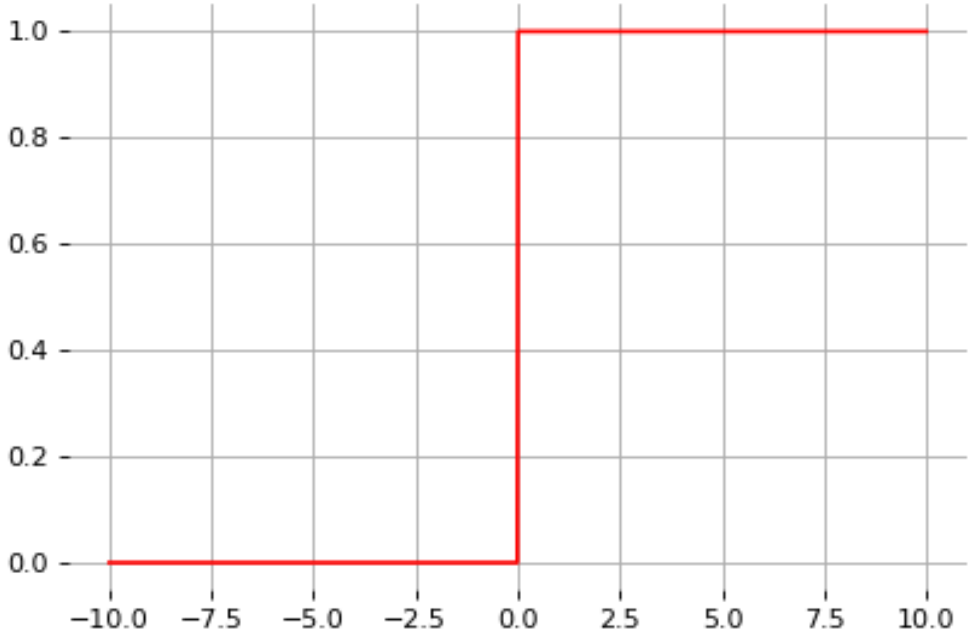
Heavyside Activation function

- Let us define the activation function:

$$\sigma(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Then, the perceptron classification rule can be rewritten as

$$f(x) = \sigma(w^T x + b)$$

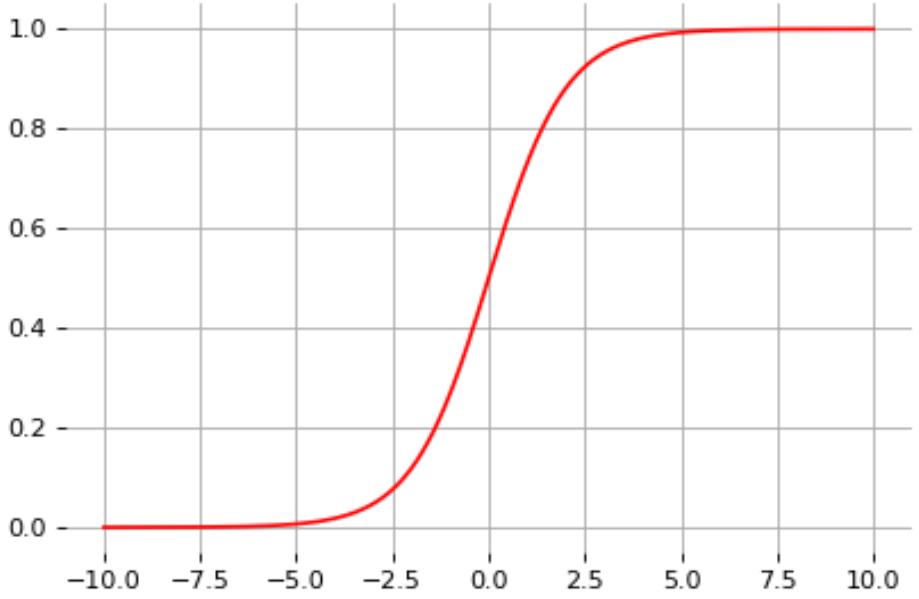


Sigmoid Activation Function

- Note that the sigmoid function

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

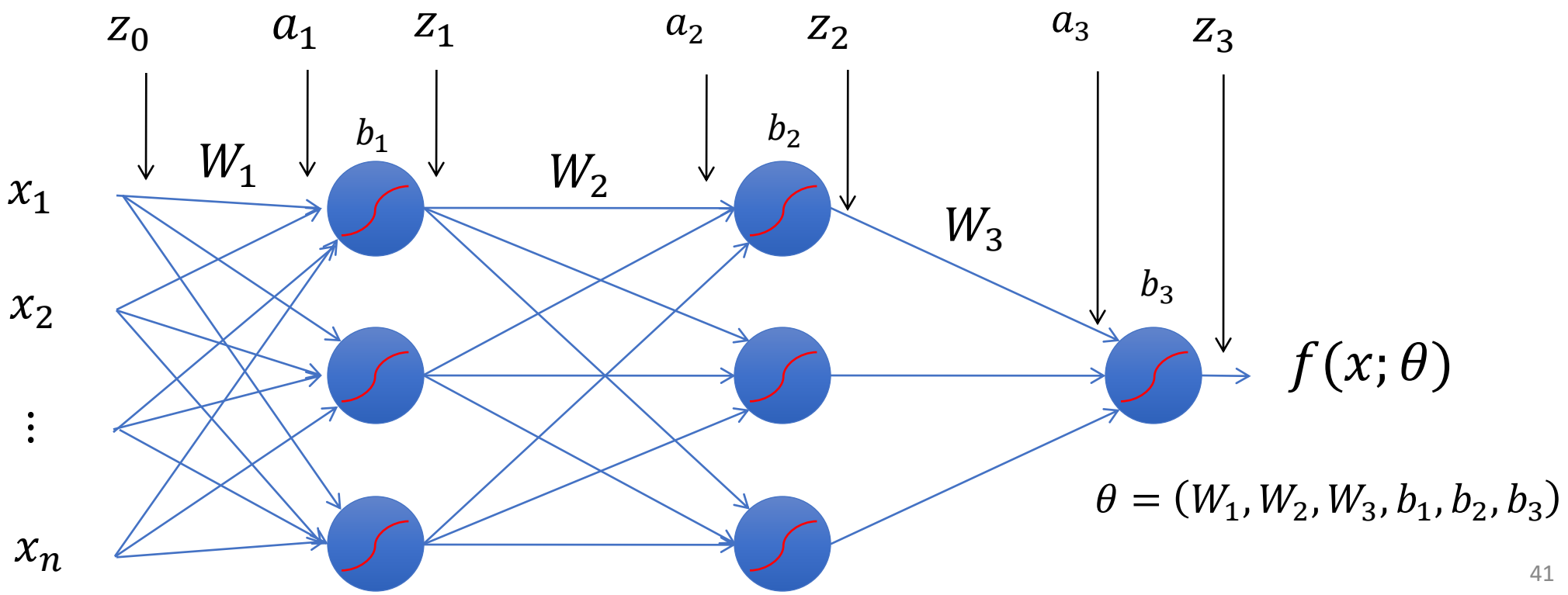
looks like a soft heavyside:



- Therefore, the overall model $f(x) = \sigma(w^T x + b)$ is very similar to the perceptron but it is differentiable with respect to w and b

Feed-Forward Networks and All Its Variables

- Predictions are fed forward through the network to classify:
  - $z_0 = x$
  - $a_k = W_k z_{k-1} + b_k,\ k = 1, 2, 3$
  - $z_k = \sigma(a_k),\ k = 1, 2, 3$
  - $f(x) = f(x; \theta) = z_3$



Terminology

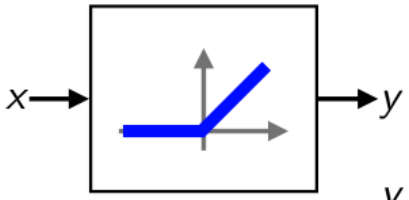
- A feedforward net is a nonlinear function composed of repeated affine transformation followed by a nonlinear action:

$$\begin{cases} f_0(x) = x \\ f_h(x) = \sigma_h(W_h f_{h-1}(x) + b_h), & 1 \leq h \leq H-1 \\ \hat{y} = f(x) = f_H(x) = \sigma_H(W_H f_{H-1}(x) + b_H). \end{cases}$$

- We say that the netwoks contains H layers and $H-1$ hidden layers
- The size of the layers, $n_0, n_1, \ldots, n_H$, corresponds to the number of neurons
- The weights of layer h are given by matrix $W_h \in \mathbb{R}^{n_h \times n_{h-1}}$
- The biases of layer h are given by $b_h \in \mathbb{R}^{n_h}$
- The nonlinear activation function of layer h is $\sigma_h(\quad)$

Element-wise activation functions

- Each neuron/unit is followed by a dedicated activation function
- Historically the sigmoid and tanh have been the most popular
- Many other activation functions available



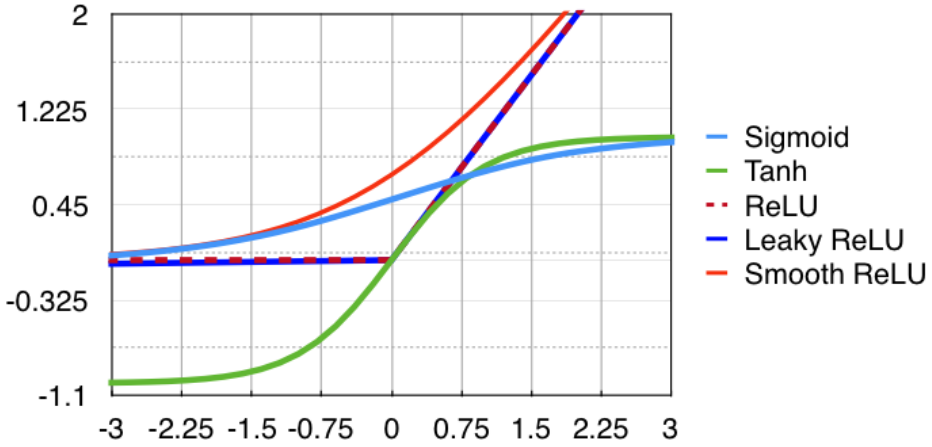
$y = \frac{1}{1+e^{-x}}$ sigmoid

$y = \tanh(x)$ hyperb. tan

$y = \max\{0, x\}$ ReLU

$y = \log(1+e^{x})$ Soft ReLU

$y = \epsilon x + (1-\epsilon)\max\{0, x\}$ Leaky ReLU



One hidden layer network

- We can already use this to perform logistic regression.
- For model $\hat{y}(x; w, b) = \Pr(Y = 1|x) = \sigma(w^T x + b)$, find (w, b) that maximizes the likelihood of the dataset $D = \{(x_i, y_i)\}$:

$$\underset{w,b}{\operatorname{argmax}} \Pr(D|w, b) =$$

$$\underset{w,b}{\operatorname{argmin}} \sum_{(x_i,y_i)\in D} -y_i \log \sigma(w^T x_i + b) - (1 - y_i) \log\big(1 - \sigma(w^T x_i + b)\big)$$

- More general notations, we minimize

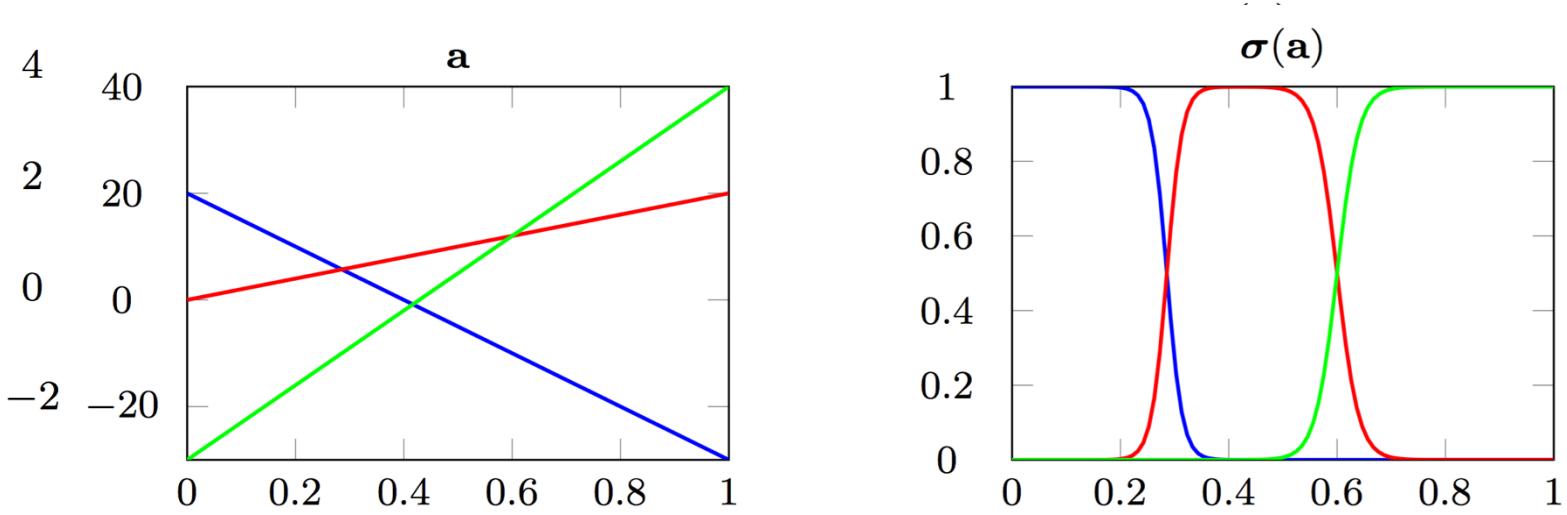
$$\mathcal{L}(W, b) = \sum_{(x_i,y_i)\in D} \ell\big(y_i, \hat{y}_i(x_i; w, b)\big)$$

Softmax for classification

- How to transform the vector output of a neural network into a probability distribution?
- The softmax function, σ: $\mathbb{R}^K \rightarrow \mathbb{R}^K$, generalizes the sigmoid function and yields a vector of K values in $[0,1]$ by exponentiating and then normalizing

$$\sigma(a) = \text{softmax}(a) = \frac{1}{\sum_{j=1}^{K} e^{a_j}} (e^{a_1}, \cdots, e^{a_K})$$

- As activation values increase softmax tends to focus on the maximum



Pytorch implementation (one-hidden layer)

This implementation defines the model as a custom Module subclass.

```
class TwoLayerNet(torch.nn.Module):

    def __init__(self, D_in, H, D_out):
        super(TwoLayerNet, self).__init__()
        self.linear1 = torch.nn.Linear(D_in, H)
        self.linear2 = torch.nn.Linear(H, D_out)

    def forward(self, x):
        h_relu = self.linear1(x).clamp(min=0)
        y_pred = self.linear2(h_relu)
        return y_pred

twolayer_net = TwoLayerNet(2, 20, 1)
```

Inheriting from nn.Module

In the constructor, we instantiate two nn.Linear modules and assign them as member variables.

In the forward function we accept a Tensor of input data and we must return a Tensor of output data.

We can use Modules defined in the constructor as well as arbitrary operators on Tensors.

Instantiates the class defined above

IT Tools

Useful Information Technology Tools

- Python and Jupyter notebook

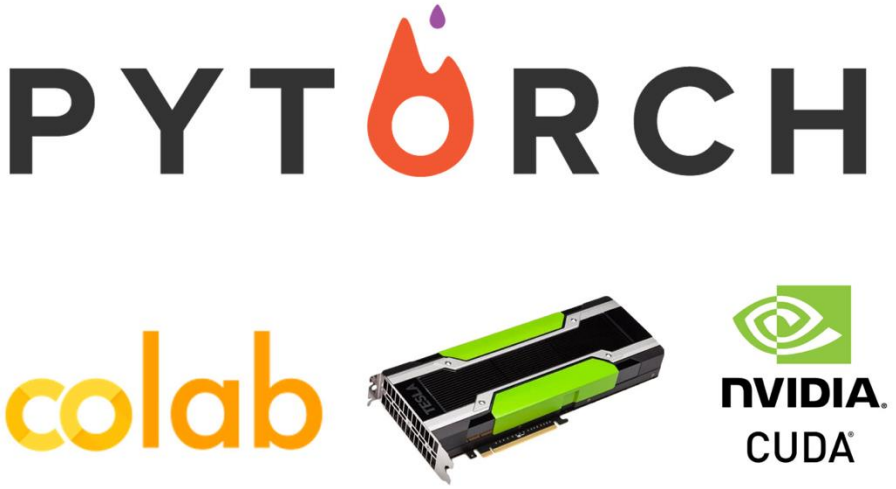
https://www.anaconda.com/download/

- Pytorch

https://pytorch.org/

- Access to a GPU

Google Colab



- Google Colab is a free Jupyter notebook environment that supports both CPU and GPU usage for free.
- It is extremely convenient to share notebooks and also it is good for self-study.
- Some key features
  - It is a Linux-like directory system
  - The current directory is « /content »
  - You can upload data files
  - Python is already installed; missing libraries can be installed

Useful commands : https://www.analyticsvidhya.com/blog/2020/03/google-colab-machine-learning-deep-learning/

Deep Learning Frameworks

- TensorFlow
  - TensorFlow was created by Google and is one of the most popular deep learning frameworks.

- Keras
  - Keras is another open source deep learning framework that is widely used.
  - Keras is remarkably simple to use

- PyTorch (used in this course)
  - Open source, primarily developed by Facebook, and is known for its simplicity, flexibility, and customizability

- Microsoft Cognitive Toolkit
  - Microsoft Cognitive Toolkit is an open source deep learning framework to train deep learning models.

Why do we need Deep Learning frameworks?

- Speed:
  - Fast GPU/CPU implementation of matrix multiplication, convolutions and backpropagation
- Automatic differentiations:
  - Pre-implementation of the most common functions and it's gradients.
- Reuse:
  - Easy to reuse other people's models
- Less error prone:
  - The more code you write yourself, the more errors

Why Pytorch?

- Python API
- Can use CPU, GPU
- Supports common platforms:
  - Windows, Mac, Linux
- Pytorch is a thin framework which lets you work closely with programming the neural network
- Focus on the machine learn part not the framework itself
- Pythonic control flow
  - Flexible
  - Cleaner and more intuitive code
  - Easy to debug
- Python debugger
  - With Pytorch we can use the python debugger

Pytorch API (just the main packages)

Package	Description
torch	The torch package contains data structures for multi-dimensional tensors and mathematical operations over these are defined.
torch.Tensor	A torch.tensor is a multi-dimensional matrix containing elements of a single data type.
torch.nn	A package that contains modules and extensible classes for building neural networks
torch.autograd	A package that provides classes and functions implementing automatic differentiation of arbitrary scalar valued functions
torch.nn.functional	A functionnal interface that contains typical operations used for building neural networks like loss functions, activation functions, and convolution operations
torch.optim	A package that implements various optimization algorithms like SGD and Adam.
torchvision	The package that provides popular datasets, model architectures, and common image transformations for computer vision.

torch.Tensor class

- Pytorch's tensors are similar to NumPy's ndarrays

```
import torch
x = torch.tensor([5.5, 3])
print(x)
```

```
Tensor([5.5000, 3.0000])
```

```
x = torch.rand(5, 3)
print(x)
```

```
tensor([[0.6915, 0.2723, 0.4345],
        [0.2429, 0.4125, 0.5318],
        [0.1012, 0.3451, 0.3916],
        [0.8321, 0.7369, 0.2084],
        [0.3251, 0.5779, 0.3637]])
```

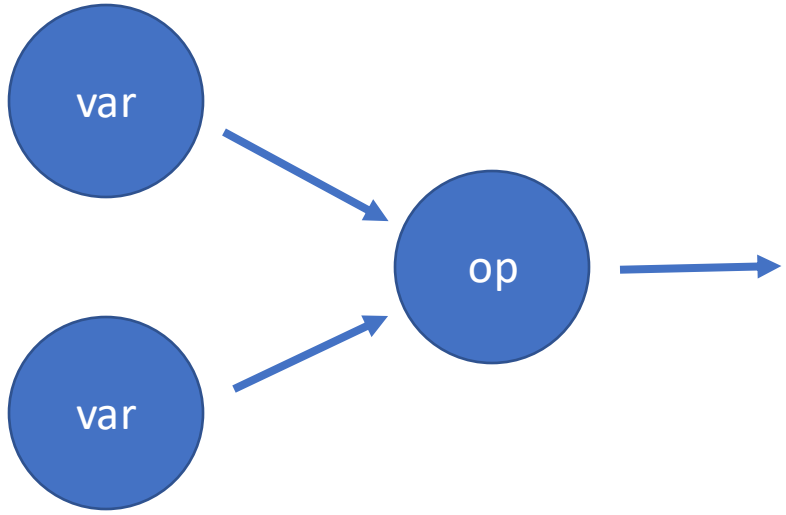
Torch.tensor attributes

Attribute	Data type	Description
data	array_like	list, tuple, NumPy ndarray, scalar
dtype	torch.dtype	The tensor's data type
requires_grad	bool	Should autograd record operation
device	torch.device	Allocated on CPU or CUDA (GPU)

```
torch.tensor(data=[1,2,3], dtype=torch.float32, device='cpu', requires_grad=False)
```

Computational graph

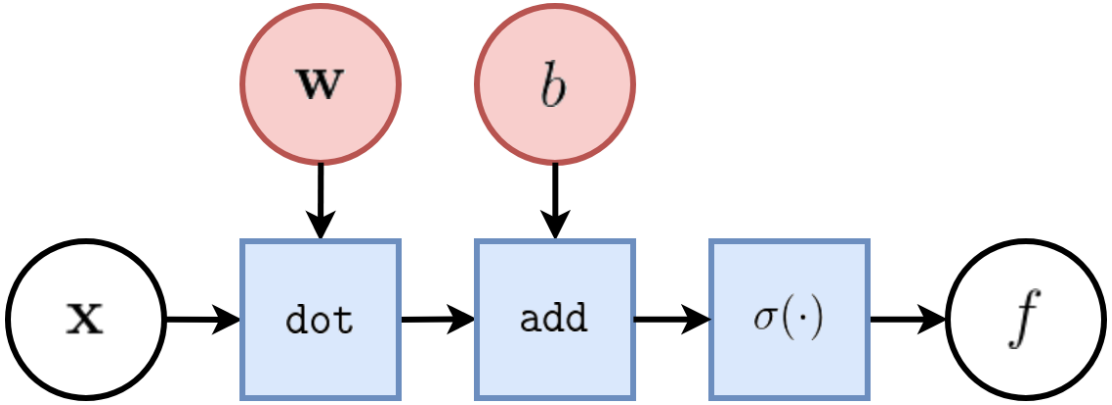
- Computational graphs
  - Help break down large computations into smaller ones by visualizing them as a graph of smaller computations,
  - Help in calculating derivatives of functions,
  - Modeled as Directed Acyclic Graph (DAG)
  - Nodes
    - Variables (tensors in Pytorch)
    - Mathematical operations
  - Edges
    - Feeding input



- Example:
  - $g(x) = wx + b$
  - $f(x) = \max(wx + b, 0) = \sigma(wx + b)$
  - $\ln(x) = 2\,\mathrm{arctanh}\left(\frac{x-1}{x+1}\right) = 2\left(\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right)$

Tensor Operations

- In terms of tensor operations, the computational graph of f can be represented as:

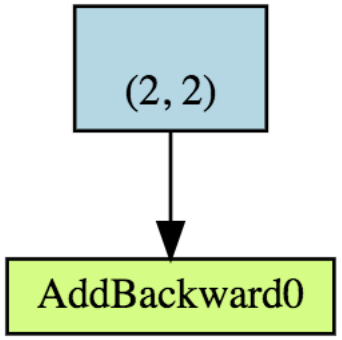


- Where
  - White nodes correspond to inputs and outputs;
  - Red nodes correspond to model parameters;
  - Blue nodes correspond to intermediate operations, which themselves produce intermediate output values (not represented).
  - This unit is the core component all neural networks!

Example in Pytorch

```
import torch
import torchviz

x = torch.ones(2, 2)
x.requires_grad_(True)
y = x + 2
torchviz.make_dot(y)
```



https://github.com/szagoruyko/pytorchviz

Conclusion

Conclusion

- A vast number of applications
- A vast number of topics and architectures
- Deep Learning is based on some main concepts
- What are these concepts?
- It is crucial to be able to implement these concepts, for example in Python with a dedicated framework
- Deep learning is essentially based on deep neural networks
- Gradient descent is a key step for training deep neural networks
- Do not underestimate the study of the generalization error