

Understanding Multiple Linear Regression

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0.1 Multiple linear regression

What is Linear Regression?

Linear regression is a predictive analysis that predicts the value on a dependent variable based on an independent variable or multiple independent variables. The predictive analysis where the dependent variable is predicted from one independent variable is Simple linear regression, and when multiple independent variable are used the it is called 'Multiple linear regression'.

Multiple linear regression

This analysis predicts the value of dependent variable (Y) by using the line of best fit, which can be defined by the equation :

$$i. E(y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + + \beta_n X_n$$

Where $E(y)$ is the predicted value of y.

$X_1, X_2, X_3, \dots, X_n$ are the values of independent variables.

β_0 is the intercept and

$\beta_1, \beta_2, \beta_3$ are the regression coefficients of X_1, X_2, X_3, X_n .

Looking at an example of Multiple linear regression model

```
[1]: # import the required libraries

import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
import pylab as pl
import missingno as msno
import stat
from scipy.stats.stats import pearsonr
import scipy.stats as stats

from sklearn.metrics import mean_squared_error, r2_score

# import train test split for splitting the data in train and test
from sklearn.model_selection import train_test_split
```

```
from sklearn.linear_model import LinearRegression
```

```
[2]: # importing the data
data = "https://raw.githubusercontent.com/Blackman9t/Machine_Learning/master/
↳Original_2000_2014_Fuel_Consumption_Ratings.csv"
df = pd.read_csv(data)
```

```
[3]: #look at the shape of the data
df.shape
```

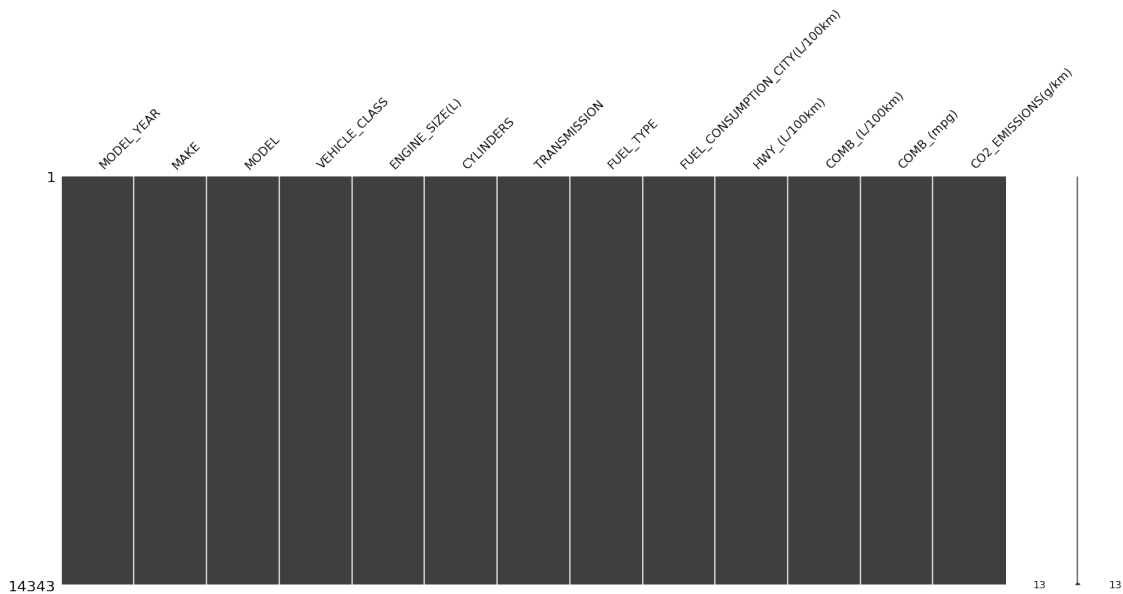
```
[3]: (14343, 13)
```

```
[6]: # see if any data is missing
df.isna().any()
```

```
[6]: MODEL_YEAR                False
MAKE                          False
MODEL                        False
VEHICLE_CLASS                False
ENGINE_SIZE(L)              False
CYLINDERS                   False
TRANSMISSION                False
FUEL_TYPE                   False
FUEL_CONSUMPTION_CITY(L/100km) False
HWY_(L/100km)               False
COMB_(L/100km)              False
COMB_(mpg)                  False
CO2_EMISSIONS(g/km)         False
dtype: bool
```

```
[12]: # Visualize missing data using missingno
msno.matrix(df)
```

```
[12]: <matplotlib.axes._subplots.AxesSubplot at 0x1a1985d0b8>
```



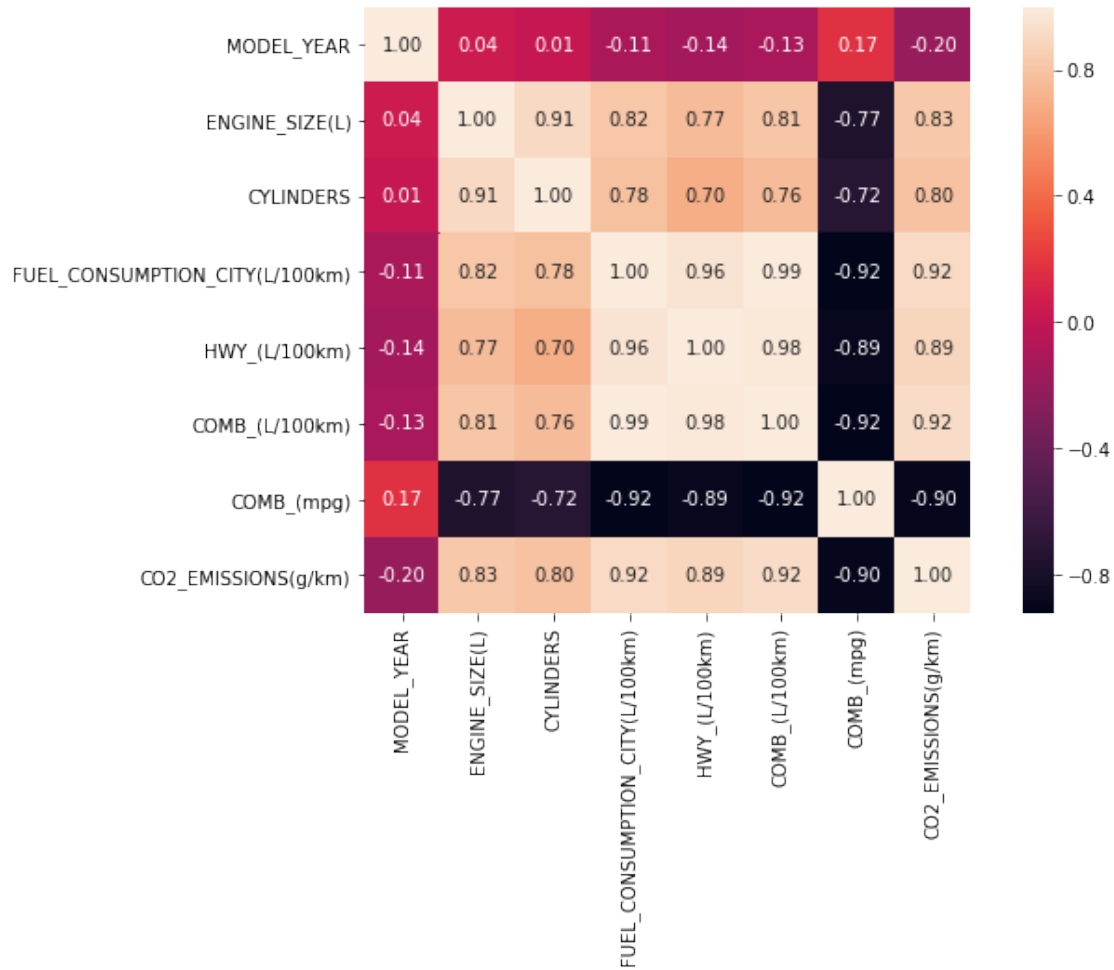
We can see therefore by visualization that no data points are missing.

We want to identify the rate of change emission with respect with other independent variables.
In this case CO2 Emission(g/km) is our dependent variable.

Let us look at the correlation between the dependent and the independent variables.

```
[13]: #outputs a correlation pandas dataframe
corrmat = df.corr()
plt.figure(figsize=(10,6))
# sns.heatmap(corrmat,vmax=1, square=True
sns.heatmap(corrmat, cbar=True, annot=True, square=True, fmt='.2f',
↪annot_kws={'size': 10})
```

```
[13]: <matplotlib.axes._subplots.AxesSubplot at 0x1a197a1e80>
```



From the correlation heatmap we can see the Engine_Size(L), Cylinders, Fuel_consumption in city, highway(HWY_) are positively correlated with CO2 Emission. We can also see the the COMB(MPG) or combined miles per gallon is negatively correlated to the CO2 emission.

For this project we will take our Engine_size, cylinders and COMB_(mpg) as our independent variables.

Why?

We can see that the Fuel consumption in city, HWY_(L100) and comb_(L/100) have a strong correlation between themselves. This is called multi-collinearity. Moderate multicollinearity might not pose a problem, however, like in our case where a colinearity of 0.99 is seen this might cause problems, such as:

- Estimates for regression coefficients to be unreliable
- Multicollinearity saps the statistical power of the analysis can cause the coefficients to switch signs, distorting the correct model.

One can test multi-collinearity with Variance Inflation Factors (VIF) however, we won't delve into that in this paper. I will delve into VIF in another paper in more detail.

Therefore we get a subtle suggestion that the larger the engine size and cylinders the greater CO2 emission, and greater the mileage per gallon for a vehicle the lesser the CO2 emission.

```
[24]: # taking a subset of our data. (Only using independent variables )
X = df[['ENGINE_SIZE(L)', 'CYLINDERS', 'COMB_(mpg)']]
# CO2 emission
y = df[['CO2_EMISSIONS(g/km)']]

[26]: # Separating the dataset into training and test dataset
# Splitting the data into training and testing dataset
X_train, X_test, y_train, y_test = train_test_split(X,y,test_size = 0.25,
→shuffle = True)

[60]: # print(type(df_pred_co2),type(y_test))
df_pred_co2.head(), y_test.head()
```

```
[60]: (
0 0
0 308.451612
1 294.718224
2 209.642996
3 196.662431
4 250.532403, CO2_EMISSIONS(g/km)
9430 299
10334 269
12099 209
9641 196
2238 242)
```

```
[27]: # Display the shape of the training and test data set
print('X train shape is', X_train.shape)
print('X test shape is', X_test.shape)
print('Y train shape is', y_train.shape)
print('y test shape is', y_test.shape)
```

```
X train shape is (10757, 3)
X test shape is (3586, 3)
Y train shape is (10757, 1)
y test shape is (3586, 1)
```

We can use the least squares estimates(LSE) to find the intercepts. The LSE works very well for computing the coefficients of linear regression, whether simple or multiple linear regression. The only drawback to the LSE is if the independent variable are linearly-dependent.

Mathematically we can get the values of β by

$$E(y) = 0+11+22+33+....+$$

However, we are not going into solving the equation algebraically, and will use the scikit's Linear Regression model instead.

```
[28]: # instantiate a linear regressin model
model = LinearRegression()
#Next train the model
model.fit(X_train, y_train)
```

```
[28]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

```
[123]: coefficients_X = model.coef_  
beta_0 = model.intercept_  
print(f"Slope{coefficients_X}, Intercept{beta_0}")  
engine_size = coefficients_X[0][0]  
cylinders = coefficients_X[0][1]  
mpg = coefficients_X[0][2]  
intercept = beta_0[0]
```

```
Slope[[ 7.52822873  6.33490332 -4.98463704]], Intercept[320.75938234]
```

```
[125]:
```

```
[125]: 320.7593823352039
```

Here we can see the coefficients of Engine_Size, cylinders and COMB_mpg is 7.52, 6.33 and -4.98 respectively. Also, the intercept value of β_0 is 320.75

Therefore our equation is:

$$i. E(y) = 320.75 + 7.52 (\text{Engine_size}) + 6.33(\text{Cylinders}) + (-4.98)\text{COMB_mpg}$$

```
[78]: # Using the model to predict values of X_test
```

```
predicted_C02_em = model.predict(X_test)  
y_test_df = y_test.reset_index()
```

```
[116]: pred_co2_df = pd.DataFrame(predicted_C02_em.reshape(1,-1)).T
```

```
[80]: # Combining the actual values and predicted values to
```

```
co2_predicted_test = pd.concat([y_test_df, pd.DataFrame(predicted_C02_em)],  
                                ↪axis = 1)
```

```
[81]: # We can see the values are close to one another. But how accurate is how model?
```

```
co2_predicted_test.head(10)
```

```
[81]:   index  CO2_EMISSIONS(g/km)    0  
0   9430                299  308.451612  
1  10334                269  294.718224  
2  12099                209  209.642996  
3   9641                196  196.662431  
4   2238                242  250.532403  
5   6202                285  265.641693  
6   7140                242  288.903853  
7   1126                315  278.934579  
8   4942                315  306.660843  
9   7025                193  194.689085
```

```
[82]: accuracy = model.score(X_test, y_test)
```

```
[84]: round(accuracy,2)
```

```
[84]: 0.86
```

We can see our model is 86% confident with our predictions.

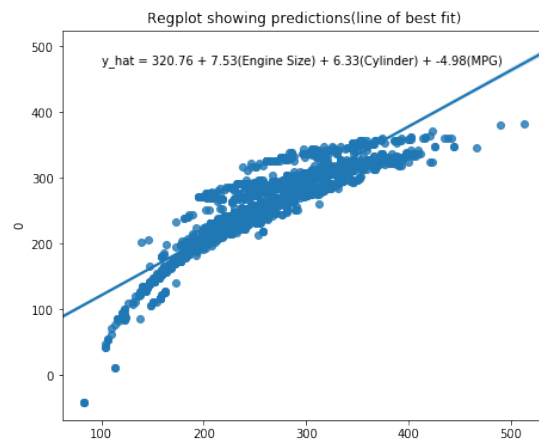
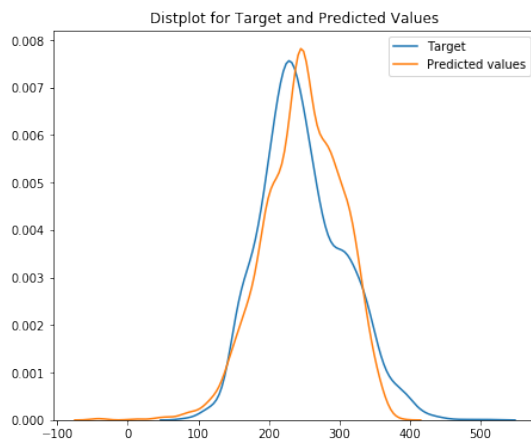
Furthermore, we can graph a distplot to visualize how our predicted values might slightly differ from actual values.

```
[129]: # Visualization:
# plt.figure(figsize=(10,6))

fig, axs = plt.subplots(1, 2, figsize=(16, 6))
# fig.suptitle ('Plotting the ')

sns.distplot(y_test, hist = False, label = 'Target', ax = axs[0])
axs[0].set_title('Distplot for Target and Predicted Values')
sns.distplot(predicted_CO2_em, hist = False, label = 'Predicted values', ax =
→axs[0])

sns.regplot(y_test, pred_co2_df[0], ax = axs[1])
axs[1].set_title('Regplot showing predictions(line of best fit)')
plt.text(100, 475, f"y_hat = {round(intercept,2)} +
→{round(engine_size,2)}(Engine Size) + {round(cylinders,2)}(Cylinder) +
→{round(mpg,2)}(MPG)")
plt.savefig('regression.png')
```



[]: