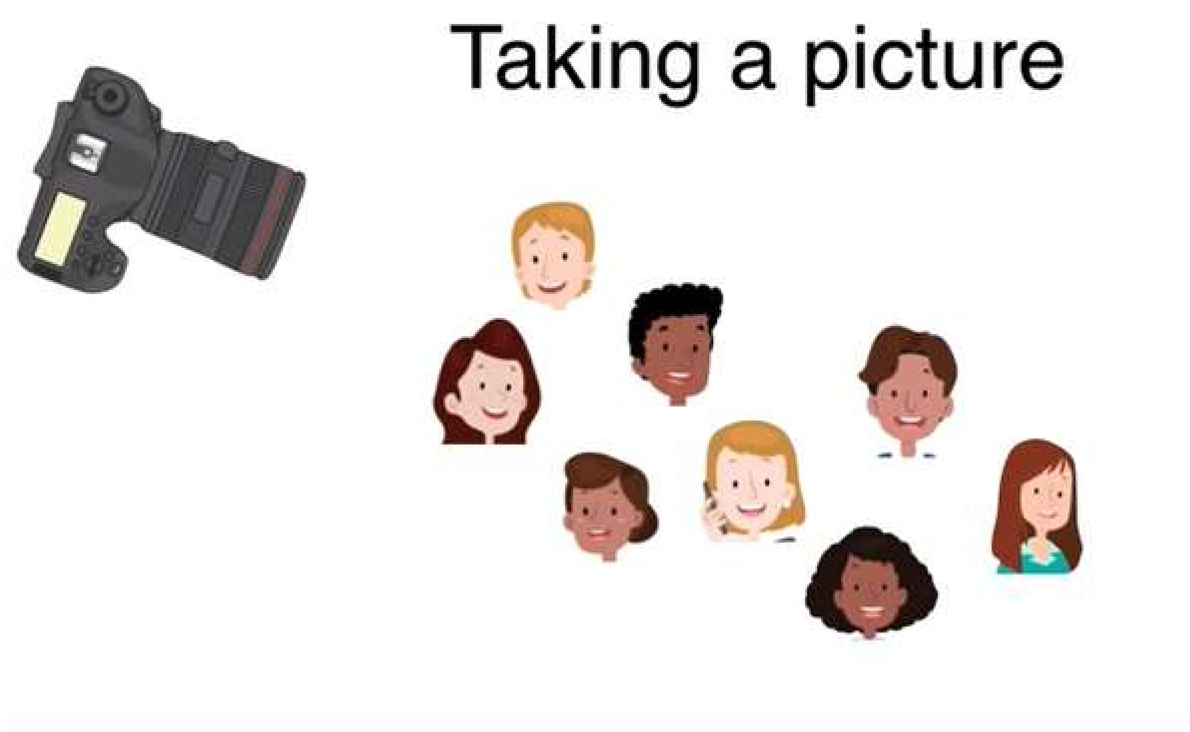
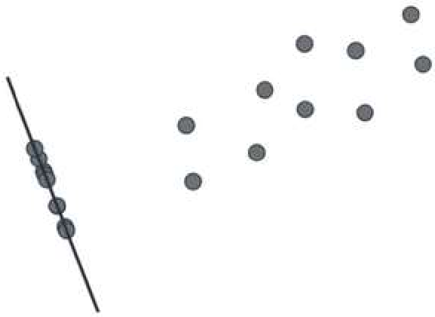
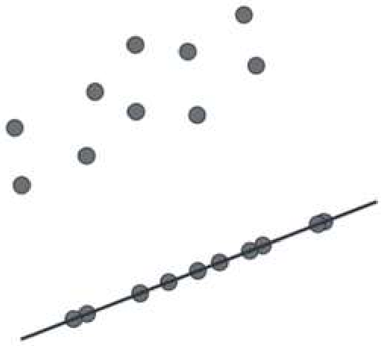
PCA – Luiss Serrano



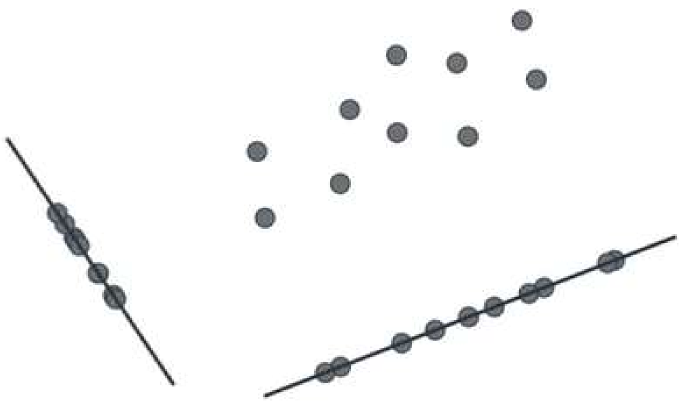
Dimensionality Reduction



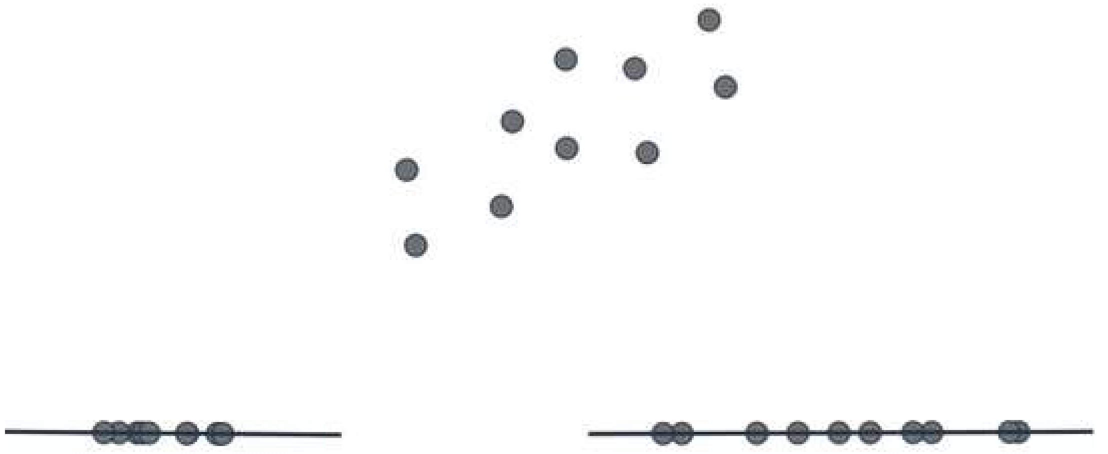
Dimensionality Reduction



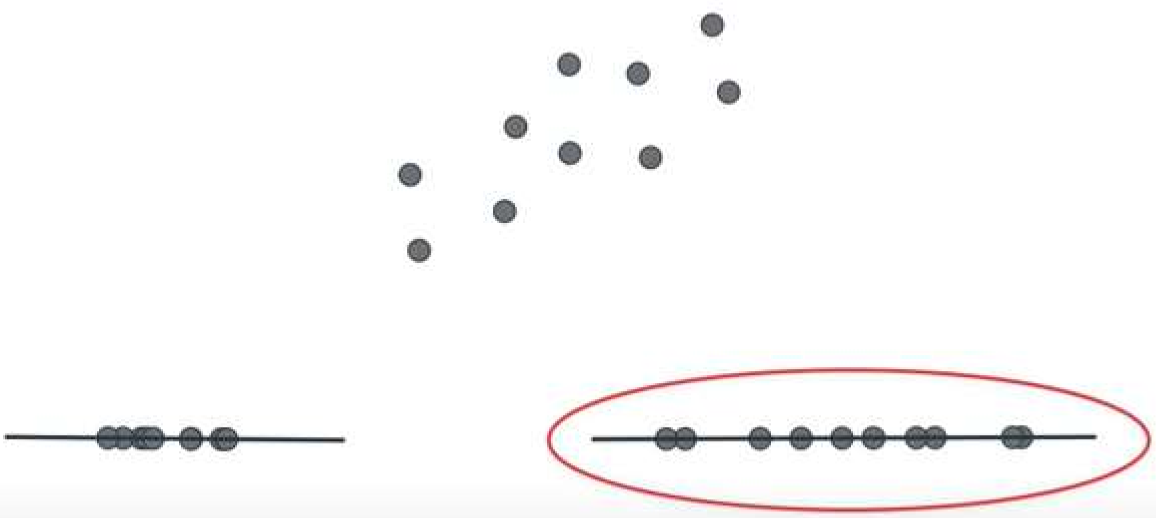
Dimensionality Reduction



Dimensionality Reduction



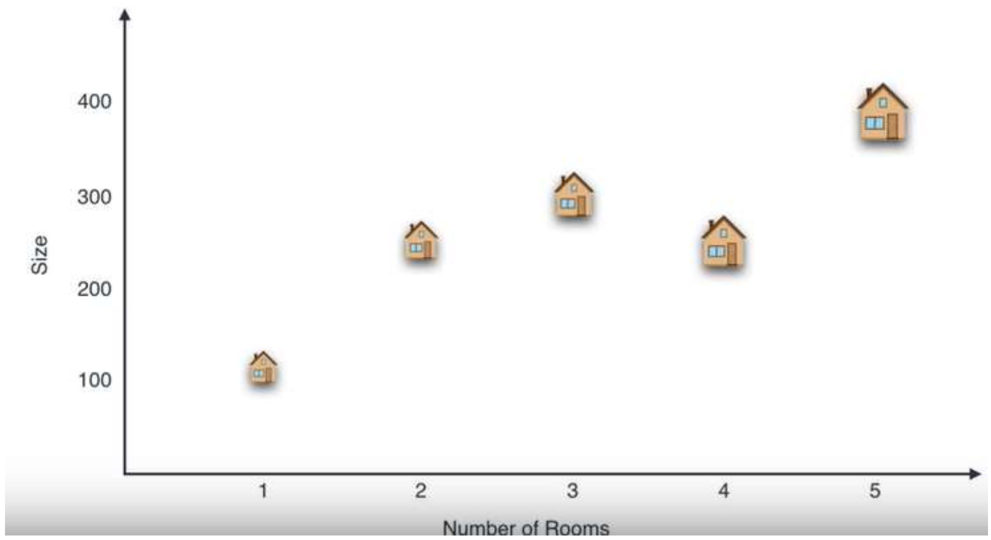
Dimensionality Reduction

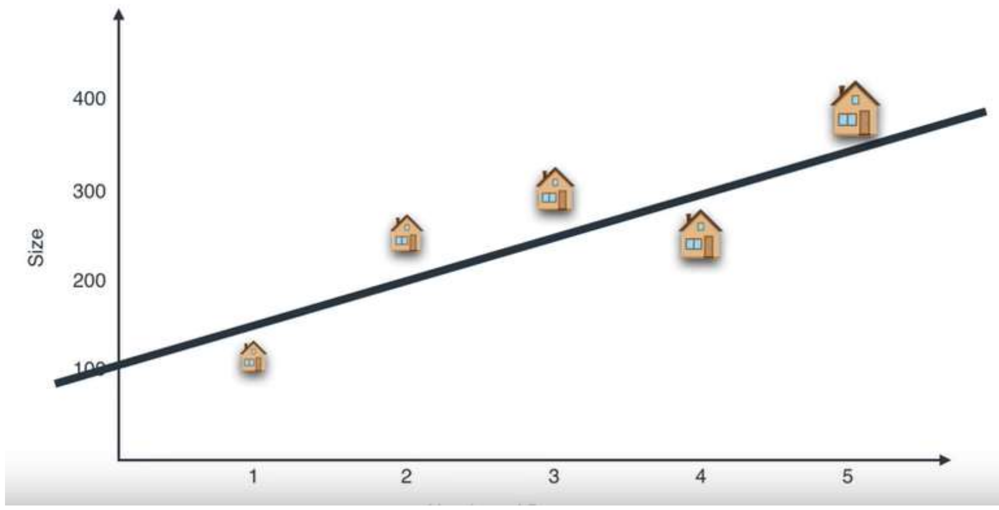


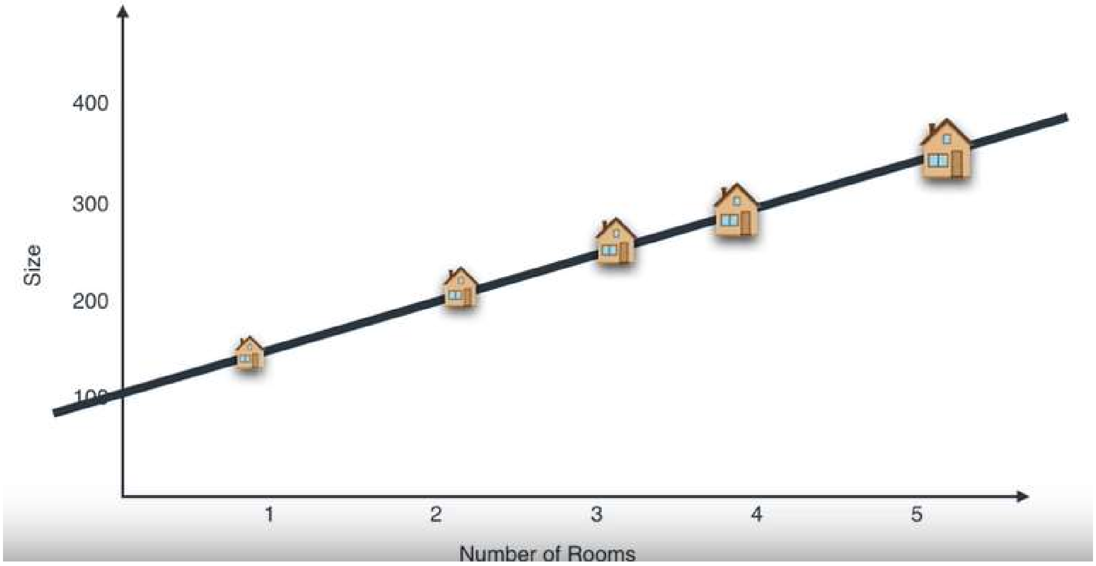
Housing Data

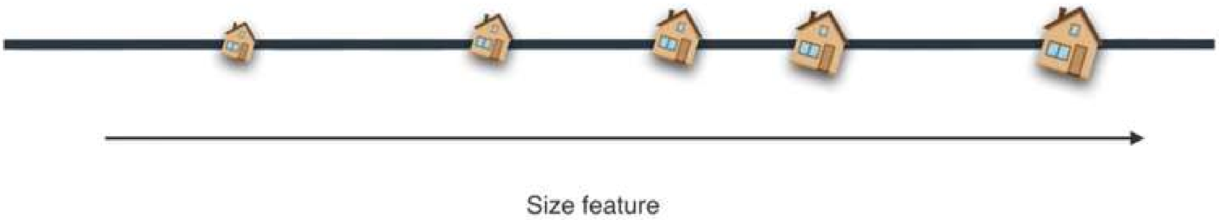
Size
Number of rooms → Size feature
Number of bathrooms

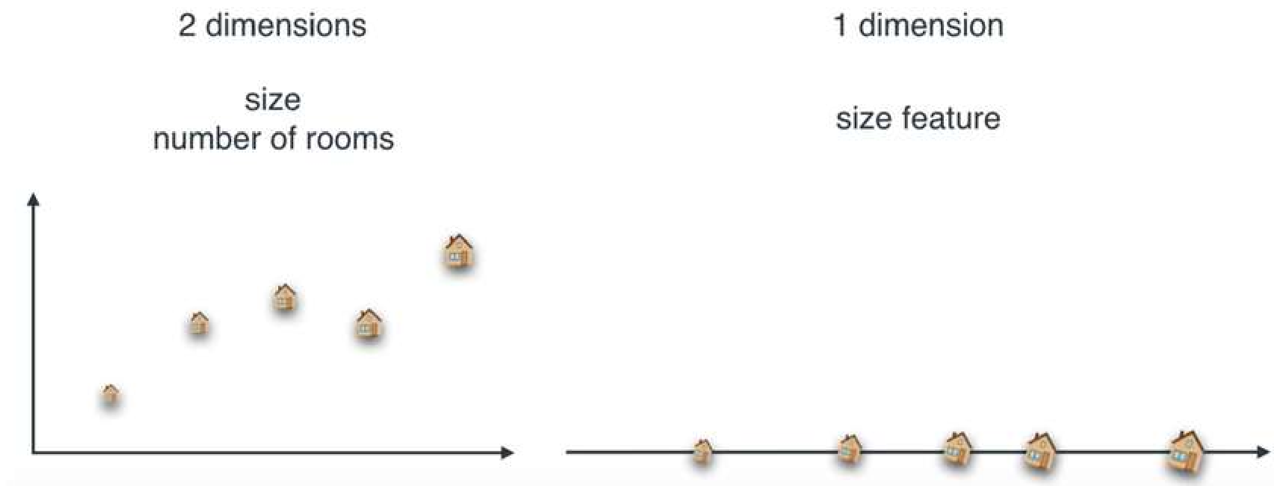
Schools around → Location feature
Crime rate






400
300
200
100
Size
1
2
3
4
5
Number of Rooms

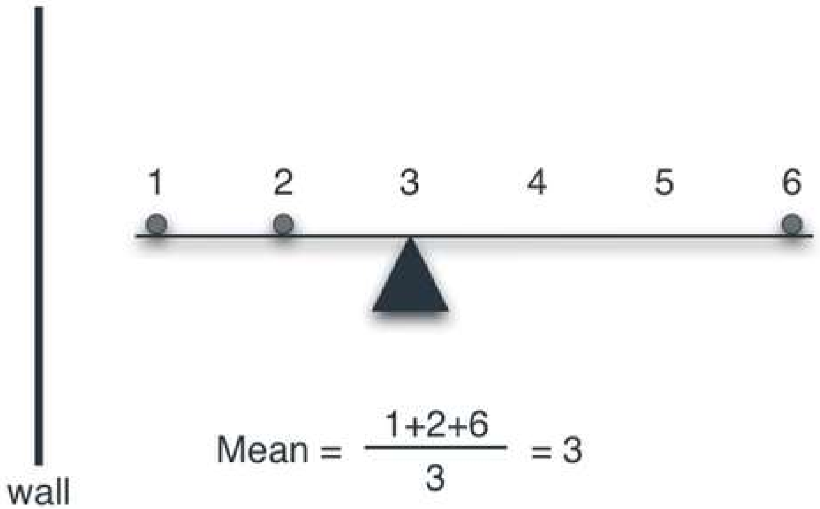

Size feature


2 dimensions
size
number of rooms
1 dimension
size feature

Housing Data

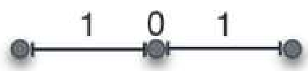
5 dimensions	**2 dimensions**
Size Number of rooms Number of bathrooms	Size feature
Schools around Crime rate	Location feature

Mean



$$\text{Mean} = \frac{1+2+6}{3} = 3$$

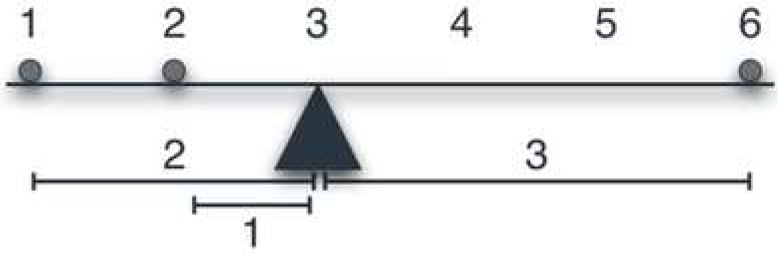
Variance



$$\text{Variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$

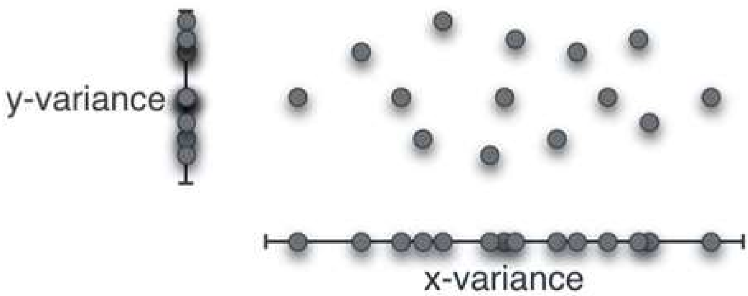
$$\text{Variance} = \frac{5^2+0^2+5^2}{3} = 50/3$$

Mean



$$\text{Variance} = \frac{2^2+1^2+3^2}{3} = 14/3$$

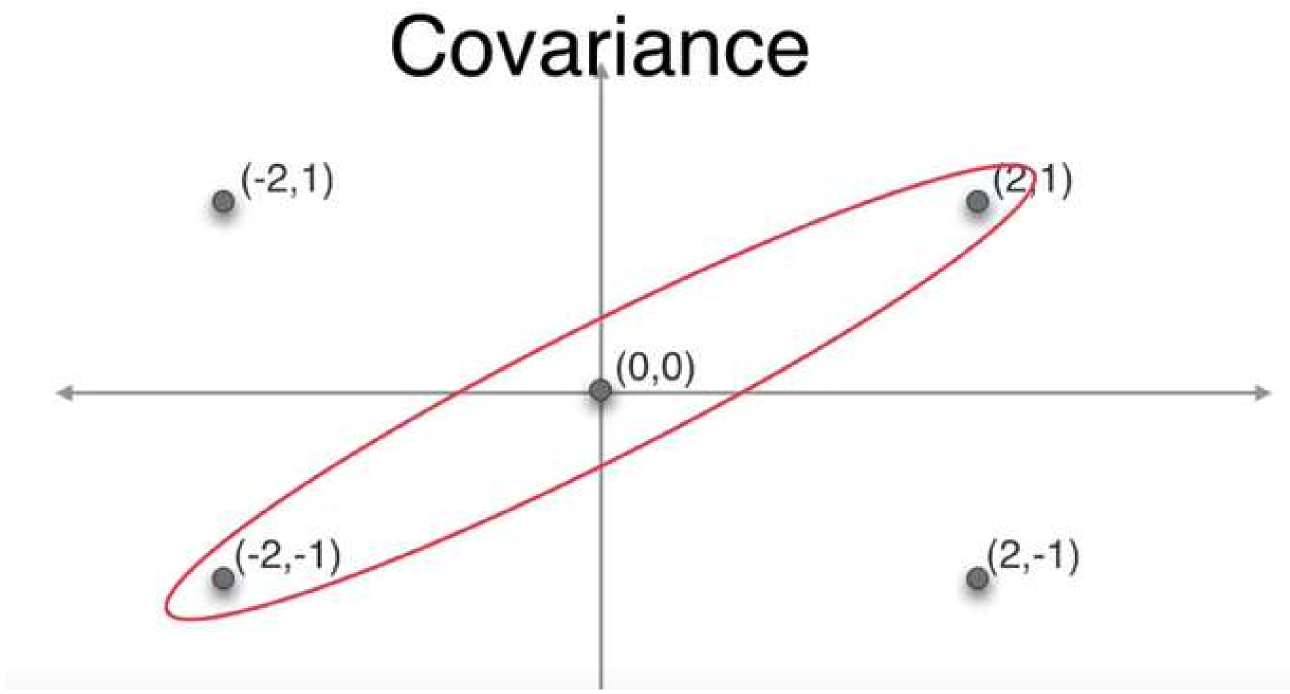
Variance?

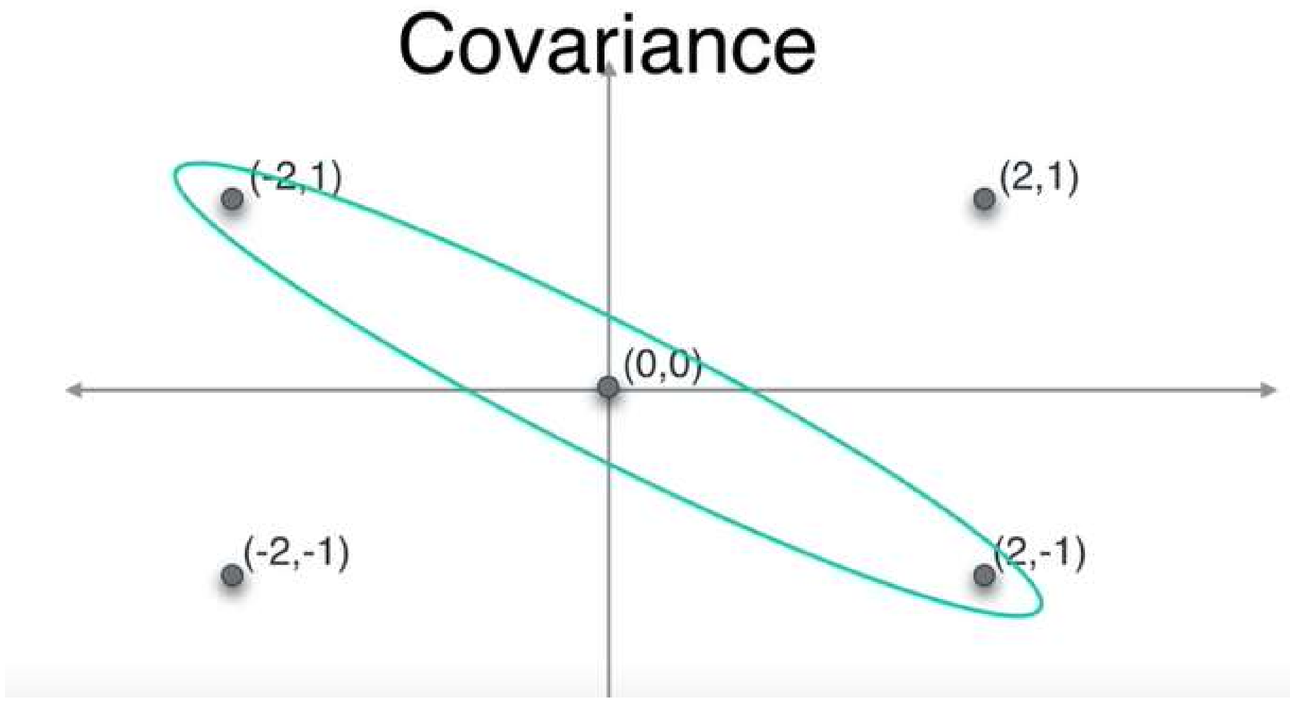


Variance?

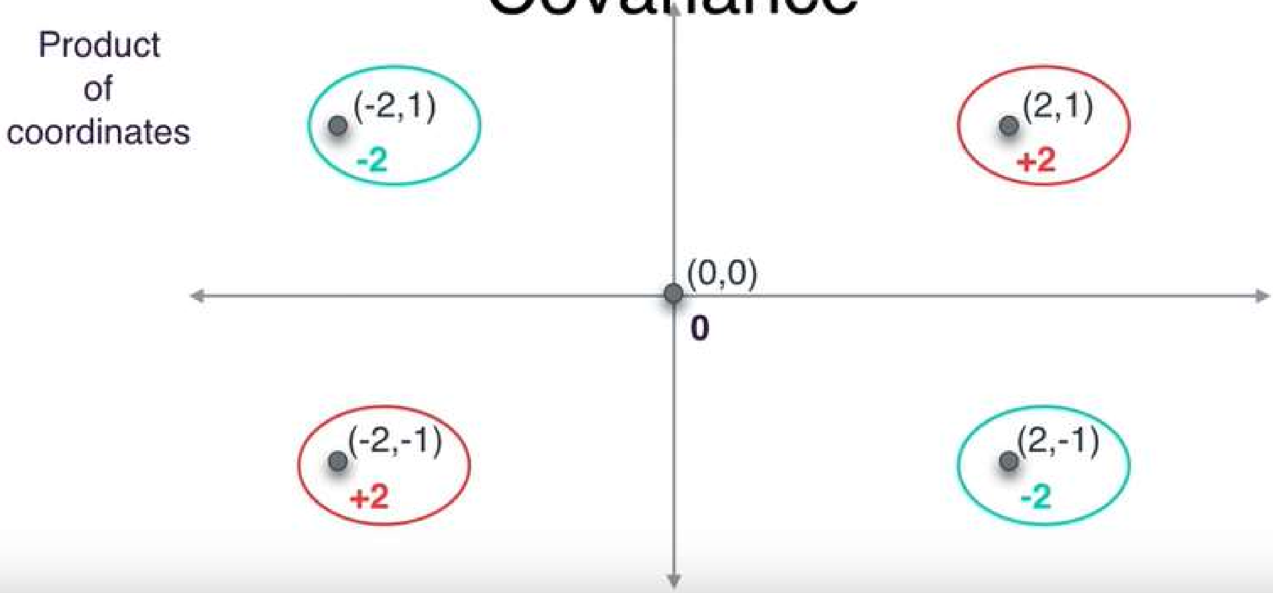
$$\text{x-variance} = \frac{2^2+0^2+2^2}{3} = 8/3$$

$$\text{y-variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$

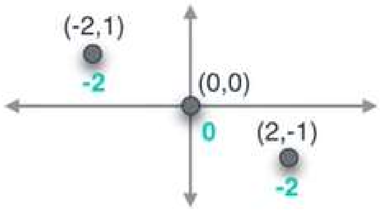

Covariance
(-2,1)
(2,1)
(0,0)
(-2,-1)
(2,-1)

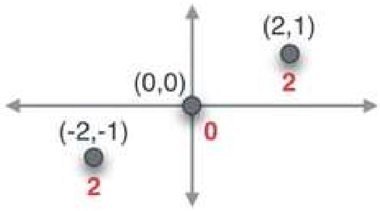

Covariance
(-2,1)
(2,1)
(0,0)
(-2,-1)
(2,-1)

Covariance



Covariance

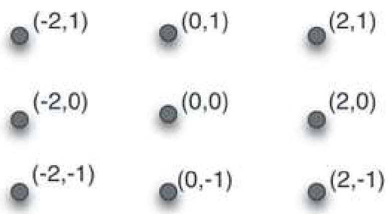




$$\text{covariance} = \frac{(-2) + 0 + (-2)}{3} = -4/3$$

$$\text{covariance} = \frac{2 + 0 + 2}{3} = 4/3$$

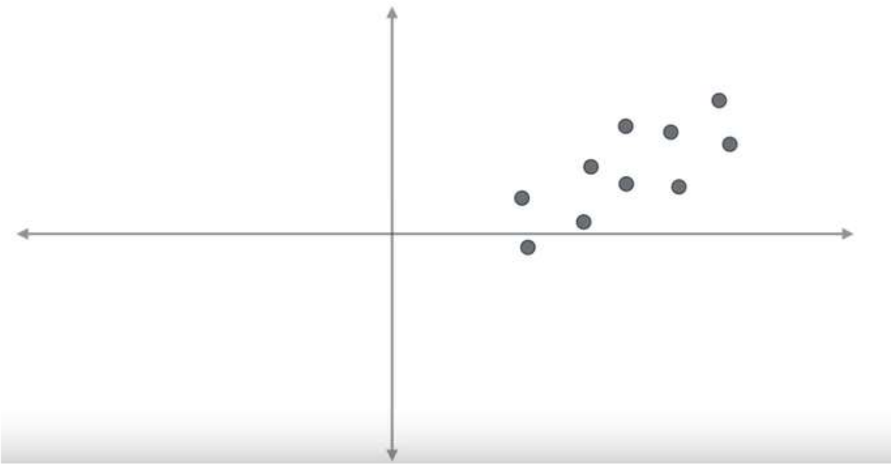
Covariance

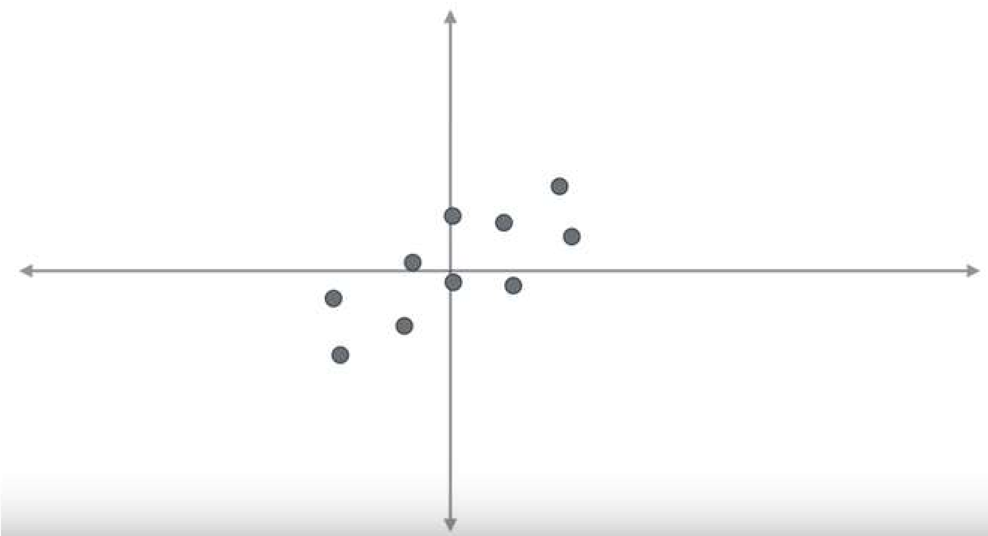


$$\text{covariance} = \frac{-2 + 0 + 2 + 0 + 0 + 0 + 2 + 0 + -2}{9}$$

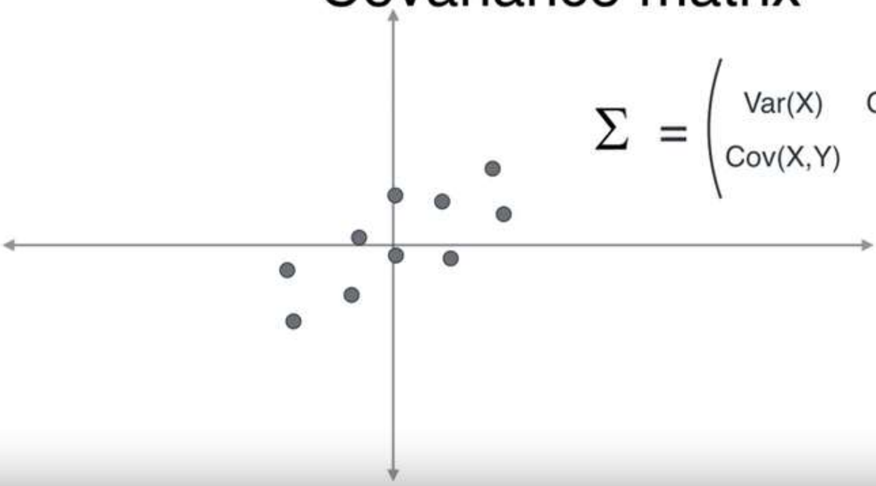

Covariance
negative covariance
covariance zero (or very small)
positive covariance

PCA – inizio





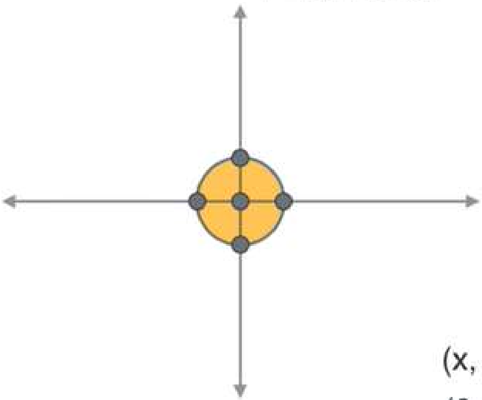
Covariance matrix



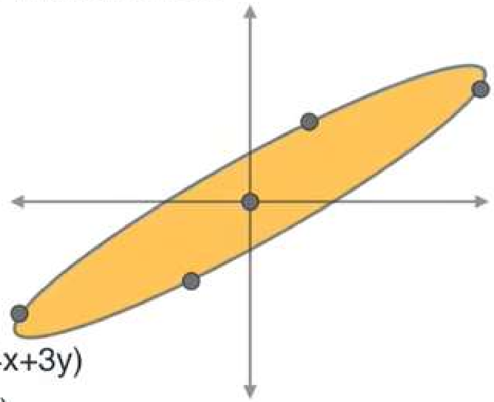
$$\Sigma = \begin{pmatrix} \mathrm{Var}(X) & \mathrm{Cov}(X,Y) \\ \mathrm{Cov}(X,Y) & \mathrm{Var}(Y) \end{pmatrix}$$

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

Linear Transformations



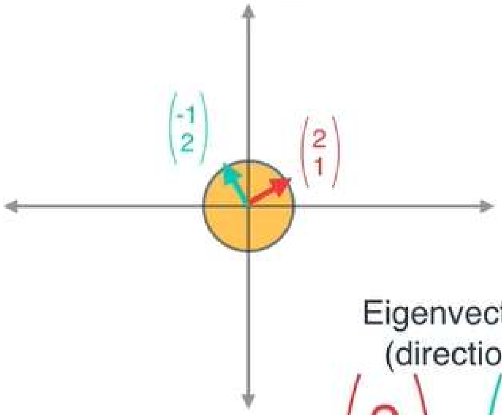
$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} \longrightarrow$$



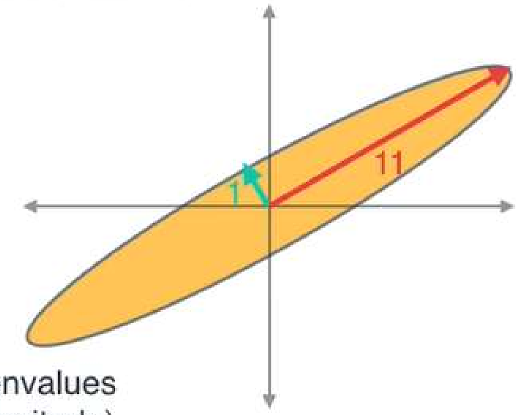
$(x, y) \longrightarrow (9x+4y, 4x+3y)$

(0,0)	(0,0)
(1,0)	(9,4)
(0,1)	(4,3)
(-1,0)	(-9,-4)
(0,-1)	(-4,-3)

Linear Transformations



$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} \longrightarrow$$



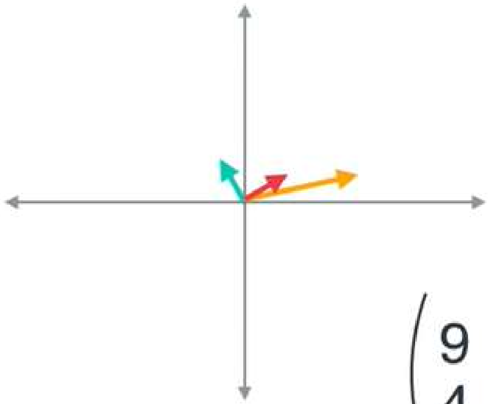
Eigenvectors
(direction)

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

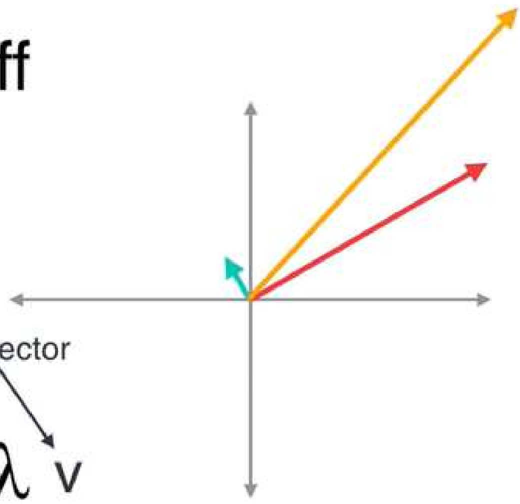
Eigenvalues
(magnitude)

11 1

Eigenstuff



$\longrightarrow$



Eigenvector

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} v = \lambda\, v$$

Eigenvalue

Eigenvalues

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

Characteristic Polynomial

$$\begin{vmatrix} x-9 & -4 \\ -4 & x-3 \end{vmatrix} = (x-9)(x-3) - (-4)(-4) = x^2 - 12x + 11$$

$$= (x-11)(x-1)$$

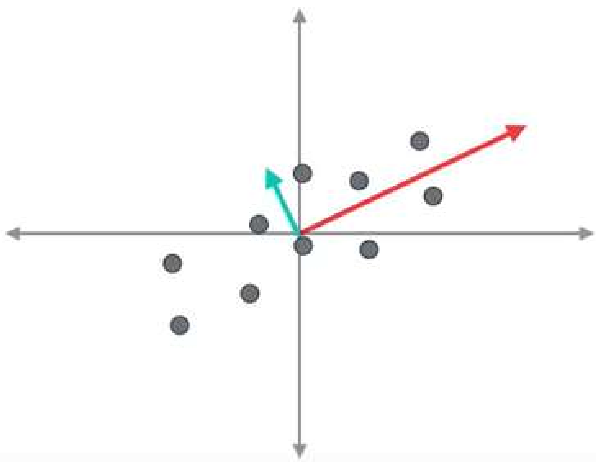
Eigenvalues 11 and 1

Eigenvectors

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 11\begin{pmatrix} u \\ v \end{pmatrix} \qquad \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 1\begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Principal Component Analysis (PCA)

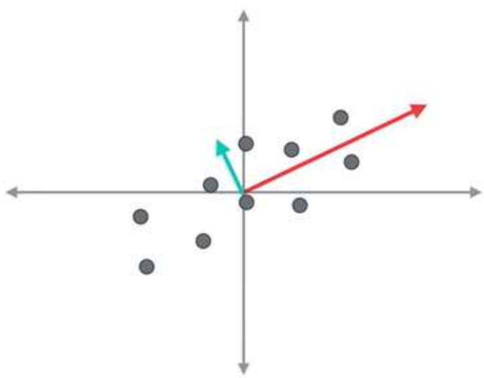


$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ Eigenvectors (direction)

11 1 Eigenvalues (magnitude)

Principal Component Analysis (PCA)

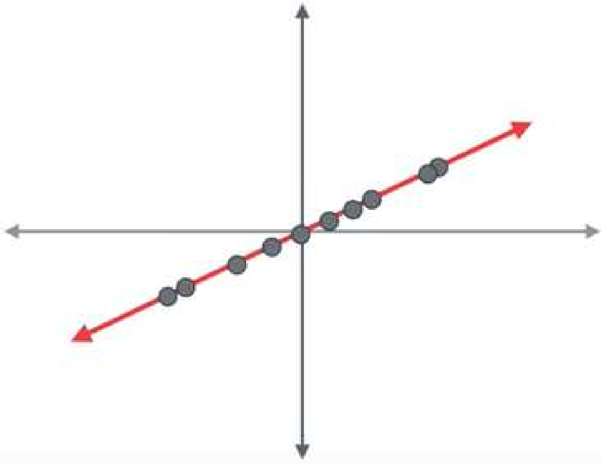


$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ Eigenvectors (direction)

11 1 Eigenvalues (magnitude)

Principal Component Analysis (PCA)

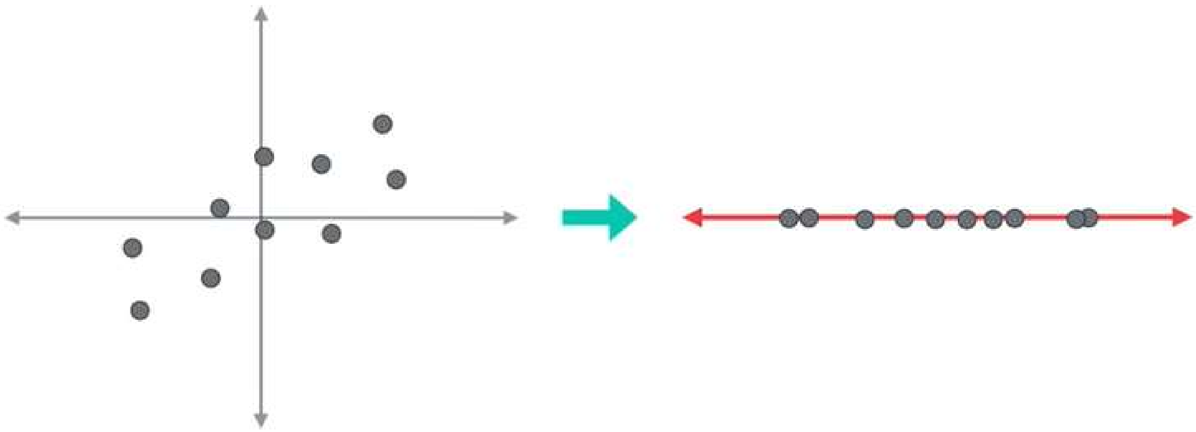


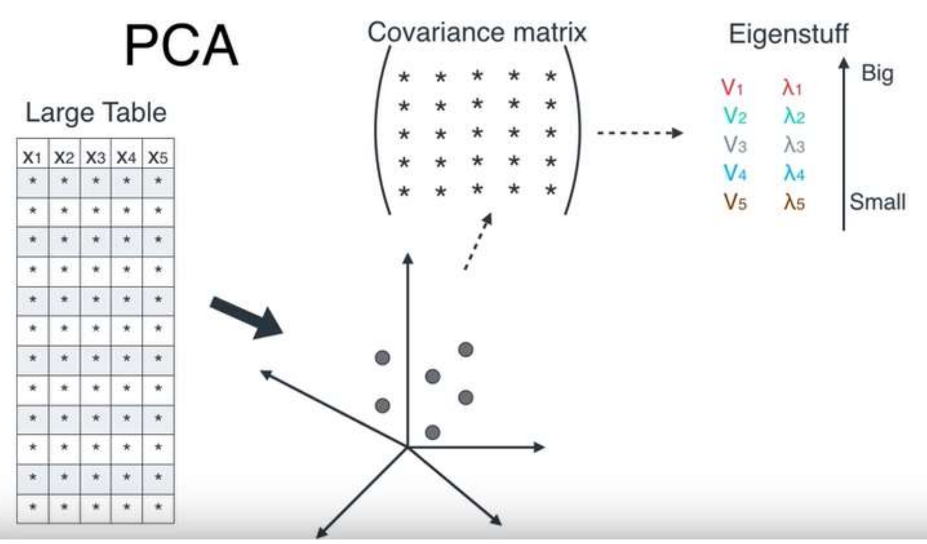
$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

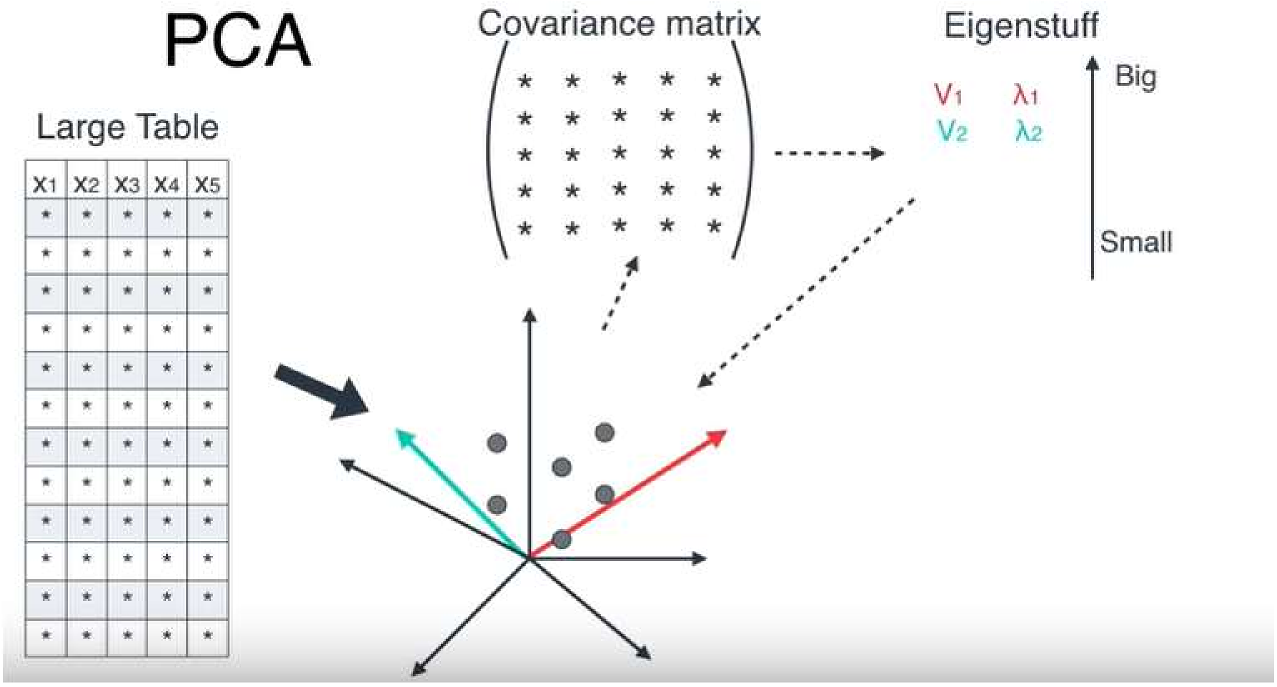
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$ Eigenvectors (direction)

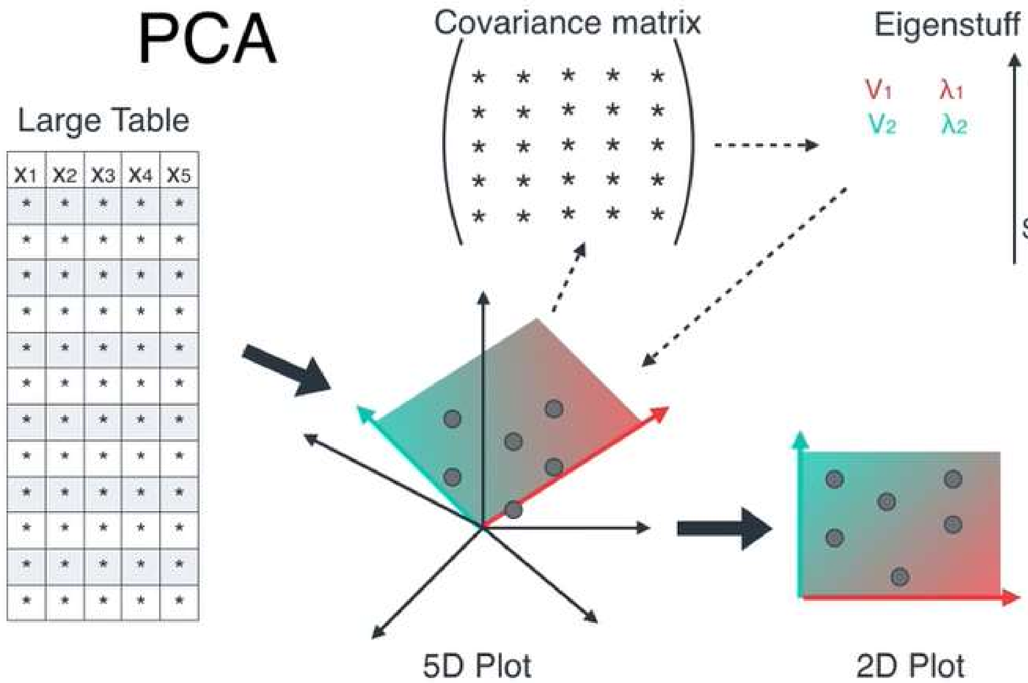
11 Eigenvalues (magnitude)

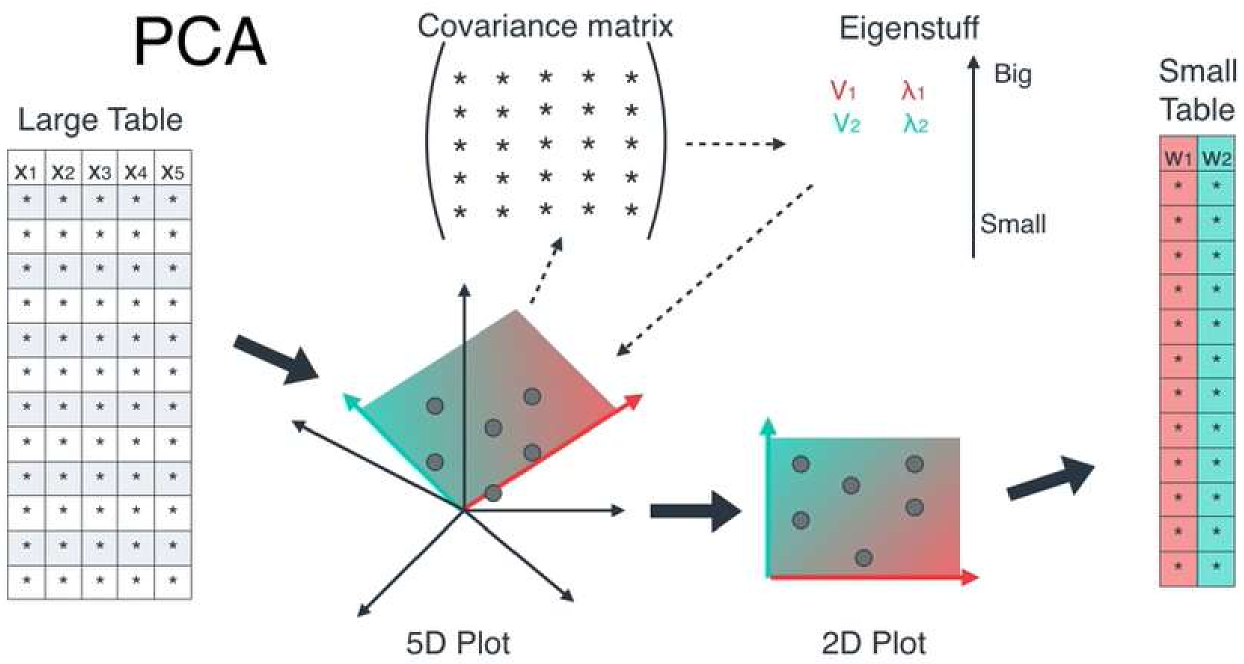
Principal Component Analysis (PCA)






PCA
Covariance matrix
Eigenstuff
Big
V1 λ1
V2 λ2
Small
Large Table
X1 X2 X3 X4 X5


PCA
Covariance matrix
Eigenstuff
Big
V1 λ1
V2 λ2
Small
Large Table
X1 X2 X3 X4 X5
5D Plot
2D Plot


PCA
Large Table
x_1 x_2 x_3 x_4 x_5
Covariance matrix
Eigenstuff
v_1 λ_1
v_2 λ_2
Big
Small
Small Table
w_1 w_2
5D Plot
2D Plot