

INTELLIGENT SYSTEMS

ITCS 6150/8150, Fall 2023

PROJECT 1

Solving the 8-puzzle using A* algorithm

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1. ABSTRACT

The classic 8-puzzle problem involves a 3x3 grid with eight numbered tiles and one empty space. The challenge is to rearrange these tiles to achieve a goal state configuration. The objective is to manipulate these tiles within the grid to ultimately achieve the goal arrangement. This report explains the application of the A* search algorithm to effectively address and solve the 8-puzzle problem.

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2. INTRODUCTION

The A* search algorithm is used to determine the best series of actions to take from the starting point to the desired goal state. The algorithm continuously explores many states, keeps both open and closed lists, and prioritizes states with the best g-cost and h-cost. The value of $f(n)$ is the sum of path cost and heuristic cost. $f(n) = g(n) + h(n)$ $g(n)$ is the cost of the current path. $h(n)$ is a heuristic function that finds the shortest route from current node to the target node.

3. ALGORITHM

3.1 DESCRIPTION :

First , we start by expanding the initial state ie., generating every possible move from that state and place these new nodes on the frontier . We compute heuristics and path cost , $h(n)$ and $g(n)$ respectively for the nodes on the frontier. Being a greedy algorithm , A * expands the node with least $f(n) = g(n) + h(n)$. This is done recursively , excluding the duplicate nodes as we progress and terminates when we reach the goal state.

3.2 HEURISTICS :

Misplaced Tile Heuristic :

In this method we count the number of misplaced tiles as $h(n)$. A * search algorithm chooses the node with lowest $f(n)$. So we have a good chance of expanding the node with fewer misplaced tiles .

Manhattan Distance Heuristic :

In this method instead of just counting the misplaced tiles , we count the number of tiles if each misplaced tile is away from its position in the goal state. So we try to choose for expansion , the node that has fewer steps leading towards the goal state.

4. IMPLEMENTATION

GLOBAL VARIABLES

- (i) **initial_board** - input given by user; contains 3 x 3 grid of values in initial state.
- (ii) **goal** - expected output specified by user. 3 x 3 grid of values in expected goal state

METHODS

`manhattan_distance(initial_board, goal_board) -> int`: This function calculates the Manhattan distance heuristic between the `initial_board` and the `goal_board`. It measures the total number of moves (both horizontal and vertical) required to move each tile from its current position in the `initial_board` to its correct position in the `goal_board`. The function returns an integer representing the Manhattan distance.

`count_misplaced(initial_board, goal_board) -> int`: This function calculates the Misplaced Count heuristic between the `initial_board` and the `goal_board`. It counts the number of tiles in the `initial_board` that are not in their correct positions in the `goal_board`. The function returns an integer representing the count of misplaced tiles.

`boardGood(board) -> bool`: This function checks if the board is a valid 8-puzzle board. It ensures that the board contains the numbers 1 to 8 (or 0 for the blank tile) exactly once. It returns True if the board is valid, and False otherwise.

`showBoard(board: list[list[int]]) -> None`: This function displays the content of the board in a 3x3 format, printing each element row by row.

`replaceEmptySpace(board: list[list[int]]) -> list[list[int]]`: This function replaces any empty spaces in the board (represented as spaces) with '0' to make it consistent for processing.

`findElement(board: list[list[int]], ele) -> list[int]`: This function finds the indices of a given element `ele` in the board and returns them as a list of two integers [row, column].

`findEmptySpace(board: list[list[int]]) -> list[int]`: This function finds the indices of the blank (0) tile in the board and returns them as a list of two integers [row, column].

`cloneBoard(board: list[list[int]]) -> list[list[int]]`: This function creates a deep copy of the input board and returns the copy.

`goalTest(board: list[list[int]], goal_board: list[list[int]]) -> bool`: This function checks if the board matches the goal_board, indicating whether the puzzle has been solved. It returns True if the board is equal to the goal_board, and False otherwise.

`attachManhattan(boards: list[list[list[int]]], goal: list[list[int]], count: int) -> list`: This function calculates the Manhattan distance for a list of boards and attaches it to each board along with the current count. It returns a list of tuples, where each tuple contains the Manhattan distance plus the count and the corresponding board.

`attachMisplaced(boards: list[list[list[int]]], goal: list[list[int]], count: int) -> list`: Similar to `attachManhattan`, this function calculates the Misplaced Count heuristic for a list of boards and attaches it to each board along with the current count. It returns a list of tuples, where each tuple contains the Misplaced Count plus the count and the corresponding board.

`actionList(board, prev_boards)`: This function generates a list of successor boards that can be reached from the given board by making valid moves. It considers moving the blank tile (0) up, down, left, or right. It ensures that the generated boards are not duplicates of previously visited boards (in `prev_boards`).

`aStarSearchManhattan(initial_board, goal_board) -> list`: This function performs an A* search using the Manhattan distance heuristic to find the solution to the 8-puzzle problem. It returns a list of tuples, where each tuple contains a board and the corresponding count of moves.

`aStarSearchMisplaced(initial_board, goal_board) -> list`: Similar to `aStarSearchManhattan`, this function performs an A* search using the Misplaced Count heuristic to find the solution to the 8-puzzle problem. It also returns a list of tuples, where each tuple contains a board and the corresponding count of moves.

These functions collectively implement the 8-puzzle problem-solving algorithm using two different heuristic methods. The program allows users to input initial and goal states and then selects the desired heuristic to find the solution to the puzzle.


```

def showBoard(board: list[list[int]]) -> None:
    for i in range(3):
        for j in range(3):
            print(board[i][j], end=" ")
        print()
    print()

# replaces empty space " ", with zero
def replaceEmptySpace(board: list[list[int]]) -> list[list[int]]:
    for i in range(3):
        for j in range(3):
            if str(board[i][j]).isspace():
                board[i][j] = '0'
    return board

def findElement(board: list[list[int]], ele) -> list[int]:
    for i in range(3):
        for j in range(3):
            if board[i][j] == ele:
                return [i, j]

def findEmptySpace(board: list[list[int]]) -> list[int]:
    for i in range(3):
        for j in range(3):
            if str(board[i][j]).isspace() or board[i][j] == '0':
                return [i, j]

    raise Exception("No space left in board, made an illegal move")
    return [0, 0]

def cloneBoard(board: list[list[int]]) -> list[list[int]]:
    newboard = [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
    for i in range(3):
        for j in range(3):
            newboard[i][j] = board[i][j]

    return newboard

def goalTest(board: list[list[int]], goal_board: list[list[int]]) -> bool:
    for i in range(3):
        for j in range(3):
            if goal_board[i][j] != board[i][j]:
                return False

    return True

```

```

def attachManhattan(boards: list[list[list[int]]], goal: list[list[int]], count: int) -> list:
    d = []

    for i in boards:
        d.append((manhattan_distance(i, goal) + count, i))

    return d

def attachMisplaced(boards: list[list[list[int]]], goal: list[list[int]], count: int) -> list:
    d = []

    for i in boards:
        d.append((count_misplaced(i, goal) + count, i))

    return d

def actionList(board, prev_boards):
    boards = []
    x, y = findEmptySpace(board)

    # Check if there is a legal move in the direction - up
    if x > 0:
        mvUpBoard = cloneBoard(board)
        mvUpBoard[x][y] = mvUpBoard[x-1][y]
        mvUpBoard[x-1][y] = '0'

        if mvUpBoard not in prev_boards:
            boards.append(mvUpBoard)

    # Check if there is a legal move in the direction - left
    if y > 0:

```

```

main.py M X 1 Readme U
main.py > manhattan_distance
You, 6 minutes ago | 1 author (You)
1 from queue import PriorityQueue
2
3 # Manhattan distance
4 def manhattan_distance(initial_board, goal_board)-> int:
5
6     distance = 0
7
8     for i in range(3):
9         for j in range(3):
10
11             You, 16 hours ago • init:heuristics
12             if initial_board[i][j] != '0':
13
14                 # Calculate Manhattan distance
15                 x_diff = abs(j - findElement(goal_board, initial_board[i][j])[0])
16                 y_diff = abs(1 - initial_board.index(initial_board[i]))
17                 distance += x_diff + y_diff
18
19     return distance
20
21 # Misplaced count
22 def count_misplaced(initial_board, goal_board)-> int:
23
24     count = 0
25
26     for i in range(3):
27         for j in range(3):
28
29             if initial_board[i][j] != '0':
30
31                 if initial_board[i][j] != goal_board[i][j]:
32                     count += 1
33
34     return count
35
36 #Test statements for heuristics
37 #initial_board = [[1, 2, 3], [4, 5, 6], [0, 7, 8]]
38 #goal_board = [[1, 2, 3], [4, 5, 6], [7, 8, 0]]
39
40 #print(manhattan_distance(initial_board, goal_board))
41 #print(count_misplaced(initial_board, goal_board))
42
43 def boardGood(board)->bool:
44     l=['1','2','3','4','5','6','7','8','9','0']
45     l1=[]
46     for i in range(3):
47         for j in range(3):
48             l1.append(board[i][j])

```

```

def actionList(board, prev_boards):
    boards=[]
    x,y = findEmptySpace(board)

    #Check if there is a legal move in the direction - up
    if x>0:
        mvUpBoard = cloneBoard(board)
        mvUpBoard[x][y] = mvUpBoard[x-1][y]
        mvUpBoard[x-1][y] = '0'

        if mvUpBoard not in prev_boards:
            boards.append(mvUpBoard)

    #Check if there is a legal move in the direction - left
    if y>0:
        mvleftBoard = cloneBoard(board)
        mvleftBoard[x][y] = mvleftBoard[x][y-1]
        mvleftBoard[x][y-1] = '0'

        if mvleftBoard not in prev_boards:
            boards.append(mvleftBoard)

    #Check if there is a legal move in the direction - right
    if y<2:
        mvrightBoard = cloneBoard(board)
        mvrightBoard[x][y] = mvrightBoard[x][y+1]
        mvrightBoard[x][y+1] = '0'

        if mvrightBoard not in prev_boards:
            boards.append(mvrightBoard)

    #Check if there is a legal move in the direction - down
    if x<2:
        mvdownBoard = cloneBoard(board)
        mvdownBoard[x][y] = mvdownBoard[x+1][y]
        mvdownBoard[x+1][y] = '0'

        if mvdownBoard not in prev_boards:
            boards.append(mvdownBoard)

    return boards

def aStarSearchManhattan(initial_board, goal_board)->list:
    path = []
    fringe=[]
    visited=[]
    count=0
    fringe = PriorityQueue()
    fringe.put((0, initial_board))
    while not fringe.empty():
        curr_board=fringe.get_nowait()[1]
        if curr_board in visited:
            pass
        elif boardGood(curr_board):

```

```

def aStarSearchManhattan(initial_board, goal_board)->list:
    path = []
    fringe=[]
    visited=[]
    count=0
    fringe = PriorityQueue()
    fringe.put((0,initial_board))
    while not fringe.empty():
        curr_board=fringe.get_nowait()[1]
        if curr_board in visited:
            pass
        if boardGood(curr_board):
            raise Exception(" Bad board")

        visited.append(curr_board)
        if goalTest(curr_board,goal_board):
            path.append((curr_board,count))
            return path
        prospectiveBoards = actionList(curr_board,visited)
        d = attachManhattan(prospectiveBoards,goal_board,count+1)
        #updating the fringe, if board/node already exists, check if we have a better heuristic to update it in the frontier
        for key in d:
            fringe.put(key)
        path.append((curr_board,count))
        count+=1
    return path

```

```

def aStarSearchMisplaced(initial_board, goal_board)->list:
    path = []
    fringe=[]
    visited=[]
    count=0
    fringe = PriorityQueue()
    fringe.put((0,initial_board))
    while not fringe.empty():
        curr_board=fringe.get_nowait()[1]
        if curr_board in visited:
            pass
        if boardGood(curr_board):
            raise Exception(" Bad board")

        visited.append(curr_board)
        if goalTest(curr_board,goal_board):
            path.append((curr_board,count))
            return path
        prospectiveBoards = actionList(curr_board,visited)

```

```

def aStarSearchMisplaced(initial_board, goal_board)->list:
    path = []
    fringe=[]
    visited=[]
    count=0
    fringe = PriorityQueue()
    fringe.put((0,initial_board))
    while not fringe.empty():
        curr_board=fringe.get_nowait()[1]
        if curr_board in visited:
            pass
        if boardGood(curr_board):
            raise Exception(" Bad board")

        visited.append(curr_board)
        if goalTest(curr_board,goal_board):
            path.append((curr_board,count))
            return path
        prospectiveBoards = actionList(curr_board,visited)
        d = attachMisplaced(prospectiveBoards,goal_board,count+1)
        #updating the fringe, if board/node already exists, check if we have a better heuristic to update it in the frontier
        for key in d:
            fringe.put(key)
        path.append((curr_board,count))
        count+=1
    return path

```

```
print("##### Welcome to 8 Puzzle Solver #####")

# Input for the initial state
print(f"Enter Initial State Board.")
init_board = []
for i in range(3):
    row = input().split()
    if len(row) != 3:
        print("Invalid input. Enter 3 elements for each row.")
    init_board.append(row)

# Input for the goal state
print(f"Enter Goal State Board.")
goal = []
for i in range(3):
    row = input().split()
    if len(row) != 3:
        print("Invalid input. Enter 3 elements for each row.")
    goal.append(row)

print("Misplaced")
print(aStarSearchMisplaced(init_board,goal))

print("Manhattan")

print(aStarSearchManhattan(init_board,goal))
```

5. RESULTS

INITIAL	GOAL	Misplaced tiles heuristic	Manhattan distance heuristic																		
<div>Example 1</div> <table><tr><td>4</td><td>1</td><td>3</td></tr><tr><td></td><td>2</td><td>5</td></tr><tr><td>7</td><td>8</td><td>6</td></tr></table>	4	1	3		2	5	7	8	6	<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>5</td><td>6</td></tr><tr><td>7</td><td>8</td><td></td></tr></table>	1	2	3	4	5	6	7	8		12 nodes generated 6 nodes expanded Solution at depth 5	12 nodes generated 6 nodes expanded Solution at depth 5
4	1	3																			
	2	5																			
7	8	6																			
1	2	3																			
4	5	6																			
7	8																				
<div>Example 2</div> <table><tr><td>7</td><td>1</td><td>8</td></tr><tr><td></td><td>6</td><td>2</td></tr><tr><td>5</td><td>4</td><td>3</td></tr></table>	7	1	8		6	2	5	4	3	<table><tr><td>7</td><td>8</td><td></td></tr><tr><td>6</td><td>1</td><td>2</td></tr><tr><td>5</td><td>4</td><td>3</td></tr></table>	7	8		6	1	2	5	4	3	9 nodes generated 4 nodes expanded Solution at depth 3	9 nodes generated 4 nodes expanded Solution at depth 3
7	1	8																			
	6	2																			
5	4	3																			
7	8																				
6	1	2																			
5	4	3																			
<div>Example 3</div> <table><tr><td>1</td><td>0</td><td>3</td></tr><tr><td>4</td><td>2</td><td>6</td></tr><tr><td>7</td><td>5</td><td>8</td></tr></table>	1	0	3	4	2	6	7	5	8	<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>5</td><td>6</td></tr><tr><td>7</td><td>8</td><td></td></tr></table>	1	2	3	4	5	6	7	8		9 nodes generated 4 nodes expanded Solution at depth 3	9 nodes generated 4 nodes expanded Solution at depth 3
1	0	3																			
4	2	6																			
7	5	8																			
1	2	3																			
4	5	6																			
7	8																				
<div>Example 4</div> <table><tr><td>4</td><td>6</td><td>8</td></tr><tr><td>2</td><td>5</td><td>3</td></tr><tr><td></td><td>7</td><td>1</td></tr></table>	4	6	8	2	5	3		7	1	<table><tr><td>4</td><td>8</td><td>0</td></tr><tr><td>2</td><td>6</td><td>3</td></tr><tr><td>7</td><td>5</td><td>1</td></tr></table>	4	8	0	2	6	3	7	5	1	10 nodes generated 5 nodes expanded Solution at depth 4	10 nodes generated 5 nodes expanded Solution at depth 4
4	6	8																			
2	5	3																			
	7	1																			
4	8	0																			
2	6	3																			
7	5	1																			
<div>Example 5</div> <table><tr><td>8</td><td>7</td><td>6</td></tr><tr><td>2</td><td>5</td><td>4</td></tr><tr><td></td><td>1</td><td>3</td></tr></table>	8	7	6	2	5	4		1	3	<table><tr><td>8</td><td>7</td><td>6</td></tr><tr><td>5</td><td>4</td><td>3</td></tr><tr><td>2</td><td>1</td><td></td></tr></table>	8	7	6	5	4	3	2	1		10 nodes generated 5 nodes expanded Solution at depth 4	10 nodes generated 5 nodes expanded Solution at depth 4
8	7	6																			
2	5	4																			
	1	3																			
8	7	6																			
5	4	3																			
2	1																				

Example 6

4	7	5
8	1	3
2		6

4	7	5
8	3	6
2	1	

9 nodes generated

4 nodes expanded

Solution at depth 3

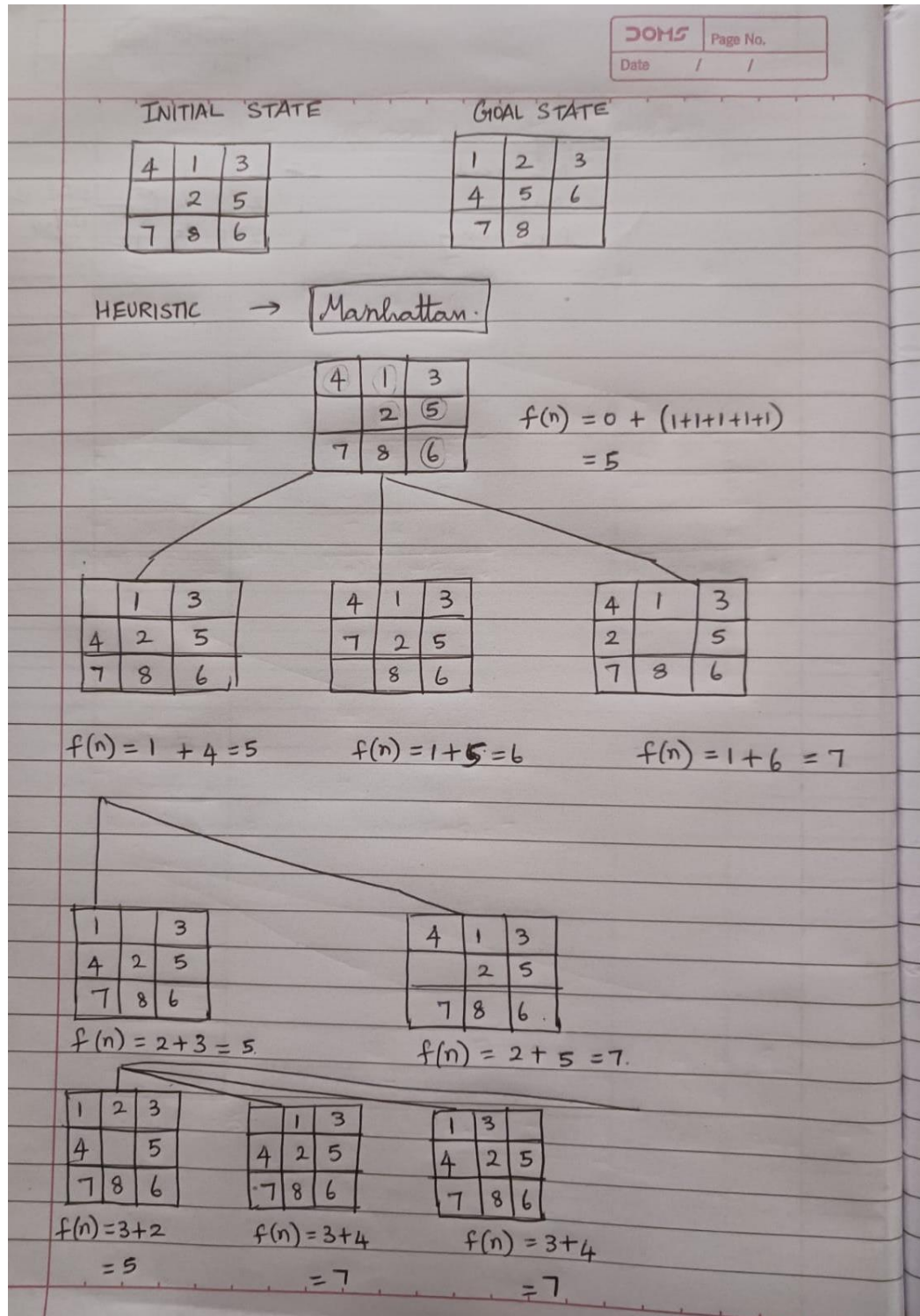
9 nodes generated

4 nodes expanded

Solution at depth 3

6. EXAMPLES

Example 1 :



1	2	3
4		5
7	8	6

1		3
4	2	5
7	8	6

$$f(n) = 4 + 3$$

$$= 7$$

1	2	3
	4	5
7	8	6

$$f(n) = 4 + 3$$

$$= 7$$

1	2	3
4	5	
7	8	6

$$f(n) = 4 + 1$$

$$= 5$$

1	2	3
4	8	5
7		6

$$f(n) = 4 + 3$$

$$= 7$$

1	2	
4	5	3
7	8	6

$$f(n) = 5 + 2$$

$$= 7$$

1	2	3
4		5
7	8	6

$$f(n) = 5 + 2$$

$$= 7$$

1	2	3
4	5	6
7	8	

$$f(n) = 5 + 0$$

(GOAL STATE)

Heuristic - Misplaced count heuristic

4	1	3
	2	5
7	8	6

$$f(n) = 0 + 5 = 5$$

4	1	3
7	2	5
8	6	

$$f(n) = 1 + 3 = 4$$

4	1	3
2		5
7	8	6

$$f(n) = 1 + 5 = 6$$

4	1	3
7	2	5
	8	6

$$f(n) = 1 + 6 = 7$$

1		3
4	2	5
7	8	6

$$f(n) = 2 + 3 = 5$$

4	1	3
	2	5
7	8	6

$$f(n) = 2 + 5 = 7$$

1	2	3
4		5
7	8	6

$$f(n) = 3 + 2 = 5$$

	1	3
4	2	5
7	8	6

$$f(n) = 3 + 4 = 7$$

1	3	
4	2	5
7	8	6

$$f(n) = 3 + 4 = 7$$

1	2	3
4		5
7	8	6

1	2	3
4	5	
7	8	6

$$f(n) = 4 + 1 = 5$$

1	2	3
	4	5
7	8	6

$$f(n) = 4 + 3 = 7$$

1		3
4	2	5
7	8	6

$$f(n) = 4 + 3 = 7$$

1	2	3
4	8	5
7		6

$$f(n) = 4 + 3 = 7$$

1	2	8
4	5	3
7	8	6

$$f(n) = 5 + 2 = 7$$

1	2	3
4	5	6
7	8	

$$f(n) = 5 + 0 = 5$$

(Goal state).

1	2	3
4		5
7	8	6

$$f(n) = 5 + 2 = 7$$

Example 2 :

INITIAL

7	1	8
	6	2
5	4	3

GOAL.

7	8	
6	1	2
5	4	3

Heuristic : Manhattan distance

7	①	⑧
	⑥	2
5	4	3

$$f(n) = 0 + 3$$

7	①	⑧
6		2
5	4	3

$$f(n) = 1 + 2 = 3$$

7	①	⑧
⑤	⑥	2
	4	3

$$f(n) = 1 + 4 = 5$$

	①	⑧
⑦	⑥	2
5	4	3

$$f(n) = 1 + 4 = 5$$

7		⑧
6	1	2
5	4	3

$$f(n) = 2 + 1 = 3$$

7	①	⑧
	⑥	2
5	4	3

$$f(n) = 2 + 3 = 5$$

7	①	⑧
6	②	
5	4	3

$$f(n) = 2 + 3 = 5$$

7	①	⑧
6	④	2
5		3

$$f(n) = 2 + 3 = 5$$

7		⑧
6	1	2
5	4	3

7	8	
6	1	2
5	4	3

$$f(n) = 3 + 0 = 3$$

(Goal state)

	⑦	⑧
6	1	2
5	4	3

$$f(n) = 3 + 2 = 5$$

7	①	⑧
6		2
5	4	3

$$f(n) = 3 + 2 = 5$$

Heuristic : Misplaced cost.

7	①	⑧
	⑥	2
5	4	3

$$f(n) = 0 + 3 = 3$$

	①	⑧
⑦	⑥	2
5	4	3

$$f(n) = 1 + 4 = 5$$

7	①	⑧
6		2
5	4	3

$$f(n) = 1 + 2 = 3$$

7	①	⑧
⑤	⑥	2
	4	3

$$f(n) = 1 + 4 = 5$$

7		⑧
6	1	2
5	4	3

$$f(n) = 2 + 1 = 3$$

7	①	⑧
	⑥	2
5	4	3

$$f(n) = 2 + 3 = 5$$

7	①	⑧
6	②	
5	4	3

$$f(n) = 2 + 3 = 5$$

7	①	⑧
6	④	2
5		3

$$f(n) = 2 + 3 = 5$$

7	①	⑧
6		2
5	4	3

$$f(n) = 3 + 2 = 5$$

	⑦	⑧
6	1	2
5	4	3

$$f(n) = 3 + 2 = 5$$

7	8	
6	1	2
5	4	3

$$f(n) = 3 + 0 = 3$$

(Goal state)

Example 3

Example :- (Manhattan)

Initial :-

1	0	3
4	2	6
7	5	8

$$f(n) = 0 + 3$$

Goal :-

1	2	3
4	5	6
7	8	0

1	3	0
4	2	6
7	5	8

$$f(n) = 1 + 4$$

1	2	3
4	0	6
7	5	8

$$f(n) = 1 + 2$$

0	1	3
4	2	6
7	5	8

$$f(n) = 1 + 4$$

1	0	3
4	2	6
7	5	8

$$f(n) = 2 + 3$$

1	2	3
0	4	6
7	5	8

$$f(n) = 2 + 3$$

1	2	3
4	6	0
7	5	8

$$f(n) = 2 + 3$$

1	2	3
4	5	6
7	0	8

$$f(n) = 2 + 1$$

1	2	3
4	0	6
7	5	8

$$f(n) = 3 + 2$$

1	2	3
4	5	6
0	7	8

$$f(n) = 3 + 2$$

1	2	3
4	5	6
7	8	0

$$f(n) = 3 + 0$$

5

Using Misplaced Tiles Heuristic :-

Initial:-

1	0	3
4	2	6
7	5	8

$$f(n) = 0 + 3$$

Goal:-

1	2	3
4	5	6
7	8	0

&

0	1	3
4	2	6
7	5	8

$$f(n) = 1 + 4$$

1	3	0
4	2	6
7	5	8

$$f(n) = 1 + 4$$

1	2	3
4	0	6
7	5	8

$$f(n) = 1 + 2$$

1	0	3
4	2	6
7	5	8

$$f(n) = 2 + 3$$

1	2	3
0	4	6
7	5	8

$$f(n) = 2 + 3$$

1	2	3
4	5	6
7	0	8

$$f(n) = 2 + 1$$

1	2	3
4	6	0
7	5	8

$$f(n) = 2 + 3$$

1	2	3
4	0	6
7	5	8

$$f(n) = 3 + 2$$

1	2	3
4	5	6
0	7	8

$$f(n) = 3 + 2$$

1	2	3
4	5	6
7	8	0

$$f(n) = 3 + 0$$

Goal

Example 4

Example :- (Manhattan)

Initial :-

4	6	8
2	5	3
0	7	1

Goal :-

4	8	0
2	6	3
7	5	1

$$f(n) = 0 + 4$$

4	6	8
2	5	3
7	0	1

$$f(n) = 1 + 3$$

4	6	8
0	5	3
2	7	1

$$f(n) = 1 + 5$$

4	6	8
2	0	3
7	5	1

$$f(n) = 2 + 2$$

4	6	8
2	5	3
0	7	1

$$f(n) = 2 + 4$$

4	6	8
2	5	3
7	1	0

$$f(n) = 2 + 4$$

4	0	8
2	6	3
7	5	1

$$f(n) = 3 + 1$$

4	6	8
0	2	3
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	5	3
7	0	1

$$f(n) = 3 + 3$$

4	6	8
2	3	0
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	0	3
7	5	1

$$f(n) = 4 + 2$$

4	8	0
2	6	3
7	5	1

$$f(n) = 4 + 0$$

0	4	8
2	6	3
7	5	1

$$f(n) = 4 + 2$$

1	2	3
4	6	0
7	5	8

$$f(n) = 2 + 3$$

2	3
5	6
8	0

$$= 3 + 0$$

(Misplaced Tiles)

I:-

4	6	8
2	5	3
0	7	1

$$f(n) = 0 + 4$$

G:-

4	8	0
2	6	3
7	5	1

4	6	8
0	5	3
2	7	1

$$f(n) = 1 + 5$$

4	6	8
2	5	3
7	0	1

$$f(n) = 1 + 3$$

4	6	8
2	0	3
7	5	1

$$f(n) = 2 + 2$$

4	6	8
2	5	3
0	7	1

$$f(n) = 2 + 4$$

4	6	8
2	5	3
7	1	0

$$f(n) = 2 + 3$$

4	0	8
2	6	3
7	5	1

$$f(n) = 3 + 1$$

4	6	8
0	2	3
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	5	3
7	0	1

$$f(n) = 3 + 3$$

4	6	8
2	3	0
7	5	1

$$f(n) = 3 + 3$$

0	4	8
2	6	3
7	5	1

$$f(n) = 4 + 3$$

4	8	0
2	6	3
7	5	1

$$f(n) = 4 + 0$$

4	6	8
2	0	3
7	5	1

$$f(n) = 4 + 2$$

Example 5

(Misplaced Tiles)

I:-

4	6	8
2	5	3
0	7	1

$$f(n) = 0 + 4$$

G:-

4	8	0
2	6	3
7	5	1

4	6	8
0	5	3
2	7	1

$$f(n) = 1 + 5$$

4	6	8
2	5	3
7	0	1

$$f(n) = 1 + 3$$

4	6	8
2	0	3
7	5	1

$$f(n) = 2 + 2$$

4	6	8
2	5	3
0	7	1

$$f(n) = 2 + 4$$

4	6	8
2	5	3
7	1	0

$$f(n) = 2 + 3$$

4	0	8
2	6	3
7	5	1

$$f(n) = 3 + 1$$

4	6	8
0	2	3
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	5	3
7	0	1

$$f(n) = 3 + 3$$

4	6	8
2	3	0
7	5	1

$$f(n) = 3 + 3$$

0	4	8
2	6	3
7	5	1

$$f(n) = 4 + 3$$

4	8	0
2	6	3
7	5	1

$$f(n) = 4 + 0$$

4	6	8
2	0	3
7	5	1

$$f(n) = 4 + 2$$

Goal.

Example:- (Manhattan)

Initial:-

4	6	8
2	5	3
0	7	1

Goal:-

4	8	0
2	6	3
7	5	1

$$f(n) = 0 + 4$$

4	6	8
2	5	3
7	0	1

$$f(n) = 1 + 3$$

4	6	8
0	5	3
2	7	1

$$f(n) = 1 + 5$$

4	6	8
2	0	3
7	5	1

$$f(n) = 2 + 2$$

4	6	8
2	5	3
0	7	1

$$f(n) = 2 + 4$$

4	6	8
2	5	3
7	1	0

$$f(n) = 2 + ~~4~~$$

4	0	8
2	6	3
7	5	1

$$f(n) = 3 + 1$$

4	6	8
0	2	3
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	5	3
7	0	1

$$f(n) = 3 + 3$$

4	6	8
2	3	0
7	5	1

$$f(n) = 3 + 3$$

4	6	8
2	0	3
7	5	1

$$f(n) = 4 + 2$$

4	8	0
2	6	3
7	5	1

$$f(n) = 4 + 0$$

0	4	8
2	6	3
7	5	1

$$f(n) = 4 + 2$$

Example 6

INITIAL

4	7	5
8	1	3
2	0	6

GOAL

4	7	5
8	3	6
2	1	0

Heuristic : Misplaced count.

4	7	5
8	(1)	(3)
2	0	(6)

$$f(n) = 0 + 3 = 3$$

4	7	5
8		(3)
2	1	(6)

$$f(n) = 1 + 2 = 3$$

4	7	5
8	(1)	(3)
	(2)	(6)

$$f(n) = 1 + 4 = 5$$

4	7	5
8	(1)	(3)
2	(6)	

$$f(n) = 1 + 3 = 4$$

4	7	5
8	(1)	(3)
2		(6)

$$f(n) = 2 + 3 = 5$$

4	7	5
8	3	
2	1	(6)

$$f(n) = 2 + 1 = 3$$

4		5
8	(7)	(3)
2	1	(6)

$$f(n) = 2 + 3 = 5$$

4	7	5
	(8)	(3)
2	1	(6)

$$f(n) = 2 + 3 = 5$$

4	7	5
8		(3)
2	1	(6)

$$f(n) = 3 + 2 = 5$$

4	7	
8	3	(5)
2	1	(6)

$$f(n) = 3 + 2 = 5$$

4	7	5
8	3	6
2	1	

$$f(n) = 3 + 0 = 3$$

(Goal state).

Heuristic : Manhattan distance

Goal

4	7	5
8	①	③
2		⑥

$$f(n) = 0 + 3 = 3$$

4	7	5
8	3	6
2	1	

4	7	5
8		③
2	①	⑥

$$f(n) = 1 + 2 = 3$$

4	7	5
8	①	③
	②	⑥

$$f(n) = 1 + 4 = 5$$

4	7	5
8	①	③
2	⑥	

$$f(n) = 1 + 3 = 4$$

4	7	5
8	①	③
2		⑥

$$f(n) = 2 + 3 = 5$$

4	7	5
8	3	
2	1	⑥

$$f(n) = 2 + 1 = 3$$

4		5
8	⑦	③
2	1	⑥

$$f(n) = 2 + 3 = 5$$

4	7	5
	⑧	③
2	1	⑥

$$f(n) = 2 + 3 = 5$$

4	7	5
8	3	6
2	1	

$$f(n) = 3 + 0 = 3$$

(Goal state)

4	7	5
8		③
2	①	⑥

$$f(n) = 3 + 3 = 6$$

4	7	●
8	3	⑤
2	1	⑥

$$f(n) = 3 + 2 = 5$$

7 . Conclusion :

The A * search algorithm has been implemented and applied to solve 8 puzzle problem , using two different heuristic functions - Manhattan distance and Misplaced tiles count.