Q1)

(a) $A(4,-4,1)$, $B(-4,3,-4)$, $C(4,-1,-2)$

$$AB = \begin{bmatrix}-4\\3\\-4\end{bmatrix} - \begin{bmatrix}4\\-4\\1\end{bmatrix} \Rightarrow \begin{bmatrix}-8\\7\\-5\end{bmatrix}.$$

$$AC = \begin{bmatrix}4\\-1\\-2\end{bmatrix} - \begin{bmatrix}4\\-4\\1\end{bmatrix} \Rightarrow \begin{bmatrix}0\\3\\-3\end{bmatrix}$$

$$n = AB \times AC = \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

$$n = i(-21+15) - j(24) + k(-24)$$

$$n = -6i - 24j - 24k \quad \text{or}$$

$$n = +i + 4j + 4k.$$

$$d = a \cdot n$$

$$(4i - 4j + k) \cdot (i + 4j + k4).$$

$$4 - 16 + 4 \Rightarrow -8.$$

Equation of plane

$$r \cdot n = d.$$

$$(xi + jy + zk) \cdot (i + 4j + 4k) = d.$$

$$x + 4y + 4z + 8 = 0.$$

(c) **Perpendicullar Distance:**

$$P.D = \frac{d}{|n|} = \frac{8}{\sqrt{1+16+16}} = \frac{8}{\sqrt{33}}.$$

(c) line OD : $r = a + \lambda b$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

for some value of λ

$r \cdot n = d.$

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$$2\lambda + \cancel{12}\lambda - \cancel{12}\lambda = -8$$

$$\lambda = -4.$$

$$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix} = -8i - 12j + 12k$$

Q2)

$A = (7i + 4j - k)$, $B(11i + 3j)$, $C(2i + 6j + 3k)$

$D(2i + 7j + \lambda k)$.

i) $AB = +4i - 1j + k.$

$CD = 0i - j + (3 - \lambda)k$

L_1 : Line $AB = OA + \lambda AB$

L_2 : Line $CD = OC + \mu CD.$

$$L_1 : \vec{r}_1 = \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}.$$

$$L_2 : \vec{r}_2 = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -1 \\ 3 - \lambda \end{pmatrix}$$

$$D = \frac{(b_1 \times b_2)\cdot(a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$b_1 \times b_2 = i(-3+\lambda) - j(12-4\lambda) + k(-4).$$

$$|b_1 \times b_2| = \sqrt{\lambda^2 + 9 - 2\lambda + 144 + 16\lambda^2 - 24\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 26\lambda + 169}$$

$$a_2 - a_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}.$$

$$= \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$9(17\lambda^2 - 26\lambda + 169) = (-15 + 5\lambda + 24 - 8\lambda - 8)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda + 24 - 23)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda - 1)^2$$

$$= 49\lambda^2 + 1 - 24\lambda$$

$$\lambda =$$

Then $\lambda^2 - 5\lambda + 4 = 0$

(b) Let Π_1 be plane ABD when $\lambda = 1$.

Let Π_2 be plane ABD when $\lambda = 4$.

i Write equ of Π_1 in form $r = a + sb + tc$

ii Write equ of Π_2 in form $ax + by + cz = d$.

Plane ABD when $\lambda = 1$.

so $AD = 9i + 7j + k$

$AB \times AD$.

$$= \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}.$$

$$= i(-2-3) - j(8+5) + k(12-5).$$

$$= -5i - 13j + 7k$$

Equ: $-5(2) - 13(7)s + 7(1)t$

$$= -10 - 91s + 7t.$$

For Plane Π_2, when $\lambda = 4$.

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 5 \end{vmatrix}.$$

$$= i(-5-3) - j(20-5) + k(12+5).$$

$$= -8i - 15j + 17k.$$

Equ $= -8(x-11) - 15(y-3) + 17(z-0)$.

$$= -8x + 88 - 15y + 45 + 17z$$

$$= 8x + 15y - 17z - 133$$

(c) Angle b/w π_1 & π_2.

$$\varphi = \cos^{-1}\left(\frac{|n_1 \cdot n_2|}{|n_1|\,|n_2|}\right)$$

$$n_1 \cdot n_2 = 4 + 195 + 119$$

$$\varphi = \cos^{-1}\left(\frac{318}{\sqrt{25+169+49}\sqrt{64+225+289}}\right)$$

$$\cos^{-1}\left(\frac{318}{(15.58)(24.04)}\right)$$

$$\cos^{-1}\left(\frac{318}{374.56}\right)$$

$$\varphi = 31.89^\circ$$

Q3)

a) Find value of t.

$L_1 : ti + j \quad -2i - j.$

$l_2 : j + tk \quad -2j + k$

shortest distance b/w lines is $\sqrt{21}$.

$$\vec{r}_1 = OA + \lambda AB$$

$$\vec{r}_2 = OA + \mu AB$$

$$l_1 = \vec{r}_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$l_2 : \vec{r}_2 = \begin{pmatrix} 0 \\ 1 \\ t \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$.

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$= i(1-0) - j(\cancel{t}2) + k(4)$

$= i + j2 + 4k$

$a_2 - a_1 = -ti + kt.$

$$\sqrt{21} = \frac{(i + j2 + 4k) \cdot (-ti + 0j + kt)}{\sqrt{21}}$$

$21 = -t + 4t.$

$t = 7.$

3) $\vec{\wedge}_1 = \vec{r} = OA + AB\lambda + \mu AC$

$= 7i + j + \lambda(-2i - j) + \mu(-2j + k).$

$5x - 6y + 7z = 0.$

$\theta = ?$ b/w l_2 & Π_2.

n_1 of line : $(0, -2, 1)$

n_2 of $\Pi_2 = (5, -6, 7)$

$$\theta = \cos^{-1} \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$$= \cos^{-1}\left(\frac{0 + 12 + 7}{(\sqrt{4+1})(\sqrt{25+36+49})}\right)$$

$$= \cos^{-1}\left(\frac{19}{\sqrt{5}\sqrt{110}}\right)$$

$$= \cos^{-1}\left(\frac{19}{23.45}\right)$$

$$\theta = 35.813^\circ$$

(d) $\pi_1 n = \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix}, \; \pi_2 = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$$\theta = \cos^{-1}\left[\frac{35 - 6 + 0}{\sqrt{49+1}\sqrt{25+36+49}}\right]$$

$$= \cos^{-1}\left(\frac{29}{\sqrt{50}\sqrt{110}}\right)$$

$$\theta = 67.09^\circ$$

Q5)

(a) $P(-2,-1)$, $Q(-6,-3)$ are end points of circle.

$$M.P = \frac{-6-2}{2}, \frac{-3-1}{2}$$

$$= -4, -2.$$

$$(x-h)^2 + (y-k)^2 = r^2.$$

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$r^2 = 5.$$

$$(x+4)^2 + (y+2)^2 = 5.$$

(b)

Let Y be centre of

at point $4, 0$.

$$(4)^2 + (0-b)^2 = r^2.$$

$$16 + b^2 = r^2.$$

at point $(0, 2)$.

$$(0)^2 + (2-b)^2 = r^2$$

$$4 + b^2 - 4b = r^2$$

$$16 + \cancel{b^2} = 4 + \cancel{b^2} - 4b$$

$$12 = -4b$$

$$b = -3$$

$$16 + 9 = r^2$$

$$r = 5.$$

Scanned with CamScanner

(Part c)

$\dot{y} = 100x.$

$y^2 = 4ax.$

$4ax = 100x$

$a = 25.$

$x = -a.$

$-25 = x$

(d) $x^2 = 24y.$

$x^2 = 24y.$

$x^2 = 4ay.$

$\cancel{4ay} = \cancel{24}\cancel{y}.$

$a = +6.$

$y = -6.$

(e) $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$a = 5, \quad b = 4.$

$c = \sqrt{a^2 - b^2} \Rightarrow \sqrt{25 - 16} \Rightarrow 3.$

$F_1 = (-3, 0) \; ; \; F_2 = (3, 0).$

Length $= 2a \Rightarrow 2(5) = 10.$