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Question #1

(a) $\lim_{x,y \to (2,1)} \dfrac{x^2 - 2xy}{x^2 - 4y^2}$

$x^2 = 4y^2$

$x = 2y.$

$y = x/2.$

$$\frac{x^2 - \cancel{2}(x)(x/\cancel{2})}{x^2 - \cancel{4}\left(\frac{x^2}{\cancel{4}}\right)}.$$

$$\frac{\cancel{x^2} - \cancel{x^2}}{\cancel{x^2} - \cancel{x^2}} \Rightarrow \frac{0}{0}.$$

(b) $\lim_{(x,y) \to (0,0)} \dfrac{x - 4y}{6y + 7x}$

let $6y = 7x$

$\Rightarrow y = 7x/6.$

putting y in limit.

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$$\frac{x-4(7x/6)}{7x+7x}$$

$$\frac{x-28x/6}{14x}$$

$$\frac{6x-28x}{6(14x)} \Rightarrow \frac{-\cancel{22}^{11}x}{3\cancel{6}(14)\cancel{x}} \Rightarrow \frac{-11}{52}.$$

(c) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

let $y=mx$

$$\frac{x^2-m^6x^6}{x\,m^3x^3}$$

$$\frac{\cancel{x^2}(1-m^6x^3)}{\cancel{x^4}^{2}m^3}$$

$$\frac{1-m^6x^3}{m^3x^2}$$

d) $\lim\limits_{(x,y,z)\to(-1,0,4)} \frac{x^3-ze^2y}{6x+2y-3z}$

$$\frac{(-1)^3-4(0)e^2}{6(-1)+2(0)-3(4)}$$

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$$\frac{-1-0}{-6+2-12} \Rightarrow \frac{\cancel{-}1}{\cancel{-}16} \Rightarrow 1/16.$$

Question # 2

(a) $f(x,y) = \cos(x/y)$ in $v = (3,-4)$.

$$f\nabla = \frac{\partial}{\partial x}(\cos(x/y))\hat{i} + \frac{\partial}{\partial y}(\cos(x/y))\hat{j}$$

$$= \frac{-1\sin(x/y)}{y}\hat{i} + x(-\sin x/y)\left(\frac{-1}{y^2}\right)\hat{j}.$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)\hat{i} + \frac{x}{y^2}\sin(x/y)\hat{j}.$$

unitvector :

$$\frac{3\hat{i}-4\hat{j}}{\sqrt{9+16}} \Rightarrow \frac{3}{\sqrt{25}}\hat{i} - \frac{4}{\sqrt{25}}\hat{j}$$

$$D_{\vec{u}}f = -\frac{3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y^2}\sin(x/y).$$

$$D_{\vec{u}}f = \frac{1}{5y}\sin(x/y)\left(\frac{4x}{y} - 3\right).$$

(b) $f(x,y,z) = x^2y^3 - 4xz$; $\vec{v} = (-1,2,0)$

$$\nabla f = (2y^3x\hat{i} + (-4z)\hat{i}) + 3y^2x^2\hat{j} - 4x\hat{k}.$$

$$\hat{v} = \frac{-\hat{i}+2\hat{j}+0\hat{k}}{\sqrt{1+4}} \Rightarrow \frac{-1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j} + 0\hat{k}$$

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$D\vec{u}f = -\frac{1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^2) + 0.$

Question #3

$f(x,y,z) = 4x - y^2 e^{3xz}$

$\nabla f = 4 - y^2 e^{3xz}(3z)\,i - 2y e^{3xz_0}\,j - y^2 e^{3xz} \cdot 3x\,k$

$\nabla f|_{(3,-1,0)} = 4 - (-1)^2 e^{3(3)(0)}(2)(3)\,i - 2(-1)e^{3(3)(0)}\,j - (-1)^2 e^{3(3)(0)} \cdot 3(3)$

$= (4-0)\,i - 2j - 9k$

$N = (-1, 4, 2)$

$\vec{u} = \frac{-1i + 4j + 2k}{\sqrt{1+16+4}} \Rightarrow \frac{-1}{\sqrt{21}}i + \frac{4}{\sqrt{21}}j + \frac{2}{\sqrt{21}}k$

$\frac{-1}{\sqrt{21}}(4) - \frac{2(4)}{\sqrt{21}} - \frac{9(2)}{\sqrt{21}}$

$\frac{-4}{21} - \frac{8}{\sqrt{21}} - \frac{18}{\sqrt{21}} \Rightarrow \frac{-4-8-18}{\sqrt{21}} \Rightarrow \frac{-32}{\sqrt{21}}$

Question # 4

(a) $f(x,y) = \sqrt{x^2+y^3}$ at $(-2,3)$

$\nabla f = \frac{1}{\cancel{2}}(x^2+y^3)^{-1/2}(\cancel{2}x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)j$

$\nabla f(-2,3) = (4+9)^{-1/2}(-2)i + \frac{1}{2}(4+9)^{-1/2}(3(9))j$

$= -\frac{2}{\sqrt{13}}i + \frac{27}{2\sqrt{13}}j$

(b) $f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$\nabla f = (e^{2x}.2\cos(y-2z))i + e^{2x}(-\sin(y-2z)(1))j$
$+ e^{2x}(-\sin(y-2z)(-2)k$

$\nabla f(4,-2,0) = e^{4x}.2\cos(-2-0)i + e^{4x}(-\sin(-2))j$
$+ e^{4x}(-\sin(-2)(-2)$

$\nabla f(4,-2,0) = e^{4x}(-2\cos 2\, i - \sin(-2)j + 2\sin(-2)k)$

Question #5

(a) $F = x^2 y \hat{i} - (z^3 - 3x)\hat{j} + 4y^2 \hat{k}$

$\nabla = 2xy\,i - 0j + 0.$

Div $f = \nabla f \cdot f$.

$= (2xy\,i) \cdot (x^2 y\, i - (z^3 - 3x) j + 4y^2 k)$.

Div $f = 2x^3 y^2 + 0$

curve $= \nabla f \times f$.

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -(z^3 - x3) & 4y^2 \end{vmatrix}$$

$= i(8y + 3z^2) - j(0-0) + k(+3x^2 - x^2)$.

$= (8y + 3z^2)\,i - 0j + (2x^2)k$.

$F = (2x + 2z^2)\,i + \frac{x^3 y^2}{z} j - (z \cdot 7x) k$

div $= \nabla f \cdot f$

$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot \left((2x + 2z^2)i + \frac{x^3 y^2}{2} j - (z\right.$

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$$= z + \frac{3x^3}{z}y - (1)$$

$$= z + \frac{9x^3}{z}y - 1.$$

Curve $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x + 9x^2 & \frac{x^3y^2}{z} & -(z - 7x) \end{vmatrix}$$

$$i\left(0 + \frac{x^3y^2}{z^2}\right) - j(7 - 4x) + k\left(\frac{3x^2y^2}{z} - 0\right)$$

$$\frac{x^3y^2}{z^2}i - (7 - 4z)j + \left(\frac{3x^2y^2}{z}\right)k.$$

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Question # 6

(a) $\vec{F} = \left(4y^2 + \frac{3x^2 y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)j + \left(11 - \frac{2x^3 y}{z^3}\right)k.$

vectors are conservative only if ;

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z^2}, \quad \frac{\partial N}{\partial x} = 8y + \frac{3x^2}{z^2}$$

$$\frac{\partial P}{\partial x} = -\frac{6x^2 y}{z^3}, \quad \frac{\partial M}{\partial z} = -\frac{6x^2 y}{z^3}$$

$$\frac{\partial M}{\partial z} = -\frac{8x^2 y}{z^3}.$$

Hence vector is conservative.

(b) $F = \underline{6x}\,i + \underline{(2x - y^2)}\,j + \underline{(6z - x^3)}\,k$

$\quad M \qquad N \qquad P.$

$$\frac{\partial M}{\partial y} = 0 \; ; \; \frac{\partial N}{\partial x} = 2 \; ; \; \frac{\partial N}{\partial z} = 0$$

$$\frac{\partial P}{\partial y} = 0 \; ; \; \frac{\partial M}{\partial z} = 0 \quad \frac{\partial P}{\partial x} = -3x^3$$

Since $\partial P/\partial x$ & $\partial N/\partial x$ are not equal

it is non-conservative.

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Question # 7

(a) $z = \dfrac{x^2 - \omega}{y^4}$; $x = t^3 + 7$; $y = \cos(2t)$

$\omega = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial \omega} + \frac{d\omega}{\partial t}.$$

$$= \left(\frac{2x}{y^4} \cdot 3t^2\right) + 4\left(-\frac{1}{y^5}(x^2 - \omega)\right) \cdot (-\sin 2t)(2)$$

$$+ \frac{-1}{y^4} \cdot 4.$$

$$= \frac{6xt^2}{y^4} + \frac{8 \sin 2t\,(x^2 - \omega)}{y^5} - \frac{4}{y^4}$$

$$= \frac{2}{y^4}\left(3xt^2 + \frac{4 \sin 2t\,(x^2 - \omega)}{y} - 2\right)$$

(b) $z = x^2 y^4 - 2y$, $y = \sin(x^2)$

$$\frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}.$$

$$= (4y^3 x^2 - 2) \cdot \cos(x^2) \cdot 2x$$

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$8y^3x^3\cos(x^2) - 4x\cos(x^2)$.

(c) Find $\frac{dy}{dx}$ for

$$x^2y^4 - 3 = \sin(xy)$$

$$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}(\sin(xy))$$

$$2xy^4 + 4y^3\frac{dy}{dx}x^2 = \cos(xy)\left(1y + \frac{dy}{dx}x\right).$$

$$2xy^4 + 4y^3\frac{dy}{dx}x^2 = y\cos xy + \frac{dy}{dx}x\cos xy.$$

$$2xy^4 - y\cos xy = x\frac{dy}{dx}\cos xy - 4y^3x^2\frac{dy}{dx}.$$

$$2xy^4 - y\cos xy = \frac{dy}{dx}(x\cos xy - 4y^3x^2).$$

$$\frac{dy}{dx} = \frac{2xy^4 - y\cos xy}{x\cos xy - 4y^3x^2}.$$

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