IE524 HW1 Solution

1 Problem 1

There are infinitely many possible solutions for this problem. We will write an example for each.

1.1

$$\begin{array}{ll} \max & x_1 + x_2 + x_3 \\ \text{s.t.} & 2x_1 + 3x_2 + x_3 \leq 5 \\ & 3x_1 + 4x_2 + x_3 \leq 7 \\ & x_1,\, x_2,\, x_3 \geq 0 \end{array}$$

The optimal solution is $(0, 0, 5)$.

1.2

$$\begin{array}{ll} \max & x_1 + x_2 + x_3 + x_4 \\ \text{s.t.} & x_1 + x_2 + x_3 + x_4 \leq 1 \\ & x_1,\, x_2,\, x_3 \geq 0 \\ & x_4 \text{ unconstrained} \end{array}$$

1.3

$$\begin{array}{ll} \min & -x_1 - 2x_2 - 3x_3 \\ \text{s.t.} & x_1,\, x_2,\, x_3 \geq 0 \end{array}$$

1.4

$$\begin{array}{ll} \max & x_1 + x_2 + x_3 + x_4 \\ \text{s.t.} & x_1 + x_2 - x_3 = 1 \\ & x_1 - x_2 + x_3 = -2 \\ & x_1,\, x_2,\, x_3,\, x_4 \geq 0 \end{array}$$

2 Problem 2

2.1

There are 5 constraints so $\binom{5}{2} = 10$ different choices.

2.2

The feasible solutions are $(0, 0)$, $(0, 4)$, $(2, 2)$, $(8/3, 1)$, $(3, 0)$.

2.3

The value of objective function are 0, 12, 14, 41/3, 12 respectively. So the optimal solution is $(2, 2)$.

3 Problem 3

3.1

The optimal value is -9. Because $(1) + (2)$ can give us $x_1 - 2x_2 \leq -3$, so

$$2x_1 - 6x_2 \leq 3x_1 - 6x_2 \leq 3(x_1 - 2x_2) \leq -9,$$

i.e., the optimal value has to be no greater than -9. And it can be attained when $x = (0, \frac{3}{2}, \frac{5}{2})$.

3.2

We can think of this problem as having four types of resources, with total capacity of 1. Each resource can bring us some amount of profit. To maximize the total profit, it's straightforward that we should hold only the most profitable resource, which means the optimal solution is attained when $x = (0, 0, 0, 1)$ and the optimal value is 9.

3.3

This problem is infeasible. Because $(1) + (3)$ can give us $x_2 \leq -1$, which cannot hold with $x_2 \geq 0$ simultaneously.