Homework 5: Due on Oct 5, Thursday: 50 points

Problem: (50 points) The Excel spreadsheet uploaded to Canvas with this homework shows historical returns (1961-2003) of Stock (S & P 500), Bond (10-year treasury), Money Market fund, and NASDAQ Composite Index.

1. (10 points) Use the data to estimate the (arithmetic) mean return of each asset and the covariance matrix. Recall that the covariance of assets i and j can be estimated by

$$\sigma_{ij} = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j),$$

   where T is the number of years, $\bar{r}_i$ and $\bar{r}_j$ are estimated mean returns of assets i and j, and r_{it} and r_{jt} are actual returns of these assets in year t $(1 \leq t \leq T)$.

2. (25 points) Let the risk-free return be 3%, solve a nonlinear optimization model to construct a portfolio of these four assets to maximize the Sharpe Ratio.

3. (15 points) Use the same data, solve a linear program model to construct a portfolio of these four assets to minimize the Mean Absolute Deviation under the condition that the mean return of the portfolio is at least 9%.