**Homework 1: Due on August 31, Thursday (50 points)**

*Problem 1* (20 points, 5 points for each question) Develop linear program models for each of the following four cases:

1. case 1: has at least three decision variables and two constraints (excluding non-negativity constraints), and has one unique optimal solution;
2. case 2: has at least four decision variables, one of them can be negative, and the problem has more than one optimal solutions;
3. case 3: has at least three decision variables, none of them can be negative, the goal is to minimize an objective function that is unbounded (i.e., the optimal objective value is $\infty$);
4. case 4: has at least four variables, and the problem has no feasible solution.

In each case, the formulation should be in one of the two standard forms.

*Problem 2* (15 points, 5 points for each question): Consider the following linear program:

$$\begin{aligned} & \max\{4x_1 + 3x_2\} \\ \text{subject to:} \quad & 3x_1 + x_2 \leq 9 \\ & 3x_1 + 2x_2 \leq 10 \\ & x_1 + x_2 \leq 4 \\ & x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

1. (3 points): how many solutions you can get by selecting two constraints and solving them as equalities?
2. (4 points): which of these solutions are feasible solutions for the problem?
3. (3 points): which solution is optimal?

*Problem 3* (15 points, 5 points for each question): Solve the following Linear Programs (you can use any method you think is the easiest. In fact, by making clever observations, you may be able to arrive at the answers quickly without doing anything complicated and/or tedious):

1.

$$\begin{aligned} & \max\{2x_1 - 6x_2\} \\ \text{subject to:} \quad & -x_1 - x_2 - x_3 \leq -4 \\ & 2x_1 - x_2 + x_3 \leq 1 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

2.

$$\begin{aligned} & \max\{6x_1 + 8x_2 + 5x_3 + 9x_4\} \\ \text{subject to:} \quad & x_1 + x_2 + x_3 + x_4 \leq 1 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$

3.

$$\begin{aligned} & \max\{2x_1 + 3x_2\} \\ \text{subject to:} \quad & -x_1 - x_2 \leq -3 \\ & -x_1 + x_2 \leq -1 \\ & x_1 + 2x_2 \leq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$