Homework 6: Integer Programming Model (optional)

Problem 1: Solve the following ILP model by logic reasoning:

$$\max_{\mathbf{x}} \{3x_1 - 2x_2 - 4x_3 + 6x_4 + x_5 - x_6\}$$

subject to:
$$x_1 + 2x_2 - x_3 \leq 1$$
$$x_2 + x_4 \geq 2$$
$$x_4 + 2x_5 \leq 2$$
$$x_1 + x_5 + x_6 \geq 1$$
$$x_2, x_3, x_4, x_6 \in \{0, 1\} \quad \text{and} \quad x_1, x_5 \text{ are non-negative integers}$$

Problem 2: You have a $250,000 budget to invest in the following assets. The cash inflows/outflows are in the table below (notice that -1.00 represents the initial investment) and reinvestment of profit is allowed.

	Year 1	Year 2	Year 3	Year 4
asset 1	-1.00		1.18	
aseet 2		-1.00		1.22
aseet 3			-1.00	1.10
asset 4	-1.00	0.14	0.14	1.00
asset 5		-1.00	0.20	1.00

(For example, the table shows that if you invest one dollar in asset 1 (-1.00) at the beginning of Year 1, you will receive $1.18 at the beginning of Year 3). **If you invest in any of these assets, the required minimum level is $100,000 in each case**. Any or all the available funds at the beginning of a year can be placed in a money market account that yields 3% per year.

Question: Formulate a mixed integer linear program to maximize the amount of money available at the end (i.e., beginning of Year 4) and use AMPL/Gurobi to solve your model.

Problem 3: You have a current portfolio of eight stocks. You have also identified a better portfolio with optimized mean/variance ratio. Investment allocations of these two portfolios are shown in the following table:

stock	A	B	C	D	E	F	G	H
current portfolio	0.12	0.15	0.13	0.10	0.20	0.10	0.12	0.08
portfolio with opt. M/V	0.02	0.05	0.25	0.06	0.18	0.10	0.22	0.12

You would like to rebalance your portfolio in order to be closer to the optimal M/V portfolio. To avoid excessively high transaction costs, you decide to rebalance only three stocks from your portfolio. Let x_i denote the weight of stock i in our replaced portfolio. The objective is to minimize

$$|x_1 - 0.02| + |x_2 - 0.05| + |x_3 - 0.25| + \cdots + |x_8 - 0.12|,$$

to make the rebalanced portfolio to match closely with the better portfolio.

Question: Formulate this problem as a mixed integer linear problem.

hint: You may want to consider the techniques of handling absolute values that we discussed in previous lectures.