Homework 2: Due on September 7, Thursday (50 points)

Problem 1: (10 points) Formulate a Linear Program for a portfolio management problem. Different from the formulation in the lecture notes, the objective here is to minimize MAD. Regarding the mean turn, the decision is subject to the constraint that it has to reach a target level $\bar{R}$.

Problem 2: (40 points) The table below shows a company's cash flow in the next six months

1	2	3	4	5	6
-200	100	-150	50	-300	600

The company needs to borrow from an existing credit line and issue commercial papers to keep its net cash position in each month from falling below zero. Decisions are made under the following conditions:

1. The company starts from a clean slate, i.e., the cash position, outstanding commercial papers, and credit line balance are all zero at the beginning of month 1.

2. The credit line has a borrowing limit of 100 and charges an interest 2% each month.

3. Five different commercial papers can be issued. Their (issue date, retirement date, coupon rate) are respectively: $(1, 2, 1\%)$, $(1, 4, 5\%)$, $(2, 5, 4\%)$, $(3, 6, 5\%)$, and $(4, 6, 3\%)$.

4. The company gets 0.5% interest on positive cash balance carried over from one month to the next.

5. The company plans to pay off all its debts at the end of month 6.

6. The company wants to maximizes its net cash position after month 6.

Questions:

1. (10 points) Develop a linear program formulation for this problem. You can do that either by giving mathematical expressions or writing a .mod file in AMPL.

   useful hint: in AMPL, you can use equality constraint directly without transform into the standard form. You can also use conditional statement to simplify the expression of constraints. For instance, if we require $x(t) \leq y(t)$ for all $t = 2, \cdots, T$ ($t = 1, \cdots, T$), rather than writing separate constraints for each t, we can write in .mod file:

   set Time;

param x[Time];

param y[Time];

subject to constrainyt-on-xy $\{t$ in Time $: t > 1\} : x[t] <= y[t]$;

Of course, elements in Time and values of x[Time] and y[Time] are specified in .dat file.

2. (20 points) solve the LP to determine 1) the amount of each commercial paper to be issued, 2) when and by how much the company should draw from the credit line, and 3) the resulting cash position in each month. (You may use other software or methods to solve the problem as long as the procedure and solutions are well documented).

3. (10 points) use the LP solution to answer the following questions

   - (5 points) Let the interest rate of the credit line to increase at 1% increment. Determine the minimum value of the interest under which the company never uses the credit line and borrows only by issuing commercial papers.

   - (5 points) How will the solution change if the commercial paper $(2, 5, 4\%)$ is no longer available?