Q1

1) $\max_x \{3x_1 - 2\overset{(1)}{x_2} - 4x_3 + 6\overset{(1)}{x_4} + \overset{0}{x_5} - x_6\}$

s.t. : $x_1 + 2x_2 - x_3 \le 1 \rightarrow x_1 = 0,\ x_3 = 1$

$x_2 + x_4 \ge 2 \rightarrow x_2 = 1,\ x_4 = 1$

$x_4 + 2x_5 \le 2 \qquad x_5 \le 0 = 0$

$0 \leftarrow x_1 + x_5 + x_6 \ge 1 \qquad x_6 \ge 1 \rightarrow [1, \infty)$

$x_2, x_3, x_4, x_6 \in \{0,1\}$ and x_1, x_5 are non neg int

opt

$\max_x \{3\overset{(0)}{x_1} - 2\overset{(1)}{x_2} - 4\overset{(1)}{x_3} + 6\overset{(1)}{x_4} + \overset{(0)}{x_5} - \overset{[1,\infty)}{x_6}\}$

$0 - \cancel{2} \; 4 + \cancel{6} + 0 - 1 = -7$

Q2

2)

$Y_1 = 250\,K$; invest x_1, x_4

$Y_2 = (Y_1 - x_1 - x_4) \times 0.03 + 0.14 x_4$; invest x_2, x_5

$Y_3 = (Y_2 - x_2 - x_5) \times 0.03 + 1.18 x_1 + 0.14 x_4 + 0.20 x_5$ invest x_3

$Y_4 = (Y_3 - x_3) \times 0.03 + 1.22 x_2 + 1.1 x_3 + 1.00 x_4 + 1.00 x_5$

$\max_x \{Y_4\}$

s.t. $z_i \in \{0,1\}$; $i \in \{1,..5\}$; $x_1 + x_4 \le Y_1$

$x_i \ge 100{,}000 \times z_i$; $\forall\, i \in \{1,..5\}$; $x_2 + x_5 \le Y_2$

$x_3 \le Y_3$

```
In [ ]: from amplpy import AMPL, Environment

# Define the AMPL model
model = """
param Y1 := 250000;

var x{1..5} >= 0;
var z{1..5} binary;

subject to con1: x[1] + x[4] <= Y1;
subject to con2: x[2] + x[5] <= (Y1 - x[1] - x[4]) * 0.03 + 0.14 * x[4];
subject to con3: x[3] <= ((Y1 - x[1] - x[4]) * 0.03 + 0.14 * x[4] - x[2] - x[5]) * 0.03 + 1.18 * x[1] + 0.14 * x[4] + 0.2 * x[5];

subject to con4{i in 1..5}: x[i] >= 100000 * z[i];

maximize obj: (((Y1 - x[1] - x[4]) * 0.03 + 0.14 * x[4] - x[2] - x[5]) * 0.03 + 1.18 * x[1] + 0.14 * x[4] + 0.2 * x[5] - x[3]) * 0.03 + 1.22 * x[2] + 1.1 * x[3] + x[4] + x[5];
"""

# Initialize AMPL environment
ampl = AMPL()

# Load the model into AMPL
ampl.eval(model)

# Choose a solver, for instance, 'cplex'
ampl.setOption('solver', 'gurobi')

# Solve the problem
ampl.solve()

# Display the results
print("Objective value:", ampl.getObjective('obj').value())
for i in range(1, 6):
    print(f"x[{i}] =", ampl.getVariable('x').get(i).value())
    print(f"z[{i}] =", ampl.getVariable('z').get(i).value())
```

```
Gurobi 10.0.3:Gurobi 10.0.3: optimal solution; objective 331200
0 simplex iterations

Objective value: 331200.0
x[1] = 0.0
z[1] = 0.0
x[2] = 0.0
z[2] = 0.0
x[3] = 42000.0
z[3] = 0.0
x[4] = 250000.0
z[4] = 0.0
x[5] = 35000.00000000001
z[5] = 0.0
```

Q3

3) minimize MAD

$$\min \left\{ \sum_{i=1}^{8} z_i^+ + z_i^- \right\};$$

Let
x_i: weight
$y_i = \{0,1\}$ for choosing stock

s.t. $z_i^+ \geq x_i -$ optimal weight$_i$
$\quad z_i^- \geq$ optimal weight$_i$ $- x_i$ $\quad \forall i \in \{1,\ldots,8\}$

$\sum_{i=1}^{8} y_i = 3$; $x_i =$ current weight$_i$ + (rebalancing weight$_i$) × y_i

$x_i \geq 0$

$\sum_{i=1}^{8} x_i \leq 1$ $\quad \boxed{z^+, z^- \geq 0}$ *