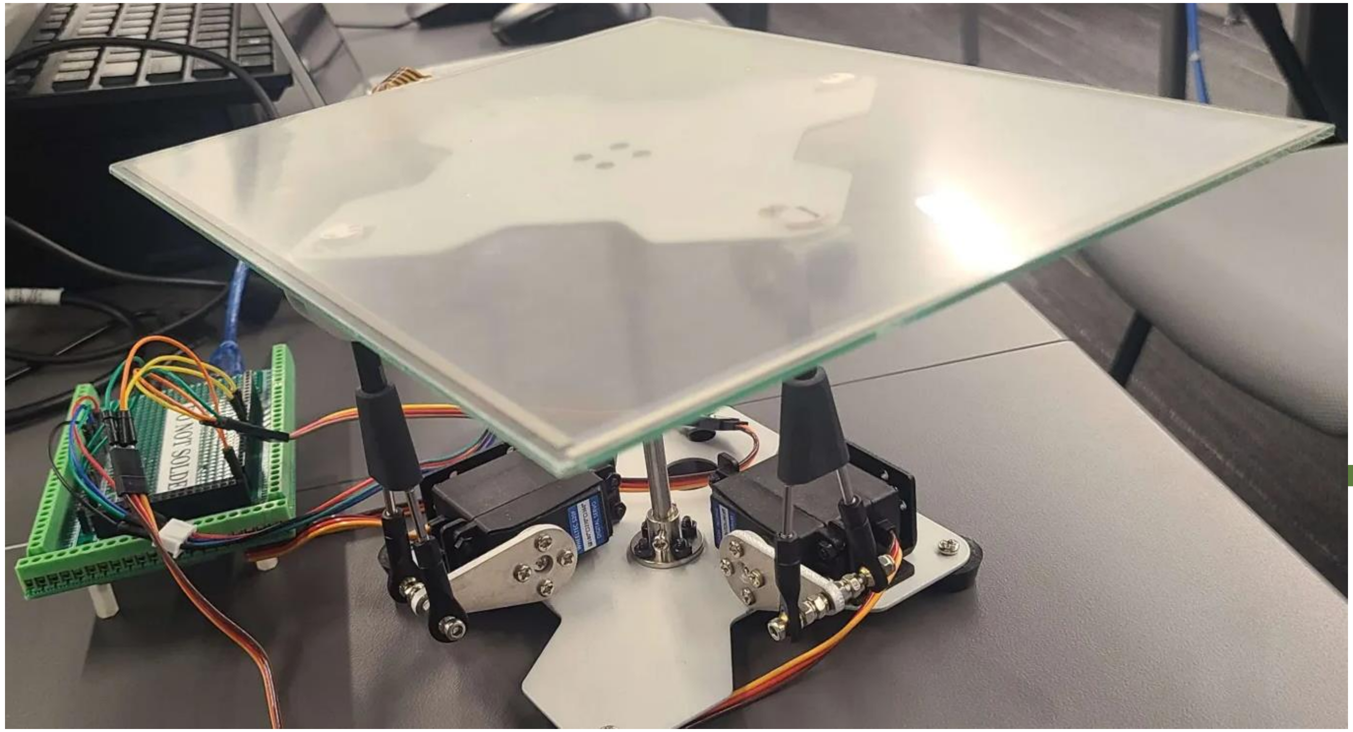
MSE483
Ball balancing platform

Agenda

- State-space formulation
- Linearization
- Controllability and observability
- State feedback control/PID control
- Implementation
- Demo



Introduction

Ball balancing platform.

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State-space formulation

$$\therefore\ (I_p + I_b)\ddot{\alpha} + m_b\left(x_b^2\ddot{\alpha} + 2\dot{\alpha}x_b\dot{x}_b + \dot{x}_b y_b\dot{\beta} + x_b\dot{y}_b\dot{\beta} + x_b y_b\ddot{\beta}\right) + m_b g x_b \cos\alpha = 0$$

Similarly:

$$(I_p + I_b)\ddot{\beta} + m_b\left(y_b^2\ddot{\beta} + 2\dot{\beta}y_b\dot{y}_b + \dot{y}_b x_b\dot{\alpha} + y_b\dot{x}_b\dot{\alpha} + x_b y_b\ddot{\alpha}\right) + m_b g y_b \cos\beta = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_b}\right) - \frac{\partial L}{\partial x_b} = 0$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b - m_b\dot{\alpha}\left(x_b\dot{\alpha} + y_b\dot{\beta}\right) + m_b g \sin\alpha = 0$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b - m_b\dot{\beta}\left(x_b\dot{\alpha} + y_b\dot{\beta}\right) + m_b g \sin\beta = 0$$

4 equations.

Linearization

Assume $\left.\begin{array}{l}\alpha \ll 1 \\ \beta \ll 1\end{array}\right| \Rightarrow \begin{array}{l}\sin\alpha \approx \alpha \\ \sin\beta \approx \beta\end{array}$

$\left.\begin{array}{l}\dot{\alpha} \ll 0 \\ \dot{\beta} \ll 0\end{array}\right| \Rightarrow \begin{array}{l}\dot{\alpha}\dot{\beta} = 0 \\ \dot{\alpha}^2, \dot{\beta}^2 = 0\end{array}$

$, B = m_b\left(m_b + I_b/r_b^2\right)^{-1}$

Assume, $u_1 = \ddot{\alpha}$, $u_2 = \ddot{\beta}$

$\longrightarrow B = \dfrac{m_b}{m_b + I_b/r_b^2} = \dfrac{\cancel{m_b}}{\cancel{m_b}\left(1 + \frac{2}{5}\cancel{r^2}/\cancel{r^2}\right)}$

$I = \frac{2}{5} M r^2$

$= \dfrac{1}{(7/5)} = 5/7$

$$\dot{\tilde{X}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \\ \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\beta} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -gB & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -Bg & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tilde{X} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tilde{u}$$

```
A = [0 1 0 0 0 0 0 0;
     0 0 0 0 g*Bi 0 0 0;
     0 0 0 1 0 0 0 0;
     0 0 0 0 0 0 g*Bi 0;
     0 0 0 0 0 1 0 0;
     0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 1;
     0 0 0 0 0 0 0 0;
     ]
```

```
B = [0 0;
     0 0;
     0 0;
     0 0;
     0 0;
     1 0;
     0 0;
     0 1;
     ]
```

```
C = [1 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 ]

D = [0 0 ; 0 0]

Co = ctrb(A,B);
rank(Co)
```

rank of $P = 8 = n$

the system is Controllable

```
ans =

     8
```

```
Ob = obsv( A , C )
rank(Ob)
```

Controllability and Observability

```
p1 = -2 + 1i;
p2 = -2 - 1i;
p3 = -40; %%as far to lft as possible so they can be neglected
p4 = -80; %%as far to lft as possible so they can be neglected

p5 = -2 + 1i;
p6 = -2 - 1i;
p7 = -40; %%as far to lft as possible so they can be neglected
p8 = -80; %%as far to lft as possible so they can be neglected

%State feedback gain K
K = place(A,B,[p1, p2, p3, p4, p5, p6, p7, p8]);
A_cl = A-B*K;
sysk = ss(A_cl,B,C,D);
sysk_tf = tf(sysk);
```

Pole placement

2.2834e+03	2.9559e-12
-1.4612e-11	2.2834e+03

$\bar{N}$ (reference gain matrix)

1	2.2834e+03	1.9123e+03	2.9559e-12	5.1159e-13	3.6850e+03	124.0000	-3.3945e-13	2.4399e-15
2	-1.4612e-11	-1.3400e-11	2.2834e+03	1.9123e+03	-2.7151e-11	-4.5233e-13	3.6850e+03	124.0000

K (gain matrix)

State feedback control

Implementation

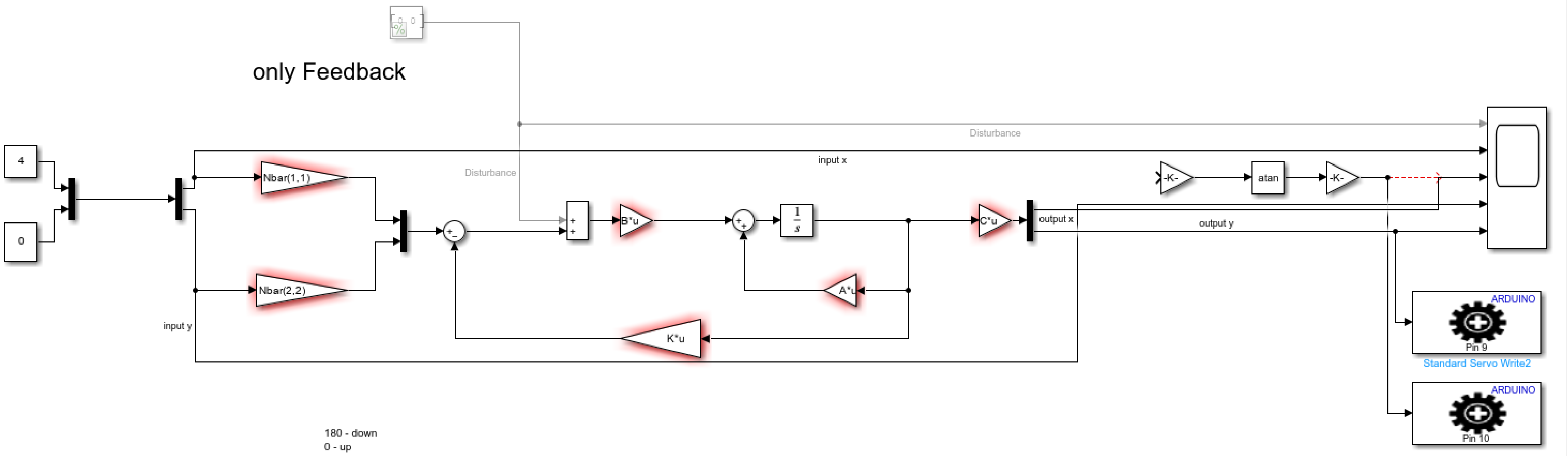
Arduino Mega

SIMULINK

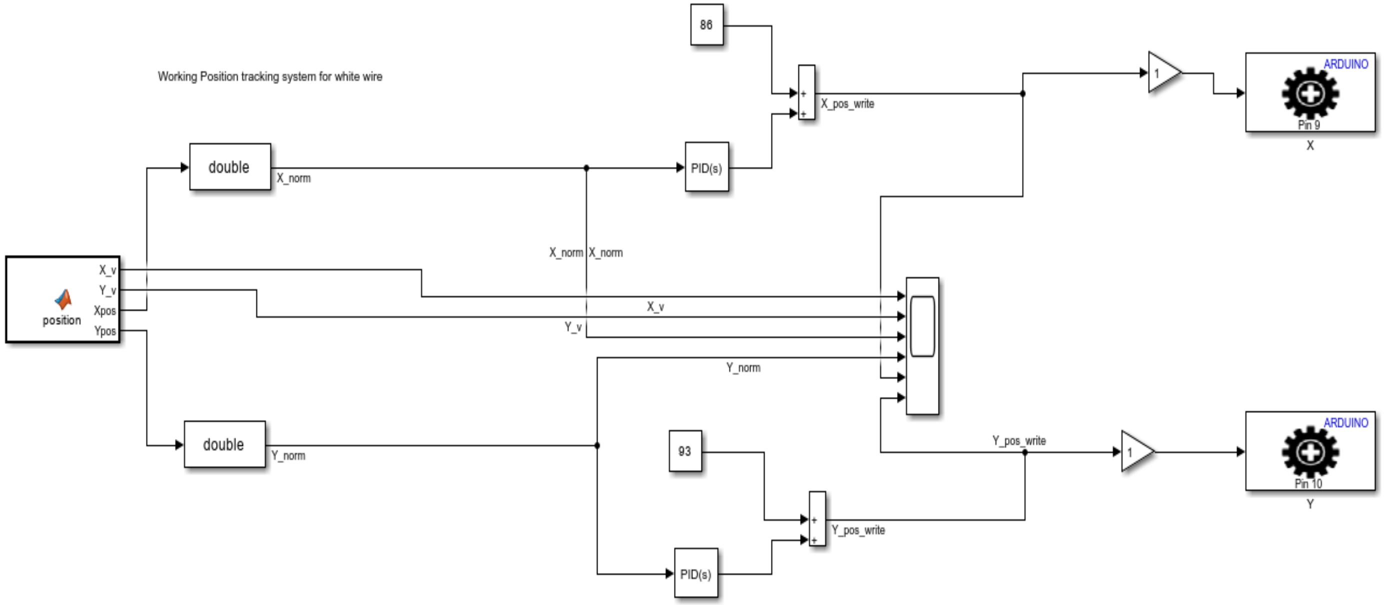
Servo

4-wire resistive
touch screen

Simulink state feedback approach



Simulink PID model approach



Challenges

1. Unable to simultaneously control both outputs.
2. Memory issues.
3. Unable to tune PID unless both servos work simultaneously.

Demo Video

https://drive.google.com/file/d/1iGkBZLbsMibquxD0axVuU7I9PWc2iqFJ/view?usp=sharing

Thank you

Questions?