A new SymPy backend for Vector: uniting experimental and theoretical physicists

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Vector

A Python library for JIT-compilable mathematical manipulations of Lorentz vectors, especially arrays of vectors, in a NumPy-like way.

```
vector.MomentumObject4D(pt=0.3, phi=0.5, eta=3.3, m=0.1)
```

Pure Python Objects

```
vector.VectorNumpy4D(
    {
        "x": [1.1, 1.2, 1.3, 1.4, 1.5],
        "y": [2.1, 2.2, 2.3, 2.4, 2.5],
        "z": [3.1, 3.2, 3.3, 3.4, 3.5],
        "t": [4.1, 4.2, 4.3, 4.4, 4.5],
    }
)
```

NumPy arrays

```
vector.Array(
    [
        [
            {"x": 1, "y": 1.1, "z": 0.1},
            {"x": 2, "y": 2.2, "z": 0.2}
        ],
        [],
        [{"x": 3, "y": 3.3, "z": 0.3}],
        [
            {"x": 4, "y": 4.4, "z": 0.4},
            {"x": 5, "y": 5.5, "z": 0.5},
            {"x": 6, "y": 6.6, "z": 0.6},
        ],
    ]
)
```

Awkward arrays

12 coordinate systems - cartesian, cylindrical, pseudorapidity, and any combination of these with time or proper time for 4D vectors.

```
v1.to_4D().like(v2).boost(v3).deltaR(v4).px
v1.rotate_axis(...).rotate_euler(...).transform4D(...)
```

Same API for every type of vector

Uses conventions set up by ROOT's TLorentzVector and Math::LorentzVector.

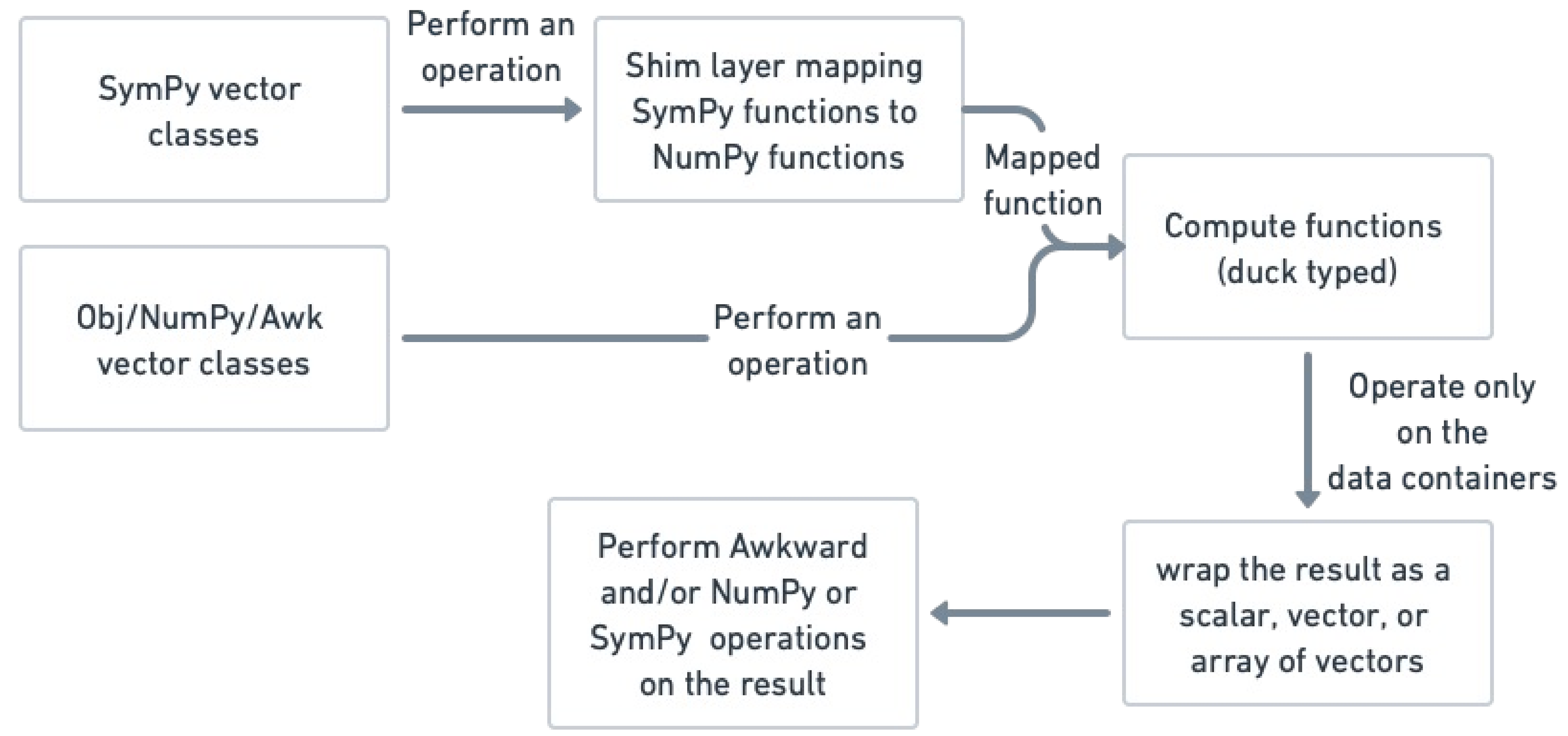
Integrations

Motivation

Symbolic computations are useful for theorists; now the same library can be used by both experimentalists and theorists.

A SymPy backend is a very different kind of backend because it's not numerical. We wanted to show that Vector is generic enough to support such far-flung use-cases.

Architecture



Results

```
v = vector.MomentumObject(pt=1, phi=2, eta=3, M=4)
v.boost(v.to_beta3()).px
np.float64(-2.2540970733043526)
```

Computations on Object type vectors

```
pt, phi, eta, M = sympy.symbols("pt phi eta M")
v = vector.MomentumSympy4D(pt=pt, phi=phi, eta=eta, M=M)
```

Sympy vector classes as drop-in replacement

```
v.boost(v.to_beta3()).px
```

$$\frac{pt^3\sin^2(\phi)\cos(\phi)}{\left(1+\frac{1}{\sqrt{-\frac{pt^2\sin^2(\phi)}{M^2+0.25pt^2\left(1+e^{-2\eta}\right)^2e^{2\eta}}-\frac{pt^2\cos^2(\phi)}{M^2+0.25pt^2\left(1+e^{-2\eta}\right)^2e^{2\eta}}-\frac{pt^2\sinh^2(\eta)}{M^2+0.25pt^2\left(1+e^{-2\eta}\right)^2e^{2\eta}}+1}}\right)\left(M^2+0.25pt^2(1+e^{-2\eta})^2e^{2\eta}\right)\left(-\frac{pt^2\sin^2(\phi)}{M^2+0.25pt^2(1+e^{-2\eta})^2e^{2\eta}}-\frac{pt^2\cos^2(\phi)}{M^2+0.25pt^2(1+e^{-2\eta})^2e^{2\eta}}-\frac{pt^2\sinh^2(\eta)}{M^2+0.25pt^2(1+e^{-2\eta})^2e^{2\eta}}+1\right)}+\cdots$$

Symbolic calculations with the same API

```
v.boost(v.to_beta3()).px.simplify()
```

$$\frac{pt\left(1.0M^2\sqrt{(M^2e^{2\eta}+0.25pt^2e^{4\eta}+0.5pt^2e^{2\eta}+0.25pt^2)e^{-2\eta}}e^{2\eta}+0.25pt^2\sqrt{(M^2e^{2\eta}+0.25pt^2e^{4\eta}+0.5pt^2e^{2\eta}+0.25pt^2)e^{-2\eta}}e^{4\eta}+0.5pt^2\sqrt{(M^2e^{2\eta}+0.25pt^2e^{4\eta}+0.5pt^2e^{2\eta}+0.25pt^2)e^{-2\eta}}e^{2\eta}+\cdots\right.}{\sqrt{(M^2e^{2\eta}+0.25pt^2e^{4\eta}+0.5pt^2e^{2\eta}+0.25pt^2)e^{-2\eta}}\left(1.0M^2\sqrt{\frac{1.0M^2e^{2\eta}+\cdots}{M^2}}\cdots\right.}\cdots$$

Results compatible with SymPy functions and methods

```
v.boost(v.to_beta3()).px.subs({"pt": 1, "phi": 2, "eta": 3, "M": 4})
```

$$\frac{\sqrt{\cos^2(2)+\sin^2(2)+16+\sinh^2(3)}\cos(2)}{\sqrt{1+0.015625(e^{-6}+1)^2e^6}\sqrt{-\frac{\sinh^2(3)}{16+0.25(e^{-6}+1)^2e^6}-\frac{\sin^2(2)}{16+0.25(e^{-6}+1)^2e^6}-\frac{\cos^2(2)}{16+0.25(e^{-6}+1)^2e^6}+1}}+\frac{\cos(2)\sinh^2(3)}{\left(1+\frac{1}{\sqrt{-\frac{\sinh^2(3)}{16+0.25\left(e^{-6}+1\right)^2e^6}-\frac{\sin^2(2)}{16+0.25\left(e^{-6}+1\right)^2e^6}-\frac{\cos^2(2)}{16+0.25\left(e^{-6}+1\right)^2e^6}+1}}\right)\left(16+0.25(e^{-6}+1)^2e^6\right)\left(-\frac{\sinh^2(3)}{16+0.25(e^{-6}+1)^2e^6}\cdots\right.}\cdots$$

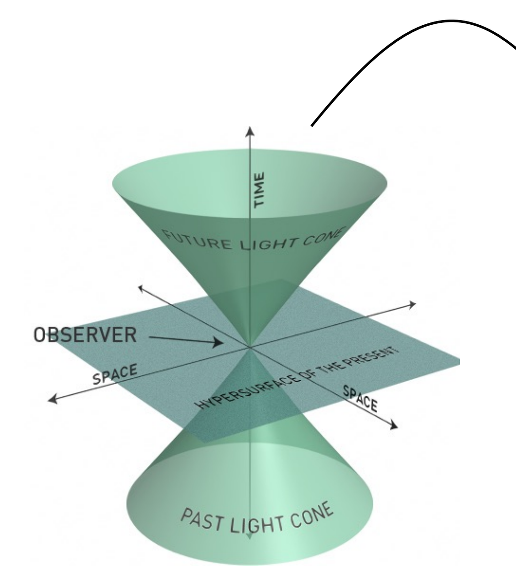
Use any SymPy functionality on the expressions

```
v.boost(v.to_beta3()).px.subs({...}).evalf()
```

$$-2.25409707330435$$

Evaluated results consistent with numerical backends

Caveats



Operations only on positive time like vectors are 100% compatible with numerical backends.

mpmath $\to \infty$ NumPy

SymPy uses mpmath for numerical computations which has more floating point precision than NumPy, producing slightly different numerical results.