Project Report for II.6
Mathematical Visualizations

"*4th Dimension, Topology, and the Grand Unified Theory*"

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Certificate of Completion

This is to certify that the following persons: **Saransh Chopra, Parth Tripathi, and Naman Priyadarshi** have completed this project titled "*4th Dimension, Topology and the Grand Unified Theory*" under my guidance and supervision as per the contentment of the requirements of the second semester in the course B.Tech (Information Technology and Mathematical Innovations) at the Cluster Innovation Centre, University of Delhi.

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Acknowledgment

It is a great pleasure for us to present this project titled "*4th Dimension, Topology and the Grand Unified Theory*" as a part of the second semester's curriculum in the B.Tech (Information Technology and Mathematical Innovations) course at the Cluster Innovation Centre, University of Delhi.

We would like to thank our teacher and mentor, Professor Shobha Bagai who helped render this project an adequate success.

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Abstract

We live in a 3-dimensional world and hence we are bound by these 3 dimensions. This project aims to give our viewers an **intuition of the 4th dimension**, and most of it will be achieved using analogies. After diving deep into the fourth dimension, we will play with some topological mathematics, discussing the **Möbius strip** and its 4th-dimensional counterpart **Klein bottle** and their weirdest properties. In the end, we will wrap our project by discussing the **Grand Unified Theory**, the need for 11 dimensions, and if they exist.

Introduction and Technologies used

FFmpeg - for processing media
MikTex - for writin LaTeX in videos
Python - for coding the mathematics into videos
Manim - binds all of these technologies together

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2 Dimensional World

Imagine that you are a 2-dimensional creature, completely flat. You know what is up, down, left and right but you have never heard of up and down.

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Kidnapped by a 3D creature

Now, let's say, a 3-dimensional creature tries to talk to you from above, you obviously can't see it hence you think it's your inner voice. So, he decided to pick you up from your homeland and takes you up, but you have no idea where the creature took you as you don't know what is up. Suddenly you tilt your body and now you can see your homeland from above, something that you have never seen.

The creature decides to drop you now, at a place where your friends are hanging out and you suddenly appear before them. You tell them that you went to a new dimension called "up" today and they take it as a joke. You try to convince them but they ask you to point out up which you obviously can't.

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The W axis

W-axis is the axis that extends to the 4th dimension. As the 3 axes, that we know off, are perpendicular to each other, the w axis is perpendicular to all 3 of them simultaneously which is something that we cannot visualize. Hence, we draw it in a 3D space the same way we draw the z-axis in a 2D space.

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Creating a tesseract

Going by the analogy -
A 1-dimensional line needs 2, 0-dimensional points for its creation.
A 2-dimensional square needs 4, 1-dimensional lines for its creation. (lines are the sides of the formed square and a single line is always perpendicular to the adjacent lines)
A 3-dimensional cube needs 6, 2-dimensional squares for its creation. (squares are the sides of the formed cube and a single square is always perpendicular to the adjacent squares).

Hence, a 4-dimensional tesseract will need 8, 3-dimensional cubes for its creation. (where cubes will be the sides of the formed tesseract and a single cube will always be perpendicular to the adjacent cube)

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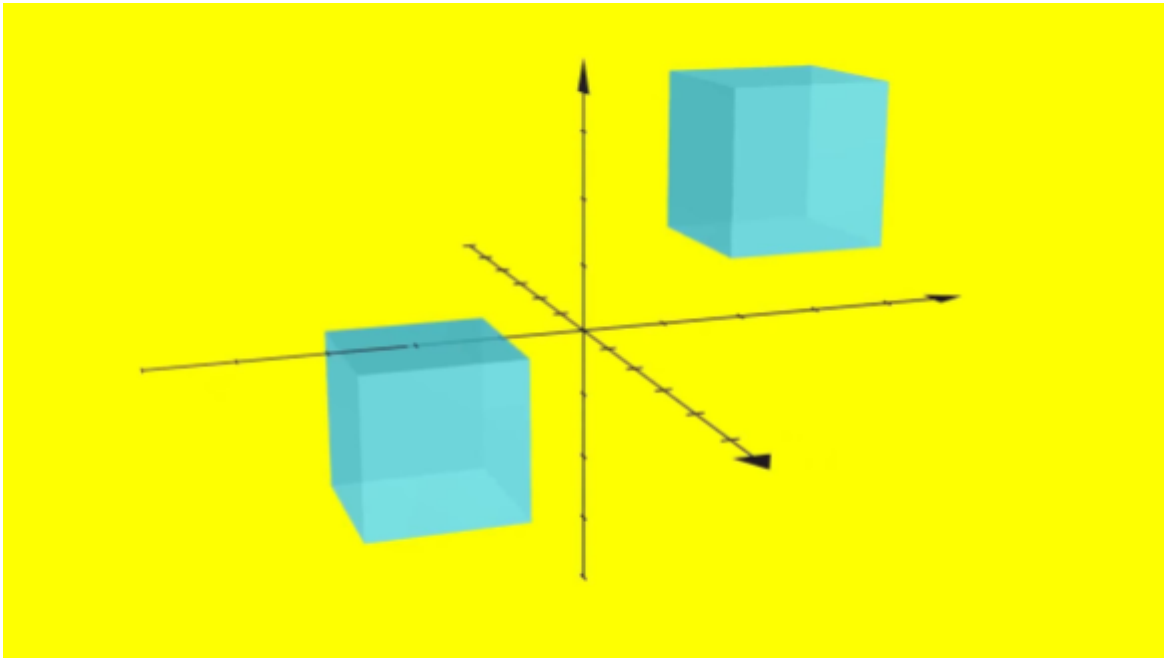
Shadow of a tesseract

Generating a 3-dimensional shadow of a tesseract.



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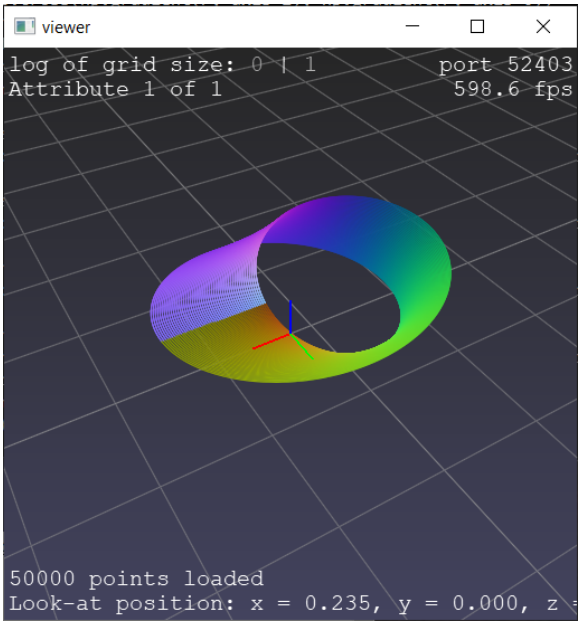
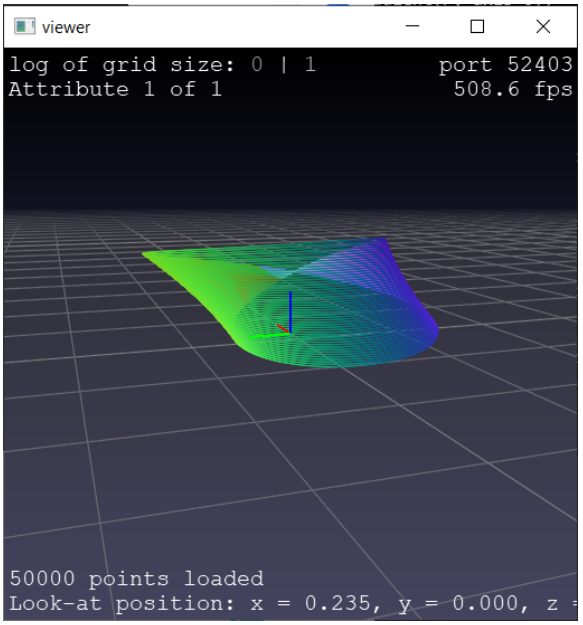
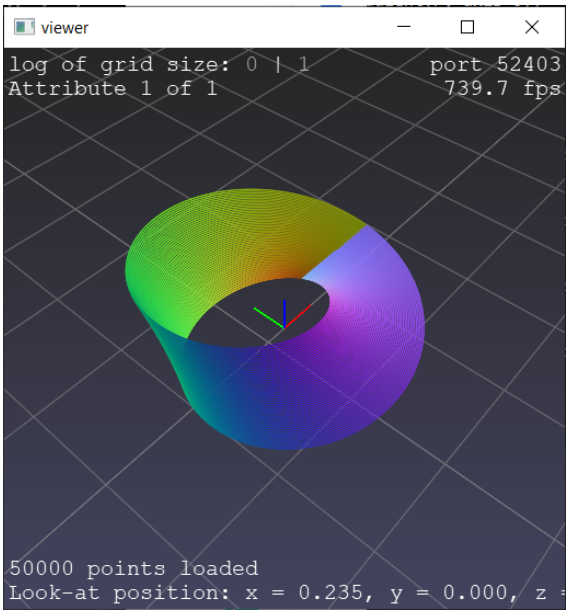
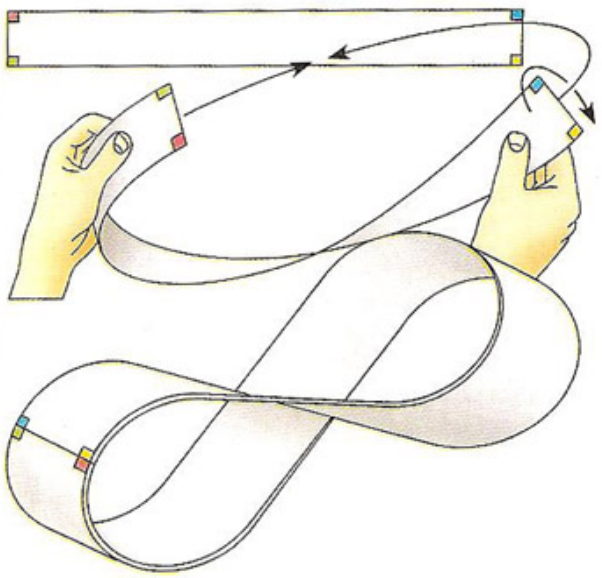
The Möbius Strip

A mobius strip is a weird topological shape that has only 1 side and 1 edge. Let's see one using code and one using some craft -





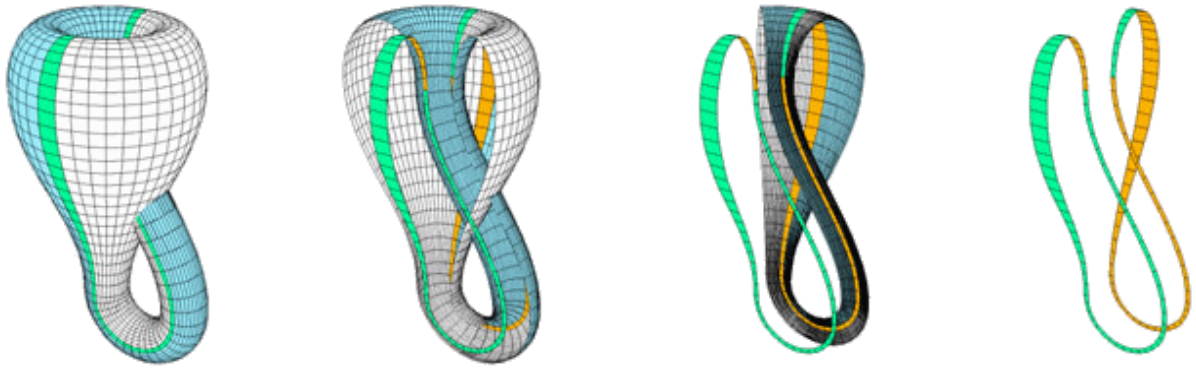




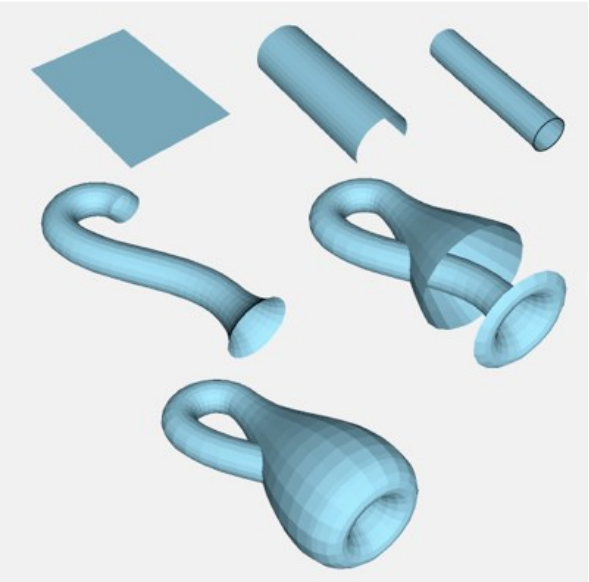
It all starts by taking a **2-dimensional strip** (a hollow circle), cutting it along its radius, and then going to the **3rd dimension** to rotate the piece of paper to join it along with that cut again. Remember this, as this will come in handy when we turn the dimensions up by 1.

A weird property of the Möbius Strip

What if we try to **sew 2 Mobius strips** together using their **edges**? We know that sewing 2 squares will **bring down the total number of edges**. Joining 2 squares (with 4 edges each, hence 8 edges) through their edges will result in a rectangle with 4 edges which is **less than 8.** But what about the strip? Will it result in a shape with **1 edge** again? Or something with 0 edges? Turns out, joining 2 Mobius strips **requires 4 dimensions** and results in a shape called **Klein Bottle**, which has **no edges**.



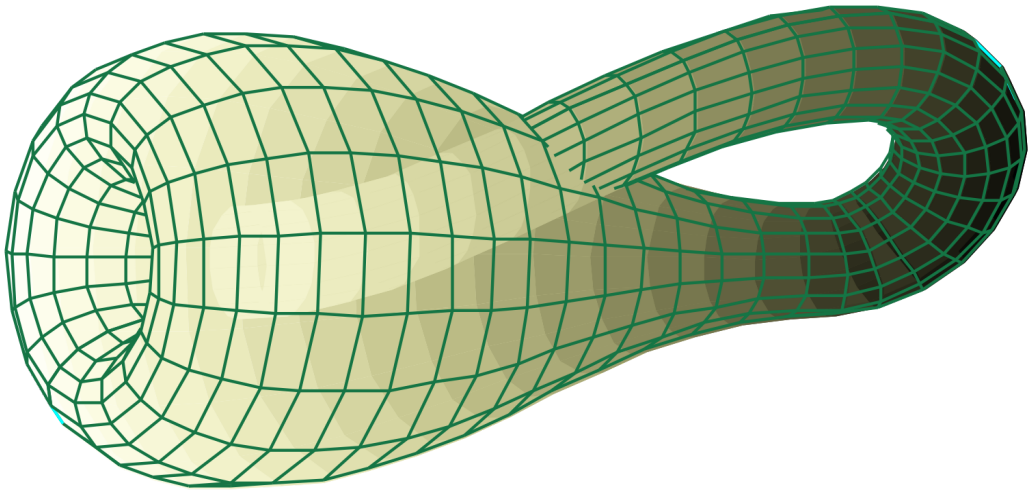
Now that we are talking about Klein bottles, there is another elegant way to create one -



The Klein bottle

A mathematician named Klein
Thought the Mobius strip was divine
Said he: “If you glue two
The edges of two,
You’ll get a weird bottle like mine”
- Leo Moser

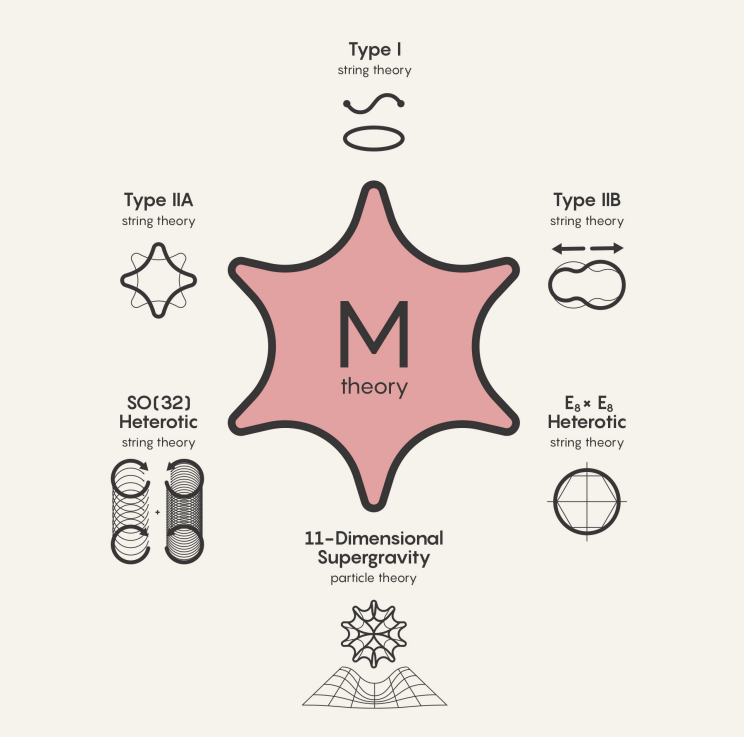
A Klein bottle is a **closed 4-dimensional figure** with **only 1 surface** and **no boundaries**. It was first formulated by mathematician **Felix Klein** in 1882 and since then has been studied in great depth by mathematicians. If you look closely at a Klein bottle, you will find that it does have an edge at the **intersection** but, the thing you are looking at is the **3-dimensional shadow** of a Klein bottle. A Klein bottle **requires 4 dimensions to exist** and this intersection never occurs in the 4th dimension. Imagine that the tube is about to hit the bottle’s surface but just before hitting it, it goes into the fourth dimension. Now again, you can’t point out the fourth dimension, just like a 2-dimensional creature can’t point out the 3rd dimension. After going into the 4th dimension, the tube crosses the surface without touching it. You can think of this as a **4th-dimensional foot-over bridge**, a 3rd-dimensional foot-over bridge helps us in crossing a 2-dimensional road without touching the road or the road divider, which is achieved by going into the 3rd dimension. Keeping in mind the analogy, the tube crosses the surface in the 4th dimension and then returns back to the 3rd dimension inside the blob (technically **there is no inside** w.r.t a Klein bottle but this will help in visualization).



The Grand Unified Theory

In the early 1900s, physicists were struggling to **unite or integrate Albert Einstein's General Theory of Relativity with Quantum Mechanics** (or uniting the gravitational force with other 3 forces, i.e. strong nuclear force, weak nuclear force, and the electromagnetic force). This was the part where the **4th physical dimension** came into play. It all started in 1921 with **Theodor Kaluza** solving Einstein's General Theory of Relativity in **5 dimensions** (4 spatial dimensions and 1 time dimension). The solution that came out of this was a **mixture of the field equations of Einstein and Maxwell's equations of electromagnetism**, thus he succeeded in uniting two different yet confirmed theories in 5 dimensions. This work was later continued by **Oscar Klein** in 1926, who gave the theory a **quantum interpretation** (which was recently introduced by **Heisenberg** and **Schrödinger**). This resulted in a theory that could (well only theoretically) **combine Quantum Mechanics with General Theory of Relativity** in a **5 dimensional world.**

There have been significant advances in this theory over the last century, the most famous one being the **String Theory** or the **M-theory** which requires 11 dimensions to exist. The research is being carried out by some renowned scientists like **Michio Kaku** and **Brian Greene**. Though the theory is not anywhere close to an actual proof, it is still very promising and is one of the only theories that can be termed as the **Grand Unified Theory** (A theory that unites all the forces together).



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6. https://www.youtube.com/watch?v=AAsICMPwGPY
7. https://www.youtube.com/watch?v=UnURElCzGc0
8. Exploring Spiral Inflation in String Theory. Pontus Ahlqvist, Brian Greene (ISCAP, New York & Columbia U.), David Kagan (Massachusetts U., North Dartmouth). Aug 2, 2013. 17 pp.

Appendix

Code for all the animations -
https://gist.github.com/Saransh-cpp/cc74f3df6aa1455f78e09fff759ab37b