

# Frugal Byzantine Computing\*

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## Abstract

Traditional techniques for handling Byzantine failures are expensive: digital signatures are too costly, while using  $3f+1$  replicas is uneconomical ( $f$  denotes the maximum number of Byzantine processes). We seek algorithms that reduce the number of replicas to  $2f+1$  and minimize the number of signatures. While the first goal can be achieved in the message-and-memory model, accomplishing the second goal simultaneously is challenging. We first address this challenge for the problem of broadcasting messages reliably. We consider two variants of this problem, Consistent Broadcast and Reliable Broadcast, typically considered very close. Perhaps surprisingly, we establish a separation between them in terms of signatures required. In particular, we show that Consistent Broadcast requires at least 1 signature in some execution, while Reliable Broadcast requires  $O(n)$  signatures in some execution. We present matching upper bounds for both primitives within constant factors. We then turn to the problem of consensus and argue that this separation matters for solving consensus with Byzantine failures: we present a practical consensus algorithm that uses Consistent Broadcast as its main communication primitive. This algorithm works for  $n = 2f+1$  and avoids signatures in the common-case—properties that have not been simultaneously achieved previously. Overall, our work approaches Byzantine computing in a frugal manner and motivates the use of Consistent Broadcast—rather than Reliable Broadcast—as a key primitive for reaching agreement.

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## 1 Introduction

Byzantine fault-tolerant computing is notoriously expensive. To tolerate  $f$  failures, we typically need  $n = 3f + 1$  replica processes. Moreover, the agreement protocols for synchronizing the replicas have a significant latency overhead. Part of the overhead comes from network delays, but digital signatures—often used in Byzantine computing—are even more costly than network delays. For instance, signing a message can be 28 times slower than sending it over a low-latency Infiniband fabric (Appendix A shows the exact measurements).

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In this work, we study whether Byzantine computing can be *frugal*, meaning if it can use few processes and few signatures. By Byzantine computing, we mean the classical problems of broadcast and consensus. By frugality, we first mean systems with  $n = 2f + 1$  processes, where  $f$  is the maximum number of Byzantine processes. Such systems are clearly preferable to systems with  $n = 3f + 1$ , as they require 33–50% less hardware. However, seminal impossibility results imply that in the standard message-passing model with  $n = 2f + 1$  processes, neither consensus nor various forms of broadcast can be solved, even under partial synchrony or randomization [30]. To circumvent the above impossibility results, we consider a message-and-memory (M&M) model, which allows processes to both pass messages and share memory, capturing the latest hardware capabilities of enterprise servers [1, 2]. In this model, it is possible to solve consensus with  $n = 2f + 1$  processes and partial synchrony [2].

Frugality for us also means the ability to achieve *low latency*, by minimizing the number of digital signatures used. Mitigating the cost of digital signatures is commonly done by replacing them with more computationally efficient schemes, such as message authentication codes (MACs). For instance, with  $n = 3f + 1$ , the classic PBFT replaces some of its signatures with MACs [20], while Bracha’s broadcast algorithm [14] relies exclusively on MACs. As we show in the paper, when  $n = 2f + 1$ , the same techniques for reducing the number of signatures are no longer applicable.

The two goals—achieving high failure resilience while minimizing the number of signatures—prove challenging when combined. Intuitively, this is because with  $n = 2f + 1$  processes, two quorums may intersect only at a Byzantine process; this is not the case with  $n = 3f + 1$ . Thus, we cannot rely on quorum intersection alone to ensure correctness; we must instead restrict the behavior of Byzantine processes to prevent them from providing inconsistent information to different quorums. Signatures can restrict Byzantine processes from lying, but only if there are enough correct processes to exchange messages and cross-check information. The challenge is to make processes prove that they behave correctly, based on the information they received so far, while using as few signatures as possible.

We focus initially on the problem of broadcasting a message reliably—one of the simplest and most widely used primitives in distributed computing. Here, a designated sender process  $s$  would like to send a message to other processes, such that all correct processes deliver the same message. The difficulty is that a Byzantine sender may try to fool correct processes to deliver different messages. Both broadcast variants, Consistent and Reliable Broadcast, ensure that (1) if the sender is correct, then all correct processes deliver its message, and (2) any two correct processes that deliver a message must deliver the *same* message. Reliable Broadcast ensures an additional property: if any correct process delivers a message, then all correct processes deliver that message.

Perhaps surprisingly, in the M&M model we show a large separation between the two broadcasts in terms of the number of signatures (by correct processes) they require. We introduce a special form of indistinguishability argument for  $n = 2f + 1$  processes that uses signatures and shared memory in an elaborate way. With it, we prove lower bounds for deterministic algorithms. For Consistent Broadcast, we prove that any solution requires one correct process to sign in some execution, and provide an algorithm that matches this bound. In contrast, for Reliable Broadcast, we show that any solution requires at least  $n - f - 2$  correct processes to sign in some execution. We provide an algorithm for Reliable Broadcast based on our Consistent Broadcast algorithm which follows the well-known Init-Echo-Ready pattern [14] and uses up to  $n + 1$  signatures, matching the lower bound within a factor of 2.

To lower the impact of signatures on the latency of our broadcast algorithms, we introduce the technique of background signatures. Given the impossibility of completely eliminating

signatures, we design our protocols such that signatures are not used in well-behaved executions, i.e., when processes are correct and participate within some timeout. In other words, both broadcast algorithms generate signatures in the background and also incorporate a fast path where signatures are not used.

We next show how to use our Consistent Broadcast algorithm to improve consensus algorithms. The algorithm is based on PBFT [19], and maintains *views* in which one process is the *primary*. Within a view, agreement can be reached by simply having the primary consistent-broadcast a value, and each replicator respond with a consistent broadcast. When changing views, a total of  $O(n^2)$  calls to Consistent Broadcast may be issued. The construction within a view is similar to our Reliable Broadcast algorithm. Interestingly, replacing this part with the Reliable Broadcast abstraction does *not* yield a correct algorithm; the stronger abstraction hides information that an implementation based on Consistent Broadcast can leverage. For the correctness of our algorithm, we rely on a technique called *history validation* and on *cross-validating* the view-change message. Our consensus algorithm has four features: (1) it works for  $n = 2f + 1$  processes, (2) it issues no signatures on the fast path, (3) it issues  $O(n^2)$  signatures on a view-change and (4) it issues  $O(n)$  background signatures within a view. As far as we know, no other algorithm achieves all these features simultaneously. This result provides a strong motivation for the use of Consistent Broadcast—rather than Reliable Broadcast—as a first-class primitive in the design of agreement algorithms.

To summarize, we quantify the impossibility of avoiding signatures by proving lower bounds on the number of signatures required to solve the two variants of the broadcast problem—*Consistent* and *Reliable Broadcast*—and provide algorithms that match our lower bounds. Also, we construct a practical consensus algorithm using the Consistent Broadcast primitive. In this work, we consider the message-and-memory model [1, 2], but our results also apply to the pure shared memory model: our algorithms do not require messages so they work under shared memory, while our lower bounds apply *a fortiori* to shared memory.

## 2 Related Work

**Message-and-memory models.** We adopt a message-and-memory (M&M) model, which is a generalization of both message-passing and shared-memory. M&M is motivated by enterprise servers with the latest hardware capabilities—such as RDMA, RoCE, Gen-Z, and soon CXL—which allow machines to *both* pass messages and share memory. M&M was introduced by Aguilera et al. in [1], and subsequently studied in several other works [2, 5, 32, 46]. Most of these works did not study Byzantine fault tolerance, but focused on crash-tolerant constructions when memory is shared only by subsets of processes [1, 5, 32, 46]. In [2], Aguilera et al. consider crash- and Byzantine- fault tolerance, as well as bounds on communication rounds on the fast path for a variant of the M&M model with dynamic access permissions and memory failures. However, they did not study any complexity bounds off the fast path, and in particular did not consider the number of signatures such algorithms require.

**Byzantine Fault Tolerance.** Lamport, Shostak and Pease [39, 45] show that Byzantine agreement can be solved in synchronous message-passing systems iff  $n \geq 3f + 1$ . In asynchronous systems subject to failures, consensus cannot be solved [31]. However, this result is circumvented by making additional assumptions for liveness, such as randomization [9, 44] or partial synchrony [22, 30]. Even with signatures, asynchronous Byzantine agreement can be solved in message-passing systems only if  $n \geq 3f + 1$  [16]. Dolev and Reischuk [29] prove a lower bound of  $n(f + 1)/4$  signatures for Byzantine agreement, assuming that every message

carries at least the signature of its sender.

**Byzantine Broadcast.** In the message-passing model, both Consistent and Reliable Broadcast require  $n \geq 3f + 1$  processes, unless (1) the system is synchronous and (2) digital signatures are available [16, 28, 49]. Consistent Broadcast is sometimes called Crusader Agreement [28]. The Consistent Broadcast abstraction was used implicitly in early papers on Byzantine broadcast [15, 51], but its name was coined later by Cachin et al. in [18]. The name “consistent broadcast” may also refer to a similar primitive used in synchronous systems [41, 49]. Our Reliable Broadcast algorithm shares Bracha’s Init-Echo-Ready structure [14] with other broadcast algorithms [16, 47, 49], but is the first algorithm to use this structure in shared memory to achieve Reliable Broadcast with  $n = 2f + 1$  processes.

**BFT with stronger communication primitives.** Despite the known fault tolerance bounds for asynchronous Byzantine Failure Tolerance (BFT), Byzantine consensus can be solved in asynchronous systems with  $2f + 1$  processes if stronger communication mechanisms are assumed. Some prior work solves Byzantine consensus with  $2f + 1$  processes using specialized trusted components that Byzantine processes cannot control [23, 24, 25, 26, 33, 52]. These trusted components can be seen as providing a broadcast primitive for communication. These works assume the existence of such primitives as black boxes, and do not study the cost of implementing them using weaker hardware guarantees, as we do in this paper. We achieve the same Byzantine fault-tolerance by using the shared memory to prevent the adversary from partitioning correct processes: once a correct process writes to a register, the adversary cannot prevent another correct process from seeing the written value.

It has been shown that shared memory primitives can be useful in providing BFT if they have *access control lists* or *policies* that dictate the allowable access patterns in an execution [2, 4, 10, 12, 42]. Alon et al. [4] provide tight bounds for the number of strong shared-memory objects needed to solve consensus with optimal resilience. They do not, however, study the number of signatures required.

**Early termination.** The idea of having a *fast path* that allows early termination in well-behaved executions is not a new one, and has appeared in work on both message-passing [2, 3, 6, 27, 34, 35, 38] and shared-memory [7, 50] systems. Most of these works measure the fast path in terms of the number of message delays (or network rounds trips) they require, but some also consider the number of signatures [6]. In this paper, we show that a signature-free fast path does not prevent an algorithm from having an optimal number of overall signatures.

### 3 Model and Preliminaries

We consider an asynchronous message-and-memory model, which allows processes to use both message-passing and shared-memory [1]. The system has  $n$  processes  $\Pi = \{p_1, \dots, p_n\}$  and a shared *memory*  $M$ . Throughout the paper, the term *memory* refers to  $M$ , not to the local state of processes. We sometimes augment the system with eventual synchrony (§3.2).

**Communication.** The memory consists of single-writer multi-reader (SWMR) read/write atomic registers. Each process can read all registers, and has access to an unlimited supply of registers it can write. If a process  $p$  can write to a register  $r$ , we say that  $p$  *owns*  $r$ . This model is a special case of access control lists (ACLs) [42], and of dynamically permissioned memory [2]. Additionally, every pair of processes  $p$  and  $q$  can send messages to each other

over links that satisfy the *integrity* and *no-loss* properties. Integrity requires that a message  $m$  from  $p$  be received by  $q$  at most once and only if  $m$  was previously sent by  $p$  to  $q$ . No-loss requires that a message  $m$  sent from  $p$  to  $q$  be eventually received by  $q$ .

**Signatures.** Our algorithms assume digital signatures: each process has access to primitives to *sign* and *verify* signatures. A process  $p$  may sign a value  $v$ , producing  $\sigma_{p,v}$ ; we drop the subscripts when it is clear from context. Given  $v$  and  $\sigma_{p,v}$ , a process can verify whether  $\sigma_{p,v}$  is a valid signature of  $v$  by  $p$ .

**Failures.** Up to  $f$  processes may fail by becoming Byzantine, where  $n = 2f + 1$ . Such a process can deviate arbitrarily from the algorithm, but cannot write on a register that is not its own, and cannot forge the signature of a correct process. As usual, Byzantine processes can collude, e.g., by using side-channels to communicate. The memory  $M$  does not fail; such a reliable memory is implementable from a collection of fail-prone memories [2]. We assume that these individual memories may only fail by crashing.

### 3.1 Broadcast

We consider two broadcast variants: Consistent Broadcast [17, 18] and Reliable Broadcast [13, 17]. In both variants, broadcast is defined in terms of two primitives: *broadcast( $m$ )* and *deliver( $m$ )*. A designated *sender* process  $s$  is the only one that can invoke *broadcast*. When  $s$  invokes *broadcast( $m$ )* we say that  $s$  *broadcasts*  $m$ . When a process  $p$  invokes *deliver( $m$ )*, we say that  $p$  *delivers*  $m$ .

► **Definition 3.1.** Consistent Broadcast *has the following properties:*

**Validity** *If a correct process  $s$  broadcasts  $m$ , then every correct process eventually delivers  $m$ .*

**No duplication** *Every correct process delivers at most one message.*

**Consistency** *If  $p$  and  $p'$  are correct processes,  $p$  delivers  $m$ , and  $p'$  delivers  $m'$ , then  $m=m'$ .*

**Integrity** *If some correct process delivers  $m$  and  $s$  is correct, then  $s$  previously broadcast  $m$ .*

► **Definition 3.2.** Reliable Broadcast *has the following properties:*

**Validity, No duplication, Consistency, Integrity** *Same properties as in Definition 3.1.*

**Totality** *If some correct process delivers  $m$ , then every correct process eventually delivers a message.*

We remark that both broadcast variants behave the same way when the sender is correct and broadcasts  $m$ . However, when the sender is faulty Consistent Broadcast has no delivery guarantees for correct processes, i.e., some correct processes may deliver  $m$ , others may not. In contrast, Reliable Broadcast forces every correct process to eventually deliver  $m$  as soon as one correct process delivers  $m$ .

### 3.2 Consensus

► **Definition 3.3.** Weak Byzantine agreement [36] *has the following properties:*

**Agreement** *If correct processes  $i$  and  $j$  decide  $val$  and  $val'$ , respectively, then  $val = val'$ .*

**Weak validity** *If all processes are correct and some process decides  $val$ , then  $val$  is the input of some process.*

**Integrity** *No correct process decides twice.*

**Termination** *Eventually every correct process decides.*

Our consensus algorithm (§6) satisfies agreement, validity, and integrity under asynchrony, but requires eventual synchrony for termination. That is, we assume that for each execution there exists a *Global Stabilization Time (GST)*, unknown to the processes, such that from GST onwards there is a known bound  $\Delta$  on communication and processing delays.

## 4 Lower Bounds on Broadcast Algorithms

We show lower bounds on the number of signatures required to solve Consistent and Reliable Broadcast with  $n = 2f+1$  processes in our model. We focus on signatures by correct processes because Byzantine processes can behave arbitrarily (including signing in any execution).

### 4.1 High-Level Approach

Broadly, we use indistinguishability arguments that create executions  $E_v$  and  $E_w$  that deliver different messages  $v$  and  $w$ ; then we create a composite execution  $E$  where a correct process cannot distinguish  $E$  from  $E_v$ , while another correct process cannot distinguish  $E$  from  $E_w$ , so they deliver different values, a contradiction. Such arguments are common in message-passing system, where the adversary can prevent communication by delaying messages between correct processes. However, it is not obvious how to construct this argument in shared memory, as the adversary cannot prevent communication via the shared memory, especially when using single-writer registers that cannot be overwritten by the adversary. Specifically, if correct processes write their values and read all registers, then for any two correct processes, at least one sees the value written by the other [8]. So, when creating execution  $E$  in which, say  $E_v$  occurs first, processes executing  $E_w$  will know that others executed  $E_v$  beforehand.

We handle this complication in two ways, depending on whether the sender signs its broadcast message. If the sender does not sign, we argue that processes executing  $E_w$  cannot tell whether  $E_v$  was executed by correct or Byzantine processes, and must therefore still output their original value  $w$ . This is the approach in the lower bound proof for Consistent Broadcast (Lemma 4.1).

However, once a signature is produced, processes can save it in their memory to prove to others that they observed a valid signature. Thus, if the sender signs its value, then processes executing  $E_w$  cannot be easily fooled; if they see two different values signed by the sender, then the sender is provably faulty, and correct processes can choose a different output. So, we need another way to get indistinguishable executions. We rely on a *correct bystander* process. We make a correct process  $b$  in  $E$  sleep until all other correct processes decide. Then  $b$  wakes up and observes that  $E$  is a composition of  $E_v$  and  $E_w$ . While  $b$  can recognize that  $E_v$  or  $E_w$  was executed by Byzantine processes, it cannot distinguish which one. So  $b$  cannot reliably output the same value as other correct processes. We use this construction for Reliable Broadcast, but we believe it applies to other agreement problems in which all correct processes must decide.

The proof is still not immediate from here. In particular, since  $f < n/2$ , correct processes can wait until at least  $f+1$  processes participate in each of  $E_v$  and  $E_w$ . Of those, in our proof we assume at most  $f-1$  processes sign values. Since we need a bystander later, only  $2f$  processes can participate. Thus, the sets executing  $E_v$  and  $E_w$  overlap at two processes; one must be the sender, to force decisions in both executions. Let  $p$  be the other process and  $S_v$  and  $S_w$  be the set that execute  $E_v$  and  $E_w$  respectively, without the sender and  $p$ . Thus,  $|S_v| = |S_w| = f-1$ .

The key complication is that if  $p$  signs its values in one of these two executions, we cannot compose them into an execution  $E$  in which the bystander  $b$  cannot distinguish which value it

should decide. To see this, assume without loss of generality that  $p$  signs a value in execution  $E_w$ . To create  $E$ , we need the sender  $s$  and the set  $S_w$  to be Byzantine. The sender will produce signed versions of both  $v$  and  $w$  for the two sets to use, and  $S_w$  will pretend to execute  $E_w$  even though they observed that  $E_v$  was executed first. Since  $|S_w| + |\{s\}| = f$ , all other processes must be correct. In particular,  $p$  will be correct, and will not produce the signature that it produces in  $E_w$ . Thus, the bystander  $b$  will know that  $S_v$  were correct. More generally, the problem is that, while we know that at most  $f - 1$  processes sign, we do not know *which* processes sign. A clever algorithm can choose signing processes to defeat the indistinguishability argument—in our case, this happens if  $p$  is a process that signs.

Due to this issue, we take a slightly different approach for the Reliable Broadcast lower bound, first using the bystander construction to show that any Reliable Broadcast algorithm must produce *a single non-sender* signature. To strengthen this to our bound, we construct an execution in which this signature needs to be repeatedly produced. To make this approach work, we show not just that *there exists* an execution in which a non-sender signature is produced, but that *for all* executions of a certain form, a non-sender signature is produced. This change in quantifiers requires care in the indistinguishability proof, and allows us to repeatedly apply the result to construct a single execution that produces many signatures.

## 4.2 Proofs

In all proofs in this section, we denote by  $s$  the designated sender process in the broadcast protocols we consider. We first show that Consistent Broadcast requires at least one signature.

► **Lemma 4.1.** *Any algorithm for Consistent Broadcast in the M&M model with  $n = 2f + 1$  and  $f \geq 1$  has an execution in which at least one correct process signs.*

**Proof.** By contradiction, assume there is some algorithm  $A$  for Consistent Broadcast in the M&M model with  $n = 2f + 1$  and  $f \geq 1$  without any correct process signing. Partition processes in  $\Pi$  into 3 subsets:  $S_1$ ,  $S_2$ , and  $\{p\}$ , where  $S_1$  contains the sender,  $|S_1| = f$ ,  $|S_2| = f$ , and  $p$  is a single process. Let  $v, w$  be two distinct messages. Consider the following executions.

EXECUTION  $E_{\text{CLEAN-V}}$ . Processes in  $S_1$  and  $p$  are correct (including the sender  $s$ ), while processes in  $S_2$  are faulty and never take a step. Initially,  $s$  broadcasts  $v$ . Since  $s$  is correct, processes in  $S_1$  and  $p$  eventually deliver  $v$ . By our assumption that correct processes never sign, processes in  $S_1$  and  $p$  do not sign in this execution; processes in  $S_2$  do not sign either, because they do not take any steps.

EXECUTION  $E_{\text{DIRTY-W}}$ . Processes in  $S_1$  and  $S_2$  are correct but  $p$  is Byzantine. Initially,  $p$  sends all messages and writes to shared memory as it did in  $E_{\text{CLEAN-V}}$  (it does so without following its algorithm;  $p$  is able to do this since no process signed in  $E_{\text{CLEAN-V}}$ ). Then, the correct sender  $s$  broadcasts  $w$  and processes in  $S_1$  and  $S_2$  execute normally, while  $p$  stops executing. Then, by correctness of the algorithm, eventually all correct processes deliver  $w$ . By our assumption that correct processes never sign, processes in  $S_1$  and  $S_2$  do not sign in this execution;  $p$  does not sign either, because it acts as it did in  $E_{\text{CLEAN-V}}$ .

EXECUTION  $E_{\text{BAD}}$ . Processes in  $S_1$  are Byzantine, while processes in  $S_2$  and  $p$  are correct. Initially, processes in  $S_2$  sleep, while processes in  $S_1$  and  $p$  execute, where processes in  $S_1$  send the same messages to  $p$  and write the same values to shared memory as in  $E_{\text{CLEAN-V}}$  (but they do not send any messages to  $S_2$ ), so that from  $p$ 's perspective the execution is indistinguishable from  $E_{\text{CLEAN-V}}$ .  $S_1$  are able to do this because no process signed in  $E_{\text{CLEAN-V}}$ . Therefore,  $p$  eventually delivers  $v$ . Next, processes in  $S_1$  write the initial values to their

registers<sup>1</sup>. Now, process  $p$  stops executing, while processes in  $S_1$  and  $S_2$  execute the same steps as in  $E_{\text{DIRTY-W}}$ —here, note that  $S_2$  just follows algorithm  $A$  while  $S_1$  is Byzantine and pretends to be in an execution where  $s$  broadcasts  $w$  ( $S_1$  is able to do this because no process signed in  $E_{\text{DIRTY-W}}$ ). Because this execution is indistinguishable from  $E_{\text{DIRTY-W}}$  to processes in  $S_2$ , they eventually deliver  $w$ . At this point, correct process  $p$  has delivered  $v$  while processes in  $S_2$  (which are correct) have delivered  $w$ , which contradicts the consistency property of Consistent Broadcast. ◀

An algorithm for Reliable Broadcast works for Consistent Broadcast, so Lemma 4.1 also applies to Reliable Broadcast.

We now show a separation between Consistent Broadcast and Reliable Broadcast: any algorithm for Reliable Broadcast has an execution where at least  $f-1$  correct processes sign.

The proof for the Reliable Broadcast lower bound has two parts. First, we show that intuitively there are many executions in which some process produces a signature: if  $E$  is an execution in which (1) two processes never take steps, (2) the sender is correct, and (3) processes fail only by crashing, then some non-sender process signs. This is the heart of the proof, and relies on the indistinguishability arguments discussed in Section 4.1. Here, we focus only on algorithms in which at most  $f$  correct processes sign, otherwise the algorithm trivially satisfies our final theorem.

► **Lemma 4.2.** *Let  $A$  be an algorithm for Reliable Broadcast in the M&M model with  $n = 2f + 1$  and  $f \geq 2$  processes, such that in any execution at most  $f$  correct processes sign. In all executions of  $A$  in which at least 2 processes crash initially, processes fail only by crashing, and the sender is correct, at least one correct non-sender process signs.*

**Proof.** By contradiction, assume some algorithm  $A$  satisfies the conditions of the lemma, but there is some execution of  $A$  where the sender  $s$  is correct, processes fail only by crashing, and at least 2 processes crash initially, but no correct non-sender process signs. Let  $E_{\text{CLEAN-V}}$  be such an execution,  $D$  be a set with two processes that crash initially in  $E_{\text{CLEAN-V}}$ <sup>2</sup>,  $C = \Pi \setminus D$ , and  $v$  be the message broadcast by  $s$  in  $E_{\text{CLEAN-V}}$ . Consider the following executions:

EXECUTION  $E_{\text{CLEAN-W}}$ . The sender  $s$  broadcasts some message  $w \neq v$ ,  $D$  crashes initially, and  $C$  is correct. Since  $s$  is correct, eventually all correct processes deliver  $w$ . By assumption, at most  $f$  processes sign. Let  $S \subset C$  contain all processes that sign, augmented with any other processes so that  $|S| = f$ . Let  $T = C \setminus S$ . Note that (1)  $|T| = f - 1$  and (2) if  $s$  signed, then  $s \in S$ , otherwise  $s \in T$ .

EXECUTION  $E_{\text{CLEAN-V}}$ . This execution was defined above (where  $s$  broadcasts  $v$ ). Since  $s$  is correct, eventually all correct processes deliver  $v$ . At least one process in  $T$  is correct—call it  $p_t$ —since processes in  $D$  are faulty and there are at least  $f + 1$  correct processes. Note that  $p_t$  delivers  $v$ . We refer to  $p_t$  in the next execution.

EXECUTION  $E_{\text{MIXED-V}}$ . Processes in  $S$  are Byzantine and the rest are correct. Initially, the execution is identical to  $E_{\text{CLEAN-V}}$ , except that (1) processes in  $D$  are just sleeping not crashed, and (2) processes in  $S$  do not send messages to processes in  $D$  (this is possible because processes in  $S$  are Byzantine). The execution continues as in  $E_{\text{CLEAN-V}}$  until  $p_t$  delivers  $v$ . Then, processes in  $S$  misbehave (they are Byzantine) and do three things: (1) they change their states to what they were at the end of  $E_{\text{CLEAN-W}}$  (this is possible because no process

<sup>1</sup> Recall that registers are single-writer. By “their registers”, we mean the registers to which the processes can write.

<sup>2</sup> If more than two processes crashed initially, pick any two arbitrarily.

in  $T$  signed in  $E_{\text{CLEAN-W}}$ ), (2) they write to their registers in shared memory the same last values that they wrote in  $E_{\text{CLEAN-W}}$ , and (3) they send the same messages they did in  $E_{\text{CLEAN-W}}$ . Intuitively, processes in  $S$  pretend that  $s$  broadcast  $w$ . Let  $t$  be the time at this point; we refer to time  $t$  in the next execution. Now, we pause processes in  $S$  and let all other processes execute, including  $D$  which had been sleeping. Since  $p_t$  delivered  $v$  and processes in  $D$  are correct, they eventually deliver  $v$  as well.

**EXECUTION  $E_{\text{BAD}}$ .** Processes in  $T \cup \{s\}$  are Byzantine and the rest are correct. Initially, the execution is identical to  $E_{\text{CLEAN-W}}$ , except that (1) processes in  $D$  are sleeping not crashed, and (2) processes in  $T \cup \{s\}$  do not send messages to processes in  $D$ . Execution continues as in  $E_{\text{CLEAN-W}}$  until processes in  $S$  (which are correct) deliver  $w$ . Then, processes in  $T \cup \{s\}$  misbehave and do three things: (1) they change their states to what they were in  $E_{\text{MIXED-V}}$  at time  $t$ —this is possible because in  $E_{\text{CLEAN-V}}$  (and therefore in all values and messages they had by time  $t$  in  $E_{\text{MIXED-V}}$ ), no non-sender process signed, and in particular, there were no signatures by any process in  $S \setminus \{s\}$ ; (2) they write to the registers in shared memory the same values that they have in  $E_{\text{MIXED-V}}$  at time  $t$ ; and (3) they send all messages they did in  $E_{\text{MIXED-V}}$  up to time  $t$ . Intuitively, processes in  $T \cup \{s\}$  pretend that  $s$  broadcast  $v$ . Now, processes in  $D$  start executing. In fact, execution continues as in  $E_{\text{MIXED-V}}$  from time  $t$  onward, where processes in  $S$  are paused and all other processes execute (including  $D$ ). Because these processes cannot distinguish the execution from  $E_{\text{MIXED-V}}$ , eventually they deliver  $v$ . Note that processes in  $D$  are correct and they deliver  $v$ , while processes in  $S$  are also correct and deliver  $w$ —contradiction.  $\blacktriangleleft$

In the final stage of the proof, we leverage Lemma 4.2 to construct an execution in which many processes sign. This is done by allowing some process to be poised to sign, and then pausing it and letting a new process start executing. Thus, we apply Lemma 4.2  $f - 1$  times to incrementally build an execution in which  $f - 1$  correct processes sign.

► **Theorem 4.3.** *Every algorithm that solves Reliable Broadcast in the M&M model with  $n = 2f + 1$  and  $f \geq 1$  has some execution in which at least  $f - 1$  correct non-sender processes sign.*

**Proof.** If  $f = 1$ , the result is trivial; it requires  $f - 1 = 0$  processes to sign.

Now consider the case  $f \geq 2$ . If  $A$  has an execution in which at least  $f + 1$  correct processes sign, then we are done. Now suppose  $A$  has no execution in which at least  $f + 1$  correct processes sign. Consider the following execution of  $A$ .

All processes and  $s$  are correct. Initially,  $s$  broadcasts  $v$ . Then processes  $s, p_1 \dots p_f$  participate, and the rest are delayed. This execution is indistinguishable to  $s, p_1 \dots p_f$  from one in which the rest of the processes crashed. Therefore, by Lemma 4.2, some process in  $p_1 \dots p_f$  eventually signs. Call  $p_1$  the first process that signs. We continue the execution until  $p_1$ 's next step is to make its signature visible. Then, we pause  $p_1$ , and let  $p_{f+1}$  begin executing. Again, this execution is indistinguishable to processes  $s, p_2 \dots p_{f+1}$  from one in which the rest of the processes crashed, so by Lemma 4.2, eventually some process in  $p_2 \dots p_{f+1}$  creates a signature and makes it visible. We let the first process to do so reach the state in which it is about to make its signature visible, and then pause it, and let  $p_{f+2}$  start executing.

We continue in this way, each time pausing  $p_i$  as it is about to make its signature visible, and letting  $p_{f+i}$  begin executing. We can apply Lemma 4.2 as long as two processes have not participated yet. At that point,  $f - 1$  processes are poised to make their signatures visible. We then let these  $f - 1$  processes each take one step. This yields an execution of  $A$  in which  $f - 1$  correct non-sender processes sign.  $\blacktriangleleft$

## 5 Broadcast Algorithms

In this section we present solutions for Consistent and Reliable Broadcast. We first implement Consistent Broadcast in Section 5.1; then we use it as a building block to implement Reliable Broadcast, in Section 5.2. We prove the correctness of our algorithms in Appendix B and C. For both algorithms, we first describe the general execution outside the common case, which captures behavior in the worst executions; we then describe how delivery happens fast in the common case (without signatures).

**Process roles in broadcast.** We distinguish between three process roles in our algorithms: sender, receiver, and replicator. This is similar in spirit to the proposer-acceptor-learner model used by Paxos [37], and any process may play any number of roles. If all processes play all three roles, then this becomes the standard model. The sender calls *broadcast*, the receivers call *deliver*, and the replicators help guarantee the properties of broadcast. By separating replicators (often servers) from senders and receivers (often clients or other servers), we improve the practicality of the algorithms: clients, by not fulfilling the replicator role, need not remain connected and active to disseminate information from other clients. Unless otherwise specified,  $n$  and  $f$  refer only to replicators; independently, the sender and any number of receivers can also be Byzantine. Receivers cannot send or write any values, as opposed to the sender and replicators, but they can read the shared memory and receive messages.

**Background signatures.** Our broadcast algorithms produce signatures in the background. We do so to allow the algorithms to be signature-free in the common-case. Indeed, in the common-case, receivers can deliver a message without waiting for background signatures. However, outside the common case, these signatures must still be produced by the broadcast algorithms in case some replicators are faulty or delayed. Both algorithms require a number of signatures that matches the bounds in Section 4 within constant factors.

### 5.1 Consistent Broadcast

We give an algorithm for Consistent Broadcast that issues no signatures in the common case, when there is synchrony and no replicator is faulty. Outside this case, only the sender signs.

Algorithm 1 shows the pseudocode. The broadcast and deliver events are called *cb-broadcast* and *cb-deliver*, to distinguish them from *rb-broadcast* and *rb-deliver* of Reliable Broadcast. Processes communicate by sharing an array of *slots*: process  $i$  can write to  $\text{slots}[i]$ , and can read from all slots. To refer to its own slot, a process uses index  $me$ . The sender  $s$  uses its slot to broadcast its message while replicators use their slot to replicate the message. Every slot has two sub-slots—each a SWMR atomic register—one for a message (*msg*) and one for a signature (*sgn*).

To broadcast a message  $m$ , the sender  $s$  writes  $m$  to its *msg* sub-slot (line 6). Then, in the background,  $s$  computes its signature for  $m$  and writes it to its *sgn* sub-slot (line 9). The presence of *msg* and *sgn* sub-slots allow the sender to perform the signature computation in the background. Sender  $s$  can return from the broadcast while this background task executes.

The role of a correct replicator is to copy the sender’s message  $m$  and signature  $\sigma$ , provided  $\sigma$  is valid. The copying of  $m$  and  $\sigma$  (lines 12–19) are independent events, since a signature may be appended in the background, i.e., later than the message. The fast way to perform a delivery does not require the presence of signatures. Note that correct replicators can have mismatching values only when  $s$  is Byzantine and overwrites its memory.

■ **Algorithm 1** Consistent Broadcast Algorithm with sender  $s$

```

1 Shared:
2 slots -  $n$  array of "slots"; each slot is a 2-tuple  $(msg, sgn)$  of SWMR atomic registers, initialized
   ↳ to  $(\perp, \perp)$ .
3
4 Sender code:
5 cb-broadcast( $m$ ):
6   slots[ $me$ ].msg.write( $m$ )
7   In the background:
8      $\sigma$  = compute signature for  $m$ 
9     slots[ $me$ ].sgn.write( $\sigma$ )
10
11 Replicator code:
12 while True:
13    $m$  = slots[ $s$ ].msg.read()
14   if  $m \neq \perp$ :
15     slots[ $me$ ].msg.write( $m$ )
16   sign = slots[ $s$ ].sgn.read()
17   val = slots[ $me$ ].msg.read()
18   if val  $\neq \perp$  and sign  $\neq \perp$  and sign is a valid signature for val:
19     slots[ $me$ ].sgn.write(sign)
20
21 Receiver code:
22 while True:
23   others = scan()
24   if others[i].msg has the same value  $m$  for all  $i$  in  $\Pi$ : // Fast path
25     cb-deliver( $m$ ); break
26   if others contains at least  $n - f$  signed copies of the same value  $m$ 
27     and ( $\#i: others[i].sgn$  is a valid signature for others[i].msg and others[i].msg  $\neq m$ ):
28     cb-deliver( $m$ ); break
29
30 scan():
31   others = [slots[i].(msg, sgn).read() for i in  $\Pi$ ]
32   done = False
33   while not done:
34     done = True
35     for i in  $\Pi$ :
36       if others[i] ==  $\perp$ :
37         others[i] = slots[i].(msg, sgn).read()
38       if others[i]  $\neq \perp$ :
39         done = False
40   return others

```

A receiver  $p$  scans the slots of the replicators. It delivers message  $m$  when the content of a majority ( $n-f$ ) of replicator slots contains  $m$  and a valid signature by  $s$  for  $m$ , and no slot contains a different message  $m'$ ,  $m' \neq m$  with a valid sender signature (line 28). Slots with sender signatures for  $m' \neq m$  result in a no-delivery. This scenario indicates that the sender is Byzantine and is trying to equivocate. Slots with signatures not created by  $s$  are ignored so that a Byzantine replicator does not obstruct  $p$  from delivering.

When there is synchrony and both the sender and replicators follow the protocol, a receiver delivers without using signatures. Specifically, delivery in the fast path occurs when there is unanimity, i.e., all  $n = 2f + 1$  replicators replicated value  $m$  (line 25), regardless of whether a signature is provided by  $s$ . A correct sender eventually appends  $\sigma$ , and  $n - f$  correct replicators eventually copy  $\sigma$  over, allowing another receiver to deliver  $m$  via the slow path, even if a replicator misbehaves, e.g., removes or changes its value.

An important detail is the use of a snapshot to read replicators' slots (line 23), as opposed to a simple collect. The scan operation is necessary to ensure that concurrent reads of the replicators' slots do not return views that can cause correct receivers to deliver different messages. To see why, imagine that the scan at line 23 is replaced by a simple collect. Then, an execution is possible in which correct receiver  $p_1$  reads some (correctly signed) message  $m_1$  from  $n - f$  slots and finds the remaining slots empty, while another correct receiver  $p_2$  reads  $m_2 \neq m_1$  from  $n - f$  slots and finds the remaining slots empty. In this execution,  $p_1$  would go on to deliver  $m_1$  and  $p_2$  would go on to deliver  $m_2$ , thus breaking the consistency

property. We present such an execution in detail in Appendix B.

To prevent scenarios where correct receivers see different values at a majority of replicator slots, the *scan* operation works as follows (lines 30–40): first, it performs a collect of the slots. If all the slots are non-empty, then we are done. Otherwise, we re-collect the *empty slots* until no slot becomes non-empty between two consecutive collects. This suffices to avoid the problematic scenario above and to guarantee liveness despite  $f$  Byzantine processes.

## 5.2 Reliable Broadcast

We now give an algorithm for Reliable Broadcast that issues no signatures in the common case, and issues only  $n + 1$  signatures in the worst case. Algorithm 2 shows the pseudocode.

Processes communicate by sharing arrays *Echo* and *Ready*, which have the same structure of sub-slots as *slots* in Section 5.1. *Echo[i]* and *Ready[i]* are writable only by replicator  $i$ , while the sender  $s$  communicates with the replicators using an instance of Consistent Broadcast (CB) and does not access *Echo* or *Ready*. In this CB instance,  $s$  invokes *cb-broadcast*, acting as sender for CB, and the replicators invoke *cb-deliver*, acting as receivers for CB.

To broadcast a message,  $s$  *cb-broadcasts*  $\langle \text{INIT}, m \rangle$  (line 6). Upon delivering the sender’s message  $\langle \text{INIT}, m \rangle$ , each replicator writes  $m$  to its *Echo msg* sub-slot (line 13). Then, in the background, a replicator computes its signature for  $m$  and writes it to its *Echo sgn* sub-slot (line 16). By the consistency property of Consistent Broadcast, if two correct replicators  $r$  and  $r'$  deliver  $\langle \text{INIT}, m \rangle$  and  $\langle \text{INIT}, m' \rangle$  respectively, from  $s$ , then  $m = m'$ . Essentially, correct replicators have the same value or  $\perp$  in their *Echo msg* sub-slot.

Next, replicators populate their *Ready* slots with a *ReadySet*. A replicator  $r$  constructs such a *ReadySet* from the  $n - f$  signed copies of  $m$  read from the *Echo* slots (lines 19–28). In the background,  $r$  reads the *Ready* slots of other replicators and copies over—if  $r$  has not written one already—any valid *ReadySet* (line 36). Thus, totality is ensured (Definition 3.2), as the *ReadySet* created by any correct replicator is visible to all correct receivers.

To deliver  $m$ , a receiver  $p$  reads  $n - f$  valid *ReadySets* for  $m$  (line 45).<sup>3</sup> This is necessary to allow a future receiver  $p'$  deliver a message as well. Suppose that  $p$  delivers  $m$  by reading a single valid *ReadySet*  $R$ .<sup>4</sup> Then, the following scenario prevents  $p'$  from delivering: let sender  $s$  be Byzantine and let  $R$  be written by a Byzantine replicator  $r$ . Moreover, let a *single* correct replicator have *cb-delivered*  $m$ , while the remaining correct replicators do not deliver at all, which is allowed by the properties of Consistent Broadcast. So, the *ReadySet* contains values from a single correct replicator and  $f$  other Byzantine replicators. If  $r$  removes  $R$  from its *Ready* slot, it will block the delivery for  $p'$  since no valid *ReadySet* exists in memory.

A receiver  $p$  can also deliver the sender’s message  $m$  using a fast path. The signature-less fast path occurs when  $p$  reads  $m$  from the *Echo* slots of all replicators (line 43), and the delivery of the INIT message by the replicators is done via the fast path of Consistent Broadcast. This is the common-case, when replicators are not faulty and replicate messages timely. Note that  $p$  delivering  $m$  via the fast path does not prevent another receiver  $p'$  from delivering. Process  $p'$  delivers  $m$  via the fast path if all the *Echo* slots are in the same state as for  $p$ . Otherwise, e.g., some Byzantine replicators overwrite their *Echo* slots,  $p'$  delivers  $m$  by relying on the  $n - f$  correct replicators following the protocol (line 45).

---

<sup>3</sup> In contrast to Algorithm 1, receivers need not use the *scan* operation when gathering information from the replicators’ *Ready* slots because there can only be a single value with a valid *ReadySet* (Invariant C.1).

<sup>4</sup> A similar argument that breaks totality applies if  $p$  were to deliver  $m$  by reading  $n - f$  signed values of  $m$  in the replicators’ *Echo* slots.

**Algorithm 2** Reliable Broadcast Algorithm with sender  $s$

```

1 Shared:
2 Echo, Ready -  $n$  array of "slots"; each slot is a 2-tuple  $(\text{msg}, \text{sgn})$  of SWMR atomic registers,
   ↳ initialized to  $(\perp, \perp)$ .
3
4 Sender code:
5 rb-broadcast( $m$ ):
6   cb-broadcast( $\langle \text{INIT}, m \rangle$ )
7
8 Replicator code:
9 state = WaitForSender //  $\in \{\text{WaitForSender}, \text{WaitForEchos}\}$ 
10 while True:
11   if state == WaitForSender:
12     if cb-delivered  $\langle \text{INIT}, m \rangle$  from  $s$ :
13       Echo[me].msg.write( $m$ )
14       In the background:
15          $\sigma$  = compute signature for  $m$ 
16         Echo[me].sgn.write( $\sigma$ )
17       state = WaitForEchos
18
19   if state == WaitForEchos:
20     ReadySet =  $\emptyset$ 
21     for  $i \in \Pi$ :
22       other = Echo[i].(msg, sgn).read()
23       if other.msg ==  $m$  and other.sgn is  $m$  validly signed by  $i$ :
24         ReadySet.add( $(i, \text{other})$ )
25
26   if size(ReadySet)  $\geq n - f$ :
27     ready = True
28     Ready[me].msg.write(ReadySet)
29
30 In the background:
31   while True
32     if not ready:
33       others = [Ready[i].msg.read() for  $i \in \Pi$ ]
34       if  $\exists i$ : others[i] is a valid ReadySet:
35         ready = True
36         Ready[me].msg.write(others[i])
37
38 Receiver code:
39 while True:
40   others = [Echo[i].msg.read() for  $i \in \Pi$ ]
41   proofs = [Ready[i].msg.read() for  $i \in \Pi$ ]
42   if others contains  $n$  matching values  $m$ : // Fast path
43     rb-deliver( $m$ ); break
44   if proofs contains  $n - f$  valid ReadySet for the same value  $m$ :
45     rb-deliver( $m$ ); break

```

## 6 Consensus

We now give an algorithm for consensus using Consistent Broadcast as its communication primitive, rather than the commonly used primitive, Reliable Broadcast. Our algorithm is based on the PBFT algorithm [19, 20] and proceeds in a sequence of (consecutive) views. It has four features: (1) it works for  $n = 2f + 1$  processes, (2) it issues no signatures in the common-case, (3) it issues  $O(n^2)$  signatures on a view-change and (4) it issues  $O(n)$  required background signatures within a view.

Our algorithm uses a sequence of Consistent Broadcast instances indexed by a broadcast sequence number  $k$ . When process  $p$  broadcasts its  $k^{\text{th}}$  message  $m$ , we say that  $p$  broadcasts  $(k, m)$ . We assume the following ordering across instances, which can be trivially guaranteed: (**FIFO delivery**) For  $k \geq 1$ , no correct process delivers  $(k, m_k)$  from  $p$  unless it has delivered  $(i, m_i)$  from  $p$ , for all  $i < k$ .

Algorithm 3 shows the pseudocode. Appendix D has its full correctness proof. The protocol proceeds in a sequence of consecutive *views*. Each view has a primary process, defined as the view number mod  $n$  (line 6). A view has two phases, PREPARE and COMMIT. There is also a view-change procedure initiated by a VIEWCHANGE message.

When a process is the primary (line 9), it broadcasts a PREPARE message with its estimate  $init$  (line 11), which is either its input value or a value acquired in the previous view (line 10). Upon receiving a valid PREPARE message, a replica broadcasts a COMMIT message (line 20) with the estimate it received in the PREPARE message. We define a PREPARE to be valid when it originates from the primary and either (a)  $view = 0$  (any estimate works), or (b)  $view > 0$  and the estimate in the PREPARE message has a proof from the previous view. Appendix D.1 details the conditions for a message to be valid. When a replica receives an invalid PREPARE message from the primary or times out, it broadcasts a COMMIT message with  $\perp$ . If a replica accepts a PREPARE message with  $val$  as estimate and  $n - f$  matching COMMIT messages (line 24), it decides on  $val$ .

**Algorithm 3** Consensus protocol based on Consistent Broadcast ( $n = 2f + 1$ )

```

1  propose(vi):
2      viewi = 0; esti =  $\perp$ ; auxi =  $\perp$ 
3      proofi =  $\emptyset$ ; vci = (0,  $\perp$ ,  $\emptyset$ )
4      decidedi = False
5      while True:
6          pi = viewi % n

7          // Phase 1
8          if pi == i:
9              initi = esti if esti  $\neq \perp$  else vi
10             cb-broadcast((PREPARE, viewi, initi, proofi))
11             wait until receive valid (PREPARE, viewi, val, proof) from pi or timeout on pi
12             if received valid (PREPARE, viewi, val, proof) from pi:
13                 auxi = val
14                 vci = (viewi, val, proof)
15             else:
16                 auxi =  $\perp$ 

17             // Phase 2
18             cb-broadcast((COMMIT, viewi, auxi))
19             wait until receive valid (COMMIT, viewi, *) from n - f processes
20                 and ( $\forall j$ : receive valid (COMMIT, viewi, *) from j or timeout on j)
21              $\forall j$ : Ri[j] = val if received valid (COMMIT, viewi, val) from j else  $\perp$ 
22             if  $\exists val \neq \perp : \#val(R_i) \geq n - f$  and auxi == val:
23                 try Decide(val)

24             // Phase 3
25             cb-broadcast((VIEWCHANGE, viewi + 1, vci) $\sigma_i$ )
26             wait until receive n - f non-conflicting view-change certificates for viewi + 1
27             proofi = set of non-conflicting view-change certificates
28             esti = val in proofi associated with the highest view
29             viewi = viewi + 1

30             In the background:
31                 when cb-deliver valid (VIEWCHANGE, viewj, vcj) $\sigma_j$  from j:
32                     cb-broadcast((VIEWCHANGEACK, d) $\sigma_i$ ) // d is the view-change message being ACKed

33             try Decide(val):
34                 if not decidedi:
35                     decidedi = True
36                     decide(val)

```

The view-change procedure ensures that all correct replicas eventually reach a view with a correct primary and decide. It uses an acknowledgement phase similar to PBFT with MACs [20]. While in [20] the mechanism is used so that the primary can prove the authenticity of a view-change message sent by a faulty replica, we use this scheme to ensure that (a) a faulty participant cannot lie about a committed value in its VIEWCHANGE message and (b) valid VIEWCHANGE messages can be received by all correct replicas.

A replica starts a view-change by broadcasting a signed VIEWCHANGE message with its view-change tuple (line 28). The view-change tuple  $(view, val, proof_{val})$  is updated when a replica receives a valid PREPARE message (line 15). It represents the last non-empty value a replica accepted as a valid estimate and the view when this occurred. We use the value's

proof,  $proof_{val}$ , to prevent a Byzantine replica from lying about its value: suppose a correct replica decides  $val$  in view  $v$ , but in view  $v + 1$ , the primary  $p$  is silent, and so no correct replica hears from  $p$ ; without the proof, a Byzantine replica could claim to have accepted  $val'$  in  $v + 1$  from  $p$  during the view-change to  $v + 1$ , thus overriding the decided value  $val$ .

When a replica receives a valid VIEWCHANGE message, it responds by broadcasting a signed VIEWCHANGEACK containing the VIEWCHANGE message (line 36). A common practice is to send a digest of this message instead of the entire message [19]. We define a VIEWCHANGE message  $m$  from  $p$  to be valid when the estimate in the view-change tuple corresponds to the value broadcast by  $p$  in its latest non-empty COMMIT and  $m$ 's proof is valid. We point out that, as an optimization, this proof can be removed from the view-change tuple and be provided upon request when required to validate VIEWCHANGE messages. For instance, in the scenario described above, when a (correct) replica  $r$  did not accept  $val'$  in view  $v + 1$ , as claimed by the Byzantine replica  $r'$ ,  $r$  can request  $r'$  to provide a proof for  $val'$ .

A view-change certificate consists of a VIEWCHANGE message and  $n - f - 1$  corresponding VIEWCHANGEACK messages. This way, each view-change certificate has the contribution of at least one correct replica, who either produces the VIEWCHANGE message or validates a VIEWCHANGE message. Thus, when a correct replica  $r$  receives a view-change certificate relayed by the primary,  $r$  can trust the contents of the certificate.

To move to the next view, a replica must gather a set of  $n - f$  non-conflicting view-change certificates  $\Psi$ . This step is performed by the primary of the next view, who then includes this set with its PREPARE message for the new view. Two view-change certificates conflict if their view-change messages carry a tuple with different estimates ( $\neq \perp$ ), valid proof, and same view number. If the set  $\Psi$  consists of tuples with estimates from different views, we select the estimate associated with the highest view. Whenever any correct replica decides on a value  $val$  within a view, the protocol ensures a set of non-conflicting view-change certificates can be constructed only for  $val$  and hence the value is carried over to the next view(s).

## 6.1 Discussion

We discuss how Algorithm 3 achieves the four features mentioned at the beginning of Section 6. The first feature (the algorithm solves consensus with  $n = 2f + 1$  processes) follows directly from the correctness of the algorithm. The second feature (the algorithm issues no signatures in the common-case) holds because in the common-case, processes will be able to deliver the required PREPARE and COMMIT messages and decide in the first view, without having to wait for any signatures to be produced or verified. The third feature (the algorithm issues  $O(n^2)$  signatures on view-change) holds because, in the worst case, during a view change each process will sign and broadcast a VIEWCHANGE message, thus incurring  $O(n)$  signatures in total, and, for each such message, each other process will sign and broadcast a VIEWCHANGEACK message, thus incurring  $O(n^2)$  signatures. The fourth feature states that the algorithm issues  $O(n)$  required background signatures within a view. These signatures are incurred by *cb-broadcasting* PREPARE and COMMIT messages. In every view, correct processes broadcast a COMMIT message, thus incurring  $n - f = O(n)$  signatures in total.

To the best of our knowledge, no existing algorithm has achieved all these four features simultaneously. The only broadcast-based algorithm which solves consensus with  $n = 2f + 1$  processes that we are aware of, that of Correia et al. [26], requires  $O(n)$  calls to Reliable Broadcast before any process can decide; this would incur  $O(n^2)$  required background signatures when using our Reliable Broadcast implementation—significantly more than our algorithm's  $O(n)$  required background signatures.

At this point, the attentive reader might have noticed that our consensus algorithm uses

some techniques that bear resemblance to our Reliable Broadcast algorithm in Section 5. Namely, the primary of a view *cb-broadcasts* a PREPARE message which is then echoed by the replicas in the form of COMMIT messages. Also, during view change, a replica’s VIEWCHANGE message is echoed by other replicas in the form of VIEWCHANGEACK messages. This is reminiscent of the Init-Echo technique used by our Reliable Broadcast algorithm.

Thus, the following question arises: Can we replace each instance of the witnessing technique in our algorithm by a single Reliable Broadcast call and thus obtain a conceptually simpler algorithm, which also satisfies the three above-mentioned properties? Perhaps surprisingly, the resulting algorithm is incorrect. It allows an execution which breaks agreement in the following way: a correct replica  $p_1$  *rb-delivers* some value  $v$  from the primary and decides  $v$ ; sufficiently many other replicas time out waiting for the primary’s value and change views without “knowing about”  $v$ ; in the next view, the primary *rb-broadcasts*  $v'$ , which is delivered and decided by some correct replica  $p_2$ .

Intuitively, by using a single Reliable Broadcast call instead of multiple Consistent Broadcast calls, some information is not visible to the consensus protocol. Specifically: while it is true that, in order for  $p_1$  to deliver  $v$  in the execution above,  $n - f$  processes must echo  $v$  (and thus they “know about”  $v$ ), this knowledge is however encapsulated inside the Reliable Broadcast abstraction and not visible to the consensus protocol. Thus, the information cannot be carried over to the view-change, even by correct processes. This intuition provides a strong motivation to use Consistent Broadcast—rather than Reliable Broadcast—as a first-class primitive in the design of Byzantine-resilient agreement algorithms.

## 7 Conclusion

A common tool to address Byzantine failures is to use signatures or lots of replicas. However, modern hardware makes these techniques prohibitive: signatures are much more costly than network communication, and excessive replicas are expensive. Hence, we seek algorithms that minimize the number of signatures and replicas. We applied this principle to broadcast primitives in a system that adopts the message-and-memory model, to derive algorithms that avoid signatures in the common case, use nearly-optimal number of signatures in the worst case, and require only  $n = 2f + 1$  replicas. We proved worst-case lower bounds on the number of signatures required by Consistent Broadcast and Reliable Broadcast, showing a separation between these problems. We presented a Byzantine consensus algorithm based on our Consistent Broadcast primitive. This is the first consensus protocol for  $n = 2f + 1$  without signatures in the common case. A novelty of our protocol is the use of Consistent Broadcast instead of Reliable Broadcast, which resulted in fewer signatures than existing consensus protocols based on Reliable Broadcast.

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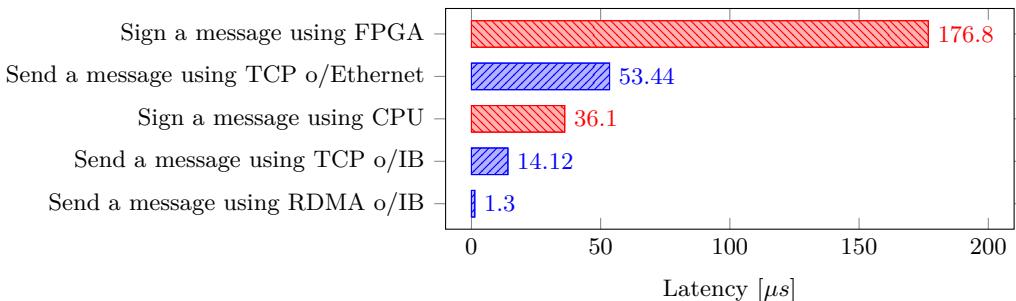
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## A APPENDIX: Latency



**Figure 1** RDMA communication is significantly faster than signature creation using CPU or hardware acceleration (FPGA). The graph shows the latency of sending or signing a 32-byte message. IB means Infiniband, a faster interconnect than Ethernet found in data centers. TCP latencies are obtained using sockperf [43]. RDMA latency is obtained using perftest [40]. Signatures use optimized implementations for CPU [11] and FPGA [48] of the ECDSA algorithm on the secp256k1 elliptic curve [21]. An FPGA improves the throughput of signature creation (not shown in figure), but not its latency, due to their relatively low clock speeds (compared to CPUs) and the non-parallelizable nature of algorithms for digital signature.

## B APPENDIX: Consistent Broadcast Correctness

We start with a simple observation:

► **Observation B.1.** *If  $p$  is a correct process, then no sub-slot that belongs to  $p$  is written to more than once.*

**Proof.** Since  $p$  is correct,  $p$  never writes to any sub-slot more than once. Furthermore, since all sub-slots are single-writer registers, no other process can write to these sub-slots. ◀

Before proving that our implementation, Algorithm 1, satisfies the properties of Consistent Broadcast, we show two intermediary results with respect to the liveness and safety of the scan operation.

► **Lemma B.2** (Termination of scan.). *Each scan operation returns.*

**Proof.** We observe the following:

1. If some slot  $S$  goes from empty to non-empty between consecutive iterations of the while loop, then some process (the writer of slot  $S$ ) wrote a value to  $S$ . This causes the while loop to continue (line 39).
2. Termination condition: the while loop exits (and thus the  $scan()$  operation returns) once either (a)  $others$  has no empty slots, or (b) no slot goes from empty to non-empty between two consecutive iterations of the while loop (line 39 is never executed).
3. Once a slot  $S$  is read and has non-empty value,  $others$  gets updated (lines 31 or 37) and the slot  $S$  is never read again.
4. the size of  $others$  is equal to the number of processes,  $n$ .

By contradiction, assume the scan function never terminates. This implies the scan function did not return after executing  $n + 1$  iterations of the while loop. This means, each iteration at least one slot went from being empty to containing a value. This value is added to the  $others$  array (line 37) and the slot is not read again in future iterations. Since the array  $others$  is bounded by  $n$ , after  $n$  iterations  $others$  contains only non-empty values. At the  $n + 1^{th}$  execution of the loop, no slot is empty,  $done$  stays true, and hence the operation returns. Contradiction.

Given every scan operation returns after executing at most  $n + 1$  times the while loop and each loop invokes at most  $n$  reads of the base registers, the complexity of the  $scan()$  operation is within  $O(n^2)$ .  $\blacktriangleleft$

► **Lemma B.3** (Non-inversion of scan). *Let  $p_1$  and  $p_2$  be correct processes who invoke  $scan$ . Let  $V_1$  and  $V_2$  be the return values of those scans, respectively. If  $V_1$  contains some value  $m_1$  in at least  $n - f$  slots and  $V_2$  contains some value  $m_2$  in at least  $n - f$  slots, then  $V_1$  contains  $m_2$  in at least one slot, or  $V_2$  contains  $m_1$  in at least one slot.*

**Proof.** Assume by contradiction that  $V_1$  does not contain  $m_2$  and  $V_2$  does not contain  $m_1$ .

Since  $V_1$  contains  $m_1$  in at least  $n - f$  slots, it must be that  $m_1$  was written by at least one correct process; call this process  $r_1$ . Similarly,  $m_2$  must have been written by at least one correct process; call this process  $r_2$ . Then the following must be true:

1.  $p_1$  must have read the slot of  $r_2$  at least twice and found it empty. Let  $t_{p_1 \leftarrow r_2}$  be the linearization point of  $p_1$ 's last read of  $r_2$ 's slot before  $p_1$  returns from the scan.
2.  $p_2$  must have read the slot of  $r_1$  at least twice and found it empty. Let  $t_{p_2 \leftarrow r_1}$  be the linearization point of  $p_2$ 's last read of  $r_1$ 's slot before  $p_2$  returns from the scan.
3.  $p_1$  must have read the slot of  $r_1$  and found it to contain  $m_1$ . Let  $t_{p_1 \leftarrow r_1}$  be the linearization point of  $p_1$ 's last read of  $r_1$ 's slot.
4.  $p_2$  must have read the slot of  $r_2$  and found it to contain  $m_2$ . Let  $t_{p_2 \leftarrow r_2}$  be the linearization point of  $p_2$ 's last read of  $r_2$ 's slot.

We now reason about the ordering of  $t_{p_1 \leftarrow r_1}$ ,  $t_{p_1 \leftarrow r_2}$ ,  $t_{p_2 \leftarrow r_1}$ , and  $t_{p_2 \leftarrow r_2}$ :

1.  $t_{p_1 \leftarrow r_1} < t_{p_1 \leftarrow r_2}$ . Process  $p_1$ 's last read of  $r_2$ 's slot must have occurred during the last iteration of the while loop before returning from the scan. Furthermore,  $p_1$ 's last read of  $r_1$ 's slot cannot have occurred on the same last iteration of the loop, otherwise the non-empty read would have triggered another iteration; thus,  $p_1$ 's last read of  $r_1$ 's slot must have occurred either in a previous iteration at line 37, or initially at line 31.
2. Similarly,  $t_{p_2 \leftarrow r_2} < t_{p_2 \leftarrow r_1}$ .
3.  $t_{p_1 \leftarrow r_2} < t_{p_2 \leftarrow r_1}$ . Process  $p_1$ 's last read of  $r_2$  returns an empty value, while  $p_2$ 's last read of  $r_2$  returns  $m_2$ . Since  $r_2$  is correct, its slot cannot go from non-empty to empty, thus the empty read must precede the non-empty read.
4. Similarly,  $t_{p_2 \leftarrow r_1} < t_{p_1 \leftarrow r_1}$ .

By transitivity, from (1)-(3), it must be that  $t_{p_1 \leftarrow r_1} < t_{p_2 \leftarrow r_1}$ . This contradicts (4). No valid linearization order exists for the four reads.  $\blacktriangleleft$

We are now ready to prove that Algorithm 1 satisfies the properties of Consistent Broadcast.

► **Lemma B.4 (Validity).** *If a correct process  $s$  broadcasts  $m$ , then every correct process eventually delivers  $m$ .*

**Proof.** Let  $s$  be a correct sender that broadcasts  $m$  and consider a correct receiver  $p$  that tries to deliver  $s$ 's message.

Since  $s$  is correct, it writes  $m$  to its message sub-slot. Therefore, all replicators read  $m$  and no other message from  $s$ , by Observation B.1.

If all replicators are correct and they copy  $m$  in a timely manner, then  $p$  is able to deliver  $m$  via the fast path (at line 25).

Otherwise  $s$ , being correct, will eventually write its (valid) signature of  $m$  to its signature sub-slot. Since we consider at most  $f$  Byzantine processes that replicate  $m$ , the  $n - f$  correct replicators are guaranteed to copy the signature of  $m$  to their slot. Moreover, since  $s$  is correct, and we assume Byzantine processes cannot forge the digital signatures of correct processes, no replicator can produce a different message  $m' \neq m$  with  $s$ 's signature. This enables receiver  $p$  to deliver  $m$  via the slow path (at line 28).  $\blacktriangleleft$

► **Lemma B.5 (No duplication).** *Every correct process delivers at most one message.*

**Proof.** Correct processes only deliver at lines 25 or 28. Immediately after a correct  $p$  process delivers a message,  $p$  exits the *while* loop and thus will not deliver again.  $\blacktriangleleft$

► **Lemma B.6 (Consistency).** *If  $p$  and  $p'$  are correct processes,  $p$  delivers  $m$  and  $p'$  delivers  $m'$ , then  $m=m'$ .*

**Proof.** Assume by contradiction that consistency does not hold; assume correct process  $p$  delivers  $m$ , while correct process  $p'$  delivers  $m' \neq m$ .

Assume first *wlog* that  $p$  delivers  $m$  using the fast path. Then  $p$  must have seen  $m$  in  $n$  replicator slots. Assume now that  $p'$  also delivers  $m'$  using the fast path; then,  $p'$  must have seen  $m'$  in  $n$  replicator slots. This means that all  $n$  replicators must have changed their written value, either from  $m$  to  $m'$ , or vice-versa; this is impossible since at least  $n - f$  of the replicators are correct and never change their written value (Observation B.1). Process  $p'$  must have then delivered  $m'$  using the slow path instead; then,  $p'$  must have seen signed copies of  $m'$  in  $n - f$  replicator slots. This means that  $n - f$  replicators, including at least one correct replicator, must have changed their value from  $m$  to  $m'$ , or vice-versa; this is impossible by Observation B.1.

So it must be that  $p$  and  $p'$  deliver  $m$  and  $m'$ , respectively, using the slow path. In this case,  $p$  sees signed copies of  $m$  in  $n - f$  slots, while  $p'$  sees signed copies of  $m'$  in  $n - f$  slots. Lemma B.3 therefore applies:  $p$  must also see a signed copy of  $m'$  or  $p'$  must also see a signed copy of  $m$ . Given there exists another validly signed value, the check at line 27 fails for  $p$  or  $p'$ . We have reached a contradiction:  $p$  does not deliver  $m$  or  $p'$  does not deliver  $m'$ .  $\blacktriangleleft$

► **Lemma B.7 (Integrity).** *If some correct process delivers  $m$  and  $s$  is correct, then  $s$  previously broadcast  $m$ .*

**Proof.** Let a correct receiver  $p$  deliver a value, say  $m \neq \perp$ . To deliver,  $m$  must either be (a) the value  $p$  reads from the slots of all replicators (line 25) or (b) the signed value  $p$  reads from the slots of at least  $n - f$  replicators (line 28). In both cases, for the delivery of  $m$  to occur, at least one correct replicator  $r$  contributes by writing value  $m$  (unsigned in case (a) or signed in case (b)) to its slot. Given  $r$  is a correct process, it must have copied the value it read from the sender's slot. Furthermore, a correct sender never writes any value unless it *cb-broadcasts* it. Therefore,  $m$  must have been broadcast by the sender  $s$ .  $\blacktriangleleft$

**Execution.** We provide an example of an execution breaking consistency when the collect operation is used instead of the scan operation in Algorithm 1. Let there be a Byzantine sender  $s$  and  $n = 3$  replicators. Let  $p_1, p_2$  be two correct receivers,  $r_1, r_2$  two correct replicators and let replicator  $r_3$  be Byzantine. Initially, let  $s$  write  $m_1$  signed in its slot. Let receiver  $p_2$  start its collect. It reads the slot of  $r_1$  which it finds empty, and sleeps. Let  $r_1, r_3$  copy  $m_1$  signed in their slot, while  $r_2$  sleeps. Let  $p_1$  perform its collect, find two signed copies of  $m_1$  and deliver  $m_1$  via the check at line 28. Let  $s$  change its value to  $m_2$  signed, while  $r_3$ , being Byzantine, changes its value to  $m_2$  signed. We resume  $r_2$  and let it copy  $m_2$  signed. We resume  $p_2$ 's collect, continuing to read  $r_2, r_3$  slots, seeing two values of  $m_2$  signed (recall it previously read  $r_1$ 's slot while it was empty) and delivering  $m_2$  via the check at line 28.

## C APPENDIX: Reliable Broadcast Correctness

► **Invariant C.1.** *Let  $S$  and  $S'$  be two valid ReadySets for  $m$  and  $m'$ , respectively. Then,  $m = m'$ .*

**Proof.** By contradiction. Assume there exist valid *ReadySets*  $S$  and  $S'$  for different values  $m \neq m'$ . Set  $S$  (resp.  $S'$ ) consists of at least  $n - f$  signed  $m$  (resp. signed  $m'$ ) messages. Then there exist correct replicators  $r$  and  $r'$  such that  $r$  writes  $m$  and its signature to its *Echo* slot and  $r'$  writes  $m'$  and its signature to its *Echo* slot. This is impossible since correct replicators only write  $*$  in their *Echo* slots once they have *cb-delivered*  $\langle \text{INIT}, * \rangle$  from the sender. By the consistency property of Consistent Broadcast,  $m$  must be equal to  $m'$ .  $\blacktriangleleft$

► **Lemma C.2 (Validity).** *If a correct process  $s$  broadcasts  $m$ , then every correct process eventually delivers  $m$ .*

**Proof.** Assume the sender  $s$  is correct and broadcasts  $m$ . Let  $p$  be a correct receiver that tries to deliver  $s$ 's message.

Since the sender is correct, it *cb-broadcasts*  $\langle \text{INIT}, * \rangle$ . By the validity property of Consistent Broadcast, all correct replicators will eventually deliver  $\langle \text{INIT}, * \rangle$  from  $s$ . Then, all correct replicators will write  $m$  to their *Echo* message sub-slots, compute a signature for  $m$  and write it to their *Echo* signature sub-slots. If all replicators are correct and they copy  $m$  in a timely manner, then  $p$  is able to deliver  $m$  via the fast path (at line 43).

All correct replicators will eventually read each other's signed messages  $m$ ; thus every correct replicator will be able to either (a) create a valid *ReadySet* and write it to its *Ready* slot or (b) copy a valid *ReadySet* to its *Ready* slot. Thus,  $p$  will eventually be able to read at least  $n - f$  valid *ReadySets* for  $m$  and deliver  $m$  via the slow path (at line 45).  $\blacktriangleleft$

► **Lemma C.3 (No duplication).** *Every correct process delivers at most one message.*

**Proof.** Correct processes only deliver at lines 43 or 45. Immediately after a correct  $p$  process delivers a message,  $p$  exits the *while* loop and thus will not deliver again.  $\blacktriangleleft$

► **Lemma C.4** (Consistency). *If  $p$  and  $p'$  are correct processes,  $p$  delivers  $m$  and  $p'$  delivers  $m'$ , then  $m=m'$ .*

**Proof.** By contradiction. Let  $p, p'$  be two correct receivers. Let  $p$  deliver  $m$  and  $p'$  deliver  $m' \neq m$ . We consider 3 cases: (1)  $p$  and  $p'$  deliver their messages via the fast path, (2)  $p$  and  $p'$  deliver their messages via the slow path, and (3) (*wlog*)  $p$  delivers via the fast path and  $p'$  delivers via the slow path.

- (1)  $p$  and  $p'$  must have delivered  $m$  and  $m'$  respectively, by reading  $m$  (resp.  $m'$ ) from the *Echo* slots of  $n$  replicators. Thus, there exists at least one replicator  $r$  such that  $p$  read  $m$  from  $r$ 's *Echo* slot and  $p'$  read  $m'$  from  $r$ 's *Echo* slot. This is impossible since correct replicators never overwrite their *Echo* slots.
- (2)  $p$  and  $p'$  must have each read  $n - f$  valid *ReadySets* for  $m$  and  $m'$ , respectively. This is impossible by Invariant C.1.
- (3)  $p'$  read at least one valid *ReadySet* for  $m'$ . To construct a valid *ReadySet*, one requires a signed set of  $n - f$  values for  $m'$ . Thus, at least one correct replicator  $r$  must have written  $m'$  to its *Echo* slot and appended a valid signature for  $m'$ . Process  $p$  delivered  $m$  by reading  $m$  from the *Echo* slots of all  $n$  replicators, which includes  $r$ . This is impossible since correct replicators never overwrite their *Echo* slots.

◀

► **Lemma C.5** (Integrity). *If some correct process delivers  $m$  and  $s$  is correct, then  $s$  previously broadcast  $m$ .*

**Proof.** Let  $p$  be a correct receiver that delivers  $m$  and let the sender  $s$  be correct. We consider 2 cases: (1)  $p$  delivers  $m$  via the fast path and (2)  $p$  delivers  $m$  via the slow path.

- (1) Fast Path.  $p$  must have read  $m$  from the *Echo* slot of at least one correct replicator  $r$ . Replicator  $r$  writes  $m$  to its slot only upon *cb-delivering*  $\langle \text{INIT}, m \rangle$  from  $s$ . By the integrity property of Consistent Broadcast,  $s$  must have broadcast  $m$ . Moreover, a correct sender only invokes *cb-broadcast*( $\langle \text{INIT}, m \rangle$ ) upon a *rb-broadcast* event for  $m$ .
- (2) Slow Path.  $p$  must have read at least one valid *ReadySet* for  $m$ . A *ReadySet* consists of a signed set of  $n - f$  values for  $m$ . Thus, at least one correct replicator  $r$  must have written  $m$  signed to its *Echo* slot. The same argument as in case (1) applies.

◀

► **Lemma C.6** (Totality). *If some correct process delivers  $m$ , then every correct process eventually delivers a message.*

**Proof.** Let  $p$  be a correct receiver that delivers  $m$ . We consider 2 cases: (1)  $p$  delivers  $m$  via the fast path and (2)  $p$  delivers  $m$  via the slow path.

- (1) Fast Path.  $p$  must have read  $m$  from the *Echo* slots of all  $n$  replicators, which include  $n - f$  correct replicators. These  $n - f$  correct replicators must eventually append their signature for  $m$ . Every correct replicator looks for signed copies of  $m$  in other replicators' *Echo* slots. Upon reading  $n - f$  such values, each correct replicator is able to construct and write a valid *ReadySet* to its *Ready* slot (or copy a valid *ReadySet* to its *Ready* slot from another replicator). Thus, every correct receiver will eventually read  $n - f$  valid *ReadySets* for  $m$  and deliver  $m$  via the slow path.
- (2) Slow Path.  $p$  must have read valid *ReadySets* for  $m$  from the slots of  $n - f$  replicators, which must include at least one correct replicator  $r$ . Since  $r$  is correct,  $r$  will never remove its *ReadySet* for  $m$ . Thus, every correct replicator will eventually either (a) copy  $r$ 's *ReadySet* to their own *Ready* slots or (b) construct and write a *ReadySet* to their *Ready* slots. Note that by Invariant C.1, all valid *ReadySets* must be for the same value  $m$ .



Thus, every correct receiver will eventually read  $n - f$  valid *ReadySets* for  $m$  and deliver  $m$  via the slow path.



## D APPENDIX: Byzantine Consensus Correctness and Additional Details

### D.1 Valid messages

A  $\langle \text{PREPARE}, view, val, proof \rangle$  message is considered valid by a (correct) process if:

- the process is part of  $view$ ,
- the broadcaster of the PREPARE is the coordinator of  $view$ , i.e.,  $view \% n$ ,
- when  $view = 0$ ,  $proof = \emptyset$  and  $val$  can be any value  $\neq \perp$
- when  $view > 0$ , the estimate matches the highest view tuple in  $proof$  and the  $proof$  set is valid, i.e., it contains a set of  $n - f$  non-conflicting view-change certificates for view  $view$ ; in case all tuples in  $proof$  are still the init value  $(0, \perp, \emptyset)$ , any estimate is a valid estimate,
- the process did not previously accept a different PREPARE in  $view$ .

A  $\langle \text{COMMIT}, view, val \rangle$  message is considered valid by a (correct) process if:

- the process is part of  $view$ ,
- $val$  can be any estimate,
- the broadcaster did not previously send a view change message for  $view' > view$ ,
- the broadcaster did not previously send another COMMIT message for  $val' \neq val$  in the same  $view$ .

A  $\langle \text{VIEWCHANGE}, view+1, (view_{val}, val, proof_{val}) \rangle$  message from process  $j$  is considered valid by a (correct) process if:

- $val \in (view_{val}, val, proof_{val})$  corresponds to the latest non-empty value broadcast in a  $\langle \text{COMMIT}, view_c, val_c \rangle$ ,  $val = val_c$  and  $view_{val} = view_c (\leq view)$  and  $proof_{val}$  is a valid proof for  $val$  (either consists of non-conflicting certificates that support  $val$  as highest view-tuple or all tuples are with their init value; all VIEWCHANGE and VIEWCHANGEACK messages must be for  $view_{val}$ ),
- $val \in (0, val, proof_{val})$ ,  $proof_{val}$  is  $\emptyset$ ,
- if for each view  $view' \leq view$ ,  $\langle \text{COMMIT}, view', \perp \rangle$  from  $j$  are empty; then  $(view_{val}, val, proof_{val})$  must be equal to  $(0, \perp, \emptyset)$ ,
- $j$  must have sent a single COMMIT message each view  $view' \leq view$ ,
- $j$  did not send another VIEWCHANGE message this view,  $view+1$ .

### D.2 Agreement

► **Lemma D.1.** *In any view  $v$ , no two correct processes accept PREPARE messages for different values  $val \neq val'$ .*

**Proof.** Let  $i, j$  be two correct processes. Any correct process accepts a PREPARE messages only from the current view's primary (line 12).

A correct primary  $p$  never broadcasts conflicting PREPARE messages (i.e., same view  $v$ , but different estimates  $val, val', val \neq val'$ ). This means,  $i, j$  must receive the same PREPARE message. By Lemma D.9, both  $i$  and  $j$  consider the PREPARE message from  $p$  valid.

A faulty primary  $p'$  may broadcast conflicting PREPARE messages. Assume the primary broadcasts  $(k, \langle \text{PREPARE}, v, val, proof \rangle)$  and  $(k', \langle \text{PREPARE}, v, val', proof' \rangle)$  where  $k, k'$  are the broadcast sequence numbers used. We distinguish between the following cases:

1.  $k < k'$ : By the FIFO property, any correct replica must process message  $k$  of  $p'$  before processing message  $k'$ . If process  $i$  accepts the  $k^{\text{th}}$  message of  $p'$ , following the consensus protocol,  $i$  will not accept a second PREPARE message in the same view  $v$ , i.e. message  $k'$ . Similarly for correct replica  $j$ .
2.  $k > k'$ : The argument is similar to (1).
3.  $k = k'$ : In this case,  $p'$  equivocates. If a message gets delivered by both  $i$  and  $j$ , then the message is guaranteed to be the same by the consistency property of Consistent Broadcast.

We conclude correct replicas agree on the PREPARE message accepted within the same view.  $\blacktriangleleft$

► **Lemma D.2.** *In any view  $v$ , no two correct processes call  $\text{try\_decide}$  with different values  $\text{val}$  and  $\text{val}'$ .*

**Proof.** By contradiction. Let  $i, j$  be two correct processes. Assume in view  $v$ , processes  $i, j$  call  $\text{try\_decide}$  with value  $\text{val}$ , respectively  $\text{val}'$ . To call  $\text{try\_decide}$ , the condition at line 24 must be true for both  $i$  and  $j$ . This means  $i$ , (resp.  $j$ ) accepts a valid PREPARE message supporting  $\text{val}$  (resp.  $\text{val}'$ ) and a set of  $n - f$  COMMIT messages supporting  $\text{val}$  (resp.  $\text{val}'$ ). By Lemma D.1, correct processes cannot accept different PREPARE messages and consequently cannot call  $\text{try\_decide}$  with different values since  $\text{aux}_i = \text{aux}_j$ .  $\blacktriangleleft$

► **Lemma D.3.** *Let a correct process  $i$  decide  $\text{val}$  in view  $v$ . For view  $v+1$ , no valid proof can be constructed for a different estimate  $\text{val}' \neq \text{val}$ .*

**Proof.** By contradiction. Let  $i$  decide  $\text{val}$  in view  $v$ . Assume the contrary and let there be a valid proof such that  $(v+1, \text{val}', \text{proof})$ .

Given  $v+1 > 0$ ,  $\text{proof}$  cannot be  $\emptyset$ . It must be the case that the proof supporting  $\text{val}'$  consists of a set of  $n - f$  non-conflicting view-change certificates. Each view-change certificate consists of a VIEWCHANGE message with format  $\langle \text{VIEWCHANGE}, v+1, (\text{view}, \text{value}, \text{proof}_{\text{val}}) \rangle$  and  $f$  corresponding VIEWCHANGEACK messages. Any view-change certificate requires the involvement of at least one correct replica, namely, either a correct replica is the broadcaster of a VIEWCHANGE message or a correct replica validates a VIEWCHANGE message, by sending a corresponding VIEWCHANGEACK.

For  $\text{val}'$  to be consistent with  $\text{proof}$ ,  $\text{proof}$  must contain either (a) at least one view-change certificate with tuple  $(v, \text{val}', \text{proof}_{\text{val}'})$  and no other view-change certificate s.t. its tuple has a different value for the same view,  $v$ , i.e.,  $\nexists (v, \text{val}, \text{proof}_{\text{val}})$ , with  $v$  the highest view among the  $n - f$  tuples or (b) only view-change certificates with tuples having the initial value  $(0, \perp, \emptyset)$  so that any value is a valid value. Let  $R_1$  denote the set of processes that contributed with a VIEWCHANGE message, which is then part of a view-change certificate in  $\text{proof}$ .

Given  $i$  decided  $\text{val}$  in view  $v$ ,  $i$  received  $n - f$  COMMIT messages for  $\text{val}$  (line 24). Such processes must have received a valid PREPARE message and updated their view-change tuple together with their auxiliary in lines 14 and 15, before sending a COMMIT message. Let  $R_2$  denote the set of processes that contributed with a COMMIT message for  $\text{val}$ .

These two sets,  $R_1$  and  $R_2$ , must intersect in at least one replica  $j$ . Replica  $j$  must have used Consistent Broadcast for its view-change message:  $(k_{vc}, \langle \text{VIEWCHANGE}, v+1, (v, \text{val}', \text{proof}_{\text{val}'}) \rangle)$ ; the argument is similar for the case  $(k_{vc}, \langle \text{VIEWCHANGE}, v+1, (0, \perp, \emptyset) \rangle)$ , where  $k_{vc}$  is the broadcast sequence number used; otherwise it could have not gathered enough VIEWCHANGEACKs, since correct replicas do not accept messages not delivered via the broadcast primitive. Similarly,  $j$  must have used Consistent Broadcast for its COMMIT

message:  $(k_c, \langle \text{COMMIT}, v, val \rangle)$ , where  $k_c$  is the broadcast sequence number used; otherwise  $i$  would not have accepted the COMMIT message.

If  $j$  is correct, and sends a COMMIT message for  $val$ , it broadcasts a VIEWCHANGE message with its true estimate,  $val$ . Hence, the  $R_1$  set of non-conflicting view-change messages must contain a tuple  $(v, val, proof_{val})$ . This yields either a set of conflicting view-change certificates if  $\exists$  another view-change certificate for  $(v, val', proof_{val'})$ , or a conflict between  $proof$  and  $val'$  as matching estimate (since  $v$  is the highest-view and the value associated with this tuple corresponds to estimate  $val$  and not  $val'$ ).

If  $j$  is Byzantine, we distinguish between the following cases:

1.  $k_c < k_{vc}$  ( $j$  broadcasts its COMMIT message before it broadcasts its VIEWCHANGE message). In this case, no correct process sends a VIEWCHANGEACK for  $j$ 's VIEWCHANGE message. By the FIFO property, a correct process first delivers the  $k_c$  message and then  $k_{vc}$  message. In order to validate a VIEWCHANGE message, the last non-empty value broadcast in a COMMIT must correspond to the value broadcast in the VIEWCHANGE. Since these do not match, no correct process sends a VIEWCHANGEACK for  $j$ 's VIEWCHANGE. Hence, the VIEWCHANGE message of  $j$  does not gather sufficient ACKs to form a view-change certificate and be included in  $proof$ .
2.  $k_{vc} < k_c$  ( $j$  broadcasts its VIEWCHANGE message before it broadcasts its COMMIT message). In this case, process  $i$  must have first delivered the VIEWCHANGE message from  $j$ . Consequently,  $i$  does not accept  $j$ 's COMMIT message as valid. This contradicts our assumption that  $i$  used this COMMIT message to decide  $val$ .
3.  $k_{vc} = k_c$  ( $j$  equivocates). By the properties of Consistent Broadcast, correct processes either deliver  $j$ 's COMMIT message, case in which the VIEWCHANGE message does not get delivered by any correct replica, and consequently does not gather sufficient VIEWCHANGEACK to form a view-change certificate (for neither  $val'$  nor  $\perp$ ); or correct processes deliver  $j$ 's VIEWCHANGE message, case in which the COMMIT message does not belong to  $R_2$ ,  $i$  does not decide.

We conclude, if  $i$  decided  $val$  in view  $v$ , no valid proof can be constructed for view  $v+1$  and  $val' \neq val$ .  $\blacktriangleleft$

► **Lemma D.4.** *Let a correct process  $i$  decide  $val$  in view  $v$ . For any subsequent view  $v' > v$ , no valid proof can be constructed for a different estimate  $val' \neq val$ .*

**Proof.** We distinguish between the following two cases: (1)  $v' = v+1$  and (2)  $v' > v$ .

**Case 1:** Follows from Lemma D.3.

**Case 2:** By contradiction. Let process  $i$  decide  $val$  in view  $v$ . Assume the contrary and let  $v' > v$  be the lowest view in which there exists a valid proof for  $val' \neq val$ , i.e.,  $(v', val', proof)$ .

A valid proof supporting  $val'$  must contain  $n-f$  non-conflicting view-change certificates out of which (a) one view-change certificate supports  $val'$  or (b) all view-change certificates claim  $\perp$ . A view-change certificate consists of a VIEWCHANGE message and  $f$  VIEWCHANGEACK messages. This means, at least one correct process must validate a VIEWCHANGE message by broadcasting a VIEWCHANGEACK message, or be the producer of a VIEWCHANGE message.

(a) For  $val'$  to be the representative value of the  $n - f$  view-change certificates in  $proof$ , one of the view-change messages must contain a tuple with the highest view among all  $n - f$  tuples. Let this tuple be  $(v' - 1, val', proof_{val'})$  such that  $v' - 1$  is the highest view possible before entering  $v'$ . This tuple must then come from view  $v' - 1$  with a valid proof,  $proof_{val'}$  supporting the fact that  $val'$  is a valid value.

By assumption, the only valid proof that can be constructed in views prior to  $v'$  but succeeding  $v$  is for estimate  $val$ . Hence, there is no valid proof,  $proof_{val'}$  for  $val'$  in view  $v' - 1$ . In the case in which the producer of the VIEWCHANGE is correct, it will not construct a VIEWCHANGE message with an invalid proof. The  $vc_i$  variable is only updated if the PREPARE message is valid. In the case in which the producer of the VIEWCHANGE is faulty, it will not gather the necessary VIEWCHANGEACK to form a view-change certificate given correct replicas do not validate a VIEWCHANGE message with an invalid proof or in which the estimate value contradicts the proof. This contradicts our initial assumption that there exists a valid view-change certificate supporting  $val'$ .

(b) All VIEWCHANGE messages in  $proof$  have tuples  $(0, \perp, \emptyset)$  so that any estimate value is valid value. Let this set be denoted by  $R_1$ . Since  $i$  decided  $val$  in view  $v$ , a set of  $R_2$  replicas contributed with a COMMIT value for  $val$ . Sets  $R_1$  and  $R_2$  must intersect in one replica, say  $j$ . If  $j$  sends COMMIT messages in subsequent views for a value  $\perp$ ,  $j$  must send a VIEWCHANGE message matching its latest non-empty COMMIT message, i.e.,  $\langle \text{VIEWCHANGE}, v', (v, val, proof_{val}) \rangle$ , in order to gather sufficient VIEWCHANGEACK and hence form a view-change certificate. If  $j$  sends a COMMIT message in any subsequent view for a value  $val' \neq \perp$ , the only possible valid proof is for value  $val$ , see case (a). Whichever the case, at least one view-change certificate in  $proof$  must contain a view-change message with a non-empty tuple which contradicts our assumption that all view-change certificates are for  $\perp$ .  $\blacktriangleleft$

► **Theorem D.5** (Agreement). *If correct processes  $i$  and  $j$  decide  $val$  and  $val'$ , respectively, then  $val = val'$ .*

**Proof.** We distinguish two cases: (1) decision in the same view (2) decision in different views.

**Case 1:** decision in the same view. Follows from Lemma D.2.

**Case 2:** decision in different views. By contradiction. Let  $i, j$  be two correct processes. Assume processes  $i$  and  $j$  decide two different values,  $val$ , respectively  $val'$ , in views  $v$ , respectively  $v'$ . Let  $v < v' wlog$ .

To decide, a correct process must receive a valid PREPARE message and  $n - f$  COMMIT messages for the same estimate, line 24. When  $i$  decides  $val$  in view  $v$ , by Lemma D.4, from view  $v+1$  onward, the only valid  $proof$  supports estimate  $val$ . Hence, a valid PREPARE message can only contain an estimate for  $val$  and at any view-change procedure, no VIEWCHANGE supporting  $val'$  is able to form a view-change certificate. Given process  $j$  only accepts valid PREPARE messages,  $j$  cannot adopt  $val'$  as its auxiliary,  $aux_j$ . This means  $j$  cannot decide  $val'$ . Given process  $j$  only collects a set of (non-conflicting) view-change certificates,  $j$  cannot adopt  $val'$  as its estimate  $est_j$ .  $\blacktriangleleft$

### D.3 Integrity

► **Theorem D.6** (Integrity). *No correct process decides twice.*

**Proof.** A correct process may call *try\_decide* (line 25) multiple times. Yet, once a correct process calls *decide* (line 41), the *decided* variable is set to *true* and hence the if statement is never entered again.  $\blacktriangleleft$

## D.4 Validity

► **Theorem D.7** (Weak validity). *If all processes are correct and some process decides  $val$ , then  $val$  is the input of some process.*

**Proof.** Assume a correct process decides  $val$ . Following the steps in the algorithm, a correct process only decides a value for which it receives a valid PREPARE message and  $n - f$  COMMIT messages, in the same view (line 24). It is either the case the value in the PREPARE message comes from the previous view or it is the input value of the current view's primary (line 10). For the latter, validity is satisfied. For the former, the value in the previous view must come from one of the VIEWCHANGE messages. Which is either an input value of a prior view's primary or the value of a previous view message. We continue by applying the same argument inductively, backward in the sequence of views, until we reach a view in which the value was the input value of a primary. This shows that  $val$  was proposed by the primary in some view. ◀

## D.5 Termination

► **Lemma D.8.** *Two correct processes cannot send conflicting VIEWCHANGE messages.*

**Proof.** Assume the contrary and let  $v$  be the earliest view in which correct processes  $i$  and  $j$  send conflicting VIEWCHANGE messages  $m_1$  and  $m_2$ , respectively. Let  $vc_i = (view_i, val_i, proof_i)$  and  $vc_j = (view_j, val_j, proof_j)$  be the view-change tuples in  $m_1$  and  $m_2$ , respectively. Since  $m_1$  and  $m_2$  conflict, it must be the case that  $view_i = view_j$  and  $\perp \neq val_i \neq val_j \neq \perp$ . Thus, in view  $view_i = view_j$ ,  $i$  and  $j$  must have received and accepted PREPARE messages for different values  $val_i$  and  $val_j$ . This contradicts Lemma D.1. ◀

► **Lemma D.9.** *A PREPARE, COMMIT or VIEWCHANGE message from a correct process is considered valid by any correct process.*

**Proof.** A correct process  $i$  only sends a PREPARE message if it is the coordinator of that view (line 9). When  $view = 0$ ,  $est_i$  is initialized to  $\perp$  which leads  $i$  to set  $init_i$  to  $v_i$  (line 10). The PREPARE message has the following format:  $\langle \text{PREPARE}, 0, v_i, \emptyset \rangle$  which matches the required specification for a valid PREPARE. When  $view > 0$ , any correct process updates its  $proof_i$  and  $est_i$  before increasing its  $view_i$  variable, i.e. moving to the next view. A correct process would update these two vars according to the protocol, lines 30 and 31. As before, in case  $est_i = \perp$ ,  $i$  to set  $init_i$  to  $v_i$ , otherwise it carries  $est_i$  (line 10). The PREPARE message has the following format:  $\langle \text{PREPARE}, view_i, init_i, proof_i \rangle$  which matches the required specification for a valid PREPARE.

A correct process  $i$  broadcasts exactly one COMMIT message in  $view$  (line 20) after it either (a) hears from the coordinator of the current view or (b) starts suspecting the coordinator. In case (a)  $i$ 's message contains the estimate of the coordinator (line 14), while in case (b) it contains  $\perp$  (line 17). In any of the two cases,  $i$ 's VIEWCHANGE message strictly follows the COMMIT message (lines 28 and 20). The behaviour is in-line with the specification.

A correct process  $i$  broadcasts exactly a single VIEWCHANGE message in one  $view$  (line 28) with its  $vc_i$ . Process  $i$  update its view-change tuple,  $vc_i$ , only when it receives a valid PREPARE message. Such message is ensured to be in accordance with the prior specifications for a valid PREPARE message. Notice that a valid PREPARE message cannot be  $\perp$ , and hence  $vc_i$  is either its initial value,  $(0, \perp, \emptyset)$  or a valid tuple  $(view, val, proof)$ . The data in  $vc_i$  is updated at the same time  $aux_i$  is updated, upon receiving a valid PREPARE, and these two variables indicate the same estimate (lines 14 and 15). The  $aux_i$  is then send

via a COMMIT message within the same view (line 20). This ensures that the broadcast of  $i$ 's latest non-empty COMMIT corresponds to the data in its  $vc_i$  variable.  $\blacktriangleleft$

Let  $i$  be a correct process. For a given execution  $E$ , we denote by  $\mathcal{V}(i)$  the set of views in which  $i$  enters. We denote  $v_{max}(i) = \max \mathcal{V}(i)$ ; by convention  $v_{max}(i) = \infty$  if  $\mathcal{V}(i)$  is unbounded from above.

► **Lemma D.10.** *For every correct process  $i$ ,  $v_{max}(i) = \infty$*

**Proof.** Assume the contrary and let *wlog*  $i$  be the process with the lowest  $v_{max}$ . Since  $i$  never progresses past view  $v_{max}(i)$ ,  $i$  must be blocked forever in one of the *wait until* statements at lines 12, 22, or 29. We now examine each such case:

1. Line 12. If the primary  $p$  of view  $v_{max}(i)$  is faulty and does not broadcast a valid PREPARE message, then eventually  $i$  times out on the primary and progresses past the *wait until* statement. If  $p$  is correct, then  $p$  eventually reaches view  $v_{max}(i)$  and broadcasts a PREPARE message  $m$ . By the validity property of Consistent Broadcast,  $i$  eventually delivers  $m$  from  $p$ . By Lemma D.9,  $i$  considers  $m$  valid and thus progresses past the *wait until* statement.
2. Line 22. By our choice of  $i$ , every correct process must eventually reach view  $v_{max}(i)$ . Given the argument at item (1) above, no correct process can remain blocked forever at the *wait until* statement in line 12, thus every correct process eventually broadcasts a COMMIT message in view  $v_{max}(i)$ . By the validity property of Consistent Broadcast and by Lemma D.9,  $i$  eventually delivers all such messages and considers them valid. Therefore,  $i$  must eventually deliver valid PREPARE messages from  $n - f$  processes and progress past the *wait until* statement.
3. Line 29. By our choice of  $i$ , every correct process must eventually reach view  $v_{max}(i)$ . Given the argument at items (1) and (2) above, no correct process can remain blocked forever at the *wait until* statements in lines 12 and 22, thus every correct process eventually broadcasts a VIEWCHANGE message in view  $v_{max}(i)$ . By the validity property of Consistent Broadcast and by Lemma D.9, every correct process eventually delivers all such VIEWCHANGE messages and considers them valid. Thus, for every VIEWCHANGE message  $m$  sent by a correct process in view  $v_{max}(i)$ , every correct process eventually broadcasts a VIEWCHANGEACK message  $mAck$  with  $m$ 's digest; furthermore,  $i$  receives and considers valid each such  $mAck$ . Thus,  $i$  eventually gathers a set of  $n - f$  view-change certificates in view  $v_{max}(i)$ , which are non-conflicting by Lemma D.8. This means that  $i$  is eventually able to progress past the *wait until* statement.

We have shown that  $i$  cannot remain blocked forever in view  $v_{max}(i)$  in any of the *wait until* statements. Thus,  $i$  must eventually reach line 32 and increase  $view_i$  to  $v_{max}(i) + 1$ . We have reached a contradiction.  $\blacktriangleleft$

We define a view  $v$  to be *stable* if in  $v$ : (1) the coordinator is correct and (2) no correct process times out on another correct process.

► **Theorem D.11 (Termination).** *Eventually every correct process decides.*

**Proof.** We will show that every correct process eventually calls *try\_decide*, which is sufficient to prove the result. By our assumption of eventual synchrony, there is a time  $T$  after which the system is synchronous. We can also assume that after  $T$ , no correct process times out on another process. Let  $i$  be a correct process. Let  $v^*$  be the earliest view such that: (1)  $i$  enters  $v^*$  after time  $T$  and (2) the primary of  $v^*$  is correct. Recall that by Lemma D.10,  $i$  and all other correct processes are guaranteed to eventually reach view  $v^*$ . Let  $p$  be the

(correct) primary of  $v^*$ . By our choice of  $v^*$ ,  $p$  broadcasts a PREPARE message  $m$  in  $v^*$ , which is received and considered valid by all correct processes (by the validity property of Consistent Broadcast and Lemma D.9). Thus all correct processes will set their  $aux$  variable to the value  $val$  contained in  $m$ , and broadcast a COMMIT message with  $val$ . Process  $i$  must eventually deliver these COMMIT messages and consider them valid, thus setting at least  $n - f$  entries of  $R_i$  to  $val$  in line 23. Therefore, the check at line 24 will succeed for  $i$  and  $i$  will call  $try\_decide$  at line 25.  $\blacktriangleleft$