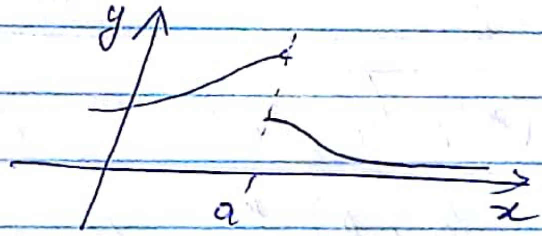
Recall

There are ~~mainly~~ 3 situations when the derivative of a function does not exist at some point (say a). Pictorially,

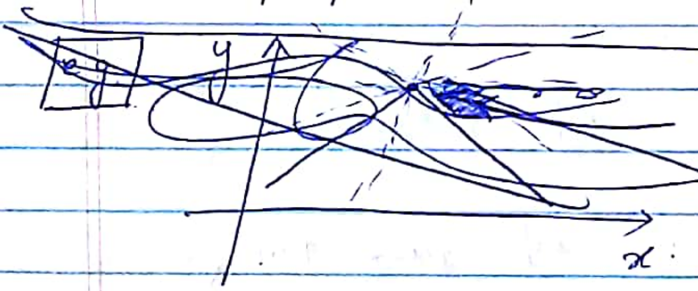
(i) The function is not continuous at $x = a$.

e.g

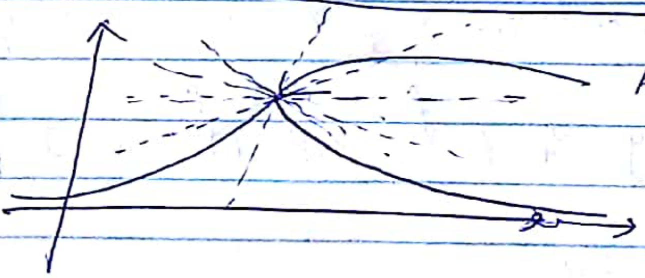



(ii) 'The ~~slope of the~~ tangent line' does not make sense. (This happens when there are usually 'sharp points' on the graph).

e.g



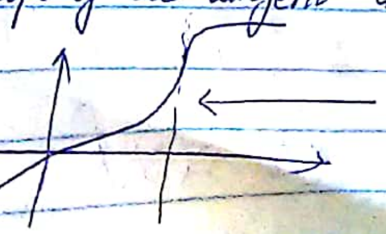

here there is no unique tangent line, so 'the tangent



At this point, there is no unique tangent line; "the tangent line" doesn't make sense.

(iii) If the slope of the tangent line is undefined.

e.g



the slope of the tangent line is undefined; i.e. when tangent lines are vertical the derivative does not exist.

e.g. $f(x) = \sqrt[3]{x}$. It is continuous at $x=0$.

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, ~~But~~ So $f'(0)$ does not make sense.

Hence, $f$ is not differentiable at $x=0$.

Remark: I thought it's good to have a discussion of these 3 cases again as they're:

i) important for understanding

ii) important for multiple choice / 'T/F' questions.

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## 3.3 Chain-Rule.

Let $h(x) = g(f(x))$, where $f$ and $g$ are differentiable. Then $h$ is differentiable and

$$\boxed{h'(x) = \frac{d}{dx}(h(x)) = \frac{d}{dx}\left(g(f(x))\right) = g'(f(x)) \cdot f'(x).}$$

Equivalently, if we write $\boxed{y = h(x) = g(u)}$ where $u = f(x)$,

then $\boxed{\frac{dy}{dx} = \frac{dg}{du} \cdot \frac{du}{dx}}$

· Consequence of Chain-Rule.

## $\boxed{\text{The General Power-Rule}}$

If $f$ is differentiable and $h(x) = [f(x)]^n$, ($n$, a real number) then $h'(x) = n\,[f(x)]^{n-1}\, f'(x)$.

$\boxed{\text{Proof}}$ Let $g(x) = x^n$. So, $g'(x) = n x^{n-1}$. ... (*)

So, $h(x) = g(f(x))$.

Thus, by chain-rule, $h'(x) = g'(f(x)) . f'(x)$.

~~Now, $g'(x) = n x^{n-1}$~~ $= n\,(f(x))^{n-1} . f'(x)$ (using (*)).

□

$\boxed{\text{e.g}}$ Suppose $f(1)=3$, $f'(1) = -2$, $f(3)=2$, $f'(3) = -3$,
and $g(1) = 3$, $g'(1) = -1$, $g(3) = 4$, $g'(3) = 0$.

If $h(x) = f(x^2 g(x))$, find $h'(1)$.

$\boxed{\text{Sol}^{\underline{n}}}$ $h'(x) = f'(x^2 g(x)) . (2x\, g(x) + x^2 g'(x))$ (Using i) Chain rule ii) Product rule on the inner function)

So, $h'(1) = f'(1\, g(1)) (2g(1) + 1\, g'(1))$
$= f'(3)\,(2 . 3 + (-1))$
$= (-3)(5) = -15$.

□