

eg Sketch:

$$g(x) = \frac{2x^2 + 5}{4 - x^2}$$

Soln. Domain =  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ .

•  $2x^2 + 5 \neq 0$  anywhere. So, there is no  $x$ -intercept;

•  $g(0) = \frac{5}{4}$ . Thus  $(0, \frac{5}{4})$  is the  $y$ -intercept.

• Now,  $4 - x^2 = 0 \Rightarrow x = \pm 2$ . and  $2x^2 + 5 \neq 0$ . Thus,

$x = 2$  and  $x = -2$  are vertical asymptotes of  $g$ .  $\left[ \begin{array}{l} \lim_{x \rightarrow 2^-} f(x) = \infty \\ \lim_{x \rightarrow 2^+} f(x) = -\infty \end{array} \right]$

•  $\lim_{x \rightarrow \infty} \frac{2x^2 + 5}{4 - x^2} = \lim_{x \rightarrow \infty} \frac{2 + \frac{5}{x^2}}{\frac{4}{x^2} - 1} = -2$ ;   
  $\lim_{x \rightarrow -\infty} \frac{2x^2 + 5}{4 - x^2} = -2$ ;   
 Thus,  $y = -2$  is the horizontal asymptote.

$$\begin{aligned} g'(x) &= \frac{(4 - x^2)4x + 2(2x^2 + 5)x}{(4 - x^2)^2} \\ &= \frac{16x - 4x^3 + 4x^3 + 10x}{(4 - x^2)^2} = \frac{26x}{(4 - x^2)^2} \end{aligned}$$

$g'(x) = 0 \Rightarrow x = 0$ .  $[2, -2 \text{ are not in domain}]$

Thus,  $x = 0$  is the critical point.

$$\begin{array}{c} - \quad + \\ \hline 0 \end{array}$$

$$\begin{aligned} g'(-1) &< 0 \\ g'(1) &> 0 \end{aligned}$$