Final Review

1. Find the domain of $f(x) = \frac{\sqrt{3-x}}{\ln(x+7)}$.
2. Evaluate the following limits.
   (a) $\lim_{x\to -3} \dfrac{x^2-9}{x^2+12x+27}$
   (b) $\lim_{x\to \infty} \dfrac{x^3+7x+9}{x+2}$
3. (a) What is the definition of continuity?
   (b) Suppose $f(x) = \begin{cases} x+7 & \text{if } x \geq 1 \\ kx-9 & \text{if } x < 1 \end{cases}$
   Find k such that $f(x)$ is continuous.
4. Find the derivative of $\sqrt{x+2}$ using the limit definition of derivative.
5. (a) State the chain rule.
   (b) Suppose $h(x) = x(f \circ g(x))$, $g(1) = 2$, $f(2) = 3$, $f'(2) = 4$, and $g'(1) = 1$. Find $h'(1)$.
6. Find $\frac{dy}{dx}$ given that x and y are related by $ye^x + y^2 + x^2 = 7$.
7. The base of a 13 ft. ladder leaning against a wall begins to slide away from the wall. When the base is 12 ft. from the wall the base is moving at a rate of 8 ft./sec. How fast is the top of the ladder sliding down the wall?
8. (a) Sketch the graph of $f(x) = \dfrac{x-1}{x+1}$.
   (b) Find the absolute extrema of $f(x)$ on $[-3, 7]$. (How do you know these extrema exist? What theorem are you appealing to?)
9. If an open box is to have a square base and a volume of 108 in^3 and is constructed from a tin sheet find the dimensions of the box assuming a minimum amount of material is used in its construction.
10. Solve for x :
    (a) $\dfrac{1}{27^{3x+1}} = 9^x$
    (b) $\ln(3x + e + 17) = 1$
11. Differentiate $f(x) = (x^2 + 3x + 1)^{x+7}$ using logarithmic differentiation.
12. A substance has a half life of 10 days. If initially there are 110 grams of the substance find the amount present after t days. How long will it take for only 20 grams of the substance to be left?
13. Find the area under $f(x) = x + \frac{1}{x} + \frac{1}{x^3}$ on $[1, 3]$.
14. Find a function $f(x)$ such that $f'(x) = (x + 1)(x^2 + 7x + 10)$ and $f(1) = 6$.
15. Evaluate the following definite and indefinite integrals.
    (a) $\displaystyle\int \frac{(\ln x)^3}{x}\, dx$
    (b) $\displaystyle\int_2^5 \frac{x^7 + 9x + 6}{x^3}\, dx$
    (c) $\displaystyle\int \frac{e^{-1/x}}{x^{-3}}\, dx$
    (d) $\displaystyle\int_0^2 x^2(x + 3 + 7)^{5/3}\, dx$
16. When evaluating an indefinite integral why do we need to add the "+ c"?