Let $f(x)$ be a function. We want to graph it! Here's how to do this in detail:

(1) Describe the **domain** of $f(x)$.

(2) Find the **x- and y-intercepts** of $f(x)$. Recall that a number x in the domain of $f(x)$ is an x-intercept if $f(x) = 0$. A number y in the range of $f(x)$ is a y-intercept if $f(0) = y$.

(3) Determine the **end behavior** of $f(x)$. That is, compute the limits

$$\lim_{x\to+\infty} f(x) \quad \text{and} \quad \lim_{x\to-\infty} f(x),$$

or state that they do not exist. If they don't exist, you may want to note whether the y-values approach $+\infty$ or $-\infty$.

(4) Find the **horizontal** and **vertical asymptotes** of $f(x)$. A function $f(x)$ has a horizontal asymptote L if $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$. A function $f(x)$ has a vertical asymptote at a number a if $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$.

(5) Determine the intervals on which $f(x)$ is **increasing** and **decreasing**.

(6) Find and classify the **relative extrema** of $f(x)$ using the critical point method.

(7) Determine the intervals on which $f(x)$ is **concave up** and **concave down**.

(8) Find the **inflection points** of $f(x)$ using the inflection point method.

(9) Plot all intercepts, critical points, inflection points and any other "interesting points" found in the previous steps. Then use the information about asymptotes, increasing/decreasing behavior and concavity to **sketch the graph** of $f(x)$.

Some notes:

- According to Tan, a rational function $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials, has a vertical asymptote at $x = a$ if $p(a) \neq 0$ and $q(a) = 0$. This should agree with your findings when solving the limits $\lim_{x\to a^+} \dfrac{p(x)}{q(x)}$ and $\lim_{x\to a^-} \dfrac{p(x)}{q(x)}$ using factor-cancel. You can directly use this to state the vertical asymptotes. However, it is crucial to identify whether function is approaching ∞ or $-\infty$ as our sketch will depend on that.
- Polynomials never have vertical or horizontal asymptotes.
- A rational function can have at most one horizontal asymptote.
- Right before Step 9, I find it useful to create a new number line in which I record all critical points, inflection points and "interesting points" and, on each interval between these, record the increasing/decreasing behavior AND concavity simultaneously. This will be a good guide as you draw the final sketch in Step 9. You may even want to sketch a little , ,  or  on each interval.

Okay, let's do it!