1) Find the domain of • $\log_{17}(3x-5)$.

& • $\ln x - \ln(2-x)$

Soln • We need $3x-5 > 0$

$\Rightarrow x > 5/3$ So, $D := (5/3, \infty)$.

• We need $x > 0$ and $2-x > 0$

$\Rightarrow x < 2$

Thus $D := (0, 2)$

(2) Find domain of $e^{\sqrt{2x-1}}$

Soln Need $2x-1 \geq 0 \rightarrow x \geq 1/2$

Thus, $D = [1/2, \infty)$

(3) Find the inflection points of $2e^{-x^2}$.

Soln $f'(x) = -4xe^{-x^2}$; $f''(x) = -4e^{-x^2} + 8x^2e^{-x^2}$

• f'' exists everywhere, as e^{-x^2} and $x^2e^{-x^2}$

So, only need to look at .

$$f''(x) = 0 \Rightarrow -4e^{-x^2} + 8x^2e^{-x^2} = 0$$

$$\Rightarrow -4e^{-x^2}\left[1 - 2x^2\right] = 0$$

$$\Rightarrow 2x^2 = 1 \quad (\text{as } e^{-x^2} > 0)$$

$$\Rightarrow x = \pm\sqrt{\tfrac{1}{2}} \quad (\text{both are in the domain})$$

	+		−		+	
-1		$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$		1

- $f''(-1) = -4e^{-1} + 8e^{-1} = 4e^{-1} > 0$.
- $f''(0) = -4 < 0$
- $f''(1) = -4e^{-1} + 8e^{-1} = 4e^{-1} > 0$.

~~(Both $-\frac{1}{\sqrt{2}}$ & $\frac{1}{\sqrt{2}}$ are in the domain of f)~~

Thus, $\left(-\frac{1}{\sqrt{2}}, 2e^{-\frac{1}{2}}\right)$, $\left(\frac{1}{\sqrt{2}}, 2e^{-\frac{1}{2}}\right)$ are the I.P.'s.

(4) ~~Calculate~~ Find abs. max/min of $f(x) = xe^{-x^2}$ on $[0, 2]$.

Sol^n: • f is continuous on $[0, 2]$. So, closed interval Method applies.

• $f'(x) = e^{-x^2} - 2x^2e^{-x^2}$.

• $f'(x)$ exists everywhere on $[0, 2)$.

Thus, critical points on $(0,2)$ are obtained from: $f'(x) = 0$.

$$\Rightarrow e^{-x^2} - 2x^2 e^{-x^2} = 0$$

$$\Rightarrow e^{-x^2}(1 - 2x^2) = 0$$

$$\Rightarrow x = \pm\sqrt{\frac{1}{2}} \quad (\text{as } e^{-x^2} > 0).$$

Thus, only crit. pt on $(0,2)$ is $x = \frac{1}{\sqrt{2}}$

• $f(0) = 0$

• $f(2) = 2e^{-4} \approx .04$ (using calculator).

• $f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} e^{-1/2} = \frac{1}{\sqrt{2}e^{1/2}} \approx 0.55$.

So, abs. min value is 0; occurs at $x = 0$

and abs. max. value is $\frac{1}{\sqrt{2}} e^{-1/2}$; occurs at $x = \frac{1}{\sqrt{2}}$.

$\boxed{OR}$ abs. min @ $(0, 0)$

abs. max @ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} e^{-1/2}\right)$

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Logarithmic differentiation

- $\boxed{\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}}$ provided $f(x) > 0$ for all x.

- $\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}$ for all $x \in (-\infty, 0) \cup (0, \infty)$

- $\frac{d}{dx}(\ln x) = \frac{1}{x}$ for all $x \in (0, \infty)$.

Algorithm

- How do we differentiate (i) $f(x) = g(x)^{h(x)}$

  e.g. $f(x) = x^{2x+1}$?

- How do we differentiate $(x^3 - 2x)(x^2 + 2x)(7x - 4)$ instead of using product rule?

Ans: Both are done using logarithmic differentiation.

- Algorithm

1) Take '$\ln$' on both sides of the equation and simplify as much as possible using 'log' properties
2) differentiate both sides w.r.t. x. ~~[illegible]~~
3) Collect $\frac{dy}{dx}$ on one side & solve.

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(1) Find $\frac{dy}{dx}$.

(a) $y = x^{2x+1}$.

$\ln y = (2x+1)\ln x$.

So, $\frac{d}{dx}(\ln y) = \frac{2x+1}{x} + 2\ln x$.

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(\frac{2x+1}{x} + 2\ln x\right)$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{2x+1}{x} + 2\ln x\right)$

[Applying chain-rule $\ln f(x) - \frac{f'(x)}{f(x)}$]

$$\boxed{\frac{dy}{dx} = x^{2x+1}\left(\frac{2x+1}{x} + 2\ln x\right)}$$

Ans:

(b) $f(x) = x(\ln x)^x$

Let $y = f(x)$. $\qquad y = x(\ln x)^x$

$$\ln y = \ln x + \ln\left((\ln x)^x\right)$$

$$\Rightarrow \ln y = \ln x + x\ln(\ln x).$$

Thus, $\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} + \ln(\ln x) + x\left(\dfrac{1}{\ln x}\right)\cdot\dfrac{1}{x}$.

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x} + \ln(\ln x) + \frac{1}{\ln x}\right]$$

$$\boxed{\frac{dy}{dx} = x(\ln x)^x\left[\frac{1}{x} + \ln(\ln x) + \frac{1}{\ln x}\right]}$$

c) $\quad y = \dfrac{(x^3-2x)(x^2+2x)}{(7x-4)}$

n: $\ln y = \ln(x^3-2x) + \ln(x^2+2x) - \ln(7x-4)$.

ifferentiating both sides w.r.t x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{3x^2-2}{(x^3-2x)} + \frac{2x+2}{x^2+2x} - \frac{7}{7x-4}$$

$$\boxed{\frac{dy}{dx} = \frac{(x^3-2x)(x^2+2x)}{(7x-4)}\left[\frac{3x^2-2}{x^3-2x} + \frac{2x+2}{x^2+2x} - \frac{7}{7x-4}\right]}.$$

Ans