STUDENT NAME: _____

INSTRUCTOR: _____

Please sign the pledge:
On my honor as a student, I have neither given nor received aid on this exam.

Directions

Answer each question in the space provided. Please write clearly and legibly. ***Show all of your work in order to receive full credit, and clearly identify your final answer. No books, notes or calculators are allowed.***

For instructor use only

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1. [10 pts] For each of the following two functions, find the domain. ***Record your answer in interval notation.***

   (a) $F(x) = \dfrac{\sqrt{1-x}}{\sqrt{x}}$

   (b) $R(x) = (g \circ f)(x) + h(x)$, where $f(x) = x + 1, g(x) = \frac{1}{3x+5}, h(x) = x^3$

2. [6 pts] Andy is going on a 10-day trip in a few months. He paid for 10 nights at \$100 per night for his hotel room. He is delaying his purchase of a plane ticket, hoping to buy one at a price he will find acceptable. Andy uses a simple "travel quotient" function to figure out which prices are acceptable. The travel quotient $Q(A)$ is given by the airplane ticket price A, divided by S, where S is the sum of the ticket price A and the amount Andy has already spent on lodging.

   (a) Write the rule for the travel quotient $Q(A)$ as a function of A.

   (b) For Andy, an acceptable price for an airplane ticket is any price A so that $Q(A) \leq \frac{1}{9}$. Should Andy buy when the ticket price is \$250?

3. [15 pts] Find the following limits, or explain why the limit does not exist.

(a) $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$

(b) $\lim_{x\to\infty}\dfrac{3x^3+2x+6}{-x^3+8x^2-5x+4}$

(c) $\lim_{x\to 3}\dfrac{x-3}{|2x-6|}$

4. (a) [4 pts] State precisely what it means for a function $f(x)$ to be continuous at $x = a$.

(b) [6 pts] Find the values of m and b that make the following function continuous:

$$f(x) = \begin{cases} 5 - x^2 & x \leq -1 \\ mx + b & -1 < x < 1 \\ x^2 + 1 & 1 \leq x \end{cases}$$

5. [15 pts] Find the derivatives of the following functions. You **do not** need to simplify your final answer.

(a) $f(x) = \dfrac{3x+5}{x^2-4x}$

(b) $g(x) = \sqrt{\sqrt[4]{x} - \frac{1}{x^2}}$

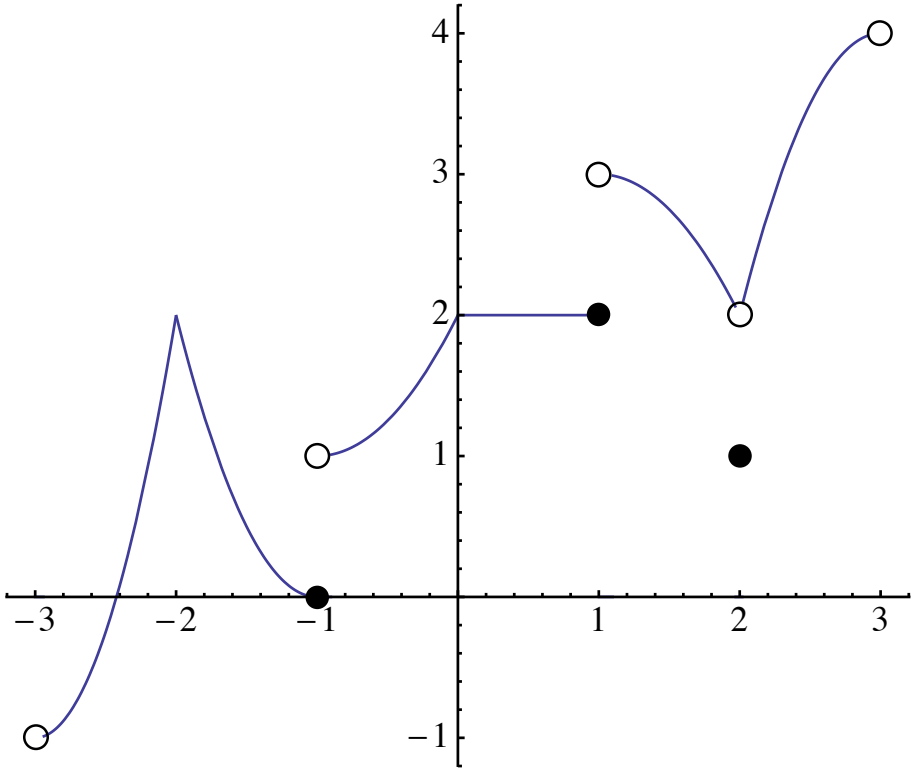
(c) $h(t) = (t^2+2t+1)(t^4-t^3-6)$

6. (a) [4 pts] State the limit definition of the derivative of a function $f(x)$.

(b) [6 pts] Using the limit definition, find the derivative of $f(x) = x^2 + \frac{1}{3}x + \frac{2}{3}$.

(c) [4 pts] The graph of $f(x) = x^2 + \dfrac{1}{3}x + \dfrac{2}{3}$ has a tangent line L that is parallel to the line $y = \dfrac{7}{3}x + 5$. Find an equation of this tangent line L.

7. [8 pts] Pictured is the graph of the function $f(x)$ for $-3 < x < 3$. Using this graph, answer the following questions about $f(x)$.



(a) For which values of x in $(-3, 3)$ is $f(x)$ not differentiable?

(b) For which values of x in $(-3, 3)$ is $f(x)$ not continuous?

(c) For which values a in $(-3, 3)$ does $\lim_{x \to a} f(x)$ not exist?

(d) What is $\lim_{x \to 2} f(x)$?

8. [8 pts] Two runners begin running from different points on a street; their respective positions at any time t, $0 \leq t \leq 1$, are given by $f(t) = t^5 + 2t - 1$ and $g(t) = 2t - t^2$.

   (a) Why are the functions f and g continuous?

   (b) Does either runner catch the other during this time? Carefully justify your answer.

9. [9 pts] Multiple-choice. Circle the correct response.

(a) Let f be a function such that $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I) f is continuous at $x = 2$

II) f is differentiable at $x = 2$

III) The derivative of f is continuous at $x = 2$

(a) I only
(b) II only
(c) I and II only
(d) II and III only
(e) I, II, and III

(b) If $\lim_{x\to 3} f(x) = 7$, which of the following must be true?

I) f is continuous at $x = 3$

II) f is differentiable at $x = 3$

III) $f(3) = 7$

(a) none
(b) II only
(c) III only
(d) I and III only
(e) I , II, and III

(c) Let f and g be differentiable functions such that $f(1) = 2$, $f'(1) = 3$, $f'(2) = -4$, $g(1) = 2$, $g'(1) = -3$, and $g'(2) = 5$. If $h(x) = f(g(x))$, what is $h'(1)$?

(a) -9
(b) -4
(c) 0
(d) 12
(e) 15

10. [5 pts] A bacterial colony, originating from a single mother cell placed at the center of a petri dish, spreads outward, maintaining the shape of a disk of radius r, where r is measured in centimeters. See the diagram below.

Growing Bacteria Colony in a Petri Dish

(a) Express the amount of area occupied by this colony as a function of its radius r.

A =

(b) Find the rate of change of area with respect to radius.