MATH 1210
Homework 1/Quiz Practice

Remark: All these problems are from past question papers.

1. Evaluate the following limits or state that they do not exist. You must show all work.

a. $\lim\limits_{x \to 4} \dfrac{\sqrt{x-3}+1}{x^2-5x+4}$

b. $\lim\limits_{x \to 2} \dfrac{\sqrt{14+t}-4}{t-2}$

c. $\lim\limits_{x \to 2} \dfrac{x^2+x-2}{x-5}$

d. $\lim\limits_{x \to \infty} \dfrac{9000x^5-30}{x^5-3000x-15}$

e. $\lim\limits_{x \to 3^-} \dfrac{3x-x^2}{|3-x|}$

f. $\lim\limits_{x \to -1} \dfrac{|1+x|}{x}$

g. $\lim\limits_{x \to \infty} \dfrac{\sqrt{30x}}{x+29}$

2. Write the domains of the following functions in interval notation:

a. $F(x) = \dfrac{\sqrt{1-x}}{\sqrt{x}}$

b. $R(x) = (g \circ f)(x) + h(x)$ where $f(x) = x+1,\ g(x) = \dfrac{1}{3x+5},\ h(x) = x^3$

c. $f(t) = \dfrac{\sqrt{t-1}}{t^2-2t-3}$

d. $g(x) = \dfrac{x^2+9}{x^2-x-6}$

e. $h(x) = g(f(x))$ where $f(x) = 2x+1,\ g(x) = \dfrac{x+3}{x-1}$

3a. Is the function $f(x) = \begin{cases} x^2+1, & x \le 3 \\ 12 - \frac{6}{x}, & x > 3 \end{cases}$ continuous at $x = 0$? Is it continuous at $x = 3$? Justify your answer.

3b. Find the value of c that makes $J(x)$ a continuous function on $\mathbb{R}$ (all real numbers)

$$J(x) = \begin{cases} 2x^2 + cx - 1, & x < 1 \\ \sqrt{x+1}, & x \geq 1 \end{cases}$$

3c. Let $f(x) = \begin{cases} \frac{10}{x-5} & x < 0 \\ x^3 + 1 & x \geq 0 \end{cases}$. Find the domain of f. Show that f is continuous at $x = 2$ but discontinuous at $x = 0$.

3d. For $f(x) = \begin{cases} 5x + 1 & x < 1 \\ k & x = 1, \\ x^2 + 5 & x > 1 \end{cases}$ determine k that makes the function continuous at $x = 1$. Justify your answer.

3e. For what value of the constant c is the following function continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} cx^2 + 2x & x < 2 \\ x^3 - cx & x \geq 2 \end{cases}$$

3f. Find where the function f is continuous.

$$f(x) = \begin{cases} -2x + 1 & x < 1 \\ 0 & x = 1 \\ \frac{1}{x-2} & x > 1 \end{cases}$$

4a. A race track with perimeter 1 mile has two identical semicircles at the ends of a rectangular area. Assuming both semicircles have radius r and the rectangle has length y, both measured in miles. Find a function f in the variable r giving the area enclosed by the race track. Find the domain of f.

4b. A rectangular box made of sheet metal is to have a square base and a volume of 100 in^3.

(i) Letting x denote the length of one side of the base, find a function $f(x)$ giving the amount (in square inches) of sheet metal needed to construct the box.

(ii) What if the domain of $f(x)$ you found in (i)?

(iii) How much sheet metal is needed to construct a box with dimensions 5 in $\times$ 5 in $\times$ 5 in?

4c. Andy is going on a 10-day trip in a few months. He paid for 10 nights at \$100 per night for his hotel room. He is delaying his purchase of a plane ticket, hoping to buy one at a price he will find acceptable. Andy uses a simple "travel quotient" function to figure out which prices are acceptable. The travel quotient $Q(A)$ is given by the airplane ticket price A, divided by S, where S is the sum of the ticket price A and the amount Andy has already spent on lodging.

(i) Write the rule for the travel quotient $Q(A)$ as a function of A.

(ii) For Andy, an acceptable price for an airplane ticket is any price A so that $Q(A) \leq 9$. Should Andy buy when the ticket price is \$250?

4d. The owner of a farm has 3000 yards of fencing with which in enclose a rectangular piece of grazing land along the side of a straight sided river. Fencing is not required next to the river. Let x be the width (perpendicular to the river) and y be the length (parallel to the river) of the enclosed land. Find a function f in terms of x for the area of the grazing land (in square yards). Find the domain of this function given the physical limitations, namely that x and y are positive numbers.

5. Make sure you understand all the problems. These are very good problems!

a. Dr. Snowworthy has constructed mathematical models for the amounts of snowfall in Duluth, MN and in Buffalo, NY. According to her model, the amount of snowfall in Duluth, in inches, t months after December, $1 \leq t \leq 4$, is predicted to be $D(t) = 13 + 12t^2 - 2t^3$ and in Buffalo is predicted to be $B(t) = 10 + 2t + 9t^2 - t^3$. According to Dr. Snowworthy's model, is there a time t in the interval $[0, 4]$ such that the amount of snow fallen in Duluth will equal the amount fallen in Buffalo? Carefully justify your answer.

b. Is there a real number that is exactly 1 more than its cube? Explain your answer. (Hint: Observe that -2 is 6 more than its cube and 0 is the cube of itself).

c. Let $f(x) = x^4 + 2x^3 + 5x + 2$. Does f have a root in the interval $(-1, 1)$? Justify.

d. Let $g(x) = x^3 - 4x^2 + x + 6$.

(i) Is g continuous? What is its domain?

(ii) Show that $g(x)$ has at least one real root.

e.. Is the difference of $f(x) = x^5 + 2x^2$ and $g(x) = x^3 + 1$ ever 0 on the interval $[0, 1]$? Explain why or why not.

6. State True/False.

a. There is a continuous function $f(x)$ defined on $[1, 3]$ such that $f(1) = 0$ and $f(3) = 5$ but $f(x) \neq 2$ for any x between 1 and 3.

If True, draw such a function. If False, state why.

b. There is a function $f(x)$ defined on $[1, 3]$ such that $f(1) = 0$ and $f(3) = 5$ but $f(x) \neq 2$ for any x between 1 and 3.

If True, draw such a function. If False, state why.