STUDENT NAME: ____________________

INSTRUCTOR: ____________________

Please sign the pledge:
*On my honor as a student, I have neither given nor received aid on this exam.*

## Directions

Answer each question in the space provided. Please write clearly and legibly. ***Show all of your work—your work must justify your answer, and clearly identify your final answer. No books, notes, or electronic devices of any kind may be used during the exam period. You must simplify results of function evaluations, when it is possible to do so. For example,*** $4^{3/2}$ ***should be evaluated*** ($\boldsymbol{replaced\ by}$ 8)**.**

**For instructor use only**

| Page | Points | Score |
|---|---|---|
| 2 | 15 | |
| 3 | 10 | |
| 4 | 11 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 12 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 6 | |
| 11 | 6 | |
| Total: | 100 | |

1. [15 pts] Find derivatives of the following functions. Do not simplify your answers.

   a) $f(x) = (x^3 + 1)^8$

   b) $g(x) = \dfrac{e^{5x}}{2x + 5}$

   c) $h(x) = x \ln \sqrt{x^2 + 1}$

2. [10 pts] Given $x^5 + x^3y = 3y^2 + 1$.

a) Find $\dfrac{dy}{dx}$

b) Find an equation of the tangent line to this curve at the point $(1, 0)$.

3. [5 pts] Suppose $g(x)$ is a differentiable function with derivative given by $g'(x) = \dfrac{x}{x^4+1}$. Suppose that the function $f$ is defined for $x \geq 0$ by $f(x) = \sqrt{3x}$. Let $h(x)$ be defined for $x \geq 0$ by $h(x) = g(f(x))$. Find $h'(2)$.

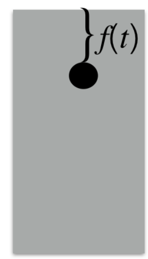
4. [6 pts] a) State the Extreme Value Theorem.

b) Complete the following definition: the line $y = b$ is a horizontal asymptote of the graph of a function $f$ if:

5. [10 pts] A steel ball drops through a fluid of varying viscosity so that its *position*, in feet below the surface of the fluid, is given by

$f(t) = t^3 - \frac{1}{4}t^4 \quad \text{for } 0 \le t \le 3,$

where time $t$ is measured in seconds. What is the **greatest velocity** that the ball attains over the time interval $0 \le t \le 3$? Carefully justify your answer.



6. [10 pts] Let $f(x) = \ln(x^2 + 4)$. You may take for granted that $f'(x) = \dfrac{2x}{x^2 + 4}$ and $f''(x) = \dfrac{2(4 - x^2)}{(x^2 + 4)^2}$.

a) Find the interval(s) where $f$ is increasing and where $f$ is decreasing.

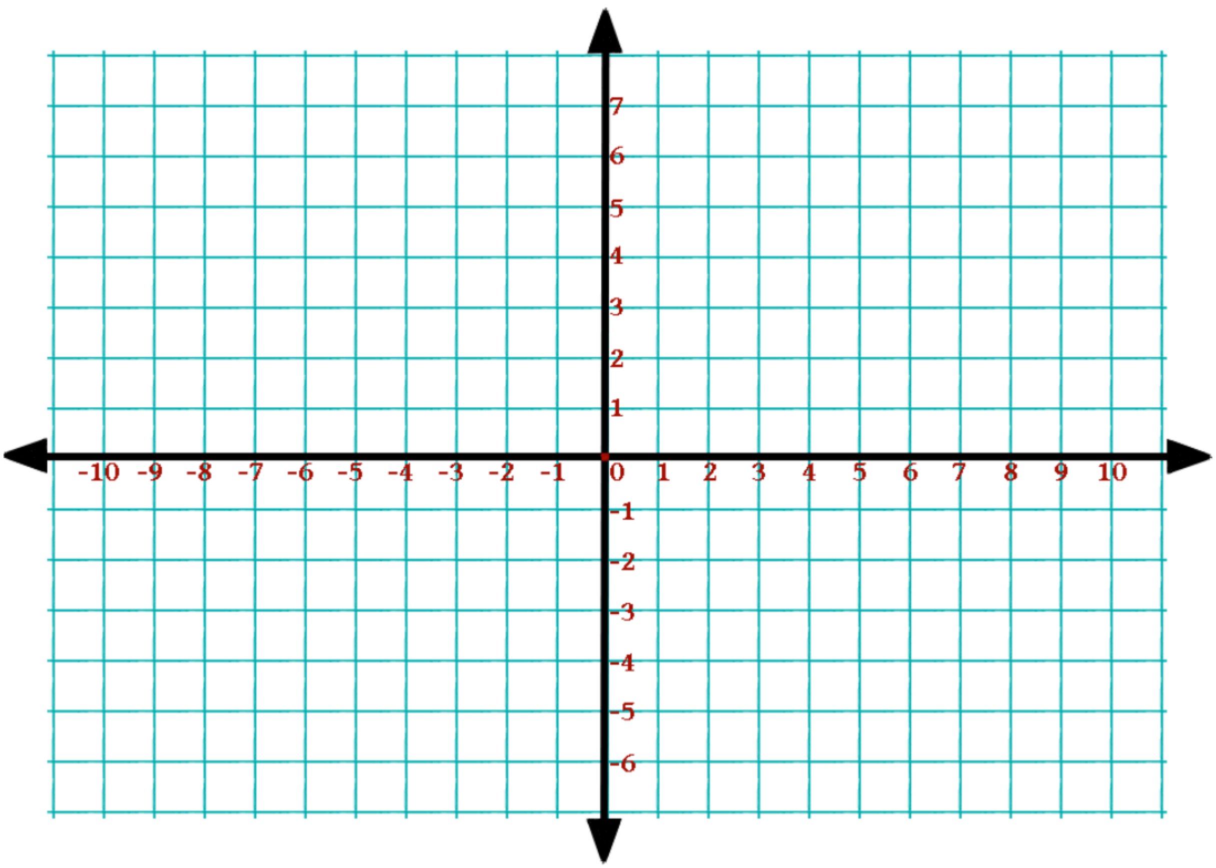
b) Classify all local maxima and minima.

c) Find the interval(s) where the graph of $f$ is concave up and those where the graph is concave down.

d) Find all the inflection points on the graph of $f$ (provide both $x$ and $y$ coordinates of all the inflection points).

7. [12 pts] Sketch a graph of $f(x)$, given the following data:

- $f(x)$ is continuous everywhere except at $x = 2$, and it is twice differentiable everywhere except at $x = 2$ and $x = -4$
- $f(x)$ has a vertical asymptote at $x = 2$.
- $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to-\infty} f(x) = 1$
- $f(x)$ has only one $x$ intercept: $(0, 0)$
- $f'(x)$ is positive on $(-\infty, -4)$ , and negative on $(-4, 2)$, $(2, \infty)$
- $f''(x)$ is positive on $(-\infty, -4)$, $(-4, -2)$ and $(2, \infty)$ , and negative on $(-2, 2)$
- $f(-4) = 5$, $f(-2) = 2$



8. [10 pts] The number of items produced by a manufacturer is given by

$$p = 100xy^3$$

where $x$ is the amount of capital and $y$ is the amount of labor, amounts that change over time. At a particular point in time:

(i) the manufacturer has 2 units of capital;

(ii) capital is increasing at a rate of 1 unit per month;

(iii) the manufacturer has 3 units of labor; and

(iv) labor is decreasing at a rate of 0.5 units per month.

Determine the rate of change in the number of items produced at this point in time. Is the number of items produced at this time increasing or decreasing?

9. [10 pts] A farmer wishes to fence in a rectangular garden of 100 $ft^2$. The north-south fences will cost \$1.50 per foot, while the east-west fences will cost \$6 per foot. Find the dimensions of the garden that will minimize the cost. You must fully justify your claim that you have found the dimensions minimizing the cost (using calculus).

10. [3 pts] True/false question: if the answer is true give a brief explanation. If false, provide an example where the statement fails.

    **Suppose that $f(x)$ is a differentiable function on $(-\infty, \infty)$ and that $f'(x)+1 > 0$ for all $x$. Then on the interval $[a, b]$ the function $g(x) = f(x)+x$ has an absolute minimum at $a$.**

11. [3 pts] Multiple choice questions: circle the correct answer

    **If $g$ is a differentiable function such that $g(x) < 0$ for all real numbers $x$ and if $f(x)$ is a function such that $f'(x) = (x^2 - 4)g(x)$, which of the following must be true?**

    a) $f$ has a relative maximum at $x = -2$ , and a relative minimum at $x = 2$

    b) $f$ has a relative minimum at $x = -2$ and a relative maximum at $x = 2$

    c) $f$ has relative minima at $x = -2$ and at $x = 2$

    d) $f$ has relative maxima at $x = -2$ and at $x = 2$

    e) it cannot be determined if $f$ has any relative extrema

12. [3 pts] **Consider the function $f(x) = 2xe^x$ . For what values of $x$ is the graph of $f(x)$ concave down?**

a) $x > 2$

b) $x > 1$

c) $x < 2$

d) $x < -1$

e) $x < -2$

13. [3 pts] **The graph of a twice-differentiable function $f$ is shown in the figure below. Which of the following appears to be true?**

a) $f(1) < f'(1) < f''(1)$

b) $f(1) < f''(1) < f'(1)$

c) $f'(1) < f(1) < f''(1)$

d) $f''(1) < f(1) < f'(1)$

e) $f''(1) < f'(1) < f(1)$