- When asked to find horizontal asymptotes, you have to clearly compute:

  - $\lim_{x \to \infty} f(x)$
  - $\lim_{x \to -\infty} f(x)$

  $\Big\}$ to get full credit.

$\boxed{\text{eg}}$ Find the h.a. of $f(x) = \dfrac{x^2+5}{x^3+4}$

Soln: • $\lim_{x \to \infty} \dfrac{x^2+5}{x^3+4} = \lim_{x \to \infty} \dfrac{\frac{x^2}{x^3} + \frac{5}{x^3}}{\frac{x^3}{x^3} + \frac{4}{x^3}} = \lim_{x \to \infty} \dfrac{\frac{1}{x} + \frac{5}{x^3}}{1 + \frac{4}{x^3}} = 0.$

• $\lim_{x \to -\infty} \dfrac{x^2+5}{x^3+4} = \lim_{x \to -\infty} \dfrac{\frac{1}{x} + \frac{5}{x^3}}{1 + 4/x^3} = 0.$

So, $y = 0$ is the only h.a.

Some facts to keep in mind:

- We talked about Extreme Value Theorem (EVT). However, EVT is not always applicable as we may not have a closed interval:

eg Find the absolute ~~max~~ extrema of $f(x) = \frac{2x}{x^2+4}$ over $[-1, \infty)$.

Soln: Here EVT can't be applied. But we can still solve this!

- $f'(x) = -\frac{2(x^2-4)}{(x^2+4)^2}$ (Check!)

- f is continuous on $[-1, \infty)$.

- ~~Sign chart of~~ $f'(x) = 0 \Rightarrow x = 2, -2$. But we need only $x = 2$

Only critical point is 2 on $(1, \infty)$

- $f(-1) = -\frac{2}{5}$; • $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x}{x^2+4} = 0$

f'

$$\underset{-1}{[}\quad + \quad \underset{2}{|} \quad -$$

Thus, f increases on $[-1, 2)$ becomes $\frac{4}{8}$ at $x = 2$

and then decreases on $(2, \infty)$

- f approaches 0 as $x \to \infty$. (it never goes below 0)

Thus, $(2, \frac{4}{8})$ is an absolute max.

$(-1, \frac{-2}{5})$ is an absolute minimum.

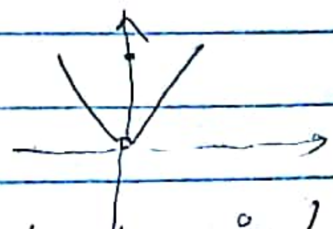
So, here we analyzed it differently using methods similar to curve sketching!

- State True/False.

(a) If f is defined on a closed interval $[a, b]$, then f has an absolute minimum.

Ans: False. [We're not given f is continuous.

$$f(x) = \begin{cases} |x| & , x \neq 0 \\ 1 & x = 0 \end{cases}$$



No abs. min.!

(b) If f is not continuous on $[a, b]$, then f cannot have an absolute max. value.

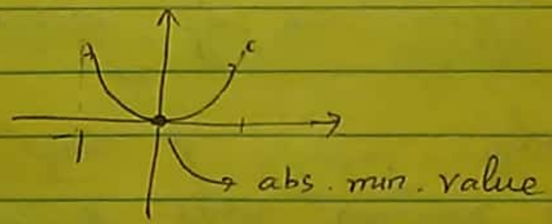
Ans: False.

Not continuous.

But 1 is an abs. max value.

(c) If f is continuous on an open interval (a, b), then f cannot have an absolute ~~maximum~~ minimum value.

Ans: False. $f(x) = x^2$ on $(-1, 1)$.



So, keep an open mind while tackling problems!