STUDENT NAME: _____

INSTRUCTOR: _____

Please sign the pledge:
On my honor as a student, I have neither given nor received aid on this exam.

Directions

Answer each question in the space provided. Please write clearly and legibly. ***Show all of your work—your work must justify your answers. Clearly identify your final answers. No books, notes, or electronic devices of any kind may be used during the exam period. You must simplify results of function evaluations when it is possible to do so. For example,*** $4^{3/2}$ ***should be evaluated*** (***replaced by*** 8)***.***

For instructor use only

Run LaTeX again to produce the table

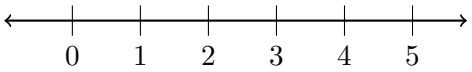
1. [15 pts] Find derivatives of the following functions. Do not simplify your answers.

   (a) $f(x) = x^5 + \ln(x^4 + 1) + e^2$

   (b) $g(x) = \dfrac{e^{x^3+5x}}{x^2+2}$

   (c) $p(x) = x^{2x+1}$

2. [5 pts] A particle is traveling along the coordinate line below; its position for $t \geq 0$ is given by $s(t) = \sqrt{3t + \sqrt{t^3}}$, where s is measured in centimeters and t is measured in seconds. Find the velocity of the particle at time $t = 1$. *Include units.*



3. [8 pts] (a) What theorem guarantees that the function $f(x) = x^3 + 3x^2 - 9x + 3$ has an absolute maximum value and an absolute minimum value on $[-4, 0]$?

(b) Use the Closed Interval Method to find the absolute maximum value and the absolute minimum value of $f(x) = x^3 + 3x^2 - 9x + 3$ on $[-4, 0]$.

4. [4 pts] Find all numbers a such that the function $f(x) = e^{ax}$ satisfies $f''(x) = f(x)$.

5. [4 pts] Solve the equation $32^{x-1} = \dfrac{2}{64^x}$ for x.

6. [5 pts] [TRUE / FALSE]. Circle your response. No work required.

   (a) **True** **False** If $x^2 + y^2 = r^2$, then $\ln(x) + \ln(y) = \ln(r)$.

   (b) **True** **False** The domain of $f(x) = b^x$ for $b > 0$ and $b \neq 1$ is $(-\infty, \infty)$.

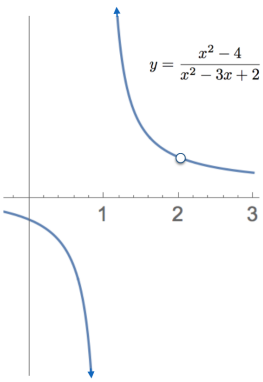
   (c) **True** **False** If $f''(2) = 0$, then $(2, f(2))$ is a point of inflection on the graph of f.

   (d) **True** **False** If $f'(c) = 0$, then $f(x)$ has a relative min or max at $x = c$.

   (e) **True** **False** If $f''(x) < 0$ for all x in $(0, \infty)$ and $f'(1) = 0$, then $f(1)$ is the absolute maximum value of f on $(0, \infty)$.

7. [3 pts] Fill-in the blank. No work required.
   (a) $\log_2\left(\frac{1}{8}\right) =$ _____.
   (b) $3^{\ln(1)} + \ln\left(\frac{1}{e}\right) =$ _____.
   (c) The graph of $f(x) = \ln(x)$ is concave _____.

8. [4 pts] Let $f(x) = \frac{x^2-4}{x^2-3x+2}$, a function whose graph is pictured at right.



   (a) Write down the vertical asymptote(s) of the graph of f. No work required.

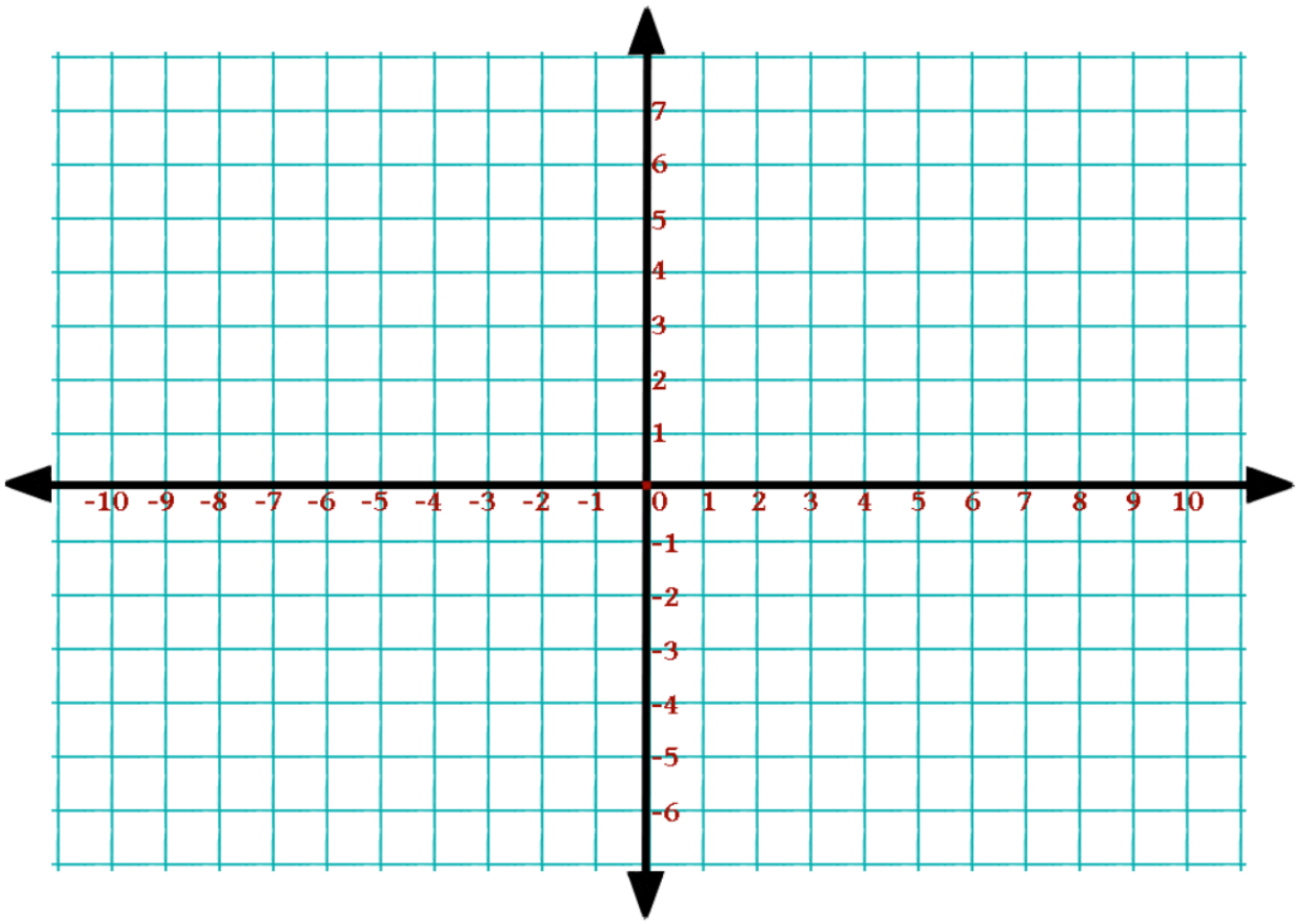
   (b) For each vertical asymptote you wrote down in response to part (a), provide a justification of why it is a vertical asymptote using the definition of vertical asymptote.

9. [4 pts] The growth rate of Escherichia coli, a common bacterium found in the human intestine, is proportional to its size. Under ideal laboratory conditions, when this bacterium is grown in a nutrient broth medium, the number of cells in a culture doubles approximately every 30 min. If the initial population is 50, determine the function $Q(t)$ that expresses the growth of the number of cells of this bacterium as a function of time t (in minutes).

10. [10 pts] A group of Canadian bandits have stolen several barrels of maple syrup and are driving east at a speed of 50 mi/h towards Town Hall. The Royal police are driving North towards Town Hall at a speed of 80 mi/h, attempting to intercept the bandits. Find the rate of change (with respect to time) of the distance between the bandits and the police at the instant when the police are 3 miles from Town Hall and Bandits are 4 miles from Town Hall.

11. [10 pts] On the grid provided, sketch the graph of a function f that satisfies all of the conditions listed below. Then answer the fill-in-the-blank question that appears below your sketch.

    1. The domain of f is $(-\infty, -3) \cup (-3, \infty)$ and f is continuous on its domain.
    2. $f(0) = 0$; $f(3) = -3$
    3. $x = -3$ is a vertical asymptote,
    4. $\lim_{x\to\infty} f(x) = 4$ and $\lim_{x\to-\infty} f(x) = -2$,
    5. $f'(x) > 0$ for x in $(-\infty, -3)$ and $(3, \infty)$, and $f'(x) < 0$ for x in $(-3, 3)$,
    6. $f''(x) > 0$ for x in $(-\infty, -3)$, $(-3, 0)$, and $(2, 5)$. $f''(x) < 0$ for x in $(0, 2)$ and $(5, \infty)$.

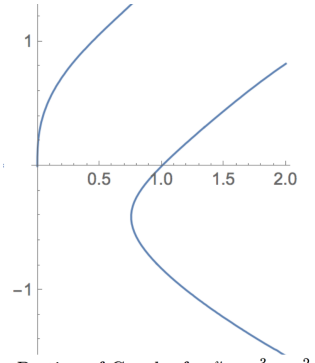


How many points of inflection are there on the graph of f above? _____.

12. [8 pts] Given that $xe^y = y^3 + x^2$.

    (a) Find $\frac{dy}{dx}$ via implicit differentiation.

    (b) Find an equation of the line tangent to the graph of $xe^y = y^3 + x^2$ at the point $(1, 0)$. Use the graph (only) to check your answer for plausibility.



Portion of Graph of $xe^y = y^3 + x^2$

13. [5 pts] Find the horizontal aysmptotes of the graph of $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ \frac{x^4+1}{3x^4+2} & \text{if } x > 0 \end{cases}$. Justify your answer using the definition of horizontal aysmptote.

14. [5 pts] Identify the intervals on which $f(x) = \dfrac{x}{x^2+9}$ is increasing and the intervals on which f is decreasing.

15. [10 pts] We are building a rectangular storage container with an open top (**no top**). The container is to have a volume of 8 cubic meters and its length should be twice its width. Materials for the sides cost \$4 per square meter; the bottom, however, is on sale at Home Depot, and so the material for it costs \$3 per square meter. Determine the dimensions of the container that costs the least to make. You must completely justify your answer; in particular, you must show that you have found the dimensions *minimizing* cost (using calculus).