MATH 1210 Test 1 Feb 24, 2015

STUDENT NAME: Spring 2015 Answer key.

INSTRUCTOR: ______________________

Please sign the pledge:
On my honor as a student, I have neither given nor received aid on this exam.

Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work in order to receive full credit, and clearly identify your final answer. No books, notes or calculators are allowed.

For instructor use only

Page	Points	Score
2	10	
3	11	
4	10	
5	17	
6	17	
7	17	
8	18	
Total:	100	

1. **If the line passing through the points (a, 1) and (2, 4) is parallel to the line passing through the points (2, a) and (4, 1), what are all possible values of a?**

ANSWER: **a=4, a = -1**

In order that the lines be parallel, we must have

$$\frac{4-1}{2-a} = \frac{1-a}{4-2}.$$

Upon cross multiplying, we obtain

$6 = (1-a)(2-a)$, or $a^2 - 3a + 4 = 0$.

Because $a^2 - 3a + 4 = (a-4)(a+1)$, we see $a=4$ or $a = -1$.

3

2

2. [6 pts] Find the domain of the function and write it as unions of intervals

$$f(x) = \frac{\sqrt{2x-4}}{x^2-2x-3}$$

ANSWER: $[2,3) \cup (3,\infty)$

We cannot take the square root of a negative number so

$2x - 4 \geq 0 \Rightarrow 2x \geq 4 \Rightarrow x \geq 2.$

We cannot divide by 0 so

$x^2 - 2x - 3 = (x-3)(x+1) \neq 0.$

This means $x \neq 3$ and $x \neq -1$

So $x \geq 2$ & $x \neq -1$ + $x \neq 3$

So our domain is

$[2,3) \cup (3,\infty)$

3. Evaluate the following limits or write DNE if the limit does not exist (please justify your answers):

(a) [3 pts] $\lim_{x\to -1} \frac{x^2-x+2}{x+3}$ ANSWER: $\boxed{2}$

$\lim_{x\to -1} x^2-x+2 = (-1)^2-(-1)+2 = 4$ since x^2-x+2 is a polynomial.

$\lim_{x\to -1} x+3 = (-1)+3 = 2$ Since $x+3$ is a polynomial.

Therefore $\lim_{x\to -1} \frac{x^2-x+2}{x+3} = \frac{\lim_{x\to -1} x^2-x+2}{\lim_{x\to -1} x+3} = \frac{4}{2} = \boxed{2}$ by limit rules.

(b) [4 pts] $\lim_{x\to -\infty} \frac{12x^4-2x^2+1}{-5x^4-30x+5}$ ANSWER: $\boxed{-\frac{12}{5}}$

$$\lim_{x\to -\infty} \frac{12x^4-2x^2+1}{-5x^4-30x+5} = \lim_{x\to -\infty} \frac{12x^4-2x^2+1}{-5x^4-30x+5} \cdot \frac{1/x^4}{1/x^4} = \lim_{x\to -\infty} \frac{12-\frac{2}{x^2}+\frac{1}{x^4}}{-5-\frac{30}{x^3}+5/x^4}$$

$$= \frac{12-0+0}{-5-0+0} = \frac{12}{-5} = \boxed{-\frac{12}{5}}$$

(c) [4 pts] $\lim_{x\to 3} \frac{|2x-6|}{x-3}$ ANSWER: DNE.

Since the limit goes to $\frac{0}{0}$ and involves absolute value we need to look at left and right sided limits.

If $x>3$ then $|2x-6| = 2x-6$ so

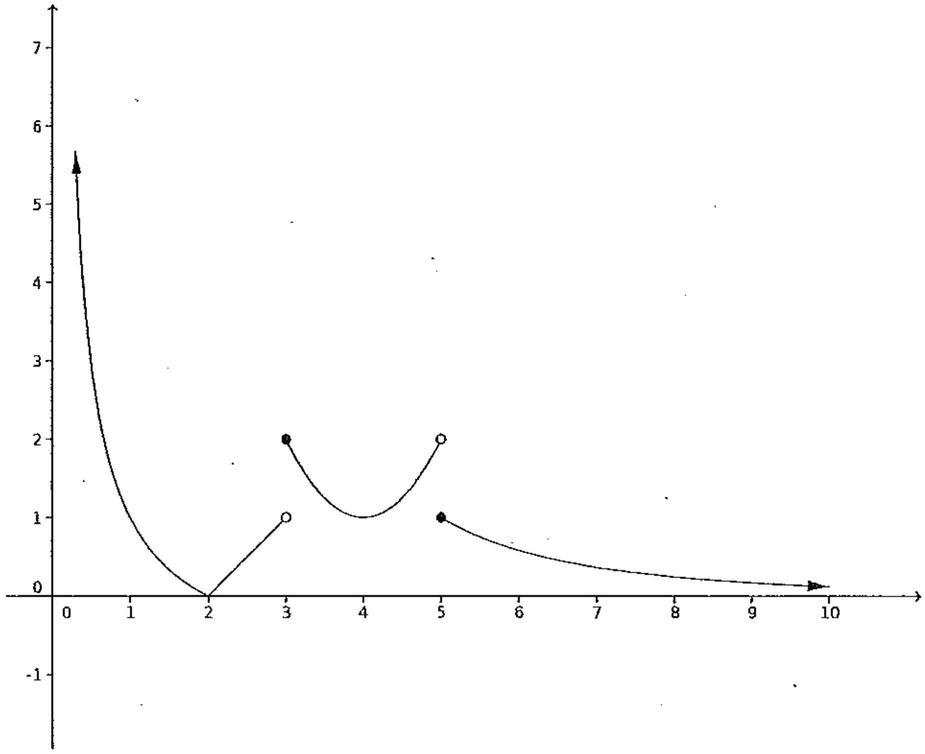
$$\lim_{x\to 3^+} \frac{|2x-6|}{x-3} = \lim_{x\to 3^+} \frac{2x-6}{x-3} = \lim_{x\to 3} \frac{2\cancel{(x-3)}}{\cancel{(x-3)}} = 2$$

but if $x<3$ then $|2x-6| = -(2x-6)$ so

$$\lim_{x\to 3^-} \frac{|2x-6|}{x-3} = \lim_{x\to 3^-} \frac{-(2x-6)}{x-3} = \lim_{x\to 3^-} \frac{-2\cancel{(x-3)}}{\cancel{(x-3)}} = -2$$

Since the one sided limits aren't equal the limit does not exist.

4. Consider the following graph of a function $f(x)$:



(a) [3 pts] What is the domain and range of $f(x)$?

The Domain is $(0, \infty)$ The range is $[0, \infty)$

(b) [2 pts] Find the points in the domain where the function is discontinuous.

Not continuous at

$x=3$ and $x=5$

(c) [2 pts] For which values of x in the domain of $f(x)$ is $f(x)$ non-differentiable?

Not differentiable at $x=3$ and $x=5$ since it isn't continuous.

Also not diffirentable at $x=2$ since the graph has a cusp.

(d) [3 pts] Evaluate the limits or explain why it doesn't exist:
$\lim_{x\to 2} f(x)$, $\lim_{x\to 5} f(x)$, $\lim_{x\to\infty} f(x)$

$\lim_{x\to 2} f(x) = 0$ since when x is close to 2 $f(x)$ is close to 0.

$\lim_{x\to 5} f(x)$ DNE. because it approaches 2 from the left and 1 from the right

$\lim_{x\to\infty} f(x) = 0$ since $f(x)$ gets close to 0 as x becomes large.

5. Compute the derivative of each of the following functions. **Do not simplify your answer. Box your final answer.**

(a) [5 pts] $f(x) = (x^3 + 2x - 1)(4x^5 + 3x^2 + 6)$

Using the product rule

$$\boxed{f'(x) = (3x^2+2)(4x^5+3x^2+6) + (x^3+2x-1)(20x^4+6x)}$$

Can also expand but it is a big pain.

(b) [6 pts] $g(x) = \dfrac{\sqrt{3-2x}}{x^2 - x}$ $= \dfrac{(3-2x)^{1/2}}{x^2-x}$

Quotient rule

$$g'(x) = \boxed{\frac{\frac{1}{2}(3-2x)^{-1/2}\cdot(-2)\cdot(x^2-x) - \sqrt{3-2x}\,(2x-1)}{(x^2-x)^2}}$$

since $\frac{d}{dx}\left[\sqrt{3-2x}\right] = \frac{d}{dx}(3-2x)^{1/2} = \frac{1}{2}(3-2x)^{-1/2}\cdot -2$ by the chain rule.

(c) [6 pts] $h(x) = (2x^3 + 3x - 1)^{\frac{1}{3}}$

Chain ~~R~~ Rule

$$\boxed{h'(x) = \frac{1}{3}(2x^3+3x-1)^{-\frac{2}{3}}\cdot(6x^2+3)}$$

6. [9 pts] Use the **limit definition of the derivative** to compute the derivative of $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \text{Conjugate trick}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

Check your answer with the power rule.

7. [8 pts] For $f(x) = \begin{cases} 5x-1 & \text{if } x<1 \\ k & \text{if } x=1 \\ x^2+3 & \text{if } x>1 \end{cases}$, determine the value of k, if any, that makes the function continuous at $x=1$. Justify your answer.

Criteria for continuity at 1. $f(1)$ exists. $\lim_{x\to 1} f(x)$ exists and $\lim_{x\to 1} f(x) = f(1)$.

if $x<1$ then $f(x) = 5x-1$ so

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 5x-1 = 5(1)-1 = 4.$$

if $x>1$ then $f(x) = x^2+3$ so

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2+3 = (1)^2+3 = 4.$$

Thus $\lim_{x\to 1} f(x) = 4$ because the left + Right side limits agree.

Thus if we define $k=4$ then $f(x)$ is continuous at 1.

8. [10 pts] A car leaves the garage at time $t = 0$ and moves in a straight line. Suppose the distance $d(t)$ between the car and the garage is given by $d(t) = 5t^4 + 3t^2$, then find both the velocity and acceleration of the car at time $t = 2$.

ANSWER: $\boxed{}$ $\underline{V(2) = 172}$ $\underline{a(2) = 246}$

Velocity is the derivative of distance so

$$V(t) = d'(t) = 20t^3 + 6t \qquad \text{so } V(2) = 20(2)^3 + 6(2)$$
$$= 20 \cdot 8 + 12 = 172$$

acceleration is the derivative of velocity

$$a(t) = \cancel{d}'(t) = d''(t) = 60(t^2) + 6 \qquad a(2) = 60(2)^2 + 6$$
$$= 246$$

9. [7 pts] Suppose $f(x)$ and $g(x)$ are differentiable functions with the following information: $f(1) = 4$, $g(1) = 2$, $f'(1) = 1$, $g'(1) = 5$, $f'(-2) = 3$, $g'(-2) = 10$
Please find the value of $h'(1)$ where $h(x) = f(g(x) - f(x))$

ANSWER: $\boxed{12}$

Using the chain rule

$$h'(x) = f'(g(x) - f(x)) \cdot (g'(x) - f'(x))$$

since

$$\frac{d}{dx}[g(x) - f(x)] = g'(x) - f'(x).$$

$$\text{So } h'(1) = f'(g(1) - f(1)) \cdot (g'(1) - f'(1)) = f'(2-4) \cdot (5-1)$$
$$= f'(-2) \cdot (4) = 3 \cdot 4 = 12$$

10. [7 pts] Does the function $f(x) = x^3 - 5x + 1$ have a root (zero) on the interval $[-1, 1]$? Explain why or why not.

$f(x) = x^3 - 5x + 1$ is continuous.

$f(-1) = (-1)^3 - 5(-1) + 1 = 5 > 0$

and $f(1) = (1)^3 - 5(1) + 1 = -3 < 0$

So By the IVT there is some x in $[-1, 1]$ so that $f(x) = 0$ so $f(x)$ has a zero in $[-1, 1]$.

11. Given the funtion $f(x) = 5x^3 + 3x^2 + 2$

(a) [6 pts] Find the line tangent to $f(x)$ at $x = 1$.

ANSWER: $\boxed{y = 21x - 11}$ or $y - 10 = 21(x-1)$

$f'(x) = 15x^2 + 6x$. The slope of the tangent line is given by

$f'(1) = 15(1)^2 + 6(1) = 21$.

Now $f(1) = 10$ so the tangent line is given by $y - 10 = 21(x-1)$

or $y = 21x - 11$

(b) [5 pts] Find all values of x that make the slope of the tangent line to $f(x)$ equal to 9.

ANSWER: $\boxed{x = -1, \frac{3}{5}}$

Slope of the tangent line is $f'(x)$. from before $f'(x) = 15x^2 + 6x$

So we need to find x so that

$f'(x) = 15x^2 + 6x = 9$.

$15x^2 + 6x - 9 = 0 \Rightarrow 5x^2 + 2x - 3 = 0$

$\Rightarrow (5x - 3)(x + 1) = 0$ so $x = -1$ or $x = \frac{3}{5}$