1a. Compute the derivative of $f(x) = 3 - \sqrt{x}$ at the point $x = 2$ from the definition. Find the equation of the tangent line to graph of f at the point $x = 2$.

1b. (i) Given a function $f(x)$ such that $f(8) = 7, f'(8) = -2$, find $g'(8)$ where $g(x) = 1 + f(x)\sqrt[3]{x}$.

(ii) Find the equation of the tangent line to the graph of $g(x)$ at the point $(8, g(8))$.

1c. Find the equations of all horizontal tangent lines to the graph of $f(x) = x^3 - 12x$.

2. Find derivatives:

a. $g(x) = \dfrac{x-2}{x^2+3x+1} + 5x$

b. $g(s) = \dfrac{-2s^2}{\sqrt{2+s^3}}$

c. $f(v) = (v^{\sqrt{5}} - \dfrac{1}{v^2})(v^3+1)$.

d. Suppose $g(x)$ is a differentiable function. Let $f(x) = 3x^2 g(x) - 5x$.

(a) Find $f'(x)$ using the rules of differentiation.

(b) Find $f'(1)$ given that $g(1) = 3$ and $g'(1) = 2$.

(c) What is the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$?

3. Write True/False. If True, provide a brief justification. If False give a counterexample or disprove it. Answer any two

a. If $f'(3)$ exists, then $\lim_{x\to 3} f(x)$ may not always exist. (Hint: If differentiable, what do we know about continuity at that point?)

Bonus Problem:*If f is differentiable at $x = a$, then f is continuous at $x = a$*

Step 1: Make a clever substitution for h in the definition of $f'(a)$ to get $f'(a) = \lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$.

Step 2: Remember that for the derivative $f'(a)$ to be defined, a is required to be in the domain of $f(x)$. So first condition in the definition of continuity of f at a is guaranteed.

Step 3: Using limit laws, show that $\lim_{x\to a} f(x) = f(a)$ is the same as $\lim_{x\to a} \Big(f(x) - f(a)\Big) = 0$.

Step 4: We will show that $\lim_{x\to a} f(x) = f(a)$. Since $f(a)$ exists by Step 2, this will also show that the limit will exist. Thus, we will satisfy the remaining two conditions for continuity.

By step 3, it is enough to show that $\lim_{x\to a} \Big(f(x) - f(a)\Big) = 0$.

Here's how you start:

Let $h(x) = \dfrac{f(x)-f(a)}{x-a}$. By Step 1, $\lim_{x\to a} h(x)$ exists. Now,

$\lim_{x\to a} (f(x) - f(a)) = \lim_{x\to a} \left[\dfrac{f(x)-f(a)}{x-a}(x-a)\right] = \lim_{x\to a} \Big(h(x)(x-a)\Big)$. Use limit laws to complete the proof.