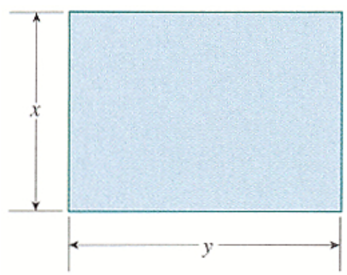
The 4 Steps of Mathematical Modelling (DUDE)

1. **D**raw a picture - if possible, draw a picture of the situation.
2. **U**se Variables - assign a letter for each variable mentioned in the problem.
3. **D**escribe the situation - use the given information to try and find equations for the desired quantity.
4. **E**valuate - solve the system of equations for the desired value.

Example 1: Patricia wishes to have a rectangular-shaped garden in her backyard. She has 72 ft of fencing material with which to enclose her garden.
a) Find a function giving the area of the garden in terms of the length of one of the sides.
b) Find the area of the garden when the length of one of the sides is 10 ft.

Solution: *(Draw a picture)* Let's draw a picture of what this garden will look like. We know that it is rectangular, so we just need to give names to the lengths of the sides of the garden. How about x and y?



(Use variables) We have already picked x and y for the dimensions of our garden. Now we need a variable for the area. How about A?
(Describe the Situation) We know that the area of the garden is $A = xy$; we also know that the total length of the fence must be 72 ft, so we get that $2x + 2y = 72$.
(Evaluate) We want to be able to write A purely in terms of one of the sides. Let's pick x (although you could just as easily pick y if you prefer). Now, we know that $2x + 2y = 72$, so $y = 36 - x$ (just by solving the equation for y in terms of x). Thus, we can substitute this into the area equation to get

$$A = xy = x(36 - x) = 36x - x^2$$

That gives us part (a); we have found an equation for A in terms of x. Now on to part (b): We want to find what A is when the length of one of the sides is 10 ft. It turns out that the area will be the same whether we pick x to be 10 ft or y to be 10 ft, so let's just pick x. Now, by part (a),we know that

$$A = 36x - x^2 = 36(10) - (10)^2 = 360 - 100 = 260$$

We get then that the area of the garden is 260 ft^2 when the length of one of the sides is 10 ft.

Example 2: A manufacturer has a fixed monthly cost of \$40,000 and a production cost of \$8 for each unit produced. The product sells for \$12 per unit.
a) Find the cost function, revenue function, and profit functions.
b) Compute the profit corresponding to monthly production at levels of 8,000 and 12,000 units.

Solution: *(Draw a picture)* Unfortunately, there isn't really a picture we can draw for this one, so step one isn't terribly helpful.
(Use variables) We know that we're going to need a variable for the number of units produced monthly, so we'll call that x. Also, we know that we'll need variables for each of the values we are looking for in (a), so let $C =$ monthly cost, $R =$ monthly revenue, and $P =$ monthly profit.
(Describe the Situation) What do we know about the situation? Well, we know that there is a flat monthly cost of \$40,000 and an additional monthly cost of \$8 per unit produced. Thus, our cost equation should look like

$$C = 40,000 + 8x$$

Now, we know that the only revenue the company has is simply the \$12 it makes for each unit it sells, giving us that the revenue equation should look like

$$R = 12x$$

Finally, we know that the total profit of the company is simply the revenue that the company makes minus the cost, so we get

$$P = R - C = 12x - 40,000 - 8x = 4x - 40,000$$

(Evaluate) Well, what we just did was all to get part (a); now on to part (b)! We're looking for the profit corresponding to a monthly production of 8,000 and 12,000 units. Since x is the number of units produced monthly, we just need to plug 8,000 and 12,000 into the profit equation:

$$P = 4(8,000) - 40,000 = 32,000 - 40,000 = -8,000 \text{ when } x = 8,000$$

$$P = 4(12,000) - 40,000 = 48,000 - 40,000 = 8,000 \text{ when } x = 12,000$$

Now let's interpret our results. When $x =$8,000, we find that $P = -$8,000, so the company actually lost \$8,000; however, when $x =$12,000, we find that $P =$8,000, so the company made \$8,000 that month.