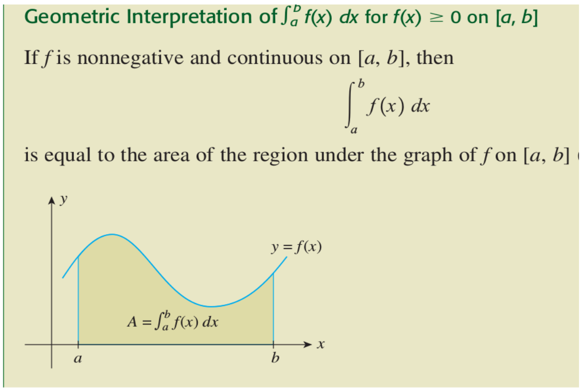
- We say f is integrable on $[a, b]$ if the above limit exists.

Theorem: If f is continuous, then f is integrable i.e. $\int_a^b f(x)\, dx$ exists.



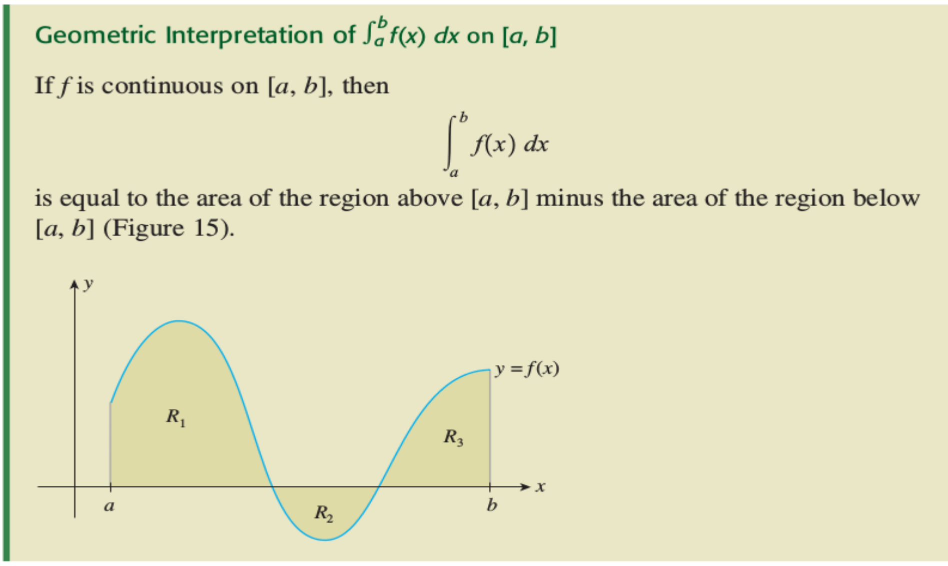
Geometric Interpretation of $\int_a^b f(x)\, dx$ for $f(x) \geq 0$ on $[a, b]$

If f is nonnegative and continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx$$

is equal to the area of the region under the graph of f on $[a, b]$

Thus, in this case, the definite integral gives the Area under the Curve.



Geometric Interpretation of $\int_a^b f(x)\, dx$ on $[a, b]$

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx$$

is equal to the area of the region above $[a, b]$ minus the area of the region below $[a, b]$ (Figure 15).

FIGURE 15
$\int_a^b f(x)\, dx =$ Area of R_1 $-$ Area of R_2 $+$ Area of R_3

Thus, here it is not exactly the area under the curve. We have to make suitable sign changes if we were to compute the definite integral geometrically.

Remark: Area must always be positive. Thus, if we were to compute an area under the $x-axis$ using integration say e.g. R_2 in the above figure, then since the $f(x)$-values are negative, the integral will come out to be negative. So, area will be negative of the integration. However, in the above scenario, we are not computing area; Thus it says minus of the area; so we computed the area R_2 and then associated a minus sign to it.