

STUDENT NAME: _____

INSTRUCTOR: _____

Please sign the pledge:

*On my honor as a student, I have neither given nor received aid on this exam.***Directions**

Answer each question in the space provided. Please write clearly and legibly. *Show all of your work—your work must justify your answers. Clearly identify your final answers. No books, notes, or electronic devices of any kind may be used during the exam period. You must simplify results of function evaluations when it is possible to do so. For example, $4^{3/2}$ should be evaluated (replaced by 8). Your work on all limit problems must be well organized; in particular, for a limit that exists, a string of equalities must connect the original limit problem with the value of the limit.*

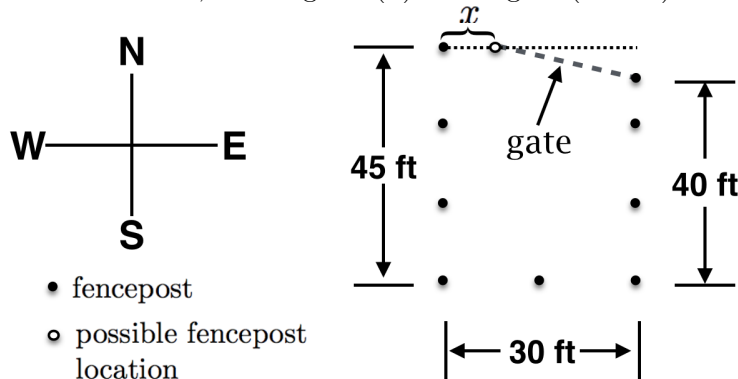
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6	10	
7	13	
8	16	
9	9	
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1. [5 pts] Find the domain of the function $f(x) = \frac{\sqrt{x-1}}{x^2 - x - 6}$, expressing your answer in interval notation. Provide work showing how you arrived at your answer.

2. [4 pts] Find an equation of the line passing through the point $(-3, 2)$ that is parallel to the line $-2x + 3y + 10 = 0$.

3. [4 pts] Sketching a design for a livestock enclosure (see below), a farmer has settled on positions for 9 of 10 fenceposts. The 10th fencepost will be set a distance x feet east of the northwestern corner post, $0 \leq x \leq 30$. A gate will span the gap between the 10th post and the northeastern-most post, as pictured. Express, as a function of x , the length $L(x)$ of the gate (in feet).



4. [4 pts] Steelworth Investment Bank is constantly buying and selling shares of AAPL stock. The number of shares of AAPL held by Steelworth Investment Bank at time t , measured in thousands, is modeled by

$$n(t) = \frac{8t^2 - t + 70}{6t + t^2 + 10}$$

How many shares of AAPL will Steelworth Investment Bank hold in the long run (i.e, as $t \rightarrow \infty$).

5. [10 pts] You need not show any work in completing this graph-based problem. Consider the function g graphed below right.

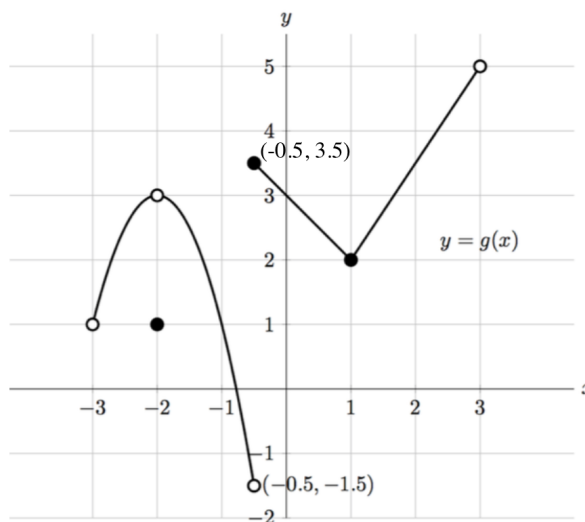
(a) Record the domain of g in interval notation.

(b) Record the range of g in interval notation.

(c) At which numbers in its domain is the function g discontinuous?

(d) At which numbers in its domain is the function g not differentiable?

(e) Find $\lim_{x \rightarrow -2} g(g(x))$ if it exists.



6. [5 pts] [TRUE/FALSE] Circle your response. No work required.

- | | | |
|---|------|-------|
| (a) If f is differentiable at $x = x_0$, then f is continuous at $x = x_0$. | True | False |
| (b) If f is continuous at $x = x_0$, then f is differentiable at $x = x_0$. | True | False |
| (c) If $\lim_{x \rightarrow x_0^+} f(x) \neq \lim_{x \rightarrow x_0^-} f(x)$, then $f(x_0)$ is not defined. | True | False |
| (d) For all real numbers $\sqrt{x^2} = x$. | True | False |
| (e) $f(x) = \frac{x^2 - 1}{x - 1}$ is not continuous at 1. | True | False |

7. [12 pts] Evaluate the following limits or write DNE if the limit does not exist. Your work must justify your answer; moreover, your work must be well organized. Work based on “l’Hospital’s Rule” will receive no credit.

(a) $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x^2 - 16}$

(b) $\lim_{x \rightarrow 0^-} \frac{x + |x|}{x}$

(c) $\lim_{x \rightarrow 0} \frac{x + 1}{g(x)}$ where $g(x) = \begin{cases} -x + 1 & x < 0 \\ \sqrt{x} + 1 & x > 0 \end{cases}$

8. [8 pts] (a) Complete the following definition:

The function f is *continuous* at the number a provided that

- (b) Is there a value of c making the function $f(x) = \begin{cases} \frac{1}{2}x^2, & \text{if } x < 1 \\ c, & \text{if } x = 1 \\ \frac{2x}{x^2+1} & \text{if } x > 1 \end{cases}$ continuous at 1? Carefully justify your answer using the definition of continuity you just stated.

9. [10 pts] (a) Complete the following “limit definition” of the derivative: the derivative of f at x , denoted $f'(x)$ is given by

$$f'(x) =$$

- (b) Let $f(x) = \sqrt{3x + 1}$. Use the definition of derivative you just stated to find $f'(x)$. *No credit will be awarded for work that is not based on the definition of derivative.

10. [7 pts] A Nuclear Regulatory Commission scientist projects that, for a certain reactor subjected to intermittent coolant-flow interruption, the temperature of the reactor core t hours after the initial interruption, $0 \leq t \leq 3$, is given by

$$f(t) = 3t^2 - t^3 + t + 1, \quad \text{in thousands of degrees Celsius.}$$

The temperature at which a core meltdown occurs is about 2.9 thousand degrees Celsius. Show there must be a time t in the modeling interval $[0, 3]$ at which the temperature of the reactor core equals the meltdown temperature 2.9. Your answer must be carefully justified.

11. [6 pts] Suppose that $p(x) = x g(f(x))$ and that $g(2) = 4$, $g'(2) = -1$, $f(3) = 2$, and $f'(3) = 5$. Find an equation of the line tangent to the graph of p at the point $(3, 12)$.

12. [10 pts] Find derivatives of the following functions. **Do not simplify your answers.**

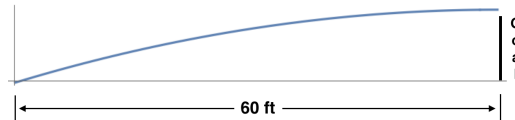
(a) $f(x) = \frac{x^3 + 4}{x^2 + 3x + \pi^2}$

(b) $g(x) = (x^2 - 20)\sqrt{5x - 1}$

13. [6 pts] Let f be the function defined by $f(x) = \frac{1}{15}(x^3 - 23)^{5/2}$. Compute $f'(3)$. **Your final answer should be in the form of an integer.**

14. [9 pts] A ball is kicked directly toward a goal as pictured below right.

The ball's initial velocity has a horizontal component a ft/sec and a vertical component b ft/sec. Here a and b are positive constants. The curve the ball follows after it is kicked is well modeled by



$$y = -\frac{16}{a^2}x^2 + \frac{b}{a}x, \text{ where } x \text{ and } y \text{ are measured in feet.}$$

- (a) Find $y'(x) = \frac{dy}{dx}$, keeping in mind that a and b are constants.
- (b) Find the point on the curve $y(x) = -\frac{16}{a^2}x^2 + \frac{b}{a}x$ at which the tangent line to the curve is horizontal. (The x -coordinate of this point depends on both a and b while the y -coordinate depends only on b .) Also note that the ball attains its maximum height at this point, which depends only on b .
- (c) Suppose the ordered pair you just found (yielding maximum height) is directly above the goal, having coordinates $(60, 9)$. Determine the corresponding initial-velocity components: a and b .