Answer each question in the space provided. Please write clearly and legibly. Show all of your work in order to receive full credit, and clearly identify your final answer. No books, notes or calculators are allowed. .

1. (4 points) Evaluate the following limit or write DNE if the limit does not exist. Your work must justify your answer; moreover, your work must be well organized. Work based on "L'Hospital's Rule" will receive no credit.

$\lim_{t\to 2} \frac{\sqrt{14+t}-4}{t-2}$

$$= \lim_{t\to 2} \frac{(\sqrt{14+t}-4)(\sqrt{14+t}+4)}{(t-2)(\sqrt{14+t}+4)} = \lim_{t\to 2} \frac{14+t-16.}{(t-2)(\sqrt{14+t}+4)}$$

$$= \lim_{t\to 2} \frac{\cancel{(t-2)}}{\cancel{(t-2)}(\sqrt{14+t}+4)} = \frac{1}{8}$$

2. (4 points) For what value of the constant c is the following function continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} cx^2 + 2x & x < 2 \\ x^3 - cx & x \geq 2 \end{cases}$$

· $f(x)$ is continuous at all points on $(-\infty, 2) \cup (2, \infty)$ as it is defined as a polynomial on these intervals

$\boxed{\text{At } x = 2}$ Need $f(2) = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$$8 - 2c = \lim_{x\to 2^-} (cx^2+2x) = \lim_{x\to 2^+} (x^3 - cx)$$

$$8 - 2c = 4c + 4 = 8 - 2c.$$

So, $4c + 4 = 8 - 2c$

$6c = 4$

$$\boxed{c = \frac{2}{3}}$$

3. (8 points) Dr. Snowworthy has constructed mathematical models for the amounts of snowfall in Duluth, MN and in Buffalo, NY. According to her model, the amount of snowfall in Duluth, in inches, t months after December, $1 \le t \le 4$, is predicted to be $D(t) = 13 + 12t^2 - 2t^3$ and in Buffalo is predicted to be $B(t) = 10 + 2t + 9t^2 - t^3$. According to Dr. Snowworthy's model, is there a time t in the interval $[0, 4]$ such that the amount of snow fallen in Duluth will equal the amount fallen in Buffalo? Carefully justify your answer.

Look at $h(t) = D(t) - B(t) = 13 + 12t^2 - 2t^3 - 10 - 2t - 9t^2 + t^3$
$= 3 + 3t^2 - t^3 - 2t$

- h is a polynomial, so it's continuous on $[0, 4]$. Hence IVT applies.
- $h(0) = 3 > 0$, $h(4) = 3 + 48 - 64 - 8 = -21 < 0$

Since, $h(0) > 0$, and $h(4) < 0$, by IVT, there is some $b \in [0, 4]$ such that $h(b) = 0$. Hence, $D(b) - B(b) = 0$ which shows that snowfall matches.

4. (4 points) A race track with perimeter 1 mile has two identical semicircles at the ends of a rectangular area. Assuming both semicircles have radius r and the rectangle has length x, both measured in miles. Find a function f in the variable r giving the area enclosed by the race track. Find the domain of f.

- $1 = 2x + 2\pi r$

$x = \dfrac{1 - 2\pi r}{2}$

- $f(r) = x \cdot 2r + \pi r^2$
$= \left(\dfrac{1 - 2\pi r}{2}\right) \cdot 2r + \pi r^2$
$= r(1 - 2\pi r) + \pi r^2$
$= r - 2\pi r^2 + \pi r^2$
$= r - \pi r^2 = r(1 - \pi r)$

Need
- $r > 0$
- $f(r) = r(1 - \pi r) > 0$ So, $1 - \pi r > 0$
$1 > \pi r$
$\frac{1}{\pi} > r$

Thus: $D = (0, \frac{1}{\pi})$