If $f(x)$ and $g(x)$ are functions such that $f(x)$ is differentiable at x and $g(x)$ is differentiable at $f(x)$, then the chain rule says that

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x).$$

Problem 1. Practice the chain rule by differentiating the following functions(no need to simplify):

(a) $\sqrt{x^3 + 2x}$

(b) $(x^2 + x + 1)^{1000}$

(c) $(x^2 + x + 1)^{1000}$

(d) $(2x^4 - 1)^2(4x + 1)^5$

(e) $\dfrac{1 - x}{(2x^2 + 7)^2}$

(f) $\sqrt{(5x^2 + 2)^4 + 3}$

(g) $\left(\dfrac{10x^2 + 3x}{x^3 - 4x^2 + 1}\right)^{3/2}$

(h) Show that $\frac{d}{dx}[f(c(x)] = cf'(cx)$ where f is differentiable, c is a constant.

(i) If f is differentiable and $f(x) > 0$ for all x, then show that $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$.

(j) Write T/F. If false write what the correct statement should be:
If f is differentiable, then $\frac{d}{dx}\left[f\left(\frac{1}{x}\right)\right] = f'\left(\frac{1}{x}\right)$.

Problem 2. Let $f(x)$ and $g(x)$ be functions with the following values:

$$\begin{aligned} f(1) &= -3 & f'(1) &= -\frac{2}{3} \\ f(4) &= 4 & f'(4) &= -5 \\ g(1) &= 0 & g'(1) &= 0 \\ g(4) &= 1 & g'(4) &= \frac{1}{2}. \end{aligned}$$

(a) Calculate $h'(4)$ when $h(x) = (f \circ g)(x)$.

(b) Calculate $j'(1)$ when $j(x) = [f(x)]^3$.

(c) Calculate $k'(4)$ when $k(x) = (g \circ f)(x)$.

Problem 3. The adiabatic law for a gas, the law that governs the behaviour of a gas that is expanding without gaining or losing heat is given by the equation

$$P(t)(V(t))^\gamma = k$$

where k, γ are constants, $P(t), V(t)$ are the pressure and volume of the gas respectively at time t. Show that

$$\frac{1}{V}\frac{dV}{dt} = -\frac{1}{\gamma}\frac{1}{P}\frac{dP}{dt}.$$

[*We often suppress the notation $V(t)$ to be just V provided that we understand that it is still a function of t; e.g. $\frac{df}{dx}$ makes sense but what we actually mean is $\frac{d}{dx}(f(x))$. Keep this in mind and do not get confused with notations.*]