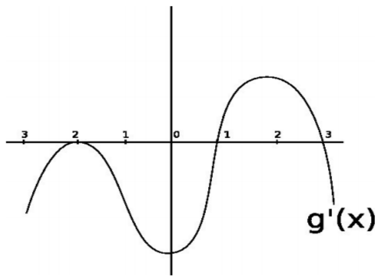
1.(6 points) The figure shows the graph of the derivative g' of some function g. Find the intervals on which the original function g is increasing and decreasing, and classify all local extrema of g.



Solution: $g' > 0$ on $(1, 3)$, $g' < 0$ on $(-\infty, -2), (-2, 1), (3, \infty)$.

Hence g is increasing on $(1, 3)$ and decreasing on $(-\infty, -2), (-2, 1), (3, \infty)$.

g' changes sign from $-$ to $+$ at $x = 1$. So at $x = 1$, there is a local minimum.

g' changes sign from $+$ to $-$ at $x = 3$. So there is a local maximum at $x = 3$.

2. (10 points) A cylinder's height is increasing at the rate of 1 inch per minute while its radius is decreasing at the rate of 1 inch per minute. Find the rate of change in the volume of this cylinder at the instant when its radius is 10 inches and its height is 8 inches. Is the volume increasing or decreasing? (Your diagram should clearly indicate the variables you are choosing)

Solution: Let r, h (in inch) be the radius and height of the cylinder. Let V be the volume (in inch^3).

$$V = \pi r^2 h$$

.

So,

$$\frac{dV}{dt} = \pi\Big(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\Big)$$

We are given $h = 8, r = 10, \dfrac{dr}{dt} = -1, \dfrac{dh}{dt} = 1$. Plugging them i the above equation, we get

$$\frac{dV}{dt} = \pi\Big(2 \times 10 \times (-1) \times 8 + (10)^2 \times 1\Big) = \pi(-160 + 100) = -60\pi \qquad (\text{inch}^3)$$

Hence, volume is decreasing at the rate of 60π inch^3 per minute.

3.(4 points) Prove or disprove:

Assume that derivatives of all high orders for both f and g exist. Suppose $f(x)$ is increasing and concave up and $g(x)$ is concave up. Then $(f \circ g)(x)$ is concave up.

Solution: We are given that $f' > 0,\ f'' > 0,\ g'' > 0$. Need to show $\Big(f \circ g\Big)'' > 0$.

Let $h(x) = (f \circ g)(x) = f(g(x))$.

$$h'(x) = f'(g(x)).g'(x).$$

$$h''(x) = \Big(f''\big(g(x)\big)g'(x)\Big).g'(x) + f'(g(x)).g''(x) = \underset{>0}{f''(g(x))}\ \underset{\geq 0}{(g'(x))^2} + \underset{>0}{f'(g(x))}\ \underset{>0}{g''(x)}$$

Since $(g'(x))^2 \geq 0$, we have right hand side of the above equation to be always > 0 from the information given to us. Hence proved.