[4e] For what value of of the constant $c$ is the following function continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$

Solution:

Note that $f$ is continuous at all points other than $x = 2$ as $f$ is a defined as a polynomial on $(-\infty, 2) \cup (2, \infty)$.

At the point $x = 2$, we need $\lim_{x \to 2^-} f(x) = f(2) = \lim_{x \to 2^+} f(x)$.

Thus, $\lim_{x \to 2^-} (cx^2 + 2x) = 8 - 2c = \lim_{x \to 2^+} (x^3 - cx)$. So, we need $4c + 4 = 8$. Hence, $c = \frac{2}{3}$.