4.4 Optimization

Absolute Extrema of a function f.

If $f(x) \leq f(c)$ (resp. $f(x) \geq f(c)$) for all x in the domain of f, then $f(c)$ is called the absolute maximum value of f. (resp. absolute minimum value).

Theorem (Extreme Value Theorem)

If a function f is continuous on a closed interval $[a,b]$, then f has both an absolute maximum value and an absolute minimum value on $[a,b]$.

Closed Interval Method: This gives us the algo for finding absolute max/min on a closed, bounded interval $[a,b]$. They always exists by the theorem above.

1. Find critical number that lie in open interval (a,b) (Next, we will treat these as well as a & b as crit. pts.)
2. Compute f at each critical number; compute $f(a)$, $f(b)$.
3. The largest out of these is absolute max.
   The least " " " is absolute min

Eg: Find absolute extrema for $g(x) = x^3 + 3x^2 - 1$ over the interval $[-3, 1]$.

Soln: $g'(x) = 3x^2 + 6x$; g' exists everywhere as g is a polynomial.

$g'(x) = 0$

$\Rightarrow 3x(x+2) = 0 \Rightarrow x = 0, -2$.

Thus, critical points on $(-3, 1)$: $x = 0, -2$.

- $f(0) = -1$;
- $f(-2) = (-2)^3 + 3(-2)^2 - 1 = -8 + 12 - 1 = 3$
- $f(-3) = -27 + 27 - 1 = -1$
- $f(1) = 1 + 3 - 1 = 3$.

Thus,

- $(0, -1)$, $(-3, -1)$ – absolute minimum
- $(-2, 3)$, $(1, 3)$ – absolute maximum.

Note: -1 is the absolute min. value whereas graph of g has an absolute min. at $(0, -1)$ and $(-3, -1)$.

3 is the absolute max. value whereas graph of g has an absolute max at $(-2, 3)$ and $(1, 3)$.

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(2) Find abs. max/min: $\sqrt{4-x^2}$.

$f(x) = \sqrt{4-x^2}$. Domain: $[-2, 2]$. So, abs. max/min exist!

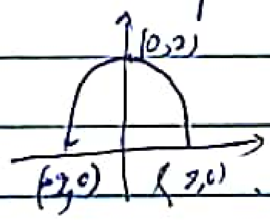
$f'(x) = \dfrac{-x}{\sqrt{4-x^2}}$. So, 0 is the only critical point on $(-2, 2)$.

- $f(0) = 2$
- $f(-2) = 0$
- $f(2) = 0$.

Thus, abs. max. value is 2. (at $(0,2)$)

abs. min value $= 0$ @ points $(-2,0)$, $(2,0)$.

Note: this is what we found out in Example 4 of curve sketching



So, while sketching curves, if you find domain is a closed interval, then you have to apply the 'closed interval test' to find max/min to help you draw.