MATH 1210 Test 2 Nov 12, 2015

STUDENT NAME: _____

INSTRUCTOR: _____

Please sign the pledge:
On my honor as a student, I have neither given nor received aid on this exam.

Directions

Answer each question in the space provided. Please write clearly and legibly. ***Show all of your work—your work must justify your answer, and clearly identify your final answer. No books, notes or calculators are allowed.***

For instructor use only

Page	Points	Score
2	10	
3	18	
4	10	
5	14	
6	16	
7	10	
8	10	
9	12	
Total:	100	

Some Formulas You May Find Useful

Description	**Formula**
Pythagoras' Theorem (right triangle with legs x, y, hypothenuse h)	$h^2 = x^2 + y^2$
Area of the surface of a sphere of radius R	$4\pi R^2$
Volume of a rectangular box, base xy, height h	xyh
Surface area of a rectangular box, base xy, height h, no top	$2xh + 2yh + xy$
Volume of a cylinder of radius r and height h	$\pi r^2 h$
Compound Interest: $A(t)$ accumulated amount after t years, P is principal, r nominal annual interest rate, m number of conversion periods per year	$A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$

1. [10 pts] Let C be the curve in the xy-plane given by the following equation.

$$x^2y + y^3 = 2$$

(a) Use the method of implicit differentiation to find $\dfrac{dy}{dx}$.

(b) Find an equation for the line tangent to C at the point $(1, 1)$.

2. [5 pts] Suppose that $f(x) = \sqrt{x + \sqrt{x^2 + 4}}$. Find $f'(x)$. (No need to simplify.)

3. Some Chapter 5 Problems

   (a) [4 pts] Find a and b, so that $f(x) = ae^{bx}$ satisfies $f(0) = 2$ and $f(3) = 6$.

   (b) [4 pts] Solve the following equation for x: $2^{\log_3 x} = \frac{1}{4}$. (Simplify your answer.)

   (c) [5 pts] How long will it take \$10000 to grow to \$18000 if the investment earns an interest rate of 10% per year compounded monthly? You need not simplify your answer (an expression you'd plug into a calculator).

4. [10 pts] Based on customer trials and surveys, CV8 Theater decides that a child's size serving of popcorn should be 32 in^3. CV8 will serve the popcorn in rectangular boxes with square base and no top. Assuming that it is to hold 32 in^3, what dimensions for the box should the theater choose in order to minimize the box's surface area (thereby minimizing its cost). You must fully justify your claim that you've found the dimensions minimizing the surface area (using calculus, of course).

5. [10 pts] (a) Why must $f(t) = t^3 - 12t^2 + 1$ have both an absolute maximum value and an absolute minimum value over the interval $[-1, 2]$?

(b) Compute the absolute maximum value and absolute minimum value of $f(t) = t^3 - 12t^2 + 1$ over $[-1, 2]$. (Show all work in order to justify your answer.)

6. [4 pts] Simplify $\sqrt[3]{\left(\sqrt{2xy^2}\right)^{12}}$ completely, writing it using only positive exponents, a single coefficient, and using each variable at most once once in your expression. (Assume x and y are positive)

7. [8 pts] The function $f(x) = \frac{1}{x(x-6)^2}$ has derivative $f'(x) = \frac{3(2-x)}{(x-6)^3x^2}$.

   (a) Identify the intervals on which f is increasing and the intervals on which f is decreasing.

   (b) Find relative maxima and relative minima of f (if any).

8. [8 pts] Where is the graph of $f(x) = x^4 - 4x^3$ concave up? Where is it concave down? Also find points of inflection on the graph of f (if any).

9. [10 pts] A cylinder's height is increasing at the rate of 1 inch per minute while its radius is decreasing at the rate of 1 inch per minute. Find the rate of change in the volume of this cylinder at the instant when its radius is 10 inches and its height is 8 inches. Is the volume increasing or decreasing?

10. [10 pts] Sketch the graph of a function f with the following properties.

(a) The domain of $f(x)$ is $(-\infty, 3) \cup (3, \infty)$ and f is continuous on its domain.

(b) y-intercept -1 and x-intercepts -2 and 1

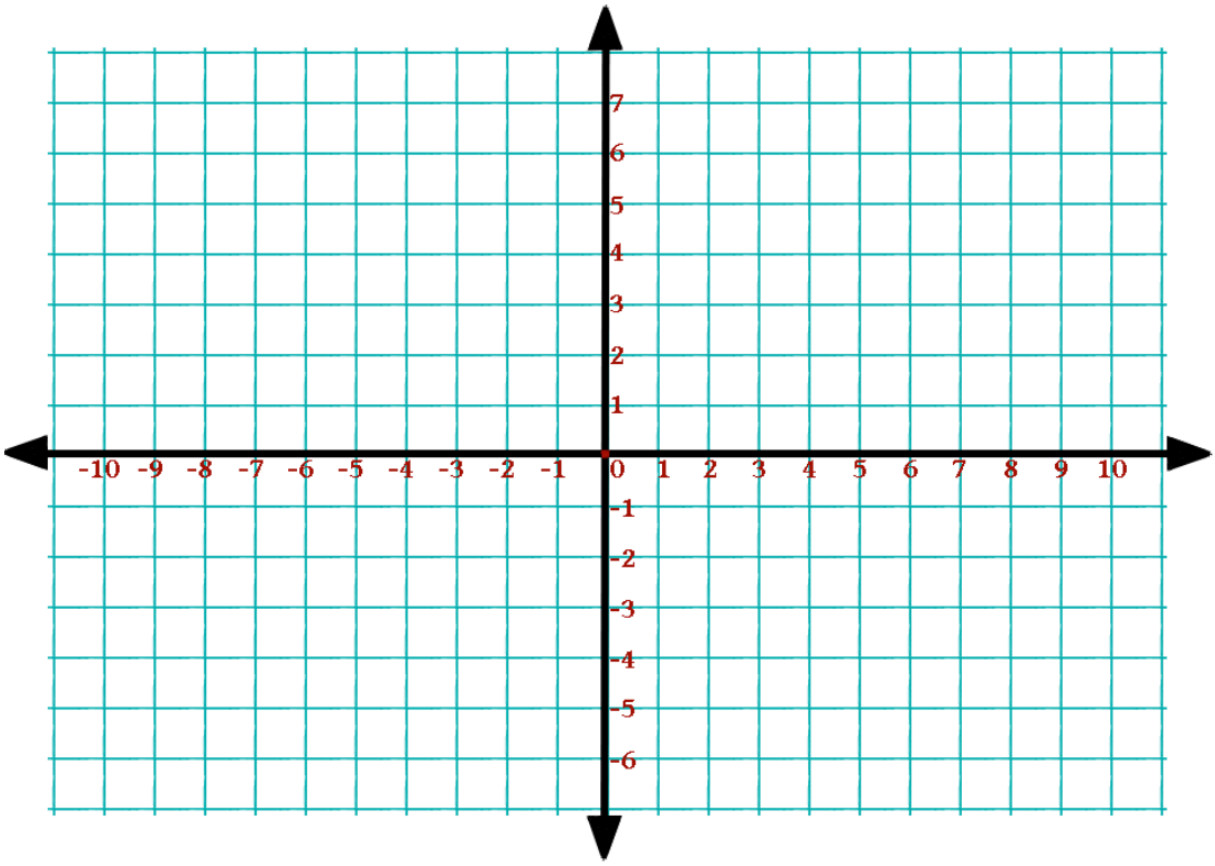
(c) $x = 3$ is a vertical asymptote

(d) $\lim_{x\to\infty} f(x) = -3$ and $\lim_{x\to-\infty} f(x) = 2$

(e) $f'(x) > 0$ for $0 < x < 3$ and for $3 < x$ and $f'(x) < 0$ for $x < 0$.

(f) $f''(x) > 0$ for $-2 < x < 3$ and $f''(x) < 0$ for $x < -2$ and for $x > 3$.

Be sure to include and label in your graph all asymptotes as well as points of inflection (if any).



11. [12 pts] Multiple-Choice: Circle the correct response.

(a) What is the maximum number of *horizontal asymptotes* that the graph of a function can have?

(a) one
(b) two
(c) three
(d) as many as we want—there is no maximium
(e) zero

(b) What is the maximum number of *vertical asymptotes* that the graph of a function can have?

(a) one
(b) two
(c) three
(d) as many as we want—there is no maximum
(e) zero

(c) Which of the following statements (I)–(III) are true:

(I) if $f(x)$ has a critical number at $x = 0$ then $f(x)$ has either a relative minimum or a relative maximum at $x = 0$.
(II) If $f(x)$ is continuous on (a, b) then $f(x)$ has an absolute maximum on (a, b).
(III) If $f''(a) = 0$ then $(a, f(a))$ is an inflection point of the graph of $f(x)$.

(a) III only
(b) I, II and III
(c) I and III
(d) II and III
(e) none

(d) For what range of values of a will $f(x) = (\log_{\frac{1}{2}} a)^x$ be a decreasing function?

(a) $0 < a < 1$
(b) $a > 1$
(c) $1 < a < 3/2$
(d) $0 < a < 1/2$
(e) $1/2 < a < 1$