Functions

A function is a rule that accepts an input and provides a unique output corresponding to the chosen input.

Ex f: ~~maps~~ sends 1 to 2
2 to 3
3 to 4.

We write

$$f : \{1, 2, 3\} \to \{2, 3, 4\}$$

$f(1) = 2$
$f(2) = 3$
$f(3) = 4$.

The set $\{1, 2, 3\}$ is called domain of the function
~~&~~ $\{2, 3, 4\}$ is called Range of the function.

In this course, we will work with domains $\mathbb{R}$, (a, b), $[a, b]$, $\{1, 2, 3, \dots\}$ mainly.

$\boxed{eg.}$ • Constant function

$f : \mathbb{R} \to \mathbb{R}.$

$f(x) = c$. for all $x \in \mathbb{R}$.

i.e. it sends all real nos to the constant real number c.

Here $\{c\}$ is the range; but we can still write

$f : \mathbb{R} \to \mathbb{R}$ ← This $\mathbb{R}$ is called the codomain.

Range is contained in the codomain.

Not A Function

$f : \{1, 2, 3\} \to \mathbb{R}.$

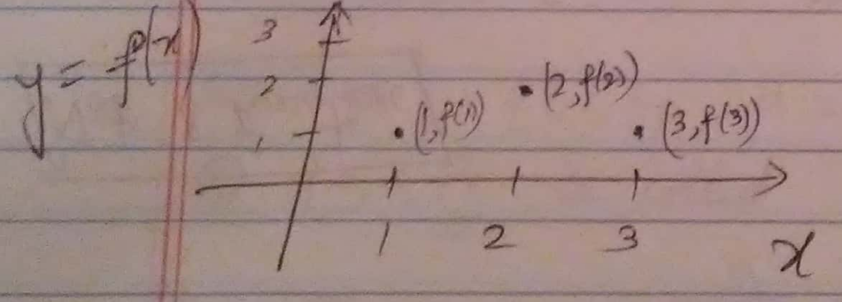
$f(1) = 2$
$f(1) = 3$ } Not a function, as it sends input 1 to different outputs.
$f(2) = 4$
$f(3) = 4.$

Given a function, we can diagramatically repersent it by plotting the output value in the Cartesian plane. We will do detailed examples later in the course as to how to ~~to~~ draw the functions.

We repersent the graph of a function as an ordered pair $(x, f(x))$ in the plane.

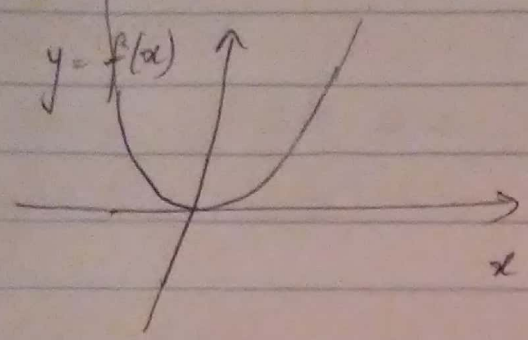
For example, $f : \{1, 2, 3\} \rightarrow \mathbb{R}$

$$f(1) = 1, \quad f(2) = 2, \quad f(3) = 1.$$



We ~~plot~~ take $f(x)$ to be our y-axis ~~in this~~ when we plot.

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$



← The graph of $f(x) = x^2$.

~~x - is the input~~

$\boxed{\text{Note}}$ - x is the input - so it is called independent variable

- y ($= f(x)$) is the output depending on input x; so it is the dependent variable.

We are often asked to find domain of a function

e.g. $f(x) = \dfrac{x^2 - 2}{2x+1}$

We need to make sure, it makes sense; i.e.

$2x+1 \neq 0$ ever

or, $x \neq -\frac{1}{2}$ Thus, we can choose domain to be $\underbrace{\mathbb{R} \setminus \{-\frac{1}{2}\}}$

This means real nos 'minus' the point $-\frac{1}{2}$.

• Polynomials, rational expressions (eg. $\frac{x^2+8}{2x-3}$)

- are always functions provided the domain has been specified carefully.

[eg] Find domain of $\sqrt{4+x}$

[Ans] (i) We can't have negative inside a square root. So, $x+4<0$ is not allowed

or, $x<-4$ " " "

Thus, $x \geq -4$ is allowed.

Hence, domain : $[-4, \infty)$

• Domain of a Polynomial function is $(-\infty, \infty)$ i.e. $\mathbb{R}$.

- Rational Functions are $h(x) = \frac{f(x)}{g(x)}$

where f & g are polynomials. The domain of this $h(x)$ is $\mathbb{R} \setminus \{\text{roots of } g(x)\}$ usually.

e.g. $\frac{x^2 - 7}{x+3}$. What is the domain?

$x = -3$ not allowed only ?!

So, $\mathbb{R} \setminus \{-3\}$.

- However, one needs to be careful.
  What is the domain of

$$\frac{x^2-9}{x-3} \,?$$

It is $\mathbb{R}$!

Why? $\frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)}$

$= x+3.$

Thus, it is a linear polynomial!

- Always be aware of such factorizations!

~~They are supposed to~~

So, when asked to find domains (even if not asked, (e.g. the question is say $f(x) = \sqrt{4+x}$ and then it asks you to show something related to the function)), always make sure you know what the domain is!

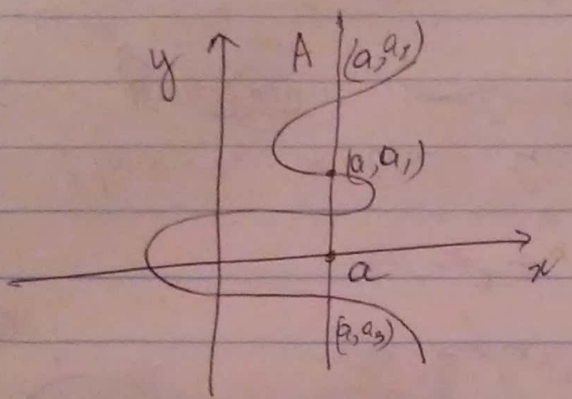
(Just check where something abnormal happens eg. $\sqrt{4+x}$ $x < -4$ absurd!).

Checking something is a function when you are given the graph of an equation (called "curve")

Vertical Line Test

Draw a vertical line. If it cuts the graph twice or more (A), then it's not a function.

eg



Is this the graph of some function?

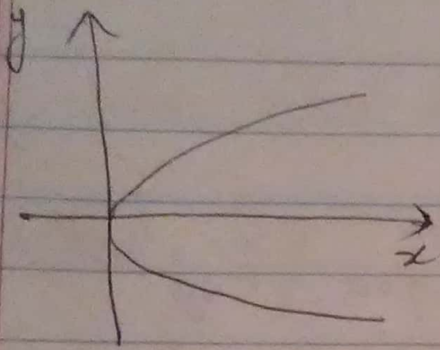
No!

The vertical line A (say) touches it more than once.

We say that it's a curve which is not a function.

$\boxed{\text{Basically}}$ The vertical line has the equation $x = a$ right?

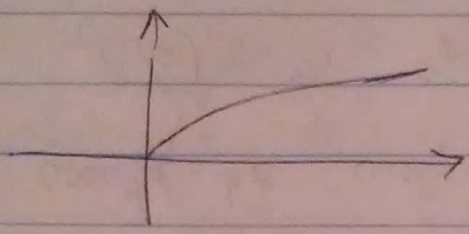
Now, had it been a function, then $f(a)$ would be unique. But, we seen $f(a) = a_1, a_2, a_3$ all different. So, not a function.



Is this a fn?

Ans. No.

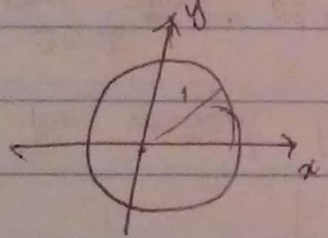
However,



This is a function.

In general, we draw graphs of equations.

e.g



This is the graph of the equation

$$x^2+y^2=1.$$

However, vertical line test tells us that ~~will not give me~~ y can't be given as a function of x.

The algebraic reason is as follows:

$$x^2+y^2=1$$

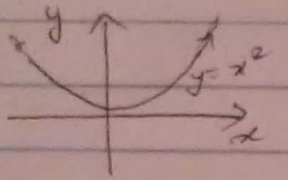
So, $y^2 = 1-x^2$

Thus, $y = \sqrt{1-x^2}$ or $y = -\sqrt{1-x^2}$

Thus, there are two choices if we want to define y as a function of x.

Hence, the confusion!

eg Is $y = x^2$ a function? Yes.

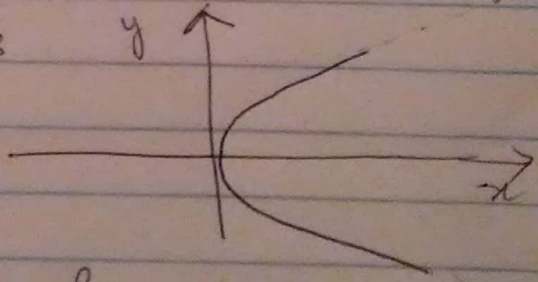


So, $y = x^2$ defines the function $f(x) = x^2$. as vert. line test shows.

~~However,~~

However, ~~is the~~

if we are given $y^2 = x$, can we express y as a function of x? No! The graph of the equation (*) $y^2 = x$ is



It fails the vert. line test.

(*) A graph of an equation is the collection of all ordered pairs that satisfies the equation.
(The picture here consists of all pts (a,b) such that $b^2 = a$).