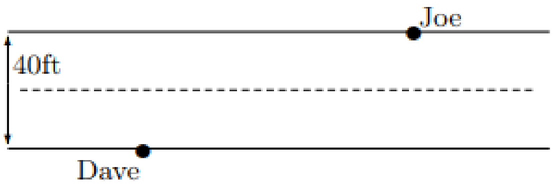
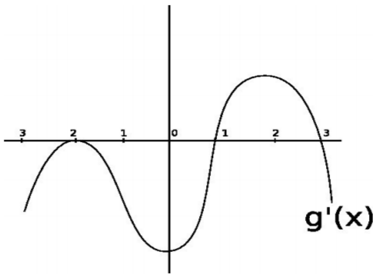
HW3/Practice for Quiz 3

1. Joe is walking east at 4 ft/s and Dave is walking west at 5 ft/sec along opposite sides of a 40 ft wide road, as in the diagram below. How fast is the distance between Joe and Dave increasing when this distance is 50 ft?



2. The figure shows the graph of the derivative g' of some function g. Find the intervals on which the original function g is increasing and decreasing, and classify all local extrema of g.



3. Suppose $f(x) = \dfrac{x+4}{(x-4)^3}$, $f'(x) = \dfrac{2(x+8)}{(x-4)^4}$, $f''(x) = \dfrac{6(x+12)}{(x-4)^5}$. State intervals of concavity. State any inflection points.

4. The equation $y^2(y^2-4) = x^2(x^2-5)$ is known as *devil's curve*. Find $\dfrac{dy}{dx}$.

5. Give an example of a function f such that $x = 1$ is a critical point of f but f has no relative max/min there.

6. A cylinder's height is changing at the rate of 1 inch per minute while its radius is decreasing at the rate of 1 inch per minute (decreasing means negative sign..careful) Find the rate of change in the volume of this cylinder at the instant when when its radius is 10 inches and its height is 8 inches. Is the volume increasing or decreasing? [Hint: Volume is $\pi r^2 h$. Now both of them are varying with time, so which rule to use to set up $\dfrac{dV}{dt}$... Hmmmm] [Note while writing solution of this problem, again describe your variables either in a diagram or in a sentence]

7. Suppose $f(x) = \dfrac{3}{5}x^5 - 4x^3$.

   (a) Find the intervals of increase/decrease of f.

   (b) Find the critical numbers of f.

   (c) Find and classify the relative extrema of f.

8. State T/F. Justify either with a proof or disproof or a result done in class, or a counterexample.

(a) If $f(x)$ has an inflection point at $x = c$, then $f''(c) = 0$.

(b) Assume that both f and g are differentiable functions. Suppose $f(x)$ is positive and increasing and $g(x)$ is negative and decreasing. Then $(fg)(x)$ (the product) is negative and decreasing.

(c) Suppose $f(x)$ is increasing and concave up and $g(x)$ is concave up. Then $(f \circ g)(x)$ is concave up. [Hint: Compute second derivative of $f \circ g$. Note that you are given that f' is always positive and f'' is positive, g'' is positive. Combine all this info into the second derivative you computed.]

(d) If a function $f(x)$ is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$, then $x = 0$ is a relative maximum for f. [Hint: Look at $f(x) = \dfrac{1}{x^2}$].