An Approach to Berger's Conjecture

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Berger's Conjecture (Around 1963)

Let k be a perfect field.

Let $R = \frac{k[[X_1 \ldots, X_n]]}{I}$ be a local k-algebra of dimension one which is a domain.

Then R is regular if and only if the module of differentials $\Omega_{R/k}$ is torsion-free.

An Invariant-Main Tool of Attack

Definition

Let R be as before. For any R-module M, let

$$\mathrm{h}(M) := \min\{\lambda(R/J) \mid M \to J \to 0, J \subset R\}.$$

Main Theorem

Theorem

Let R be as before. Assume that $I \subset (X_1, \ldots, X_n)^{s+1}$ for some $s \geq 1$. If

$$\mathrm{h}(\Omega_{R/k}) \leq \binom{n+s}{s} \left(\frac{s}{s+1}\right),$$

then Berger's Conjecture is true.

Relationship with the Conductor ideal

Theorem

Let R be as before. For any ideal J:

$$h(J) = \lambda\left(\frac{R}{J}\right) \iff \mathfrak{C}\omega \subset J\omega$$

($\mathfrak{C}-$conductor ideal, $\omega-$canonical module)

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In particular for R Gorenstein,

$$\mathrm{h}(\Omega_{R/k}) \leq \lambda(R/\mathfrak{C}).$$

—Thank You—