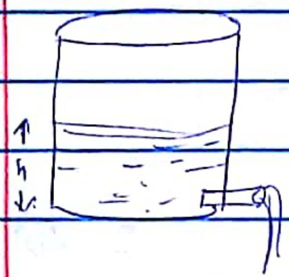
Recap: Find $\left.\frac{d^2y}{dx^2}\right|_{(5,1)}$ given $xy - y^3 = 4$.

- Water flows from a tank of constant cross-sectional area 50 sq ft through an orifice of constant cross-sectional area 1.4 ft^2 located at the bottom of the tank.



Initially the height of the water in the tank was 20 ft, and its height t sec later is given by the equation.

$$2\sqrt{h} + \frac{1}{25}t - 2\sqrt{20} = 0 \qquad (0 \leq t \leq 50\sqrt{20})$$

How fast was height of water decreasing, when its height was 8 ft?

Soln: Need to find $\frac{dh}{dt}$ @ $h = 8$

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-  Coffee pot shaped like a cylinder of radius 4 in is being filled with water flowing at a constant rate. If the water level is rising at the rate of 0.4 in/s, what is the rate at which water is flowing into the coffee pot?

Soln: Let volume at time t be V in^3.

$V = \pi r^2 h$. where $r = 4$, $\frac{dh}{dt} = 0.4$.

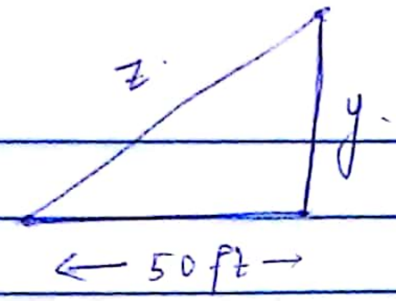
$= 16\pi h$.

$$\frac{dV}{dt} = 16\pi \frac{dh}{dt}.$$

$$= 16\pi(0.4) = 6.4\pi.$$

- At a distance of 50 ft from the pad, a man observes a helicopter taking off from a heliport. If the helicopter lifts off vertically and is rising at a speed of 44 ft/s when it is at an altitude of 120 ft, how fast is the distance between the helicopter and the man changing at that constant?

Sol^n:



$y^2+50^2=z^2$.

Need to figure out:

$\frac{dz}{dt}$ when $y=120$.

Also given $\frac{dy}{dt}=44$.

Since $y=120$, $z=\sqrt{120^2+50^2}=130$

So,

$y^2+50^2=z^2$

So, $\frac{d}{dt}(y^2+50^2)=\frac{d}{dt}(z^2)$

or, $\cancel{2}y\frac{dy}{dt}=\cancel{2}z\frac{dz}{dt}$

$\therefore$ So, $12\cancel{0}\times 44=13\cancel{0}\frac{dz}{dt}$

or, $\frac{dz}{dt}=\frac{12\times 44}{13}$.

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• Another example would be a ~~square~~ spherical example.

Fact) $V = \frac{4}{3}\pi r^2$. $\qquad \frac{dV}{dt} = \frac{4}{3}\pi 2r \frac{dr}{dt}$.

$A = 4\pi r^2$. $\qquad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$.

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