Eg: Find absolute extrema for $g(x) = x^3 + 3x^2 - 1$ over the interval $[-3, 1]$

Soln: $g'(x) = 3x^2 + 6x$; g' exists everywhere as g is a polynomial.

$g'(x) = 0$

$\Rightarrow 3x(x+2) = 0 \Rightarrow x = 0, -2$.

Thus, critical points on $(-3, 1)$: $x = 0, -2$.

- $f(0) = -1$;
- $f(-2) = (-2)^3 + 3(-2)^2 - 1 = -8 + 12 - 1 = 3$
- $f(-3) = -27 + 27 - 1 = -1$
- $f(1) = 1 + 3 - 1 = 3$.

Thus,
- $(0, -1)$, $(-3, -1)$ — absolute minimum
- $(-2, 3)$, $(1, 3)$ — absolute maximum.

Note: -1 is the absolute min. value whereas graph of g has an absolute min. at $(0, -1)$ and $(-3, -1)$.

3 is the absolute max. value whereas graph of g has an absolute max at $(-2, 3)$ and $(1, 3)$.