(Definition).

Let $f: [a,b] \to \mathbb{R}$ be a function.

If $\lim\limits_{n \to \infty} \left(\left[f(x_1) + \cdots + f(x_n) \right] \Delta_n \right)$ exists

     when — Upper Riemann Sum is considered
          — Lower Riemann Sum is considered.

and in both cases, the limit is the same,
then we call this limit the <u>definite integral</u>
of f from a to b, denoted by

$$\int_a^b f(x)\, dx \, . \quad \text{Thus,} \quad \int_a^b f(x)\, dx = \lim_{n \to \infty} \left[f(x_1)\Delta_n + f(x_2)\Delta_n + \cdots + f(x_n)\Delta_n \right]$$

where $x_1, \ldots, x_n$ can be (i) the left-end points OR

(ii) the right-end points of the subintervals.

Remark: Actually this works even for arbitrarily chosen points from each sub interval.

$\underline{a}$ — is called lower limit of integration

$\underline{b}$ — is called upper limit of integration.