Problem 1. Sketch $f(x) = x^3 - 3x^2 - 24x + 32$.

(1) The domain of $f(x)$ is: $\mathbb{R}$.

(2) The x-intercepts of $f(x)$ are: on the intervals $[-5,-4]$, $[1,2]$, $[6,7]$

The y-intercepts of $f(x)$ are: $(0, 32)$.

(3) Compute:

$\lim_{x\to+\infty} f(x) =$ ∞ DNE (∞).

$\lim_{x\to-\infty} f(x) =$ DNE $(-\infty)$.

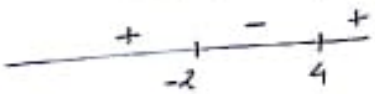
(4) Does $f(x)$ have any horizontal or vertical asymptotes? If so, list them: No. because f is a polynomial.

(5) Find the intervals on which $f(x)$ increases and decreases.

$f(x)$ is increasing on: $(-\infty, -2)$, $(4, \infty)$

$f(x)$ is decreasing on: $(-2, 4)$

$f'(x) = 3x^2 - 6x - 24 = 3(x^2 - 2x - 8) = 3(x-4)(x+2)$



(6) $f(x)$ has the following critical points: $4, -2$

$f(x)$ has the following relative maxima: $(-2, 60)$

$f(x)$ has the following relative minima: $(4, -48)$

$f(-2) = 60$

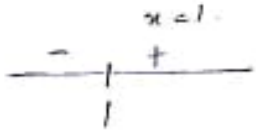
$f(4) = -48$

(7) Find the intervals on which $f(x)$ is concave up and concave down.

$f(x)$ is concave up on: $(1, \infty)$

$f(x)$ is concave down on: $(-\infty, 1)$

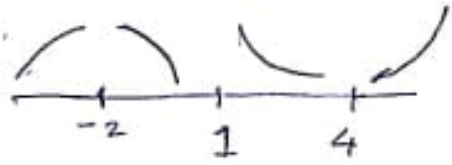
$f''(x) = 6x - 6$.



(8) For the following x, $f''(x) = 0$ or DNE:

$f(x)$ has the following inflection points (list both x and y values):

$(1, 6)$



(9) Now plot the function!