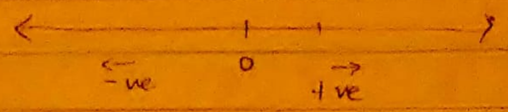
~~2.1~~

$\boxed{\text{Review}}$

- Real Line ~~[illegible]~~
  $(\mathbb{R})$



- Interval Notations: $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$
  ↳ 'belongs to'

$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$(a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

- $\boxed{\text{Exponents Rules}}$

  - $(a^x)^y = a^{xy}$
  - negative power ~~means~~ $\boxed{\text{e.g}}$ $2^{-2} = \dfrac{1}{2^2}$

$$\left(\frac{1}{2}\right)^{-3} = \left(2^{-1}\right)^{-3} = 2^{(-1)(-3)} = 2^3 = 8$$

$$\frac{1}{x} = x^{-1} \quad \text{provided } x \neq 0.$$

- $\boxed{x^0 = 1}$ → definition, provided $x \neq 0$

- $b^n = \underbrace{b\, b\, b \cdots b}_{\leftarrow n \text{ times} \rightarrow}$ where $b \in \mathbb{R}$
  n is a ~~[illegible]~~ positive integer

e.g $(-2)^8 = \underbrace{(-2)(-2)(-2) \cdots (-2)}_{\leftarrow 8 \text{ times} \rightarrow}$

$\boxed{Q}$ $(-\pi)^0 = ?$ 'pi'

- 'n-th' root $b^{\frac{1}{n}} = \sqrt[n]{b}$
  This is the notation.

$\boxed{\text{Important}}$ If n is even number, and b is negative, then $b^{1/n}$ is not what we will care about. It opens up a new area of Mathematics.

So, $(-2)^{1/2} = \sqrt[2]{-2}$ (square root of -2)

not defined for this class.

$\boxed{Q}$ Is $(-4)^{1/16}$ defined for this class?

Ans. $\boxed{\text{No.}}$ Remember we said 'n' is even and 'b' is negative

$\sqrt[16]{-4}$ not defined for us

• [Simplify]

$$9^{\frac{3}{2}}$$

[Ans] $9^{\frac{3}{2}} = (9)^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 27$

[Q] Try $\left(4^{\frac{1}{2}}\right)^4$.

[Q] $\left(-\frac{1}{9}\right)^{-3/2} = ?$

[Ans] $= \left(-1 \cdot \frac{1}{9}\right)^{-3/2} = (-1)^{-3/2} \left(\frac{1}{9}\right)^{-3/2}$

[Using $(ab)^x = a^x b^x$]

• $(ab)^x = a^x b^x$

[e.g] $(-12)^2 = (-1)^2 (12)^2$

• [Negative Powers] $2^{-n} = \frac{1}{2^n}$

$$2^{-3/4} = \left(\frac{1}{2}\right)^{3/4}$$

$$\parallel$$

$$\frac{1}{2^{3/4}} \quad //$$

• $a^x . a^y$

$= a^{x+y}$

[e.g] $4^{3/2} = 4^{1+1/2}$

$= 4 . 4^{1/2}$

$= 4 . \sqrt{4}$

$= 4\ 2 = 8.$

$\boxed{Q}$ Simplify $\left[\left(-\frac{1}{3}\right)^{-3}\right]^2$

$\boxed{Ans}$ $\left[(-1)^{-3}\cdot\left(\frac{1}{3}\right)^{-3}\right]^2$

$= \left[-1 \cdot (3^{-1})^{-3}\right]^2 = \left[-3^{-1\cdot -3}\right]^2$

$= \left(-3^3\right)^2$

$= (-27)^2 = (-27)(-27)$

$= 27\times 27 =$

$\boxed{\text{Alternately}}$

Note that $\left(\frac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = \frac{1}{1}\times 2 = 2$

- $\left(\frac{1}{x}\right)^{-2} = \left(\frac{1}{\frac{1}{x}}\right)^2 = (x)^2 = x^2$

- Basically, we are using $\frac{1}{x} = x^{-1}$

That is why $\left(\frac{1}{x}\right)^{-2} = \left(x^{-1}\right)^{-2}$

$= x^{(-1)(-2)}$

$= x^2$

$\boxed{Q}$ $4^{-5/2}$

$\boxed{Ans}$ $4^{-5/2} = (4)^{-5/2} = \left(2^2\right)^{-5/2}$

$= 2^{2\times(-5/2)}$

$= 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

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$\boxed{\text{Notation}}$ $\sqrt[2]{3}$ → is ~~written~~ read as square root of 3

We usually write $\sqrt{3}$ (omitting the 2 at the top)

So, $\sqrt{81x^{-5}}$

$$= \left(81x^{-5}\right)^{\frac{1}{2}}$$

$$= (81)^{\frac{1}{2}} \left(x^{-5}\right)^{\frac{1}{2}}$$

$$= \left(9^2\right)^{\frac{1}{2}} \left(x^{-5}\right)^{\frac{1}{2}}$$

$$= 9\,x^{-5/2} = \frac{9}{x^{5/2}} = \frac{9}{\sqrt{x^5}}$$

- $(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b)$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2$$

Thus, $(a+b)^2 = a^2 + 2ab + b^2$

- $(a-b)^2 = a^2 - 2ab + b^2$

- $a^2 - b^2 = (a+b)(a-b)$ → Start from this side

$$a(a-b) + b(a-b) = a^2 - \cancel{ab} + \cancel{ba} - b^2$$

$$= a^2 - b^2$$

There are other formulae in the book.
Don't blindly memorize! As you solve more and more problems, you will automatically remember them.

- Simplifications (i) $\frac{1}{7}x + \frac{3}{5}x$

$$= \frac{5}{35}x + \frac{21}{35}x$$

$$= \frac{5x + 21x}{35} = \frac{26x}{35}$$

(ii) $27x^{-1} + 48x^{-2}$

$$= \frac{27}{x} + \frac{48}{x^2}$$

$$= \frac{27x}{x^2} + \frac{48}{x^2} = \frac{27x + 48}{x^2}$$

(iii) $\left(\frac{1}{x}\right)^{-1} + 2x = (x^{-1})^{-1} + 2x = x + 2x$

$$= 3x.$$

Inequalities

Find all x such that $5+x \leq 2$.

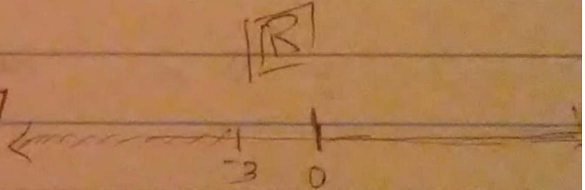
Ans: $5+x \leq 2$

or, $5+x-5 \leq 2-5$

or, $x \leq -3$.

So, $x \in (-\infty, 3]$

infinity



- Find all x such that

$$-x+6 \leqslant 2x+18.$$

Ans

$$-x+6 \leqslant 2x+18$$

or, $-x-2x+6 \leqslant 2x+18-2x.$

or, $-3x+6 \leqslant 18$

or, $-3x \leqslant 18-6$

or, $-3x \leqslant 12$

or, $\frac{1}{3}(-3x) \leqslant \frac{1}{3}\cdot(12)$

or, $-x \leqslant 4.$

or, $x \geqslant -4.$ (~~Remember~~ Note $x \geqslant a$ Then $-x \leqslant -a$ i.e. Multiplying by -1 on both sides reverses inequality).

So, $[-4, \infty)$ works.

$\boxed{Q}$ Find maximum x in $x + 99 \leqslant -5x + 105$

$\boxed{Ans}$ $x + 99 \leqslant -5x + 105$

or, $x \leqslant -5x + 105 - 99$

or, $x + 5x \leqslant 6$

or, $6x \leqslant 6$

or, $x \leqslant 1$. So, max. x is 1.

$\boxed{Q.}$ Find x s.t $x + 2 > 5$ or $x - 2 < -5$

$\boxed{Ans}$ $x + 2 > 5$

or, $x > 3$

So, $x \in (3, \infty)$

$x - 2 < -5$

or, $x < -5 + 2$

or, $x < -3$

Thus, $x \in (-\infty, -3)$

Hence, our reqd answer is $(-\infty, -3) \cup (3, \infty)$

↳ union of the two intervals

i.e any $x \in (-\infty, -3) \cup (3, \infty)$, will satisfy the hypothesis of the problem.

Q. Find x such that $x+8>5$ and $x-8<-5$.

Ans.

$x+8>5$

or, $x>5-8$

or, $x>-3$

So, $x\in(-3,\infty)$

$x-8<-5$

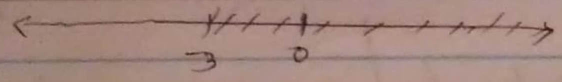
or, $x<-5+8$

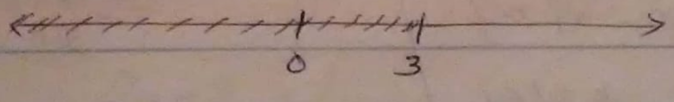
or, $x<3$.

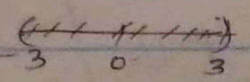
Thus, $x\in(-\infty,3)$.

Now, we need both conditions in the problem to hold true.

So, we need to look at the 'intersection' of the 2 intervals we found.

$(-3,\infty)$ 

$(-\infty,3)$ 

Thus, intersection is 

i.e. $3>x>-3$ So, $x\in(-3,3)$ is our answer

• Quadratic

$ax^2 + bx + c = 0.$

This is a quadratic polynomial. The roots are given using the formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Tip: Whenever possible, factorize

e.g. $x^2 - 5x + 6$ Find roots

You can apply the above formula.

Alternately, one can factorize:

$$x^2 - 2x - 3x + 6$$

$$= x(x-2) - 3(x-2)$$

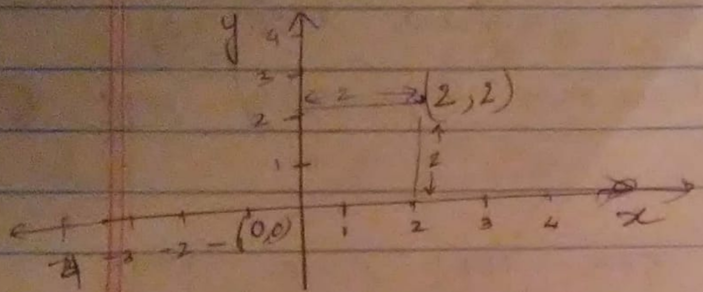
$$= (x-2)(x-3).$$

Thus, $x = 2, 3$ are roots of $x^2 - 5x + 6$.

Here, factorization was easy; choose method according to your comfort.

Cartesian

• Co-ordinate System



It's basically the plane (look at floor for example)

We call this, the x-y plane.

Any point here is an 'ordered pair' (x, y).

Geometrically, vertical distance from x-axis is the 2nd co-ordinate

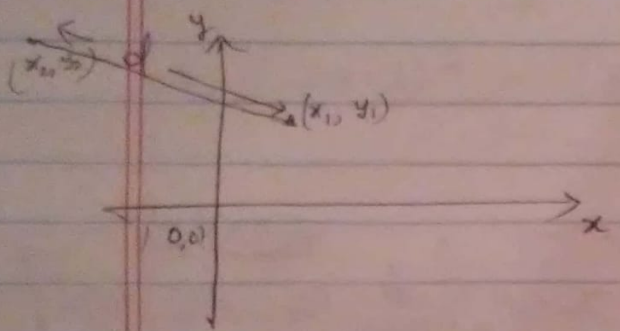
and horiz. dist from y-axis is the 1st co-ordinate.

x- is called abscissa
y - is called ordinate
(0,0) is called origin

- Suppose (x_1, y_1) & (x_2, y_2) is given.

Distance between them is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(formula)}$$



$\boxed{\text{eg}}$ Find distance between $(0,0)$ and $(2,4)$.

Ans $d = \sqrt{(2-0)^2 + (4-0)^2}$

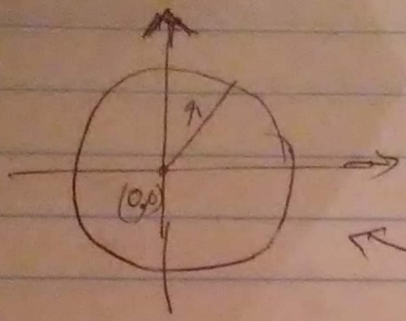
$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= \sqrt{5 \cdot 2^2}$

$= 2\sqrt{5}$

Circle



A figure where all points are at a constant distance from a fixed point. This figure is a circle with 'centre' at (0,0).

The fixed point is called 'center' of the circle.

The fixed distance is called 'radius' of the circle.

Equation

A circle with center (a,b) and radius r is given by the equation:

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$

[Note that this looks like just the square of the distance between (x,y) & (a,b)]

$\boxed{\text{e.g}}$ Find equation of circle with radius 4 & center $(2, -8)$

$\boxed{\text{Ans}}$ $(x-2)^2 + (y-(-8))^2 = 4^2$

or, $(x-2)^2 + (y+8)^2 = 16$

Ans

$\boxed{\text{Q.}}$ Does the point $(3, 4)$ lie on the above circle?

$\boxed{\text{Ans}}$ Just plug in $x = 3$, $y = 4$ and check whether you get 16.

$(3-2)^2 + (4+8)^2 \neq 16.$

Thus, $(3, 4)$ is not on the circle.

$\boxed{\text{Q}}$ Find all x-intercepts of the circle.

$\boxed{\text{Ans}}$ x-intercept means where it touches the x-axis. (Similarly y-intercept means ...)

So, when it touches the x-axis, $y = 0$. Plug $y = 0$

$(x-2)^2 + 8^2 = 16$

or, $(x-2)^2 = 16 - 64 = -48$. not possible as $a^2 \geq 0$

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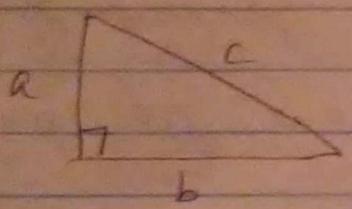
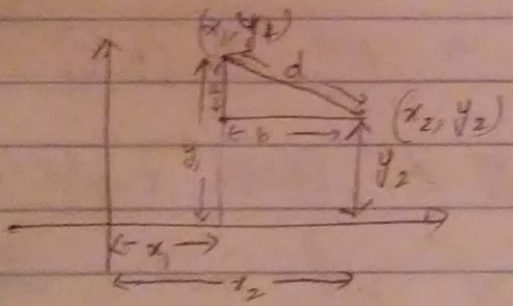
So, never crosses x-axis.

Fact The distance formula is basically a consequence of Pythagoras Theorem.



$$c^2 = a^2 + b^2$$

Why?



Thus,

$a = y_1 - y_2$

$b = x_2 - x_1$

Hence $d^2 = a^2 + b^2$

So, $d = \sqrt{a^2 + b^2}$

$= \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$

Fact Three non-collinear points determine a unique circle