Problem 1. Suppose $f(x)$ is positive and *increasing* on the interval $[a, b]$. Which of the following statements is true about Riemann approximations of $\int_a^b f(x)\,dx$?

(a) The left-endpoint approximation L_4 is an over-approximation of the integral.

(b) The left-endpoint approximation L_4 is an under-approximation of the integral.

(c) The right-endpoint approximation R_4 is an over-approximation of the integral.

(d) Both (b) and (c).

(e) None of the above – we do not have enough information.

Problem 2. What is the value of the following definite integral?:

$$\int_1^4 x\sqrt{x^2-1}\,dx.$$

(a) $\dfrac{64}{3}$ (b) $\dfrac{4}{3}\cdot 15^{3/2}$ (c) $\dfrac{1}{3}\cdot 15^{3/2}$ (d) $\dfrac{128}{3}$ (e) We cannot solve this integral.

Problem 3. Is the following u/du substitution TRUE or FALSE:

$$\int_1^4 x\sqrt{x^2-1}\,dx = \int_1^4 u^{1/2}\,\frac{du}{2}.$$

Problem 4. TRUE or FALSE: The fundamental theorem of calculus says that for any continuous function $f(x)$ on $[a, b]$,

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$ on the interval.

Problem 5. Which of the following rules for antidifferentiation is *invalid*?

(a) For any $n \neq -1$, $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$.

(b) $\displaystyle\int \ln x\,dx = \frac{1}{x} + c$.

(c) $\displaystyle\int e^x\,dx = e^x + c$.

(d) $\displaystyle\int 0\,dx = c$.

(e) All of the above are valid.

Problem 6. Let $I = \displaystyle\int_a^b f(x)\,dx$ be the definite integral of some function $f(x)$ over an interval $[a, b]$. Which of these characterizations of the definite integral is correct?

(a) I is equal to the area between the graph of $f(x)$ and the x-axis on the interval $[a, b]$.

(b) I is equal to $F(b) - F(a)$, where $F(x)$ is any antiderivative of $f(x)$ on the interval.

(c) I is equal to the net change of any antiderivative of $f(x)$ on the interval.

(d) All of the above are correct.

(e) None of the above are correct.

Problem 7. (Spring 16, Problem 15) If the function f has continuous derivative on the interval $[0, c]$, where c is a positive constant, then $\int_0^c f'(x)\,dx =$

(a) $|f(c) - f(0)|$

(b) $f(c) - f(0)$

(c) $f(c)$

(d) $f(x) + c$

(e) $f''(c) - f''(0)$

Problem 8. (Spring 16, Problem 16) Suppose f is a function for which $\int_0^{50} 3f(x)\,dx = 3$ and $\int_2^{50} f(x)\,dx = -4$. What is $\int_0^2 f(x)\,dx$?

(a) -1 (b) -3 (c) There is not enough information (d) 7 (e) 5

Problem 9. TRUE or FALSE:

$$\int x \ln x\,dx = \frac{x^2}{2} \cdot \frac{1}{x} + c.$$

Problem 10. TRUE or FALSE: For any functions $f(x)$ and $g(x)$,

$$\int f(x)g(x)\,dx = \left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right).$$

Problem 11. Let $f(x)$ be continuous on the interval $[a, b]$. Which of the following statements is correct?

(a) The net change of $f(x)$ on $[a, b]$ is $\displaystyle\int_a^b f(x)\,dx$.

(b) The total change of $f(x)$ on $[a, b]$ is $\displaystyle\int_a^b |f(x)|\,dx$.

(c) The net change of $f(x)$ on $[a, b]$ is $\displaystyle\int_a^b f'(x)\,dx$.

(d) The total change of $f(x)$ on $[a, b]$ is $|f'(b) - f'(a)|$.

(e) None of the above.

Problem 12. The average value of the function $f(x) = 3$ over the interval $[0, 10]$ is

(a) 30 (b) $\dfrac{10}{3}$ (c) $\dfrac{1}{10}\displaystyle\int_0^{10} f'(x)\,dx$ (d) 3 (e) This function has no average value.

Problem 13. Let $f(x) = \sqrt{x^4 + 1}$. Then the definite integral $\int_1^{10} f(x)\,dx$ is

(a) positive

(b) negative

(c) zero

(d) sometimes positive and sometimes negative

(e) undefined

Problem 14. A sprinter practices by running back and forth in a straight line. Her velocity after t seconds is given by $v(t)$. What does $\int_0^{60} v(t)\,dt$ represent?

(a) The total distance the sprinter ran in 1 minute.

(b) The sprinter's average velocity over 1 minute.

(c) The sprinter's displacement after 1 minute.

(d) The change in the sprinter's velocity over 1 minute.

(e) Both (b) and (d).

Problem 15. What does $\int_0^{60} |v(t)|\,dt$ represent?

(a) The total distance the sprinter ran in 1 minute.

(b) The sprinter's average velocity over 1 minute.

(c) The sprinter's displacement after 1 minute.

(d) The change in the sprinter's velocity over 1 minute.

(e) Both (b) and (d).

Problem 16. The definition of the indefinite integral of $f(x)$ is

(a) $\int f(x)\,dx = F(x)$ where $f'(x) = F(x)$.

(b) $\int f(x)\,dx = F(x)$ where $F'(x) = f(x)$.

(c) $\int_a^b f(x)\,dx$ where $f(x)$ is continuous on $[a, b]$.

(d) $\int f(x)\,dx = F(x) + c$ where $f'(x) = F(x)$.

(e) $\int f(x)\,dx = F(x) + c$ where $F'(x) = f(x)$.

Problem 17. TRUE or FALSE: The definition of the definite integral of $f(x)$ over the interval $[a, b]$ is

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where $F(x)$ is any antiderivative of $f(x)$.

Problem 18. Let $f(x)$ be a continuous function. An antiderivative is

(a) $F(x) + c$

(b) $f'(x) + c$

(c) any function $F(x)$ such that $F'(x) = f(x)$

(d) always of the form $F(x) + c$ where $F(x)$ is a known antiderivative

(e) Both (c) and (d) are correct.

Problem 19. Which of the following statements about the function $f(x) = e^x$ is correct?

(a) The domain of $f(x)$ is $(0, \infty)$.

(b) The range of $f(x)$ is $(-\infty, \infty)$.

(c) The derivative of $f(x)$ is concave up on $(-\infty, \infty)$.

(d) $f(x)$ has a horizontal asymptote.

(e) Both (c) and (d) are correct.

Problem 20. TRUE or FALSE: For a continuous function $f(x)$ on $[a, b]$, if $f(c)$ is the absolute maximum of $f(x)$ on the interval $[a, b]$, then $f'(c) = 0$.

Problem 21. TRUE or FALSE: If $f'(c) = 0$ and c is in the domain of $f(x)$, then there must be a relative maximum or minimum at $x = c$.

Problem 22. TRUE or FALSE: For any real number x, $e^{\ln x} = x$.

Problem 23. Solve for x in the following expression: $2\ln(x + 1) = \ln(2) + \ln(x + 1)$.

(a) 2 (b) 1 (c) -1 (d) 0 (e) Both (b) and (c)

Problem 24. Let $f(x)$ be a function. Which of the following is correct?

(a) A critical point of $f(x)$ is a point c in the domain of f such that $f'(c) = 0$ or DNE.

(b) An inflection point of $f(x)$ is a point c in the domain of f such that $f''(c) = 0$ or DNE.

(c) An absolute minimum of $f(x)$ is a critical point c such that f is concave up at c.

(d) A critical point of $f(x)$ is a point c in the domain of f such that $f''(c) = 0$ or DNE.

(e) Both (a) and (b).

Problem 25. Fill in the blank: A function $f(x)$ has an absolute maximum and an absolute minimum value on a closed interval $[a, b]$, provided $f(x)$ is __________ on $[a, b]$.

(a) continuous (b) differentiable (c) continuously differentiable

(d) differentiably continuous (e) a polynomial

Problem 26. Ten years ago, your uncle invested \$20,000 into a savings account with continuously compounding interest. The account now contains \$22,000. What is the interest rate on the account?

(a) $\ln\left(\frac{11}{10}\right)$ (b) $\ln\left(\frac{10}{11}\right)$ (c) $\frac{1}{10}\ln\left(\frac{11}{10}\right)$ (d) $\frac{1}{11}\ln\left(\frac{10}{11}\right)$ (e) $10\ln\left(\frac{10}{11}\right)$

Problem 27. How many years from now will it take for your uncle to have \$40,000 in his account?

(a) 10 (b) $\dfrac{10\ln(2)}{\ln(11/10)}$ (c) $\ln(2)$ (d) $\dfrac{10\ln(2)}{\ln(11/10)} - 10$

(e) He will never have \$40,000 in his account.

Problem 28. Let $q(x) = \ln\sqrt{x}$. Then $q'(1) =$

(a) $\dfrac{1}{2}$ (b) 1 (c) -1 (d) e (e) $\ln\left(\dfrac{1}{2}\right)$

Problem 29. Let $h(x) = x^x$. Then

(a) $h'(1) = 1$

(b) $h'(e) = 0$

(c) $h'(2) = \ln(16) - 4$

(d) All three are correct.

(e) Only (b) and (c) are correct.

Problem 30. The maximum number of horizontal asymptotes a function can have is

(a) 1 (b) 2 (c) 3 (d) no limit

(e) There is no such thing as a horizontal asymptote.

Problem 31. The absolute maximum of $f(x) = x$ on the interval $[0, 1]$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) DNE (e) equal to its average on $[0, 1]$

Problem 32. The absolute maximum of $f(x) = x$ on the interval $[0, 1)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) DNE (e) equal to its average on $[0, 1]$

Problem 33. (Spring 16, Problem 17) Which of the following functions has a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$?

(a) $a(x) = \dfrac{2x^2 + 1}{x^2 - 1}$

(b) $b(x) = \ln(2x + 2)$

(c) $c(x) = e^{x-1} + 2$

(d) $d(x) = \dfrac{2\sqrt{x + 1}}{x + 2}$

(e) $e(x) = \dfrac{2}{e^{x+1} - 1}$

Problem 34. (Spring 13, Problem 2) Let $f(x)$ be a function such that $f'(4) = 7$, and let $g(x) = f(x^2)$. Then $g'(2) =$

(a) 7 (b) 14 (c) 21 (d) 28 (e) DNE

Problem 35. A function $f(x)$ is differentiable at $x = a$ if

(a) $\dfrac{f(x) - f(a)}{x - a}$ exists.

(b) $\dfrac{f(x) - f(a)}{x - a} = f(a)$.

(c) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists.

(d) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a} = f(a)$.

(e) $f(x)$ is continuous at $x = a$.

Problem 36. Consider the following statement: the function $f(x) = x^3 + x^2 + x + 2$ has a root on $[-2, 0]$.

(a) The statement is false: $f(x) > 0$ on $[-2, 0]$.

(b) The statement is true: $x = -1$ is a root.

(c) The statement is true: every cubic polynomial has a root.

(d) The statement is false: $f(x)$ is only guaranteed to have an absolute maximum and minimum on a closed interval.

(e) The statement is true: $f(x)$ is continuous, $f(-2) < 0$ and $f(0) > 0$ so the intermediate value theorem applies.

Problem 37. (Spring 13, Problem 8) Find and classify all relative extrema of $f(t) = \frac{8}{t} + \frac{t^2}{2}$.

(a) $f(t)$ has a relative maximum at $t = 2$.

(b) $f(t)$ has a relative minimum at $t = 2$.

(c) $f(t)$ has a relative minimum at $t = 2$ and a relative maximum at $t = -2$.

(d) $f(t)$ has a relative maximum at $t = 2$ and a relative minimum at $t = -2$.

(e) $f(t)$ has a critical point at $t = 2$ but not a relative extremum.

Problem 38. (Spring 13, Problem 10c) $\lim_{x \to 1} \frac{3x^4}{x^5 + 3x} =$

(a) 3 (b) 0 (c) $\frac{3}{4}$ (d) 1 (e) DNE