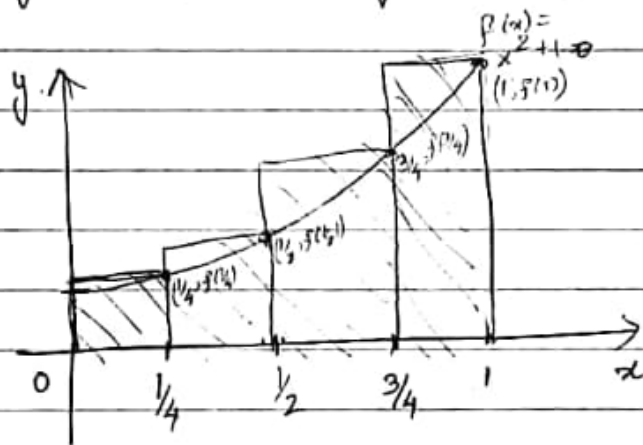


Example: Use a Riemann Sum with four subintervals ($n=4$) to approximate the area under the curve $f(x) = x^2 + 1$ over the interval $[0, 1]$.

Choose the representative points to be the right-end points of the subintervals.

Soln:



$$n = 4, b = 1, a = 0$$

$$\Delta = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$

$$x_1 = 0 + \frac{1}{4}$$

$$x_2 = 0 + 2 \cdot \frac{1}{4} = \frac{1}{2}$$

$$x_3 = 0 + 3 \cdot \frac{1}{4} = \frac{3}{4}$$

The approximate area is

$$\frac{1}{4} \cdot f\left(\frac{1}{4}\right) + f\left(\frac{1}{2}\right) \cdot \frac{1}{4} + f\left(\frac{3}{4}\right) \cdot \frac{1}{4} + f(1) \cdot \frac{1}{4}$$

$$= \frac{1}{4} \left(\frac{1}{16} + 1 \right) + \frac{1}{4} \left(\frac{1}{4} + 1 \right) + \frac{1}{4} \left(\frac{9}{16} + 1 \right) + \frac{1}{4} (1 + 1) \approx 1.4$$