$\boxed{\text{e.g.}}$ $f(x) = x^3 - 12x^2 + 36x$. Find relative max/min.

$\boxed{\text{Exercise}}$

$\boxed{4.2}$ Applications of 2nd derivative.

Note that provided f' is friendly, we can talk about f''. Hence we can apply same treatment to f' now as we did to f to figure out where f' is increasing/decreasing etc. etc.

Definition: given a function $f(x)$, we say the graph of f is concave up (respectively, concave down) on the interval (a, b) if $f'(x)$ is increasing i.e. $f''(x) > 0$ on (a, b) (resp. $f'(x)$ is decreasing, i.e. $f''(x) < 0$ on (a, b)).

$f''(x)$	$f'(x)$	Graph of f
+	increasing	concave up
−	decreasing	concave down

Definition

An inflection point $(a, f(a))$ of f is a point in the ~~domain~~ graph of f where concavity changes.

What happens ~~when~~ at an inflection point?

Theorem

If $y = f(x)$ is continuous on (a, b) and has an inflection point at $x = c \in (a, b)$, then either $f''(c) = 0$ or $f''(c)$ DNE. ~~(i.e. it is sort of a point of relative max)~~

Note $f''(c) = 0$ does not imply that c is an inflection point. We have to check concavity change.

So, here's an algorithm:

Finding Inflection Points.

① Compute $f''(x)$.

② Solve $f''(x)=0$ and also find x such that $f''(x)$ DNE.

③ ~~Determine sign of $f''(x)$~~ Draw Sign Chart for $f''(x)$:

if there is a change in sign as we move across a particular point c, then $(c, f(c))$ is an inflection point.

e.g. Find inflection point for $g(x) = \frac{1}{x-3}$

$g'(x) = -\frac{1}{(x-3)^2}$; $g''(x) = -\frac{2}{(x-3)^3}$

$g''(x) \neq 0$ for all x in the domain of g.

g'' is not defined at $x=3$. but $x=3$ is not in the domain of g. So, 3 can't be included in our discussion.

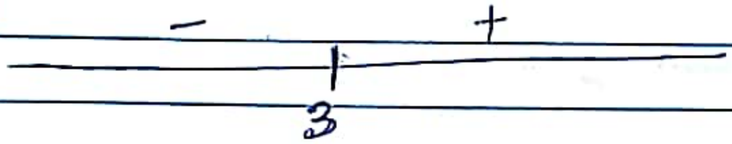
So, g has no inflection point.

e.g. Find inflection points for $f(x) = x^3 - 9x^2 + 24x - 10$.

Sln: • $f'(x) = 3x^2 - 18x + 24$

• $f''(x) = 6x - 18$.

So, $f''(x) = 0$ when $x = 3$. ; f'' exists everywhere.
(Thus 3 is the only candidate for checking)



$f''(2) = 12 - 18 < 0$

$f''(4) > 0$.

Thus, $(3, f(3))$ is an I.P. (inflection point).

Second-Derivative Test for Local Max/Min.

1. Compute $f'(x)$, $f''(x)$.
2. Compute all critical numbers of f at which $f'(x) = 0$.
3. Compute $f''(c)$ for such critical number c.
   - (a) If $f''(c) < 0$, then f has a relative maximum at c.
   - (b) If $f''(c) > 0$, then f has a relative minimum at c.
   - (c) If $f''(c) = 0$ or $f''(c)$ DNE, then Test is inconclusive.

(e.g.) Find relative max/min of the function:

$$f(x) = \frac{9}{x} + x.$$

Soln: $f'(x) = -\frac{9}{x^2} + 1.$ So, $f'(x) = 0$ at $x = 3, -3$.

$f''(x) = \frac{18}{x^3}$; $f''(3) > 0$ so, at $x = 3$ f has a local min with value $f(3) = 6$.

$f''(-3) < 0$. So, at $x = -3$ f has a local max. with value -6.

First-derivative Test is stronger than 2nd derivative Test.

e.g. $f(x) = x^{2/3}$. 2nd-derivative test can't be applied!!