

Let  $f(x)$  be a function. We want to graph it! Here's how to do this in detail:

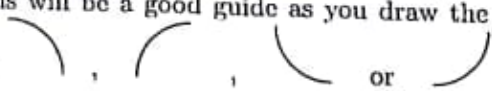
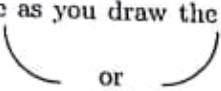
- (1) Describe the **domain** of  $f(x)$ .
- (2) Find the  **$x$ - and  $y$ -intercepts** of  $f(x)$ . Recall that a number  $x$  in the domain of  $f(x)$  is an  $x$ -intercept if  $f(x) = 0$ . A number  $y$  in the range of  $f(x)$  is a  $y$ -intercept if  $f(0) = y$ .
- (3) Determine the **end behavior** of  $f(x)$ . That is, compute the limits

$$\lim_{x \rightarrow +\infty} f(x) \quad \text{and} \quad \lim_{x \rightarrow -\infty} f(x),$$

or state that they do not exist. If they don't exist, you may want to note whether the  $y$ -values approach  $+\infty$  or  $-\infty$ .

- (4) Find the **horizontal and vertical asymptotes** of  $f(x)$ . A function  $f(x)$  has a horizontal asymptote  $L$  if  $\lim_{x \rightarrow +\infty} f(x) = L$  or  $\lim_{x \rightarrow -\infty} f(x) = L$ . A function  $f(x)$  has a vertical asymptote at a number  $a$  if  $\lim_{x \rightarrow a^+} f(x) = \pm\infty$  or  $\lim_{x \rightarrow a^-} f(x) = \pm\infty$ .
- (5) Determine the intervals on which  $f(x)$  is **increasing** and **decreasing**.
- (6) Find and classify the **relative extrema** of  $f(x)$  using the critical point method.
- (7) Determine the intervals on which  $f(x)$  is **concave up** and **concave down**.
- (8) Find the **inflection points** of  $f(x)$  using the inflection point method.
- (9) Plot all intercepts, critical points, inflection points and any other "interesting points" found in the previous steps. Then use the information about asymptotes, increasing/decreasing behavior and concavity to **sketch the graph** of  $f(x)$ .

Some notes:

- According to Tan, a rational function  $f(x) = \frac{p(x)}{q(x)}$ , where  $p(x)$  and  $q(x)$  are polynomials, has a vertical asymptote at  $x = a$  if  $p(a) \neq 0$  and  $q(a) = 0$ . This should agree with your findings when solving the limits  $\lim_{x \rightarrow a^+} \frac{p(x)}{q(x)}$  and  $\lim_{x \rightarrow a^-} \frac{p(x)}{q(x)}$  using factor-cancel. You can directly use this to state the vertical asymptotes. However, it is crucial to identify whether function is approaching  $\infty$  or  $-\infty$  as our sketch will depend on that.
- Polynomials never have vertical or horizontal asymptotes.
- A rational function can have at most one horizontal asymptote.
- Right before Step 9, I find it useful to create a new number line in which I record all critical points, inflection points and "interesting points" and, on each interval between these, record the increasing/decreasing behavior AND concavity simultaneously. This will be a good guide as you draw the final sketch in Step 9. You may even want to sketch a little  or  on each interval.

Okay, let's do it!