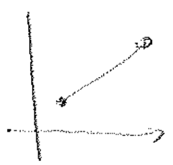
1.4 Straight Line

1.4.1 Slope

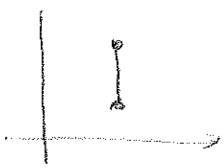
- The slope of a straight line measures its steepness / direction.
- Definition of the slope of a straight line:

(1) non-vert. line:

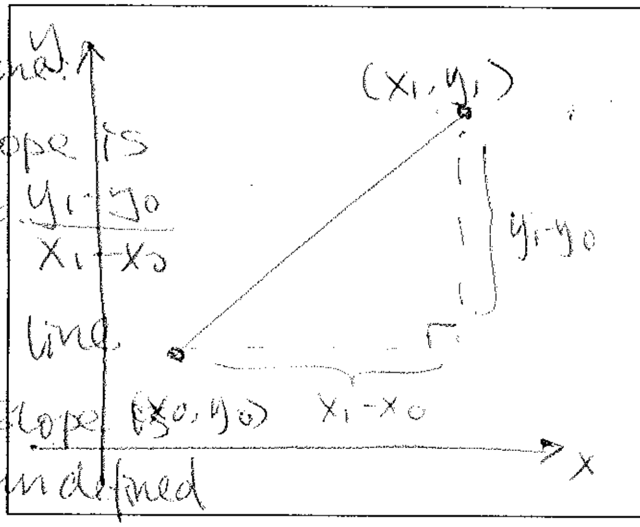


Slope is $m = \dfrac{y_1 - y_0}{x_1 - x_0}$

(2) vertical line.



Slope undefined

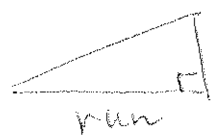


(1) Slope of a non-vertical line joining (x_0, y_0) and (x_1, y_1) is $m = \dfrac{y_1 - y_0}{x_1 - x_0}$.

(2) Slope of a vertical line is undefined.

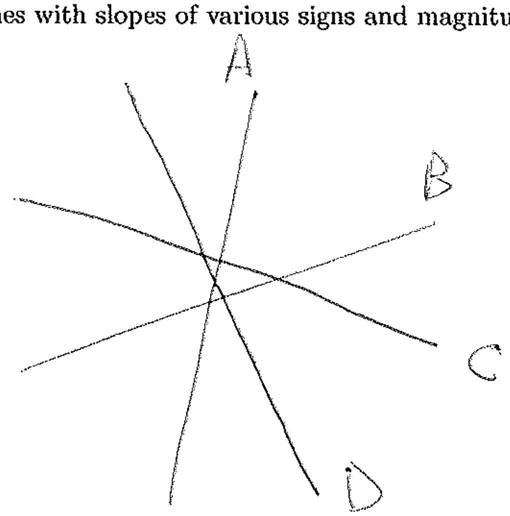
- Various interpretations of slope:

  - Slope $= \dfrac{\text{rise}}{\text{run}}$



  - Slope = The change of y-coordinate as x-coordinate increases by 1.

- Lines with slopes of various signs and magnitude:



Slopes of:

A: large positive

B: small positive

C: small negative

D: large negative

- Parallel lines and perpendicular lines:

If line L_1 has slope m_1, line L_2 has slope m_2:

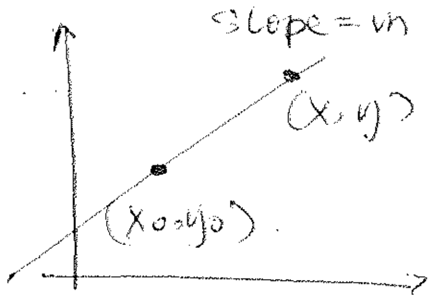
- L_1 and L_2 are parallel if $m_1 = m_2$ (or both undefined).
  (So same slope <-> same direction)
- L_1 and L_2 are perpendicular if $m_1 \cdot m_2 = -1$ (or if one line is horizontal and the other is vertical)

Example 1. *Find the slope of the straight line joining the points* $(-2, -1)$ *and* $(1, -6)$.

$$\text{Slope} = \frac{-6-(-1)}{1-(-2)} = -\frac{5}{3}$$

1.4.2 Equation of Straight Line

- Say a line is known to contain the point (x_0, y_0) and has slope m, write an equation to describe the points (x, y) on the line:



For any (x, y) on the line,

$$\frac{y-y_0}{x-x_0} = m,$$

$$\boxed{y - y_0 = m(x - x_0)}$$

This is known as the <u>point-slope</u> form of a straight line

Example 2. *Find the equation of the straight line that passes through* $(-2, 1)$ *and has slope 3.*

Equation is: $y-1 = 3(x-(-2))$.

- Other equivalent forms of equations of straight line:
  - Slope-intercept form:

$y = mx+b$ where m is the slope, b is the y-intercept

above example:

$y-1 = 3(x-(-2)) \Rightarrow y-1 = 3x+6 \Rightarrow y = 3x+7$

  - General form:

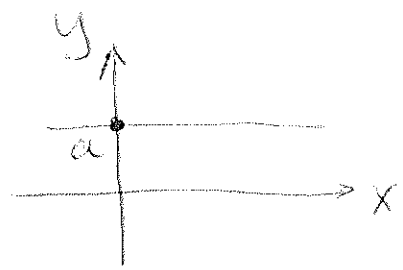
$ax+by+c=0$, where a, b, c are constants

To convert to slope-intercept form, solve for y.

e.g. $2x-3y-6=0 \Rightarrow -3y = -2x+6 \Rightarrow y = \frac{2}{3}x-2$

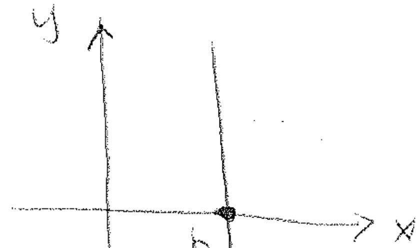
- Equations of horizontal and vertical lines:

Horizontal line:



Equation is $y = a$.

Vertical line:



Equation is $x = b$

1.4.3 Examples

Example 3. *Find the equation of the straight line which passes through (2,3) and is parallel to the straight line represented by the equation* $5x + 2y + 3 = 0$.

$5x + 2y + 3 = 0 \Rightarrow 2y = -5x - 3 \Rightarrow y = -\frac{5}{2}x - \frac{3}{2}$

$\therefore$ Slope of the line $5x+2y+3=0$ is $-\frac{5}{2}$.

$\therefore$ Slope of the line $\boxed{\text{required}}$ is $-\frac{5}{2}$.

$\therefore$ Equation of the required line is

$$y - 3 = -\frac{5}{2}(x-2)$$

Example 4. *Sketch the line in the previous example. Find the x-intercept and y-intercept of the line.*

x-intercept: put $y=0$ into the equation

$$0 - 3 = -\frac{5}{2}(x-2)$$

$$-3 = -\frac{5}{2}x + 5$$

$$-8 = -\frac{5}{2}x, \quad x = \frac{16}{5}.$$

$\therefore$ x-int. is $(\frac{16}{5}, 0)$

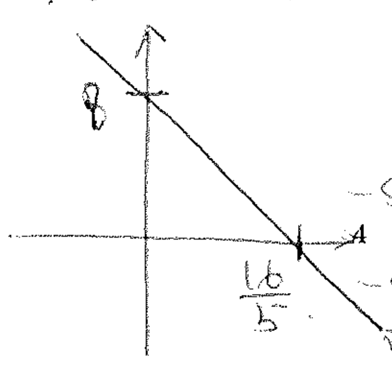
y-intercept: put $x=0$ into the equation,

$$y - 3 = -\frac{5}{2}(0-2)$$

$$y - 3 = 5, \quad y = 8.$$

$\therefore$ y-int. is $(0, 8)$.

So the line looks like



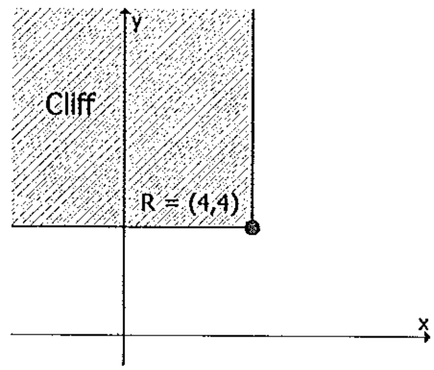
- slope is
- sign of int. is

— math info!

Verbally:

- Straight by having only two points.
- Do not plot extra p[oints]
- Scale is not nec.

Example 5. *Two guardposts of coordinates $P = (0,1)$ and $Q = (9,8)$ are situated near a cliff with corner $R = (4,4)$ represented by the shaded region below.*



(a) *Find the equation of the straight line joining P and R.*

(b) *Can the soldiers in P and Q see each other?*

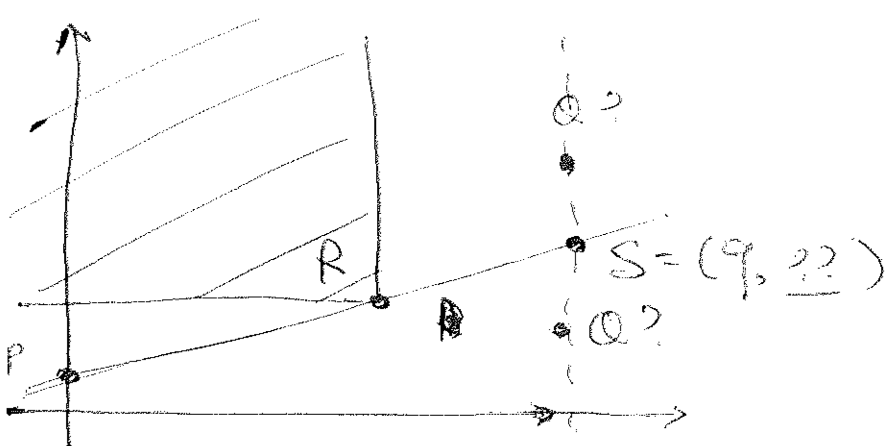
(a) Slope of segment PR

$$= \frac{4-1}{4-0} = \frac{3}{4}$$

So equation of PR is

$$\underline{y-4 = \tfrac{3}{4}(x-4)}$$

(b). We have to tell whether Q is above or below the line joining P and R.



To do so, we may compare the y-coordinates of Q and S.

y-coord of Q is 8

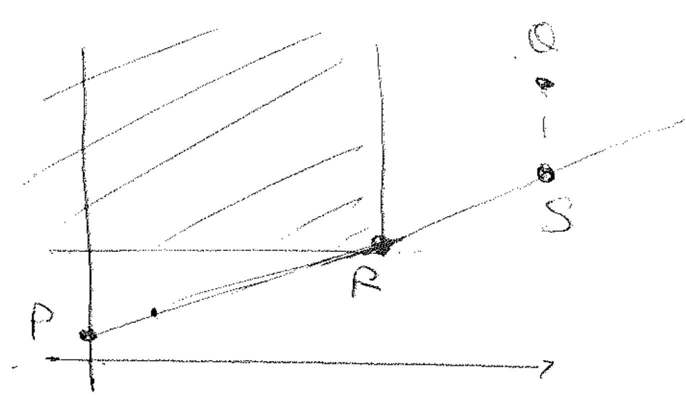
y-coord of S:

put $X = 9$ into the eq. of PR

$$y - 4 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4} \cdot 5 + 4 = \frac{31}{4}.$$

$\frac{31}{4} < 8$ so Q sits above S. So the picture is:



So P and Q cannot see each other