**Problem 1**. Suppose $g(t) = 5t^6 - 2t^4$. and upon investigation it is found that two critical points of $g(t)$ are $t = \pm\frac{2}{\sqrt{15}} \approx \pm 0.516$. Use the second derivative test to classify these critical points (hopefully no calculator is needed!)

**Problem 2:** Answer any one and do not forget to read the remark after 2b.

**(2a):** Suppose we have a function $f$ such that:

**Domain** $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$, and $f$ is twice differentiable on its entire domain

**Vertical Asymptote** $x = -3$ and $x = 3$ are vertical asymptotes

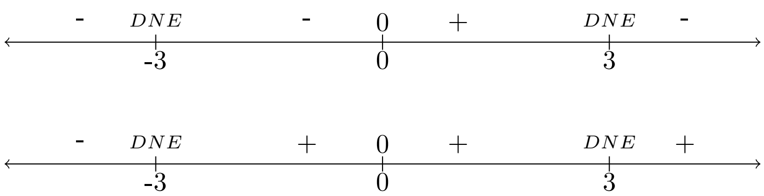
**Horizontal Asymptote** $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$

**Intercepts** $f$ has an $x$-intercept at $x = -4$, and a $y$-intercept at $y = 1$

**Sign Chart** $f$ has the following sign chart (and assume no interesting behavior is not shown on this chart):

sign of $f'$

sign of $f''$



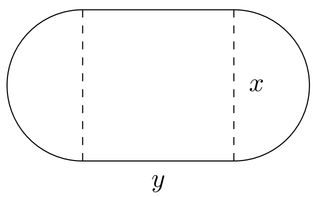

Sketch the graph of $f$. Label any asymptotes and any relevant $x-$ or $y-$coordinates. (Hence, we can actually draw a graph without even not knowing the function, if we are given the above info!)

**(2b):** Sketch the graph of $f(x) = \frac{x+3}{x-3}$. (Don't forget to show the required steps!)

*Remark:* Suppose we have a function $f(x) = \frac{x-3}{x^2-9}$. Then while computing vertical asymptotes, you have to be careful to cancel out common roots in the numerator and denominator. Thus, only $x = -3$ is the vertical asymptote here. I probably forgot to mention this in class.

**Problem 3:** Answer any one. Do think on 4b even if you don't answer it, it's a bit tricky to set up but not difficult!

**3a):** The port-side window on a submarine is to have the shape of a rectangle bordered by two semicircles (as the picture shows). Navy regulations dictate that the perimeter of the window must be 10 feet. Find the dimensions $x$ and $y$ that allow the greatest amount of light through.




**3b):**A family wants to set up a campsite next to a river and surround it with fencing to keep the animals out at night. Using the river as one side, they will fence off the other three sides of a rectangular enclosure with 100 yards of fencing. The enclosure must also be able to fit a circular firepit somewhere, which measures 20 yards in diameter. Find the length and width of the rectangular campsite which maximizes the family's camping space. (Hint: Be careful what you choose for a *closed interval.*)

**Problem 4:** Find the absolute maximum value and absolute minimum value of $f(x) = e^{x^2-x-2}$ on $[-1, 2]$ after justifying why they exist. (Hint: Closed Interval Method...)

**Problem 5:** Solve any two of the following:

(a) $16^{3x-2} = \dfrac{1}{8^x}$.

(b) $2\ln x = \ln(x+2)$.

(c) Suppose $f(x) = a + b\ln(x-1)$ and suppose $f(2) = 3,\ f(4) = 6$. Find $a, b$.

**Problem 6:** Answer any four

State $T/F$. If True, provide justification. If False, give counterexample or disprove it:

(a) If $y = 0$ is a horizontal asymptote for the function $h(x)$, then $h$ has no $x$-intercepts.

(b) If $x = 0$ is a vertical asymptote for the function $g(x)$, then $g$ has no $y$-intercepts. (In case you are crisscrossing across a vertical asymptote, please beware of the vertical line test for $g$ to be a function!)

(c) If $h$ is a continuous function, then it must have absolute extrema on its domain.

(d) If a function $f$ is not continuous on the interval $[a, b]$, then $f$ does NOT have any absolute extrema on the interval $[a, b]$.

(e) For a function $f(x)$ and a value $c$ in the domain of $f$, $f'(c) \geq 0$ implies $f(c) \geq 0$.

(f) If $f$ is concave upward on $(a, b)$, then $-f$ is concave downward on $(a, b)$.

(g) If $f(x) = \ln x,\ g(x) = e^x$, then $\dfrac{d}{dx}\Big((g \circ f)(x)\Big) = 0$.

(h) $e^{\ln x} = x$ for all real numbers $x$.