Math 1210 - Related Rates Handout

The general approach to related rates problems:

1. Draw a picture!
2. Assign variables to important quantities and label them in the diagram. Write down in words what each quantity means.
3. Write down the given information. Figure out which quantity or quantities we want to solve for.
4. Find an equation (or equations) which relate the quantities we are interested in. You might be able to use this equation to find more information.
5. Implicitly differentiate the equation. Be careful to use the chain rule when appropriate.
6. Substitute in the known information and solve for the unknown quantity.
7. Write a sentence explaining your answer.

1. A passenger ship and an oil tanker left the same port at different times in the morning. At noon, the passenger ship is 40 miles north of port, and it is traveling north at 30 mph. At noon (time $t = t_0$), the tanker is 30 miles east of port and traveling east at 20 mph. How fast is the distance between the two ships changing at noon?

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value at noon, when $t = t_0$, write it down.

      i. $x(t)$
      ii. $x'(t)$
      iii. $y(t)$
      iv. $y'(t)$
      v. $z(t)$
      vi. $z'(t)$

   (c) Find $z(t_0)$ using the Pythagorean theorem:

   (d) An equation relating our important variables is:

   (e) Implicitly differentiate the above equation. Solve for $z'(t)$:

   (f) Plug in the given information for the rest of the variables at time $t = t_0$. Then the rate of change of the distance between the ships at noon is:

2. The base of a 13 foot ladder leaning against a wall begins sliding away from the wall. When the base of the ladder is 12 feet from the wall (at the time $t = t_0$), the base is moving at 8 feet per second. How fast is the top of the ladder sliding down the wall at that instant?

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value when $t = t_0$, write it down.

      i. $a(t)$
      ii. $a'(t)$
      iii. $b(t)$
      iv. $b'(t)$
      v. $c(t)$
      vi. $c'(t)$

   (c) Find $b(t_0)$ with the Pathagorean Theorem:

   (d) An equation relating our important variables is:

   (e) Implicitly differentiate the above equation. Solve for $b'(t)$:

   (f) Plug in the given information for the rest of the variables at time $t = t_0$. Then the rate the top of the ladder is sliding *down* the wall is

3. You are blowing air into a spherical bubble at a rate of 8 cubic centimeters per second. How fast is the radius growing at the instant (time $t = t_0$) when the radius is 10 centimeters? How fast is the surface area changing at that instant?

   The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. The surface area of a sphere of radius r is $S = 4\pi r^2$

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value when $t = t_0$, write it down.

      i. $r(t)$
      ii. $r'(t)$
      iii. $V(t)$
      iv. $V'(t)$
      v. $S(t)$
      vi. $S'(t)$

   (c) We know $V(t) = \frac{4}{3}\pi[r(t)]^3$

   (d) Differentiate the above equation. Solve for $r'(t)$:

   (e) Plug in the given information for the rest of the variables at time $t = t_0$. Then the radius of the bubble is growing at:

   (f) We know $S(t) = 4\pi[r(t)]^2$.

   (g) Differentiate the above equation.

   (h) Plug in the known information at time $t = t_0$. Then the surface area of the bubble at the instant we are interested in is growing at:

4. A man walking 3 feet from the base of a lamppost casts a shadow which is 4 feet long at time $t = t_0$. If the man is 6 feet tall and is walking away from the lamppost at 400 feet per minute, at what rate does his shadow lengthen? How fast is the tip of his shadow moving (from the base of the lamppost)?

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value when $t = t_0$, write it down.

      i. $D(t)$
      ii. $D'(t)$
      iii. $L(t)$
      iv. $L'(t)$
      v. $X(t)$
      vi. $X'(t)$
      vii. H

   (c) Use similar triangles to find H.

   (d) We know $X(t) = L(t) + D(t)$. Then $X'(t) =$

   (e) Similar triangles give us a relation between $X(t)$ and $L(t)$. Implicitly differentiate this equation and solve for $L'(t)$, using the fact that $X'(t) = L'(t) + D'(t)$.

   (f) Plug in the given information for the rest of the variables at time $t = t_0$. Then the man's shadow is growing at a rate of:

   (g) Since $X'(t) = L'(t) + D'(t)$, at time $t = t_0$, find $X'(t_0)$. Then the the tip of the man's shadow is moving at:

5. Water is dripping through the bottom of a conical cup which is 4 inches across and 6 inches deep. Given that the cup loses half a cubic inch of water per minute, how fast is the water level dropping when the water is 3 inches deep (time $t = t_0$)?

   The volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value when $t = t_0$, write it down.

      i. $r(t)$
      ii. $r'(t)$
      iii. $h(t)$
      iv. $h'(t)$
      v. $V(t)$
      vi. $V'(t)$

   (c) Use similar triangles to get a relation between $r(t)$ and $h(t)$. Solve this for $r(t)$ in terms of $h(t)$:

   (d) We know $V(t) = \frac{1}{3}\pi[r(t)]^2 h(t)$. Write $V(t)$ in terms of $h(t)$ only:

   (e) Differentiate the above equation. Solve for $h'(t)$:

   (f) Plug in the given information for the rest of the variables at time $t = t_0$. Then the rate the height of the water is changing is:

6. A baseball diamond is a square with sides 90 feet long. A player is running from 2^{nd} base to 3^{rd} base at 15 feet per second. Find the rate of change of the distance from the player to home plate at the time $t = t_0$ when the player is 10 feet from 3^{rd} base.

   (a) Draw a picture:

   (b) List the important variables and their derivatives, and explain what they mean. If we know their value when $t = t_0$, write it down.

      i. $a(t)$
      ii. $a'(t)$
      iii. $b(t)$
      iv. $b'(t)$
      v. $c(t)$
      vi. $c'(t)$

   (c) Use the Pythagorean theorem to find $c(t_0)$.

   (d) An equation which relates the important variables is:

   (e) Implicitly differentiate this equation and solve for $c'(t)$.

   (f) Plug in the given information for the rest of the variables at time $t = t_0$. Then the distance between the player and home plate is changing at a rate of