3.5 Higher Order derivatives.

Note that f' is itself a function and we can talk about

i) f' is continuous or not
ii) f' is differentiable or not.

This motivates the search for "higher order derivatives." i.e. repeatedly taking derivatives of a given function f.

Answering (i) implies checking

(a) $f'(a)$ is defined for all $a \in$ Domain

(b) $\lim_{x \to a} f'(x)$ exists.

(c) $\lim_{x \to a} f'(x) = f'(a)$

Answering (ii) needs checking

$$\lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} \text{ exists or not.}$$

If it exists, it is called the 2nd derivative of f with respect to x.

Notation: $f''(x)$ OR $\frac{d^2}{dx^2}(f(x))$ or if $y = f(x)$, then $\frac{d^2y}{dx^2}$

Similarly we can talk about even higher derivatives.

- $f'''(x), f''''(x), f'''''(x)$

Instead of writing so many " ' "s. we write.

$f'''(x) = f^{(3)}(x)$; $f''''(x) = f^{(4)}(x)$ and so on.

[So, $f^{(0)}(x) = f(x)$ and $f^{(1)}(x) = f'(x)$]

Another notation is to take. $y = f(x)$.

Then. $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots, \frac{d^ny}{dx^n}$ are standard notations

Most important Practical Example of Higher-Order of derivatives is the following:

Let $s(t)$ represent position of a particle in time t.
Then. $s'(t) = \frac{d}{dt}(s(t))$ is the instantaneous rate of change of position at time t, in other words, $s'(t)$ is the velocity at time t.

$s''(t) = \frac{d^2}{dt^2}(s(t))$. is the rate of change of velocity, i.e. acceleration at time t.

$\boxed{\text{Rmk}}$ We often suppress the functional notation $s(t)$ to be s, where it is understood, that s is a function of t. Thus, ~~acceleration~~ velocity becomes $\frac{ds}{dt}$, whereas $\frac{d^2s}{dt^2}$ becomes acceleration.

$a = \frac{d}{\ldots} = \frac{d}{\ldots}\left(\frac{ds}{\ldots}\right) = \frac{d^2s}{\ldots} \ldots$

e.g. A ball is thrown upwards from the roof of a building. Let height from ground at time t be given by

$$s = -16t^2 + 24t + 120 . \quad \text{where } s \text{ is in ft, } t \text{ is in seconds.}$$

(a) Find velocity & acceleration 3 seconds after it is thrown into air.

(b) Find height of the building.

(Sol^n b) Height of the building is simply s evaluated at 0.

So, 120 ft.

(Sol^n a) $v = \frac{ds}{dt} = -32t + 24$

So, velocity (in ft/s) 3 seconds after it is thrown is

$$= -32 \times 3 + 24 = -72 .$$

So, the ball is falling downward at 72 ft/s.

$$a = \frac{dv}{dt} = -32 .$$

Thus, the acceleration downward is constant $32 \ ft/s^2$.