1) Find the domain of

- $\log_{17}(3x-5)$.
- $\ln x - \ln(2-x)$

Soln

- We need $3x-5>0$
  $\Rightarrow x > 5/3$ So, $D := (5/3, \infty)$.

- We need $x>0$ and $2-x>0$
  $\Rightarrow x<2$
  Thus $D := (0,2)$

(2) Find domain of $e^{\sqrt{2x-1}}$

Soln Need $2x-1 \geq 0 \rightarrow x \geq 1/2$
Thus, $D = [1/2, \infty)$

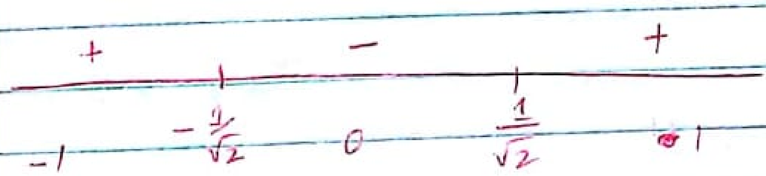
(3) Find the inflection points of $2e^{-x^2}$.

Soln $f'(x) = -4xe^{-x^2}$; $f''(x) = -4e^{-x^2} + 8x^2e^{-x^2}$

- f'' exists everywhere, as e^{-x^2} and $x^2e^{-x^2}$ are

So, only need to look at.

$$f''(x) = 0 \implies -4e^{-x^2} + 8x^2e^{-x^2} = 0$$
$$\implies -4e^{-x^2}\left[1 - 2x^2\right] = 0.$$
$$\implies 2x^2 = 1 \quad (\text{as } e^{-x^2} > 0)$$
$$\implies x = \pm\sqrt{\tfrac{1}{2}}$$ (both are in the domain)



- $f''(-1) = -4e^{-1} + 8e^{-1} = 4e^{-1} > 0.$
- $f''(0) = -4 < 0$
- $f''(1) = -4e^{-1} + 8e^{-1} = 4e^{-1} > 0.$

~~Both $-\frac{1}{\sqrt{2}}$ & $\frac{1}{\sqrt{2}}$ are in the domain of f~~

Thus, $\left(-\frac{1}{\sqrt{2}}, 2e^{-\frac{1}{2}}\right)$, $\left(\frac{1}{\sqrt{2}}, 2e^{-\frac{1}{2}}\right)$ are the I.P.'s.

(4) ~~Calculate~~ Find abs. max/min of $f(x) = xe^{-x^2}$ on $[0, 2]$.

Soln: • f is continuous on $[0, 2]$. So, closed interval Method applies.

• $f'(x) = e^{-x^2} - 2x^2e^{-x^2}.$

• $f'(x)$ exists everywhere on $[0, 2)$.

Thus, critical points on $(0,2)$ are obtained ~~from~~ ~~by~~ from: $f'(x) = 0$.

$$\Rightarrow e^{-x^2} - 2x^2 e^{-x^2} = 0$$
$$\Rightarrow e^{-x^2}(1 - 2x^2) = 0$$
$$\Rightarrow x = \pm\sqrt{\tfrac{1}{2}} \quad (\text{as } e^{-x^2} > 0).$$

Thus, only crit. pt on $(0,2)$ is $x = \frac{1}{\sqrt{2}}$

• $f(0) = 0$
• $f(2) = 2e^{-4} \approx .04$ (using calculator).
• $f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} e^{-1/2} = \frac{1}{\sqrt{2}e^{1/2}} \approx 0.55$.

So, abs. min value is 0; occurs at $x = 0$

and abs. max. value is $\frac{1}{\sqrt{2}} e^{-1/2}$; occurs at $x = \frac{1}{\sqrt{2}}$.

$\boxed{\text{OR}}$ abs. min. @. $(0,0)$

abs. max @ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} e^{-1/2}\right)$

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Logarithmic differentiation

- $\boxed{\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}}$ provided $f(x) > 0$ for all x.

- $\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}$ for all $x \in (-\infty, 0) \cup (0, \infty)$

- $\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$ for all $x \in (0, \infty)$.

Algorithm • How do we differentiate (i) $f(x) = g(x)^{h(x)}$

e.g. $f(x) = x^{2x+1}$?

• How do we differentiate $(x^3 - 2x)(x^2 + 2x)(7x - 4)$ instead of using product rule?

Ans: Both are done using logarithmic differentiation.

- Algorithm

1) Take '$\ln$' on both sides of the equation and simplify as much as possible using 'log' properties.
2) differentiate both sides w.r.t. x.
3) Collect $\frac{dy}{dx}$ on one side & solve.

(1) Find. $\frac{dy}{dx}$.

(a) $y = x^{2x+1}$.

$\ln y = (2x+1)\ln x$.

So, $\frac{d}{dx}(\ln y) = \frac{2x+1}{x} + 2\ln x$.

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(\frac{2x+1}{x} + 2\ln x\right)$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{2x+1}{x} + 2\ln x\right)$

[Applying chain-rule $\ln f(x) = \frac{f'(x)}{f(x)}$]

$$\boxed{\frac{dy}{dx} = x^{2x+1}\left(\frac{2x+1}{x} + 2\ln x\right)}$$

Ans:

(b) $f(x) = x(\ln x)^{x}$

Let $y = f(x)$. $\quad y = x(\ln x)^x$

$\ln y = \ln x + \ln((\ln x)^x)$

$\Rightarrow \ln y = \ln x + x\ln(\ln x)$.

Thus, $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \ln(\ln x) + x\left(\frac{1}{\ln x}\right)\cdot\frac{1}{x}$.

$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x} + \ln(\ln x) + \frac{1}{\ln x}\right]$

$$\boxed{\frac{dy}{dx} = x(\ln x)^x\left[\frac{1}{x} + \ln(\ln x) + \frac{1}{\ln x}\right]}$$

c) $y = \frac{(x^3-2x)(x^2+2x)}{(7x-4)}$

n: $\ln y = \ln(x^3-2x) + \ln(x^2+2x) - \ln(7x-4)$

ifferentiating both sides w.r.t x,

$\frac{1}{y}\frac{dy}{dx} = \frac{3x^2-2}{(x^3-2x)} + \frac{2x+2}{x^2+2x} - \frac{7}{7x-4}$

$$\boxed{\frac{dy}{dx} = \frac{(x^3-2x)(x^2+2x)}{(7x-4)}\left[\frac{3x^2-2}{x^3-2x} + \frac{2x+2}{x^2+2x} - \frac{7}{7x-4}\right]}$$

Ans

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$\boxed{\text{Remember:}}$

The 2 key identities:

$$\boxed{\begin{array}{ll} x = e^{\ln x} & \text{for all } x > 0 \\ x = \ln(e^{x}) & \text{for all } x \in \mathbb{R}. \end{array}}$$

Sometimes, it is more useful to use derivative of exponential, than to use logarithmic differentiation.

Trick is to remember, that for $x>0$,

$$\boxed{x = e^{\ln x}}$$ (Remember? $x = b^{\log_b x}$ when $x>0$)

For example,

$$x^{2x+1}$$

$$= e^{\ln (x^{2x+1})}$$

$$= e^{(2x+1)\ln x}$$

Let's do an example where we can use logarithmic differentiation and modify that example a bit so that log differentiation lands you in trouble., So, the above method will come in handy.

differentiate

(i) $y = x^x$

(ii) $y = x^x + 1$.

$\boxed{\text{For i)}}$ logarithmic differentiation works smoothly.

$$\ln y = x \ln x$$

$$\frac{1}{y} y' = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

$$\boxed{y' = y(\ln x + 1) = x^x(\ln x + 1)}$$

(For ii) If you just do, 'ln' on both sides, trouble!!

$$\ln y = \underbrace{\ln(x^x + 1)}$$ → Don't know how to deal with this.

In this case, the other method comes in handy:.

$$y = x^x + 1$$

$$= e^{\ln(x^x)} + 1$$

$$= e^{x \ln x} + 1.$$

Now, $\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \ln x}\right) = e^{x \ln x}\left[\ln x + x \cdot \frac{1}{x}\right]$

$$= e^{x \ln x}\left[\ln x + 1\right]$$

$$= x^x\left[\ln x + 1\right].$$

So, choose wisely.