Let us review how we solve inequalities:

• If we are trying to solve $(x-a)(x-b) \leq 0$, this means that $(x-a)$ and $x-b$ must have opposite sign.

e.g. Solve all x such that $(x-2)(x-3) \leq 0$.

Soln: We break it up into cases;

i.)suppose $x-2 \leq 0$ i.e. $x \leq 2$. Now we must have $x-3 \geq 0$ as opposite signs must be satisfied. i.e. $x \geq 3$. Now $x \leq 2$ and $x \geq 3$ is not possible. SO this case is not possible.

ii) Suppose $x-3 \leq 0$. So $x \leq 3$. We must have $x-2 \geq 0$ i.e. $x \geq 2$. Hence, combining the above results, we have $2 \leq x \leq 3$. This is the solution set.

• The same idea works when solving inequalities like $x^2-8 \leq 0$.

e.g. Find the domain of the function $f(x) = \sqrt{8-x^2}$.

Solution: We need $8-x^2 \geq 0$ i.e. $x^2-8 \leq 0$.

$$x^2 - 8 \leq 0$$
$$\text{or, } (x-2\sqrt{2})(x+2\sqrt{2}) \leq 0$$

So, $(x-2\sqrt{2})$ and $(x+2\sqrt{2})$ must have opposite signs. Break it into cases,;

i) Suppose $x+2\sqrt{2} \leq 0$ i.e. $x \leq -2\sqrt{2}$. We must have $x-2\sqrt{2} \geq 0$ (since opposite signs) i.e. $x \geq 2\sqrt{2}$ So, $x \geq 2\sqrt{2}$ and $x \leq -2\sqrt{2}$. Not possible.

ii) $x+2\sqrt{2} \geq 0$ i.e. $x \geq -2\sqrt{2}$. We must have the other $x-2\sqrt{2} \leq 0$ (since opposite signs) i.e. $x \leq 2\sqrt{2}$. Hence, $-2\sqrt{2} \leq x \leq 2\sqrt{2}$ is the required solution.

• One can similarly solve $(x-a)(x-b) \geq 0$. You can convert it into a previous model by changing the sign of one of the terms.

e.g. Solve $(x-3)(x-5) \geq 0$.

Solution: Need to solve $(3-x)(x-5) \leq 0$. (I changed the sign of one of the terms(any of the two will work) and the inequality is reversed.)

Break it into two cases as before; i. Suppose $(3-x) \leq 0$. So $(x-5) \geq 0$. Combining the two we get $x \geq 5$ and $x \geq 3$. Thus taking intersection of the two intervals, $x \geq 5$ works.

ii) Suppose $(3-x) \geq 0$ so $(x-5) \leq 0$. We have $x \leq 3$ and $x \leq 5$. Thus taking the intersection of the two intervals, we have $x \leq 3$.

Hence our final answer from these two cases is $x \in (-\infty, 3] \cup [5, \infty)$.

• So the general idea is to study the cases, and then combine the solutions in a compatible manner.

• In *Notes 2.1,2.2 .pdf*, we talked about how to solve linear inequalities involving absolute value functions. There was a question in the Quiz too. I recommend reading the notes again for those who got it wrong. We will be regularly using the ideas to solve inequalities to find domains of functions when we work on Optimization problems. Statistics shows that a lot of students mess up in this section. Hopefully, this class will be an exception if you get comfortable solving these this early on in the course.